JEE (Main)-2025 (Online) Session-2

Question Paper with Solutions

(Mathematics, Physics, And Chemistry)

8 April 2025 Shift – 2

Time: 3 hrs. **M.M : 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A : Attempt all questions.

(5) Section - B : Attempt all questions.

(6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

(7) Section - B (21 – 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

SECTION-A

1. Let the values of λ for which the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-\lambda}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ be λ_1 and λ_2. Then the radius of the circle passing through the points (0, 0), (λ_1, λ_2) and (λ_2, λ_1) is

(1) $\frac{5\sqrt{2}}{3}$ (2) 4

(3) $\frac{\sqrt{2}}{3}$ (4) 3

Ans. (1)

Sol. $\vec{p}=2\hat{i}+3\hat{j}+4\hat{k},\vec{q}=3\hat{i}+4\hat{j}+5\hat{k}$

$$\Rightarrow \vec{p}\times\vec{q}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&4\\3&4&5\end{vmatrix}=-\hat{i}+2\hat{j}-\hat{k}$$

$A \equiv (1, 2, 3)$ $B \equiv (\lambda, 4, 5)$

Shortest Distance $=\left|\frac{\overrightarrow{AB}.(\vec{P}\times\vec{q})}{|\vec{P}\times\vec{q}|}\right|$

$$\frac{1}{\sqrt{6}}=\left|\frac{\left((\lambda-1)\hat{i}+2\hat{j}+2\hat{k}\right).\left(-\hat{i}+2\hat{j}-\hat{k}\right)}{\sqrt{6}}\right|$$

$\Rightarrow|-\lambda+1+4-2|=1\Rightarrow|\lambda-3|=1$

$\Rightarrow \lambda = 3 \pm 1 = 4, 2$

Radius of circle passing through points (0, 0), (4, 2) & (2, 4)

$$=\frac{abc}{4\Delta}=\frac{\sqrt{20}\times\sqrt{20}\times\sqrt{8}}{4\times\frac{1}{2}\begin{vmatrix}1&1&1\\0&4&2\\0&2&4\end{vmatrix}}=\frac{20\times2\sqrt{2}}{2\times12}$$

$$=\frac{5\sqrt{2}}{3}$$

2. Let α be a solution of $x^2 + x + 1 = 0$, and for some a and b in $\mathbb{R}$, $[4\ \ a\ \ b]\begin{bmatrix}1&16&13\\-1&-1&2\\-2&-14&-8\end{bmatrix}=[0\ \ 0\ \ 0]$. If $\frac{4}{\alpha^4}+\frac{m}{\alpha^a}+\frac{n}{\alpha^b}=3$, then m + n is equal to ___

(1) 3 (2) 11

(3) 7 (4) 8

Ans. (2)

Sol. $x^2 + x + 1 = 0$

α is root

$\therefore \alpha^2 + \alpha + 1 = 0$

$\Rightarrow \alpha = \omega$ as ω^2 [cube root of unity]

also

$[\,4 - a - 2b\quad 64 - a - 14b\quad 52 + 2a - 8b]$
$= [0\quad 0\quad 0]$

$\therefore a + 2b = 4$

$a + 14b = 64$

$\Rightarrow 12b = 60 \Rightarrow \boxed{b=5}$

$\Rightarrow \boxed{a=-6}$

$$\therefore \frac{4}{\alpha^4}+\frac{m}{\alpha^{-6}}+\frac{n}{\alpha^5}=3$$

$$\Rightarrow \frac{4}{\omega}+\frac{m}{1}+\frac{n}{\omega^2}=3$$

$\Rightarrow 4\omega^2 + m + n\omega = 3$

$$\Rightarrow 4\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)+m+n\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=3$$

$\therefore -2+m-\frac{n}{2}=3$ ...(1)

& $\frac{-4\sqrt{3}}{2}+\frac{n\sqrt{3}}{2}=0$

$\therefore \boxed{n=4}$

$\boxed{m=7}$

$\boxed{\therefore m+n=11}$

3. Let the function $f(x) = \frac{x}{3}+\frac{3}{x}+3,\ x \neq 0$ be strictly increasing in $(-\infty,\alpha_1)U(\alpha_2,\infty)$ and strictly decreasing in $(\alpha_3,\alpha_4)U(\alpha_4,\alpha_5)$. Then $\sum_{i=1}^{5}\alpha_i^2$ is equal to :-

(1) 48 (2) 28

(3) 40 (4) 36

Ans. (4)

Sol. $f(x)=\frac{x}{3}+\frac{3}{x}+3,\ x\neq 0$

$f'(x)=\frac{1}{3}-\frac{3}{x^2}=0 \quad \Rightarrow x=\pm 3$

$f'(x)=\frac{x^2-3}{3x^2}$

$f'(x)>0\ \forall\ (-\infty,-3)\cup(3,\infty)\rightarrow$ increasing

$f'(x)<0\ \forall\ (-3,0)\cup(0,3)\rightarrow$ decreasing

$\sum_{i=1}^{5}\alpha_i^2=(-3)^2+(3)^2+(-3)^2+(0)^2+(3)^2$

$=36$

4. If A and B are two events such that P(A) = 0.7, P(B) = 0.4 and $P(A\cap\bar{B})=0.5$, where $\bar{B}$ denotes the complement of B, then $P\left(B|\left(A\cup\bar{B}\right)\right)$ is equal:-

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$

(3) $\frac{1}{6}$ (4) $\frac{1}{3}$

Ans. (1)

Sol. $P(A)=\frac{7}{10}$, $P(B)=\frac{4}{10}$

$P(A\cup\bar{B})=\frac{5}{10}$

$P\left(\frac{B}{A\cup\bar{B}}\right)=\frac{P(B\cap(A\cup\bar{B}))}{P(A\cup\bar{B})}$

$=\frac{P((B\cap\bar{B})\cup(B\cap A))}{P(A\cup\bar{B})}=\frac{P(A\cap B)}{P(A\cup\bar{B})}$

$=\frac{P(A)-P(A\cap\bar{B})}{P(A)+P(\bar{B})-P(A\cap\bar{B})}=\frac{\frac{7}{10}-\frac{5}{10}}{\frac{7}{10}+\left(1-\frac{4}{10}\right)-\frac{5}{10}}$

$=\frac{2}{8}=\frac{1}{4}$

5. If $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+.....\infty=\frac{\pi^4}{90}$,

$\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+.....\infty=\alpha$,

$\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+.....\infty=\beta$,

then $\frac{\alpha}{\beta}$ is equal to

(1) 23 (2) 18

(3) 15 (4) 14

Ans. (3)

Sol. If $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+.....\infty=\frac{\pi^4}{90}$ ....(i)

$\beta=\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+...,$

$=\frac{1}{16}\left[\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+.....\right],$

$=\frac{1}{16}\times\frac{\pi^4}{90}$ using (ii) ....(ii)

$\alpha=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+.....\infty$

$\left(\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\frac{1}{5^4}+.....\right)$

$-\left(\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+.....\right)$

$\alpha=\frac{\pi^4}{90}-\frac{1}{16}\times\frac{\pi^4}{90}$ [using (i) and (ii)]

$\alpha=\frac{16-1}{16\times 90}\times\pi^4=\frac{15}{16\times 90}\pi^4=\frac{\pi^4}{96}$

$\therefore\frac{\alpha}{\beta}=\frac{\frac{\pi^4}{96}}{\frac{\pi^4}{16\times 90}}=\frac{16\times 90}{96}=15$

6. The sum of the squares of the roots of $|x+2|^2+|x-2|-2=0$ and the squares of the roots of $x^2-2|x-3|-5=0$, is

(1) 26 (2) 36

(3) 30 (4) 24

Ans. (2)

Sol. $|x-2|^2 + 2|x-2| - |x-2| - 2 = 0$

$\Rightarrow (|x-2|+2)(|x-2|-1) = 0$

$\Rightarrow |x-2| = 1$

$\Rightarrow x = 2 \pm 1 = 3, 1$

$\Rightarrow$ sum of square of roots = 9 + 1 = 10

$x^2 - 2|x-3| - 5 = 0$

Case-I $x - 3 \geq 0$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x-1)^2 = 0$

$\Rightarrow x = 1$

But $x \geq 3$

$\Rightarrow x \in \phi$

Case-II $x - 3 < 0$

$x^2 + 2x - 11 = 0$, $D > 0 \Rightarrow$ Real & distinct roots

$f(x) = x^2 + 2x - 11$

$f(3) > 0$, $\frac{-p}{2a} = -1 < 3$

$\Rightarrow$ both roots < 3, both roots acceptable

Sum of square of roots = $(\alpha+\beta)^2 - 2\alpha\beta$

$= 4 + 22 = 26$

$\Rightarrow$ Final sum = 10 + 26 = 36

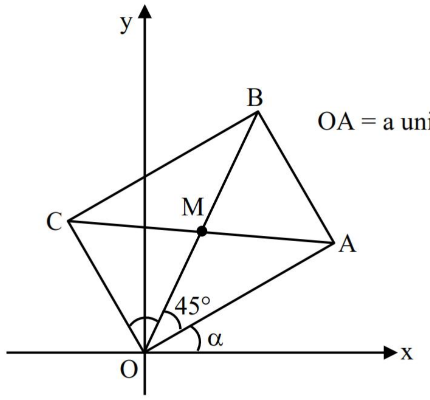
7. Let a be the length of a side of a square OABC with O being the origin. Its side OA makes an acute angle α with the positive x-axis and the equations of its diagonals are $(\sqrt{3}+1)\,x + (\sqrt{3}-1)\,y = 0$ and $(\sqrt{3}-1)x - (\sqrt{3}+1)y + 8\sqrt{3} = 0$. Then a^2 is equal to

(1) 48 (2) 32

(3) 16 (4) 24

Ans. (1)

Sol.



OA = a unit

Slope of diagonal OB $= \frac{\sqrt{3}+1}{1-\sqrt{3}}$

$= \tan 105°$

$\therefore \alpha = 60°$

$\therefore A(a\cos 60°, a\sin 60°)$

$\therefore A\left(\frac{a}{2}, \frac{\sqrt{3}a}{2}\right)$

A Lies on other diagonal

$\therefore \left(\frac{\sqrt{3}-1}{2}\right)a - \left(\frac{\sqrt{3}+1}{2}\right).\ \sqrt{3}a + 8\sqrt{3} = 0$

$a\left[\frac{\sqrt{3}-1-3-\sqrt{3}}{2}\right] = -8\sqrt{3}$

$\boxed{a = 4\sqrt{3}}$

$\boxed{\therefore a^2 = 48}$

8. Let f(x) be a positive function and $I_1 = \int_{-\frac{1}{2}}^{1} 2xf(2x(1-2x))\,dx$ and $I_2 = \int_{-1}^{2} f(x(1-x))\,dx$.

Then the value of $\frac{I_2}{I_1}$ is equal to ____

(1) 9 (2) 6

(3) 12 (4) 4

Ans. (4)

Sol. $I_1 = \int_{-\frac{1}{2}}^{1} 2xf(2x(1-2x))dx$

$\Rightarrow 2x = t \Rightarrow 2dx = dt \qquad \Rightarrow I_1 = \frac{1}{2}\int_{-1}^{2} tf(t(1-t))dt$

$\Rightarrow 2I_1 = \int_{-1}^{2} (1-t)f(1-t)(1-(1-t))dt$

$\Rightarrow 2I_1 = \int_{-1}^{2} f(t(1-t)dt - \int_{-1}^{2} tf(t(1-t)dt$

$\Rightarrow 2I_1 = I_2 - 2I_1$

$\Rightarrow 4I_1 = I_2$

$\Rightarrow \frac{I_2}{I_1} = 4$

9. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. Let $\hat{c}$ be a unit vector in the plane of the vectors $\vec{a}$ and $\vec{b}$ and be perpendicular to $\vec{a}$. Then such a vector $\hat{c}$ is :

(1) $\frac{1}{\sqrt{5}}\left(\hat{j} - 2\hat{k}\right)$ (2) $\frac{1}{\sqrt{3}}\left(-\hat{i} + \hat{j} - \hat{k}\right)$

(3) $\frac{1}{\sqrt{3}}\left(\hat{i} - \hat{j} + \hat{k}\right)$ (4) $\frac{1}{\sqrt{2}}\left(-\hat{i} + \hat{k}\right)$

Ans. **(4)**

Sol. Let vector $\vec{p}$ in plane of $\vec{a}$ & $\vec{b} = K\left(\vec{a} + \lambda\vec{b}\right)$

$\vec{p} \perp \vec{a} = \vec{p}.\vec{a} = 0$

$\Rightarrow K\left(\vec{a} + \lambda\vec{b}\right).\vec{a} = 0$

$\Rightarrow \vec{a}.\vec{a} + \lambda\vec{b}.\vec{a} = 0$

$\Rightarrow 6 + \lambda(3) = 0$

$\Rightarrow \lambda = -2$

$\Rightarrow \vec{p} = \left(-3\hat{i} + 3\hat{k}\right)$

Unit vector $\rightarrow \pm\frac{\left(-\hat{i} + \hat{k}\right)}{\sqrt{2}}$

10. Let the ellipse $3x^2 + py^2 = 4$ pass through the centre C of the circle $x^2 + y^2 - 2x - 4y - 11 = 0$ of radius r. Let f_1, f_2 be the focal distances of the point C on the ellipse. Then $6f_1f_2 - r$ is equal to

(1) 74 (2) 68

(3) 70 (4) 78

Ans. **(3)**

Sol. $E : \frac{x^2}{4/3} + \frac{y^2}{4/P} = 1$

Centre of circle (1, 2), radius

$r = \sqrt{1+4+11}$

$\boxed{r = 4}$

$\because$ E pass from centre (1, 2)

$\therefore \frac{3}{4} + P = 1$

$\boxed{P = \frac{1}{4}}$ $\therefore$ vertical ellipse

$e = \sqrt{1 - \frac{4/3}{16}} = \sqrt{1 - \frac{1}{12}} = \sqrt{\frac{11}{12}}$

$\therefore$ Focal distance of C (h, k)

$= b \pm ek$

$F_1 = 4 + \sqrt{\frac{11}{12}} \times 2$

$F_2 = 4 - \sqrt{\frac{11}{12}} \times 2$

$\therefore F_1F_2 = 16 - \frac{11}{3} = \frac{37}{3}$

$\therefore 6F_1F_2 - r = 74 - 4 = 70$

11. The integral $\int_{-1}^{\frac{3}{2}} \left(\left|\pi^2 x \sin(\pi x)\right|\right) dx$ is equal to :

(1) $3 + 2\pi$ (2) $4 + \pi$

(3) $1 + 3\pi$ (4) $2 + 3\pi$

Ans. **(3)**

Sol. Let, $I = \pi^2 \int_{-1}^{3/2} |x \sin \pi x| dx$

$$= \pi^2 \left\{ \int_{-1}^{1} x \sin \pi x dx - \int_{1}^{3/2} x \sin \pi x dx \right\}$$

$$= \pi^2 \left\{ 2\int_{0}^{1} x \sin \pi x dx - \int_{-1}^{3/2} x \sin \pi x dx \right\}$$

Consider

$$\int x \sin \pi x dx$$

$$-x.\frac{1}{\pi}\cos \pi x + \int 1.\frac{1}{\pi}\cos \pi x dx$$

$$= -\frac{x}{\pi}\cos \pi x + \frac{\sin \pi x}{\pi^2}$$

$$I = \pi^2 \left\{ 2\left(-\frac{x}{\pi}\cos \pi x + \frac{\sin \pi x}{\pi^2} \right)_0^1 - \left(-\frac{x}{\pi}\cos \pi x + \frac{\sin \pi x}{\pi^2} \right)_1^{3/2} \right\}$$

$$= \pi^2 \left\{ \frac{2}{\pi} - \left(-\frac{1}{\pi^2} - \frac{1}{\pi} \right) \right\}$$

$$= \pi^2 \left\{ \frac{3}{\pi} + \frac{1}{\pi^2} \right\}$$

$= 3\pi + 1$

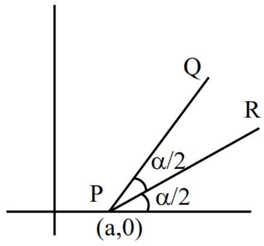
12. A line passing through the point P(a, θ) makes an acute angle α with the positive x-axis. Let this line be rotated about the point P through an angle $\frac{\alpha}{2}$ in the clock-wise direction. If in the new position, the slope of the line is $2 - \sqrt{3}$ and its distance from the origin is $\frac{1}{\sqrt{2}}$, then the value of $3a^2\tan^2\alpha - 2\sqrt{3}$ is

(1) 4 (2) 6

(3) 5 (4) 8

Ans. **(1)**

Sol.



$m_{PR} = 2 - \sqrt{3} = \tan 15°$

$\therefore \frac{\alpha}{2} = 15° \quad \Rightarrow \boxed{\alpha = 30°}$

equation of PR :

$y = \tan 15° (x - a)$

$y = (2 - \sqrt{3})(x - a)$

$\perp$ distance from origin $= \frac{1}{\sqrt{2}}$

$$\left| \frac{\sqrt{3}a - 2a}{\sqrt{4 + 3 - 4\sqrt{3} + 1}} \right| = \frac{1}{\sqrt{2}}$$

$$\frac{|a|(2-\sqrt{3})}{2\sqrt{(2-\sqrt{3})}} = \frac{1}{\sqrt{2}}$$

$$|a| = \frac{\sqrt{2}}{\sqrt{2-\sqrt{3}}} = \sqrt{2}(\sqrt{2+\sqrt{3}})$$

$a^2 = 2(2+\sqrt{3})$

$3a^2\tan^2\alpha - 2\sqrt{3}$

$3 \times (4 + 2\sqrt{3}).\frac{1}{3} - 2\sqrt{3} = 4$

13. There are 12 points in a plane, no three of which are in the same straight line, except 5 points which are collinear. Then the total number of triangles that can be formed with the vertices at any three of these 12 points is

(1) 230 (2) 220

(3) 200 (4) 210

Ans. **(4)**

Sol. ${}^{12}C_3 - {}^{5}C_3 = 210$

14. Let A = $\left\{\theta \in [0, 2\pi] : 1 + 10\,\text{Re}\left(\frac{2\cos\theta + i\sin\theta}{\cos\theta - 3i\sin\theta}\right) = 0\right\}$.

Then $\sum_{\theta \in A} \theta^2$ is equal to

(1) $\frac{21}{4}\pi^2$ (2) $8\pi^2$

(3) $\frac{27}{4}\pi^2$ (4) $6\pi^2$

Ans. **(1)**

Sol. $1 + 10\ \text{Re}\left(\frac{2\cos\theta + i\sin\theta}{\cos\theta - 3i\sin\theta}\right) = 0$

$\therefore\ z + \bar{z} = 2\text{Re}(z)$

$\frac{2\cos\theta + i\sin\theta}{\cos\theta - 3i\sin\theta} + \frac{2\cos\theta - i\sin\theta}{\cos\theta + 3i\sin\theta} = 2 \times \left(\frac{-1}{10}\right)$

$\frac{(2\cos^2\theta - 3\sin^2\theta) + (2\cos^2\theta) - (3\sin^2\theta)}{\cos^2\theta + 9\sin^2\theta} = \frac{-2}{10}$

$\Rightarrow \frac{2\cos^2\theta - 3\sin^2\theta}{\cos^2\theta + 9\sin^2\theta} = \frac{-1}{10}$

$\Rightarrow 20\cos^2\theta - 30\sin^2\theta = -\cos^2\theta - 9\sin^2\theta$

$21\cos^2\theta - 21\sin^2\theta = 0$

$\Rightarrow \cos 2\theta = 0$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\Rightarrow \sum\theta^2 = \frac{\pi^2}{16} + \frac{9\pi^2}{16} + \frac{25\pi^2}{16} + \frac{49\pi^2}{16} = \frac{84\pi^2}{16} = \frac{21\pi^2}{4}$

15. Let A = {0, 1, 2, 3, 4, 5}. Let R be a relation on A defined by (x, y) $\in$ R if and only if max {x, y} $\in$ {3, 4}. Then among the statements

(S_1) : The number of elements in R is 18, and

(S_2) : The relation R is symmetric but neither reflexive nor transitive

(1) both are true (2) both are false

(3) only (S_2) is true (4) only (S_1) is true

Ans. **(3)**

Sol. A = {0,1,2,3,4,5}

R ≡ {(0,3), (3,0), (0,4), (4,0), (1,3), (3,1), (1,4), (4,1), (2,3), (3,2), (2,4), (4,2), (3,3), (3,4), (4,3), (4,4)}

Total 16 elements

Not reflexive as (0,0), ..., (2,2) $\notin$ R

Symmetric $\because$ $\forall$ all a,b

(a, b) & (b, a) $\in$ R

Not transitive $\because$ (0,3), (3,1) $\in$ R

but (0,1) $\notin$ R

$\Rightarrow$ Only S_2 correct

16. The number of integral terms in the expansion of $\left(5^{\frac{1}{2}} + 7^{\frac{1}{8}}\right)^{1016}$ is

(1) 127 (2) 130

(3) 129 (4) 128

Ans. **(4)**

Sol. $T_r = {}^{1016}C_r (5)^{\frac{1016-r}{2}} 7^{\frac{r}{8}}$

$\Rightarrow r = 0, 8, 16, 24, \ldots, 1016$

$1016 = 0 + (n-1)8$

$\Rightarrow n - 1 = \frac{1016}{8} = 127$

So, n = 128.

17. Let f(x) = x – 1 and g(x) = e^x for x $\in \mathbb{R}$. If $\frac{dy}{dx} = \left(e^{-2\sqrt{x}} g(f(f(x))) - \frac{y}{\sqrt{x}}\right)$, y(0) = 0, then y(1) is :-

(1) $\frac{1-e^2}{e^4}$ (2) $\frac{2e-1}{e^3}$

(3) $\frac{e-1}{e^4}$ (4) $\frac{1-e^3}{e^4}$

Ans. **(3)**

Sol. f(x) = x – 1

f(f(x)) = f(x) – 1 = x – 1 – 1 = x – 2

$g(f(f(x))) = e^{x-2}$

$\therefore\ \frac{dy}{dx} = e^{-2\sqrt{x}} \times e^{x-2} - \frac{1}{\sqrt{x}}y$

$\frac{dy}{dx} + \frac{1}{\sqrt{x}}y = e^{x-2\sqrt{x}-2}$ which is L.D.E

I.F. = $e^{\int \frac{dy}{\sqrt{x}}} = e^{2\sqrt{x}}$

Its solution is

$y \times e^{2\sqrt{x}} = \int e^{2\sqrt{x}} \times e^{x-2\sqrt{x}-2}dx + c$

$y \times e^{2\sqrt{x}} = \int e^{x-2}dx + c$

$y \times e^{2\sqrt{x}} = e^{x-2} + c$

Given x = 0, y = 0 $\Rightarrow$ $0 = e^{-2} + c$; $c = -e^{-2}$

$\therefore\ y \times e^{2\sqrt{x}} = e^{x-2} - e^{-2}$

when x = 1, $y \times e^2 = e^{-1} - e^{-2}$

$y = \frac{e^{-1} - e^{-2}}{e^2} = \frac{\frac{1}{e} - \frac{1}{e^2}}{e^2} = \frac{e^2 - e}{e^5} = \frac{e-1}{e^4}$

Option (1) is correct

18. The value of $\cot^{-1}\left(\frac{\sqrt{1+\tan^2(2)}-1}{\tan(2)}\right) - \cot^{-1}\left(\frac{\sqrt{1+\tan^2\left(\frac{1}{2}\right)}+1}{\tan\left(\frac{1}{2}\right)}\right)$ is equal to

(1) $\pi - \frac{5}{4}$ (2) $\pi - \frac{3}{2}$

(3) $\pi + \frac{3}{2}$ (4) $\pi + \frac{5}{2}$

Ans. (1)

Sol. $\cot^{-1}\left(\frac{|\sec 2|-1}{\tan 2}\right) - \cot^{-1}\left(\frac{\left|\sec\frac{1}{2}\right|+1}{\tan\frac{1}{2}}\right)$

$= \cot^{-1}\left(\frac{-1-\cos 2}{\sin 2}\right) - \cot^{-1}\left(\frac{1+\cos\frac{1}{2}}{\sin\frac{1}{2}}\right)$

$= \pi - \cot^{-1}(\cot 1) - \cot^{-1}\left(\cot\frac{1}{4}\right)$

$= \pi - 1 - \frac{1}{4} = \pi - \frac{5}{4}$

19. Let $A = \begin{bmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{bmatrix}$.

If $det\ (adj\ (adj(3A))) = 2^m \cdot 3^n$, m, n $\in \mathbb{N}$, then m + n is equal to

(1) 22 (2) 24

(3) 26 (4) 20

Ans. (2)

Sol. $|A| = \begin{vmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{vmatrix}$

$C_3 \to C_3 - C_2 - C_1 \times \frac{q}{2}$

Then $C_3 \to C_2 - C_1 x \left(1+\frac{p}{2}\right)$

$\Rightarrow |A| = \begin{vmatrix} 2 & 0 & 0 \\ 4 & 2 & 2+p \\ 6 & 6 & 8+3p \end{vmatrix}$

$\Rightarrow |A| = 2(16 + 6p - 12 - 6p) = 8 = 2^3$

$|adj(adj(3A))| = |3A|^{(3-1)^2} = |3A|^4$

$= (3^3|A|)^4 = (3^3 \times 2^3)^4 = 2^{12} \times 3^{12}$

$\Rightarrow m + n = 24$

20. Given below are two statements :

Statement I :

$$\lim_{x\to 0}\left(\frac{\tan^{-1}x + \log_e\sqrt{\frac{1+x}{1-x}} - 2x}{x^5}\right) = \frac{2}{5}$$

Statement II : $\lim_{x\to 1}\left(x^{\frac{2}{1-x}}\right) = \frac{1}{e^2}$

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

(4) Both Statement I and Statement II are true

Ans. (4)

Sol. $\lim\limits_{x\to 0}\dfrac{\tan^{-1}x+\dfrac{1}{2}\left[\ell n(1+x)-\ell n(1-x)\right]-2x}{x^5}$

$=\lim\limits_{x\to 0}\dfrac{\left(x-\dfrac{x^3}{3}+\dfrac{x^5}{5}....\right)+\dfrac{1}{2}\left[x-\dfrac{x^2}{2}+\dfrac{x^3}{3}...-\left(-x-\dfrac{x^2}{2}-\dfrac{x^3}{3}...\right)\right]-2x}{x^5}$

$=\lim\limits_{x\to 0}\dfrac{2x+\dfrac{2x^5}{5}....-2x}{x^5}=\dfrac{2}{5}$

$\lim\limits_{x\to 1}x^{\frac{2}{(1-x)}}=e^{\lim\limits_{x\to 1}\left(\frac{2}{1-x}\right)(x-1)}=e^{-2}$

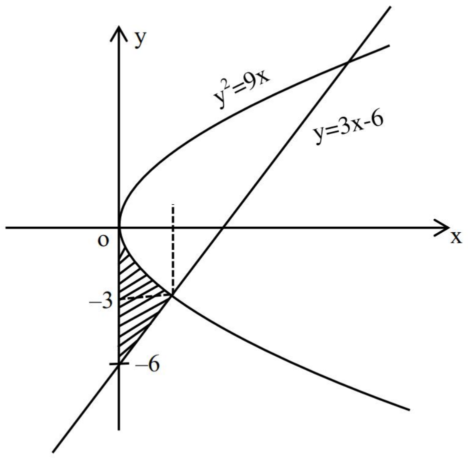
$\Rightarrow$ Both statements correct

SECTION-B

21. Let the area of the bounded region $\{(x, y) : 0 \le 9x \le y^2,\ y \ge 3x - 6\}$ be A. Then 6A is equal to ______

Ans. **(15)**

Sol. $0 \le 9x \le y^2$ & $y \ge 3x - 6$



$A = \text{Required Area} = \left[\int_0^1\left(-3\sqrt{x}\right)dx-\int_0^1(3x-6)dx\right]$

$A=-3\left(\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)\Bigg|_0^1-\left(\dfrac{3x^2}{2}-6x\right)\Bigg|_0^1$

$A=-2[1-0]\left[\dfrac{3}{2}-6\right]$

$A=-2-\dfrac{3}{2}+6=\dfrac{5}{2}$ Sq. unit

$\therefore\ 6A=6\times\dfrac{5}{2}=15$

22. Let the domain of the function

$f(x)=\cos^{-1}\left(\dfrac{4x+5}{3x-7}\right)$ be $[\alpha, \beta]$ and the domain of $g(x) = \log_2(2 - 6\log_{27}(2x + 5))$ be (γ, δ).

Then $|7(\alpha + \beta) + 4(\gamma + \delta)|$ is equal to ______

Ans. **(96)**

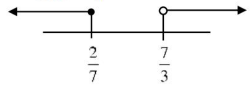
Sol. $f(x)=\cos^{-1}\left(\dfrac{4x+5}{3x-7}\right)$

$\Rightarrow -1\le\left(\dfrac{4x+5}{3x-7}\right)\le 1$

$\left(\dfrac{4x+5}{3x-7}\right)\ge -1$

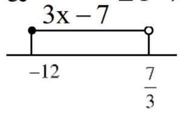
$\dfrac{4x+5+3x-7}{3x-7}\ge 0$

$\Rightarrow\dfrac{7x-2}{3x-7}\ge 0$



$x\in\left(-\infty,\dfrac{2}{7}\right]\cup\left(\dfrac{7}{3},\infty\right)$

& $\dfrac{4x+5}{3x-7}\le 1\Rightarrow\dfrac{x+12}{3x-7}\le 0$



$\therefore$ Domain of f(x) is

$\left[-12,\dfrac{2}{7}\right]$ $\boxed{\alpha=-12,\beta=\dfrac{2}{7}}$

$g(x) = \log_2(2-6\log_{27}(2x + 5))$

Domain

$2 - 6\log_{27}(2x + 5) > 0$

$\Rightarrow\quad 6\log_{27}(2x + 5) < 2$

$\Rightarrow\quad \log_{27}(2x+5)<\dfrac{1}{3}$

$\Rightarrow\quad 2x + 5 < 3$

$\Rightarrow\quad x < -1$

& $2x+5>0\Rightarrow x>-\dfrac{5}{2}$

Domain is $x\in\left(-\dfrac{5}{2},-1\right)$

$\boxed{\gamma=-\dfrac{5}{2},\delta=-1}$

$|7(\alpha+\beta)+4(\gamma+\delta)|=|7(-12+\dfrac{2}{7}+4(-\dfrac{5}{2}-1)|$

$|-82 - 14| = 96$

23. Let the area of the triangle formed by the lines $x + 2 = y - 1 = z$, $\frac{x-3}{5} = \frac{y}{-1} = \frac{z-1}{1}$ and $\frac{x}{-3} = \frac{y-3}{3} = \frac{z-2}{1}$ be A. Then A^2 is equal to ______

Ans. **(56)**

Sol. $L_1 : x + 2 = y - 1 = z = \ell$

$$L_2 : \frac{x-3}{5} = \frac{y}{-1} = \frac{z-1}{1} = m$$

$$L_3 : \frac{x}{-3} = \frac{y-3}{5} = \frac{z-2}{1} = n$$

Point of intersection of L_1 and L_2

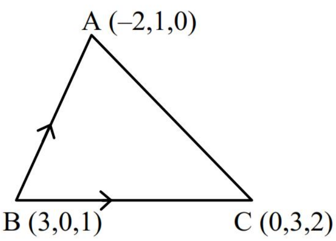
$$\left.\begin{matrix} \ell - 2 = 5m + 3 \\ \ell + 1 = -m \\ \ell = m + 1 \end{matrix}\right\} \ell = 0, m = -1 \quad A(-2, 1, 0)$$

Point of intersection of L_2 and L_3

$$\left.\begin{matrix} 5m + 3 = -3n \\ -m = 3n + 3 \\ m + 1 = n + 2 \end{matrix}\right\} m = 0, n = -1, B(3,0,1)$$

Point of intersection L_3 and L_4

$$\left.\begin{matrix} -3n = \ell - 2 \\ 3n + 3 = \ell + 1 \\ n + 2 = \ell \end{matrix}\right\} \ell = 2,\ n = 0, C(0, 3, 2)$$



$$Ar(\Delta ABC) = \left| \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 1 & -1 \\ -3 & 3 & 1 \end{vmatrix} \right|$$

$$A = \frac{1}{2} |\hat{i}(4) - \hat{j}(-8) + \hat{k}(-12)|$$

$$A = \frac{1}{2}\sqrt{16 + 64 + 144} = \sqrt{56}$$

$A^2 = 56$

24. The product of the last two digits of $(1919)^{1919}$ is ______

Ans. **(63)**

Sol. $(1919)^{1919} = (1920 - 1)^{1919}$

$= {}^{1919}C_0(1920)^{1919} - {}^{1919}C_1(1920)^{1918} + \ldots$

$+ {}^{1919}C_{1918}(1920)^1 - {}^{1919}C_{1919}$

$= 100\lambda + 1919 \times 1920 - 1$

$= 100\lambda + 3684480 - 1$

$= 100\lambda + \ldots\ldots\ldots 79$ (last two digit)

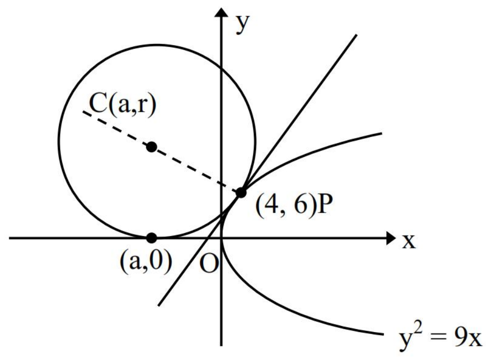
$\Rightarrow$ Number having last two digit 79

$\therefore$ Product of last two digit 63

25. Let r be the radius of the circle, which touches x-axis at point (a, 0), a < 0 and the parabola $y^2 = 9x$ at the point (4, 6). Then r is equal to ______

Ans. **(30)**

Sol.



$(x - a)^2 + (y - r)^2 = r^2$

$(4 - a)^2 + (6 - r)^2 = r^2$

$16 + a^2 - 8a + 36 + r^2 - 12r = r^2$

$a^2 - 8a - 12r + 52 = 0$

Tangent to parabola at (4, 6) is

$6.4 = 9.\left(\frac{x+4}{2}\right)$ i.e. $3x - 4y + 12 = 0$

This is also tangent to the circle

$\therefore$ CP = r

$$\frac{3a - 4r + 12}{5} = \pm r$$

$$3a + 12 = 4r \pm 5r\begin{cases} ar \\ -r \end{cases} \qquad \ldots..(1)$$

equation of circle is

$(x - a)^2 + (y - r)^2 = r^2$

satsty P(4, 6) $\Rightarrow a^2 - 8a - 12r + 52 = 0$ .....(2)

From equation (1)

If a + 4 = 3r then a = +6 (rejected)

If 3a + 12 = –r then a = –14 and r = 30

SECTION-A

26. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Work done in moving a test charge between two points inside a uniformly charged spherical shell is zero, no matter which path is chosen.

Reason R : Electrostatic potential inside a uniformly charged spherical shell is constant and is same as that on the surface of the shell.

In the light of the above statements, choose the **correct** answer from the options given below

(1) **A** is true but **R** is false

(2) Both **A** and **R** are true and **R** is the correct explanation of **A**

(3) **A** is false but **R** is true

(4) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

Ans. (2)

Sol. Conceptual

27. A rod of linear mass density 'λ' and length 'L' is bent to form a ring of radius 'R'. Moment of inertia of ring about any of its diameter is :

(1) $\frac{\lambda L^3}{16\pi^2}$ (2) $\frac{\lambda L^3}{12}$

(3) $\frac{\lambda L^3}{4\pi^2}$ (4) $\frac{\lambda L^3}{8\pi^2}$

Ans. (4)

Sol. $L = 2\pi R$

$$I = \frac{MR^2}{2} = \frac{\lambda \times L}{2} \times \left(\frac{L}{2\pi}\right)^2 = \frac{\lambda L^3}{8\pi^2}$$

28. A 3 m long wire of radius 3 mm shows an extension of 0.1 mm when loaded vertically by a mass of 50 kg in an experiment to determine Young's modulus. The value of Young's modulus of the wire as per this experiment is $P \times 10^{11} Nm^{-2}$, where the value of P is : (Take $g = 3\pi$ m/s^2)

(1) 5 (2) 10

(3) 25 (4) 2.5

Ans. (1)

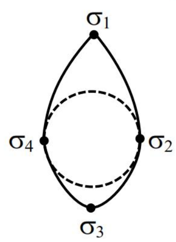
Sol. $\frac{50g}{\pi r^2} = y.\frac{\Delta \ell}{\ell}$

$$\frac{50 \times 3\pi}{\pi \times (3 \times 10^{-3})^2} = P \times 10^{11} \times \frac{0.1 \times 10^{-3}}{3}$$

$$\Rightarrow P = \frac{50 \times 3 \times 3}{3^2 \times 10^{-6} \times 10^{11} \times 0.1 \times 10^{-3}}$$

$P = 5$

29. Electric charge is transferred to an irregular metallic disk as shown in figure. If σ_1, σ_2, σ_3 and σ_4 are charge densities at given points then, choose the correct answer from the options given below:



(A) $\sigma_1 > \sigma_3$; $\sigma_2 = \sigma_4$ (B) $\sigma_1 > \sigma_2$; $\sigma_3 > \sigma_4$

(C) $\sigma_1 > \sigma_3 > \sigma_2 = \sigma_4$ (D) $\sigma_1 < \sigma_3 < \sigma_2 = \sigma_4$

(E) $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4$

(1) A, B and C Only (2) A and C Only

(3) D and E Only (4) B and C Only

Ans. (1)

Sol. $\sigma \propto \frac{1}{ROC}$

$(ROC)_1 < (ROC)_3 < (ROC)_2 = (ROC)_4$

$\sigma_1 > \sigma_3 > \sigma_2 = \sigma_4$

30. Water falls from a height of 200 m into a pool. Calculate the rise in temperature of the water assuming no heat dissipation from the water in the pool.

(Take $g = 10\ m/s^2$, specific heat of water = 4200 J/(kg K))

(1) 0.23 K (2) 0.36 K

(3) 0.14 K (4) 0.48 K

Ans. **(4)**

Sol. $mgh = ms\Delta T$

$$\Delta T = \frac{gh}{s} = \frac{10\times200}{4200}K = \frac{10}{21}K$$

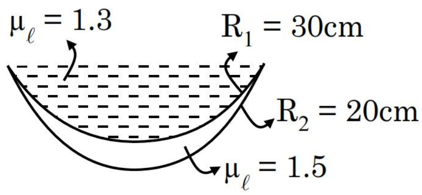
31. A concave-convex lens of refractive index 1.5 and the radii of curvature of its surfaces are 30 cm and 20 cm, respectively. The concave surface is upwards and is filled with a liquid of refractive index 1.3. The focal length of the liquid-glass combination will be

(1) $\frac{500}{11}$cm (2) $\frac{800}{11}$cm

(3) $\frac{700}{11}$cm (4) $\frac{600}{11}$cm

Ans. **(4)**

Sol.



$$\frac{1}{f} = \left(\frac{1.3-1}{1}\right)\left(\frac{1}{\infty}-\frac{1}{-30}\right)$$

$$= \left(\frac{1.5-1}{1}\right)\left(\frac{1}{-30}-\frac{1}{-30}\right)$$

$$= \frac{0.3}{30}+\frac{0.5}{60} = \frac{1}{100}+\frac{1}{120}$$

$$= \frac{6+5}{600} = \frac{11}{600}$$

$$f = \frac{600}{11}cm$$

32. An infinitely long wire has uniform linear charge density λ = 2 nC/m. The net flux through a Gaussian cube of side length $\sqrt{3}$ cm, if the wire passes through any two corners of the cube, that are maximally displaced from each other, would be xNm^2C^{-1}, where x is :

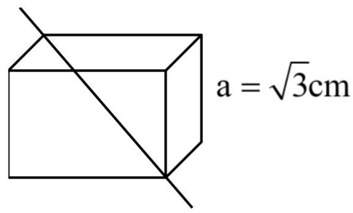
[Neglect any edge effects and use $\frac{1}{4\pi\varepsilon_0} = 9\times10^9$ SI units]

(1) 0.72 π (2) 1.44 π

(3) 6.48 π (4) 2.16 π

Ans. **(4)**

Sol.

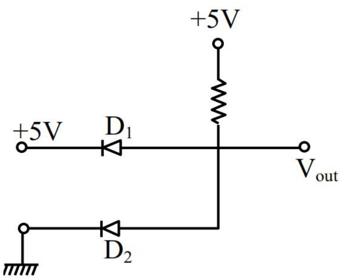


$$\phi = \frac{q_{enc}}{\varepsilon_0} = \frac{\lambda\cdot\sqrt{3}a}{\varepsilon_0}$$

$= 2\times10^{-9}\times\sqrt{3}\times\sqrt{3}\times10^{-2}\times36\pi\times10^9\ Nm^2C^{-1}$

$= 2.16\ \pi\ Nm^2C^{-1}$

33. The output voltage in the following circuit is (Consider ideal diode case)



(1) 10 V (2) 0 V

(3) + 5 V (4) – 5 V

Ans. **(2)**

Sol. Here D_1 is reverse biased and D_2 is forward biased. Therefore current flow through D_Q and 5V drop on resistor.

So, $V_{out} = 0$

34. Two metal spheres of radius R and 3R have same surface charge density σ. If they are brought in contact and then separated, the surface charge density on smaller and bigger sphere becomes σ_1 and σ_2, respectively. The ratio $\frac{\sigma_1}{\sigma_2}$ is.

(1) $\frac{1}{9}$ (2) 9

(3) $\frac{1}{3}$ (4) 3

Ans. (4)

Sol. For conducting sphere, $V = \frac{\sigma r}{\varepsilon_0}$

After contact, $V_1 = V_2$

$\sigma_1 r_1 = \sigma_2 r_2$

$\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}$

$\frac{\sigma_1}{\sigma_2} = 3$

35. A quantity Q is formulated as $X^{-2}Y^{+\frac{3}{2}}Z^{-\frac{2}{5}}$. X , Y and Z are independent parameters which have fractional errors of 0.1, 0.2 and 0.5, respectively in measurement. The maximum fractional error of Q is

(1) 0.1 (2) 0.8

(3) 0.7 (4) 0.6

Ans. (3)

Sol. Fractional error $= 2\frac{\Delta X}{X} + \frac{3}{2}\frac{\Delta Y}{Y} + \frac{2}{5}\frac{\Delta Z}{Z}$

$= 2(0.1) + \frac{3}{2}(0.2) + \frac{2}{5}(0.5)$

$= 0.2 + 0.3 + 0.2 = 0.7$

36. A monoatomic gas having $\gamma = \frac{5}{3}$ is stored in a thermally insulated container and the gas is suddenly compressed to $\left(\frac{1}{8}\right)^{th}$ of its initial volume. The ratio of final pressure and initial pressure is:

(γ is the ratio of specific heats of the gas at constant pressure and at constant volume)

(1) 16 (2) 40

(3) 32 (4) 28

Ans. (3)

Sol. $P_i V_i^{\gamma} = P_f V_f^{\gamma}$

$\frac{P_f}{P_i} = \left(\frac{V_i}{V_f}\right)^{\gamma} = (8)^{5/3}$

$\frac{P_f}{P_i} = 32$

37. A convex lens of focal length 30 cm is placed in contact with a concave lens of focal length 20 cm. An object is placed at 20 cm to the left of this lens system. The distance of the image from the lens in cm is ______

(1) 30 (2) 45

(3) $\frac{60}{7}$ (4) 15

Ans. (4)

Sol. Equivalent focal length

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

$= \frac{1}{30} + \frac{1}{-20} = \frac{2-3}{60} = -\frac{1}{60}$

$f = -60$ cm

Lens formula

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{-20} = \frac{1}{-60}$

$v = -15$ cm

38. Two strings with circular cross section and made of same material, are stretched to have same amount of tension. A transverse wave is then made to pass through both the strings. The velocity of the wave in the first string having the radius of cross section R is v_1, and that in the other string having radius of cross section R/2 is v_2. Then $\frac{v_2}{v_1} =$

(1) $\sqrt{2}$ (2) 2

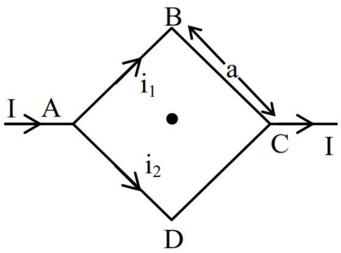
(3) 8 (4) 4

Ans. (2)

Sol. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{f\pi R^2}}$

$\frac{v_2}{v_1} = \frac{R_1}{R_2} = 2$

39. Figure shows a current carrying square loop ABCD of edge length is 'a' lying in a plane. If the resistance of the ABC part is r and that of ADC part is 2r, then the magnitude of the resultant magnetic field at centre of the square loop is

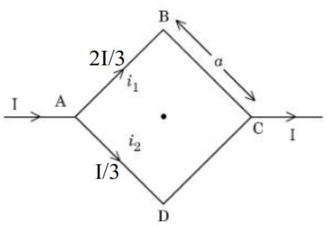


(1) $\frac{3\pi\mu_o I}{\sqrt{2}a}$ (2) $\frac{\mu_o I}{2\pi a}$

(3) $\frac{\sqrt{2}\mu_o I}{3\pi a}$ (4) $\frac{2\mu_o I}{3\pi a}$

Ans. (3)

Sol.



$\vec{B} = \vec{B}_{AB} + \vec{B}_{BC} + \vec{B}_{CD} + \vec{B}_{DA}$

$$\vec{B} = \left[\frac{-\mu_0(2I/3)}{4\pi(a/2)}\sqrt{2} - \frac{\mu_0(2I/3)}{4\pi(a/2)}\sqrt{2} + \frac{\mu_0(I/3)}{4\pi(a/2)}\sqrt{2} + \frac{\mu_0(I/3)}{4\pi(a/2)}\sqrt{2}\right]\hat{k}$$

$$\vec{B} = \left[\frac{-2\sqrt{2}\mu_0 I}{3\pi a} + \frac{\sqrt{2}\mu_0 I}{3\pi a}\right]\hat{k}$$

$$\vec{B} = \frac{-\sqrt{2}\mu_0 I}{3\pi a}\hat{k}$$

40. A body of mass 2 kg moving with velocity of $\vec{v}_{in} = 3\hat{i} + 4\hat{j}\,ms^{-1}$ enters into a constant force field of 6N directed along positive z-axis. If the body remains in the field for a period of $\frac{5}{3}$ seconds, then velocity of the body when it emerges from force field is

(1) $4\hat{i} + 3\hat{j} + 5\hat{k}$ (2) $3\hat{i} + 4\hat{j} + 5\hat{k}$

(3) $3\hat{i} + 4\hat{j} - 5\hat{k}$ (4) $3\hat{i} + 4\hat{j} + \sqrt{5}\hat{k}$

Ans. (2)

Sol. $\vec{a} = \frac{B}{2}\hat{k} = 3\hat{k}$, $t = \frac{5}{3}s$

$\vec{u} = 3\hat{i} + 4\hat{j}$

$\vec{v} = \vec{u} + \vec{a}t = 3\hat{i} + 4\hat{j} + 5\hat{k}$

41. Two balls with same mass and initial velocity, are projected at different angles in such a way that maximum height reached by first ball is 8 times higher than that of the second ball. T_1 and T_2 are the total flying times of first and second ball, respectively, then the ratio of T_1 and T_2 is :

(1) $2\sqrt{2} : 1$ (2) 2 : 1

(3) $\sqrt{2} : 1$ (4) 4 : 1

Ans. (1)

Sol. Given, $(H_{max})_1 = 8 \times (H_{max})_2$

$$\frac{u^2 \sin^2 \theta_1}{2g} = 8 \times \frac{u^2 \sin^2 \theta_2}{2g}$$

$$\Rightarrow \sin\theta_1 = 2\sqrt{2}\sin\theta_2$$

$$\frac{T_1}{T_2} = \frac{2u\sin\theta_1 / g}{2u\sin\theta_2 / g} = \frac{\sin\theta_1}{\sin\theta_2} = 2\sqrt{2}$$

42. The amplitude and phase of a wave that is formed by the superposition of two harmonic travelling waves, $y_1(x, t) = 4 \sin (kx - \omega t)$ and

$y_2(x, t) = 2 \sin (kx - \omega t + \frac{2\pi}{3})$, are :

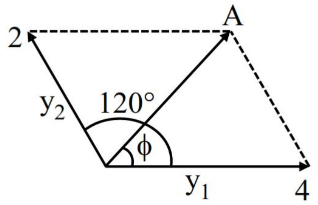
(Take the angular frequency of initial waves same as ω)

(1) $\left[6, \frac{2\pi}{3}\right]$ (2) $\left[6, \frac{\pi}{3}\right]$

(3) $\left[\sqrt{3}, \frac{\pi}{6}\right]$ (4) $\left[2\sqrt{3}, \frac{\pi}{6}\right]$

Ans. **(4)**

Sol.



$$A = \sqrt{2^2 + 4^2 + 2 \times 2 \times 4 \times \cos 120°}$$

$$= \sqrt{12} = 2\sqrt{3}$$

$$\tan\phi = \frac{2\sin 120°}{4 + 2\cos 120°} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\phi = \frac{\pi}{6}$$

43. In a Young's double slit experiment, the source is white light. One of the slits is covered by red filter and another by a green filter. In this case

(1) There shall be an interference pattern for red distinct from that for green.

(2) There shall be no interference fringes.

(3) There shall be alternate interference fringes of red and green.

(4) There shall be an interference pattern, where each fringe's pattern center is green and outer edges is red.

Ans. **(2)**

Sol. Different colours will have different fringe width. Within a few fringes of red, there will be several fringes of violet.
Also, there will be overlapping of colours.

44. For a nucleus of mass number A and radius R, the mass density of nucleus can be represented as

(1) A^3 (2) $A^{\frac{1}{3}}$

(3) $A^{\frac{2}{3}}$ (4) Independent of A

Ans. **(4)**

Sol. Conceptual

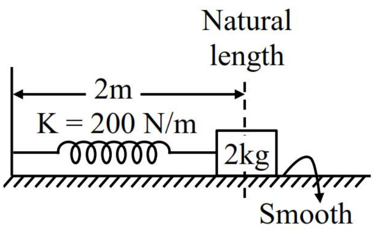
45. A block of mass 2 kg is attached to one end of a massless spring whose other end is fixed at a wall. The spring-mass system moves on a frictionless horizontal table. The spring's natural length is 2 m and spring constant is 200 N/m. The block is pushed such that the length of the spring becomes 1 m and then released. At distance x m (x < 2) from the wall. the speed of the block will be :

(1) $10[1-(2-x)]^{3/2}$ m/s (2) $10[1-(2-x)^2]^{1/2}$ m/s

(3) $10[1-(2-x)^2]$ m/s (4) $10[1-(2-x)^2]^2$ m/s

Ans. **(2)**

Sol.



Given, Natural length of spring = 2m

Initial compression in spring (x_i) = 1m

Final compression in spring (x_f) = (2 – x)m

Using energy conservation

$K_i + U_i = K_f + U_f$

$$0+\frac{1}{2}Kx_i^2=\frac{1}{2}mv^2+\frac{1}{2}Kx_f^2$$

$$\frac{1}{2}mv^2=\frac{1}{2}K(x_i^2-x_f^2)$$

$$\frac{1}{2}\times 2\times v^2=\frac{1}{2}\times 200\times(1^2-(2-x)^2)$$

$v^2 = 100[1 – (2 – x)^2]$

$v = 10[1 – (2 – x)^2]^{1/2}$

SECTION-B

46. An electron is released from rest near an infinite non-conducting sheet of uniform charge density '–σ'. The rate of change of de-Broglie wave length associated with the electron varies inversely as n^{th} power of time. The numerical value of n is _____.

Ans. **(2)**

Sol. Let the momentum of e^- at any time t is p and its de-broglie wavelength is λ.

Then, $p=\frac{h}{\lambda}$

$$\frac{dp}{dt}=\frac{-h}{\lambda^2}\frac{d\lambda}{dt}$$

$$ma=F=-\frac{h}{\lambda}\frac{d\lambda}{dt} \quad [m = \text{mass of } e^-]$$

Where, –ve sign represents decrease in λ with time

$$ma=\frac{-h}{(h/p)^2}\frac{d\lambda}{dt}$$

$$a=-\frac{p^2}{mh}\frac{d\lambda}{dt}$$

$$a=-\frac{mv^2}{h}\frac{d\lambda}{dt}$$

$$\frac{d\lambda}{dt}=-\frac{ah}{mv^2} \qquad ...(1)$$

here, $a=\frac{qE}{m}=\frac{e}{m}\frac{\sigma}{2\varepsilon_0}$

$$a=\frac{\sigma e}{2m\varepsilon_0}$$

and $v = u + at$

$v = at$

Substituting values of a & v in equation (1)

$$\frac{d\lambda}{dt}=-\frac{2h\varepsilon_0}{\sigma e t^2}$$

$$\Rightarrow \frac{d\lambda}{dt}\propto\frac{1}{t^2}$$

$\Rightarrow n = 2$

47. A sample of a liquid is kept at 1 atm. It is compressed to 5 atm which leads to change of volume of 0.8 cm^3. If the bulk modulus of the liquid is 2 GPa, the initial volume of the liquid was ______ litre. (Take 1 atm = 10^5 Pa)

Ans. **(4)**

Sol. Given, Initial pressure of liquid (P_i) = 1 atm

Final pressure of liquid (P_f) = 5 atm

Change in pressure (dP) = $P_f – P_i$ = 4 atm

= 4×10^5 Pa

Change in volume (dV) = – 0.8 cm^3

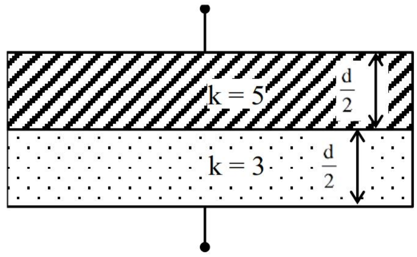
Bulk modulus (B) = 2×10^9 Pa

Now, $B=\frac{-dP}{(dV/V)}\Rightarrow V=-B\left(\frac{dV}{dP}\right)$

$$\Rightarrow V=-2\times10^9\times\frac{(-0.8\times10^{-6})}{4\times10^5}$$

$= 4 \times 10^{-3}\ m^3 = 4$ litre

48.

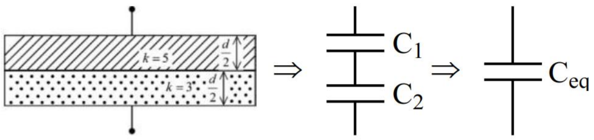


Space between the plates of a parallel plate capacitor of plate area 4 cm^2 and separation of (d) 1.77 mm, is filled with uniform dielectric materials with dielectric constants (3 and 5) as shown in figure. Another capacitor of capacitance 7.5 pF is connected in parallel with it. The effective capacitance of this combination is _____ pF.

(Given $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m)

Ans. (15)

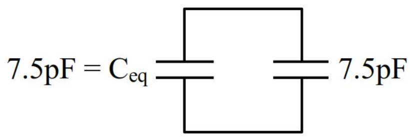
Sol.



$$C_1 = \frac{5\times4\times10^{-4}\times8.85\times10^{-12}}{\frac{1.77}{2}\times10^{-3}} = 20 \text{ pF}$$

$$C_2 = \frac{3\times4\times10^{-4}\times8.85\times10^{-12}}{\frac{1.77}{2}\times10^{-3}} = 12 \text{ pF}$$

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{12\times20}{12+20} = 7.5 \text{ pF}$$



Finally equivalent capacitance

$(C_{eq})_{final} = 7.5 + 7.5 = 15$ pF

49. A thin solid disk of 1 kg is rotating along its diameter axis at the speed of 1800 rpm. By applying an external torque of 25 π Nm for 40s, the speed increases to 2100 rpm. The diameter of the disk is ____ m.

Ans. (40)

Sol. Given, m = 1 kg

$$\omega_i = 1800 \text{ rpm} = 1800\times\frac{2\pi}{60} = 60\pi\frac{\text{rad}}{\text{sec}}$$

$$\omega_f = 2100 \text{ rpm} = 2100\times\frac{2\pi}{60} = 70\pi\frac{\text{rad}}{\text{sec}}$$

$\tau_{ext} = 25\pi$ Nm

t = 40 sec

Using equation of motion

$\omega_f = \omega_i + \alpha t$

$70\pi = 60\pi + \alpha(40)$

$$\alpha = \frac{\pi}{4} \text{ rad/sec}^2$$

Also, $\tau = I\alpha$

$$\tau = \frac{mR^2}{4}\alpha$$

$$25\pi = \frac{1\times R^2}{4}\times\frac{\pi}{4}$$

R = 20 m

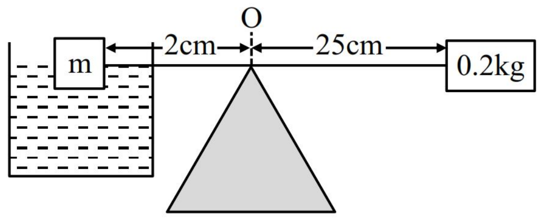
Hence, diameter of disk = 2R = 2 × 20 = 40m

50. A cube having a side of 10 cm with unknown mass and 200 gm mass were hung at two ends of an uniform rigid rod of 27 cm long. The rod along with masses was placed on a wedge keeping the distance between wedge point and 200 gm weight as 25 cm. Initially the masses were not at balance. A beaker is placed beneath the unknown mass and water is added slowly to it. At given point the masses were in balance and half volume of the unknown mass was inside the water.

(Take the density of unknown mass is more than that of the water, the mass did not absorb water and water density is 1 gm/cm^3.) The unknown mass is ____ kg.

Ans. (3)

Sol.



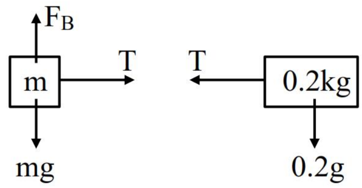
Given, volume of block = $(10 \times 10^{-2})^3 = 10^{-3}\ m^3$

Let density of block = ρ kg/m^3

mass of block = $\rho \times 10^{-3}$ kg

Buoyant Force (F_B) = $1000 \times \dfrac{10^{-3}}{2} \times 10 = 5N$

F.B.D. of blocks



Balancing torque about point O, we get

$mg(2 \times 10^{-2}) - F_B(2 \times 10^{-2}) = 0.2g(25 \times 10^{-2})$

$\rho \times 10^{-3} \times 10 \times 2 - 10 = 50$

$\rho = 3000$ kg/m^3

Hence, mass of block = $\rho \times 10^{-3}$

$= 3000 \times 10^{-3} = 3$ kg

SECTION-A

51. In a first order decomposition reaction, the time taken for the decomposition of reactant to one fourth and one eighth of its initial concentration are t_1 and t_2 (s), respectively. The ratio t_1/t_2 will :

(1) $\frac{4}{3}$ (2) $\frac{3}{2}$

(3) $\frac{3}{4}$ (4) $\frac{2}{3}$

Ans. (4)

Sol. For I[st] order reaction

When $C_t = Co/4$

$t_1 = 2t_{50\%}$.

when $C_t = Co/8$

$t_2 = 3t_{50\%}$

so $\frac{t_1}{t_2} = \frac{2}{3}$

52. Match the **LIST-I** with **LIST-II**

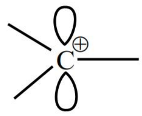
LIST-I		LIST-II	
A.	Carbocation	I.	Species that can supply a pair of electrons.
B.	C-Free radical	II.	Species that can receive a pair of electrons.
C.	Nucleophile	III.	sp^2 hybridized carbon with empty p-orbital.
D.	Electrophile	IV	sp^2/sp^3 hybridized carbon with one unpaired electron.

Choose the ***correct*** answer from the options given below :

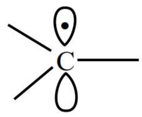
(1) A-IV, B-II, C-III, D-I
(2) A-II, B-III, C-I, D-IV
(3) A-III, B-IV, C-II, D-I
(4) A-III, B-IV, C-I, D-II

Ans. (4)

Sol. (A) Carbocation $\rightarrow sp^2$ hybridised carbon with empty P-orbital



(B) Carbon free radical $\rightarrow sp^2/sp^3$ hybridised carbon with one unpaired electron.



(C) Nuecleophile $\rightarrow$ species of that can supply a pair of electron.

(D) Electrophile $\rightarrow$ species that can receive a pair of electron.

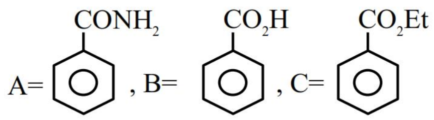
53. $A \xrightarrow[(ii)H_3O^+]{(i)NaOH} B \xrightarrow[(ii)H_2SO_4,\Delta]{(i)EtOH} C$

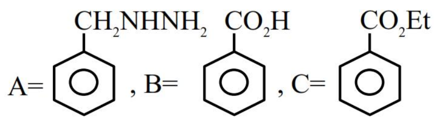
'A' shows positive Lassaign's test for N and its molar mass is 121.

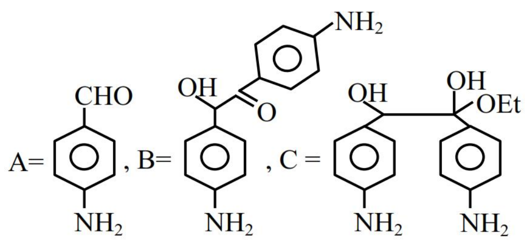
'B' gives effervescence with aq. $NaHCO_3$.

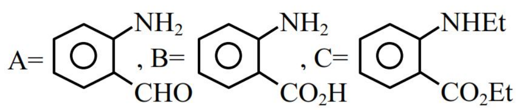
'C' gives fruity smell.

Identify A, B and C from the following.

(1) 

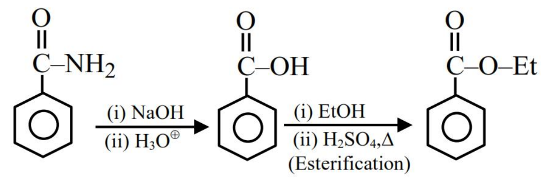
(2) 

(3) 

(4) 

Ans. **(1)**

Sol.



(A) (B) (C)

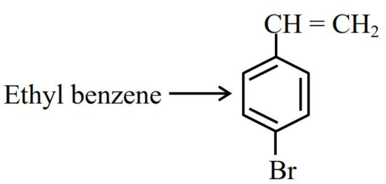
Molar mass = 121

A → Benzamide Shows positive Lassaigh's test.

B → Benzoic acid gives effervescence with aq. $NaHCO_3$.

C → Ester gives fruity smell.

54. Choose the correct set of reagents for the following conversion.



(1) Br_2/Fe ; Cl_2, Δ ; alc. KOH

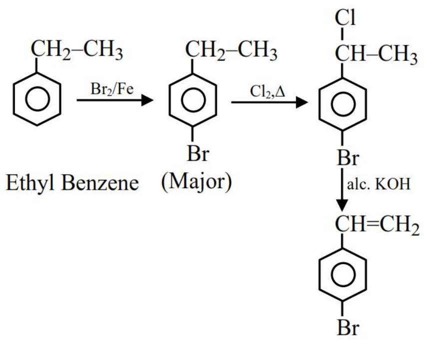
(2) Cl_2/Fe ; Br_2/anhy.$AlCl_3$; aq. KOH

(3) Br_2/anhy.$AlCl_3$; Cl_2, Δ ; aq. KOH

(4) Cl_2/anhy.$AlCl_3$; Br_2/Fe ; alc. KOH

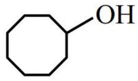
Ans. **(1)**

Sol.

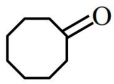


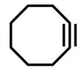
55. 1, 2-dibromocyclooctane $\xrightarrow[\text{(ii) NaNH}_2 \text{ (iii) Hg}^{2+}/\text{H}^+ \text{ (iv) Zn-Hg/H}^+]{\text{(i) KOH (alc.)}}$ P (Major product)

'P' is

(1) 

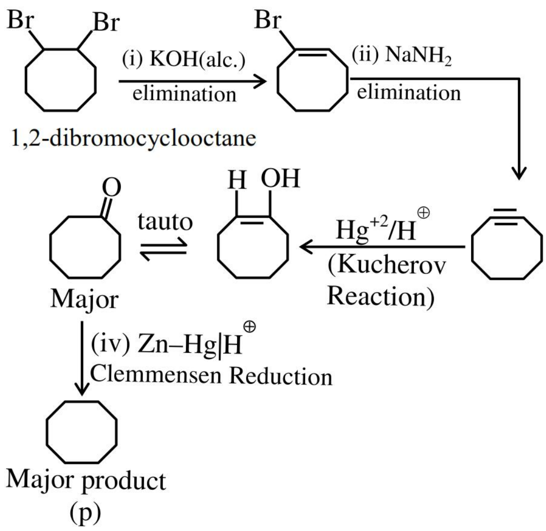
(2) 

(3) 

(4) 

Ans. **(2)**

Sol.



56. Given below are two statements :

Statement I : A homolepitc octahedral complex, formed using monodentate ligands, will not show stereoisomerism.

Statement II : cis– and trans– platin are heteroleptic complexes of Pd.

In the light of the above statements, choose the ***correct*** answer from the options given below.

(1) Both statement I and Statement II are false.

(2) Statement I is false but Statement II is true.

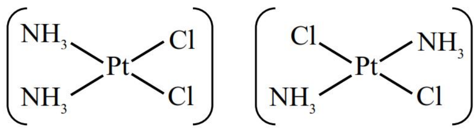
(3) Both statement I and Statement II are true.

(4) Statement I is true but Statement II is false.

Ans. **(4)**

Sol. Homoleptic complex of type $[Ma_6]$ (Where a $\Rightarrow$ monodentate ligand) cannot show geometrical as well as optical isomerism.

Cis-platin and trans-platin has formula $[Pt(NH_3)_2Cl_2]$ which is a heteroleptic complex of platinum.



Cis-platin Trans-platin

57. The atomic number of the element from the following with lowest 1st ionisation enthalpy is :

(1) 32 (2) 35

(3) 87 (4) 19

Ans. **(3)**

Sol. Atomic no. 32 $\Rightarrow$ Ge

Atomic no. 35 $\Rightarrow$ Br

Atomic no. 87 $\Rightarrow$ Fr

Atomic no. 19 $\Rightarrow$ K

Lowest first I.E. among the given element will be of Fr [87].

Fr – [Rn] $7s^1$

58. Which of the following binary mixture does not show the behaviour of minimum boiling azeotropes ?

(1) $H_2O + CH_3COC_2H_5$

(2) $C_6H_5OH + C_6H_5NH_2$

(3) $CS_2 + CH_3COCH_3$

(4) $CH_3OH + CHCl_3$

Ans. **(2)**

Sol. Binary mixture of C_6H_5OH and $C_6H_5NH_2$ shows negative deviation from Raoult's law

So vapour pressure of solution is less than V.P of pure C_6H_5OH & $C_6H_5NH_2$

So B.P. of solution is greater than boiling point of pure C_6H_5OH & $C_6H_5NH_2$

So shows maximum Boiling azeotrope

59. $HA(aq) \rightleftharpoons H^+(aq) + A^-(aq)$

The freezing point depression of a 0.1 m aqueous solution of a monobasic weak acid HA is 0.20°C. The dissociation constant for the acid is

Given :

$K_f(H_2O) = 1.8$ K kg mol^{-1}, molality $\equiv$ molarity

(1) 1.38×10^{-3}

(2) 1.1×10^{-2}

(3) 1.90×10^{-3}

(4) 1.89×10^{-1}

Ans. **(1)**

Sol. $\Delta T_f = ik_fm$

$0.2 = i \times 1.8 \times 0.1$

$$i = \frac{20}{18} = \frac{10}{9}$$

For $HA_{(aq)} \rightleftharpoons H^+_{(aq)} + A^-_{(aq)}$

t=0 1

$t = t_{eq}$ $1-\alpha$ α α

$i = 1 + \alpha$

$$\frac{10}{9} = 1 + \alpha$$

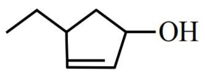
$$\alpha = \frac{1}{9}$$

$$K_{eq} = \frac{[H^+][A^-]}{[HA]} = \frac{C\alpha^2}{1-\alpha}$$

$$= \frac{0.1\left(\frac{1}{9}\right)^2}{1-\frac{1}{9}} = \frac{1}{720}$$

$K_{eq.} = 1.38 \times 10^{-3}$

60. What is the correct IUPAC name of

 ?

(1) 4-Ethyl-1-hydroxycyclopent-2-ene

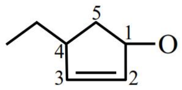
(2) 1-Ethyl-3-hydroxycyclopent-2-ene

(3) 1-Ethylcyclopent-2-en-3-ol

(4) 4-Ethylcyclopent-2-en-1-ol

Ans. **(4)**

Sol.



4-Ethylclopent-2-en-1-ol

61. The correct decreasing order of spin only magnetic moment values (BM) of Cu^+, $Cu^{2+,}$ Cr^{2+} and Cr^{3+} ions is :

(1) $Cu^+ > Cu^{2+} > Cr^{3+} > Cr^{2+}$

(2) $Cu^{2+} > Cu^+ > Cr^{2+} > Cr^{3+}$

(3) $Cr^{2+} > Cr^{3+} > Cu^{2+} > Cu^+$

(4) $Cr^{3+} > Cr^{2+} > Cu^+ > Cu^{2+}$

Ans. **(3)**

Sol. Cu^+ : [Ar] $3d^{10}$, Spin only magnetic moment = 0 B.M.

Cu^{+2} : [Ar] $3d^9$, Spin only magnetic moment = $\sqrt{3}$ B.M.

Cr^{+2} : [Ar] $3d^4$, Spin only magnetic moment = $\sqrt{24}$ B.M.

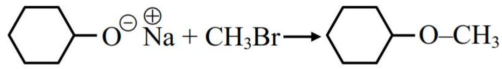
Cr^{+3} : [Ar] $3d^3$, Spin only magnetic moment = $\sqrt{15}$ B.M.

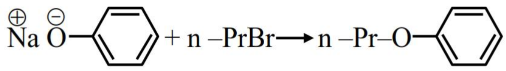
Order of μ : $Cr^{+2} > Cr^{+3} > Cu^{+2} > Cu^+$

62. Which one of the following reactions will not lead to the desired ether formation in major proportion?

(iso-Bu ⇒ isobutyl, sec-Bu ⇒ sec-butyl, nPr ⇒ n-propyl, ^{t}Bu ⇒ tert-butyl, Et ⇒ ethyl)

(1) $^tBu\overset{\ominus}{O}\overset{\oplus}{Na}$ + EtBr ⟶ ^{t}Bu–O–Et

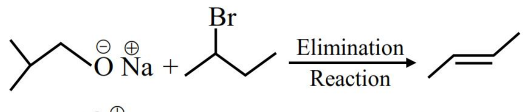
(2) 

(3) 

(4) iso-Bu$\overset{\ominus}{O}$ $\overset{\oplus}{Na}$ + sec – BuBr ⟶ Sec–Bu–O–iso–Bu

Ans. **(4)**

Sol.



(iso-Bu$\overset{\ominus}{O}$$\overset{\oplus}{Na}$) (sec-Bu Br) Major product

63. On combustion 0.210 g of an orgainc compound containing C, H and O gave 0.127 g H_2O and 0.307 g CO_2. The percentages of hydrogen and oxygen in the given organic compound respectively are:

(1) 53.41, 39.6 (2) 6.72, 53.41

(3) 7.55, 43.85 (4) 6.72, 39.87

Ans. **(2)**

Sol. In the combustion of organic compound, all "C" in CO_2 and all "H" in H_2O comes from organic compound

$$\underset{0.210gm}{C_xH_yO_z} + O_2 \longrightarrow \underset{0.307gm}{CO_2} + \underset{0.127gm}{H_2O}$$

Weight of "C" in $CO_2 = \frac{12}{44} \times 0.307$

= 0.0837 gm

Weight of "H" in $H_2O = \frac{2}{18} \times 0.127 = 0.0141g$

% ‘H’ in compound = $\frac{0.0141}{0.21} \times 100 = 6.719\ \%$

$= 6.72\ \%$

Weight of “O” in compound

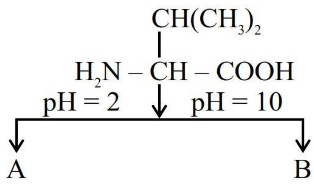
$= 0.210 - (0.0837 + 0.0141)$

$= 0.1122$

% of “O” in compound $= \frac{0.1122}{0.21} \times 100$

$= 53.41\ \%$

64.



Choose the correct option for structures of A and B, respectively.

(1) $H_3\overset{+}{N}-CH(CH(CH_3)_2)-COOH$ and $H_2N-CH(CH(CH_3)_2)-CO\bar{O}$

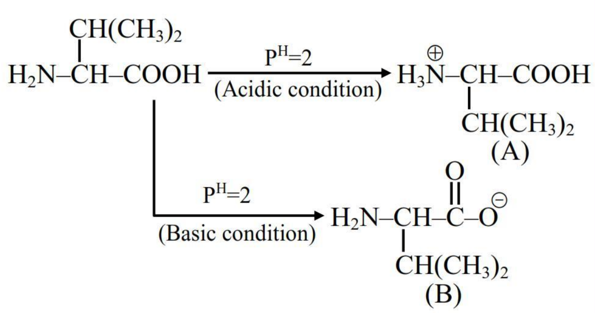
(2) $H_2N-CH(CH(CH_3)_2)-CO\bar{O}$ and $H_3\overset{+}{N}-CH(CH(CH_3)_2)-COOH$

(3) $H_2N-CH(CH(CH_3)_2)-CO\overset{\ominus}{O}$ and $H_3\overset{+}{N}-CH(CH(CH_3)_2)-CO\bar{O}$

(4) $H_3\overset{+}{N}-CH(CH(CH_3)_2)-CO\bar{O}$ and $H_3\overset{+}{N}-CH(CH(CH_3)_2)-COOH$

Ans. (1)

Sol.



65. Correct statements for an element with atomic number 9 are

A. There can be 5 electrons for which $m_s = +\frac{1}{2}$ and 4 electrons for which $m_s = -\frac{1}{2}$

B. There is only one electron in p_z orbital

C. The last electron goes to orbital with n = 2 and $l = 1$

4. The sum of angular nodes of all the atomic orbitals is 1.

Choose the correct answer from the options given below:

(1) C and D Only (2) A and C Only

(3) A, C and D Only (4) A and B Only

Ans. (2)

Sol. Element with atomic number 9 is Fluorine

$F(9) = 1s^2\ 2s^2\ 2p^5$

[⇅] [⇅] [⇅|⇅|↑]

(A) 5 electrons can be up-spin $\left[m_s = +\frac{1}{2}\right]$ and 4 electrons can be down spin $\left[m_s = -\frac{1}{2}\right]$

(B) Unpaired electron can be in anyone of p_x, p_y or p_z orbital

(C) Last electron is in 2p subshell with n = 2, $\ell = 1$

(D) Angular node for s-orbital = 0 while of each p-orbital = 1

Sum of all angular node = 3

66. The number of species from the following that are involved in sp^3d^2 hybridization is

$[Co(NH_3)_6]^{3+}$, SF_6, $[CrF_6]^{3-}$, $[CoF_6]^{3-}$, $[Mn(CN)_6]^{3-}$ and $[MnCl_6]^{3-}$

(1) 5 (2) 6

(3) 4 (4) 3

Ans. (3)

Sol. In $[Co(NH_3)_6]^{3+}$, Co^{+3} : $[Ar]3d^6$, NH_3 is S.F.L

Hybridisation state of Co^{3+} is d^2sp^3

In SF_6, Hybridisation state of sulphur is sp^3d^2

In $[CrF_6]^{3-}$, Cr^{+3} : $[Ar]3d^3$

Hybridisation state of Cr^{3+} is d^2sp^3

$[CoF_6]^{3-}$, Co^{+3} : $[Ar]3d^6$ F^- is W.F.L

Hybridisation state of Co^{3+} is sp^3d^2

$[Mn(CN)_6]^{3-}$, Mn^{+3} : $[Ar]3d^4$ CN^- is S.F.L

Hybridisation state of Mn^{3+} is d^2sp^3

$[MnCl_6]^{3-}$, Mn^{+3} : $[Ar]3d^4$ Cl^- is W.F.L

Hybridisation state of Cl^- is sp^3d^2

Total number of sp^3d^2 hybridized molecules is 3

67. Match the **LIST-I** with **LIST-II**

LIST-I (Reagent)		**LIST-II (Functional Group detected)**	
A.	Sodium bicarbonate solution	I.	double bond/unsaturation
B.	Neutral ferric chloride	II.	carboxylic acid
C.	ceric ammonium nitrate	III.	phenolic - OH
D.	alkaline $KMnO_4$	IV	alcoholic - OH

Choose the ***correct*** answer from the options given below :

(1) A-II, B-III, C-IV, D-I

(2) A-II, B-III, C-I, D-IV

(3) A-III, B-II, C-IV, D-I

(4) A-II, B-IV, C-III, D-I

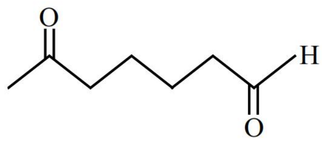
Ans. (1)

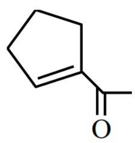
Sol. (1) Carboxylic acid gives efferve scence with sodium bicarbonate solution

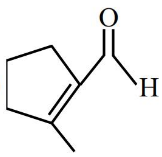
(2) Phenolic-OH gives voilet coloured complex with Neutral $FeCl_3$.

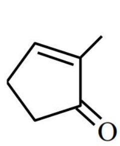
(3) Alcoholic-OH gives Red colour with cerric ammonium Nitrate.

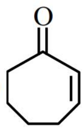
(4) When alkaline $KMnO_4$ reacts with an unsaturated compound (Alkene or alkyne) the purple colour of $KMnO_4$ solution disappears, indicating positive test for unsaturation.

68. When  undergoes intramolecular aldol condensation, the major product formed is :

(1) 

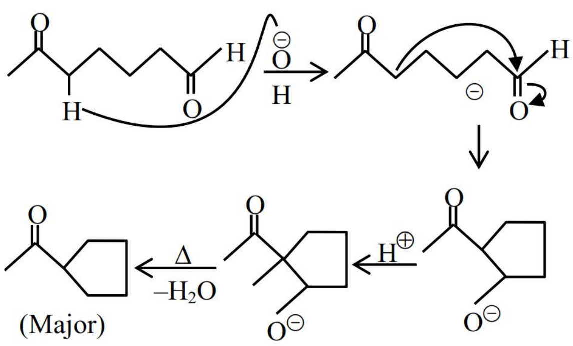
(2) 

(3) 

(4) 

Ans. (1)

Sol. Aldol condensation reaction



69. Match the **LIST-I** with **LIST-II**

LIST-I (Complex/Species)		LIST-II (Shape & magnetic moment)	
A.	$[Ni(CO)_4]$	I.	Tetrahedral, 2.8 BM
B.	$[Ni(CN)_4]^{2-}$	II.	Square planar, 0 BM
C.	$[NiCl_4]^{2-}$	III.	Tetrahedral, 0 BM
D.	$[MnBr_4]^{2-}$	IV	Tetrahedral, 5.9 BM

Choose the ***correct*** answer from the options given below :

(1) A-III, B-IV, C-II, D-I

(2) A-I, B-II, C-III, D-IV

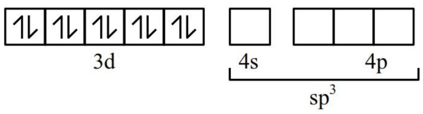
(3) A-III, B-II, C-I, D-IV

(4) A-IV, B-I, C-III, D-II

Ans. **(3)**

Sol. (A) $[Ni(CO)_4]$, Ni^0 : $[Ar]3d^8\ 4s^2$

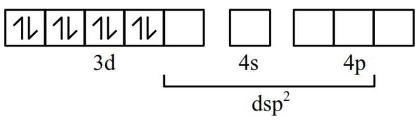
Valence orbitals of Ni^0 in pre-hybridisation state :



Tetrahedral, Diamagnetic, $\mu = 0$ B.M.

(B) $[Ni(CN)_4]^{2-}$, Ni^{+2} : $[Ar]3d^8 4s^0$

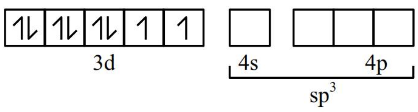
Valence orbitals of Ni^{+2} in pre-hybridisation state :



Square planar, Diamagnetic, $\mu = 0$ B.M.

(C) $[NiCl_4]^{2-}$, Ni^{+2} : $[Ar]3d^8 4s^0$

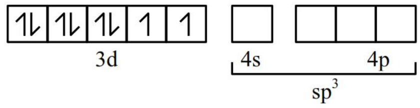
Valence orbitals of Ni^{+2} in ground state :



Tetrahedral, paramagnetic, $\mu = \sqrt{8} = 2.8$ B.M.

(D) $[MnBr_4]^{2-}$, Mn^{+2} : $[Ar]3d^5$

Valence orbitals of Mn^{+2} in ground state :



Tetrahedral, paramagnetic, $\mu = \sqrt{35} = 5.9$ B.M.

70. Given below are two statements :

Statement I : H_2Se is more acidic than H_2Te.

Statement II : H_2Se has higher bond enthalpy for dissociation than H_2Te.

In the light of the above statements, choose the ***correct*** answer from the options given below.

(1) Both statement I and Statement II are false.

(2) Both statement I and Statement II are true.

(3) Statement I is true but Statement II is false.

(4) Statement I is false but Statement II is true.

Ans. **(4)**

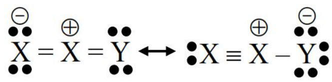
Sol. Acidic strength : $H_2Se < H_2Te$

$\Delta_{dis}H$: H_2Se > H_2Te

[276 KJ/mol] [238 KJ/mol]

SECTION-B

71. Resonance in X_2Y can be represented as



The enthalpy of formation of

$X_2Y\left(X \equiv X(g) + \frac{1}{2}Y = Y(g) \rightarrow X_2Y(g)\right)$ is 80 kJ mol^{-1}.

The magnitude of resonance energy of X_2Y is ________ kJ mol^{-1} (nearest integer value)

Given : Bond energies of $X \equiv X$, $X = X$, $Y = Y$ and $X = Y$ are 940, 410, 500 and 602 kJ mol^{-1} respectively.

valence X : 3, Y : 2

Ans. **(98)**

Sol. $\Delta H_{R.E} = \Delta H_{f(exp)} - \Delta H_{f(Theo)}$

$\Delta H_{f(exp)}$ for $X_2Y_{(g)} = 80$ kJ/mole

for $\Delta H_{f(Theo)}$

$$X_{2(g)} + \frac{1}{2}Y_{2(g)} \rightarrow X_2Y_{(g)}\ \Delta H_f = ?$$

$$\Delta H_{f(Theo)} = \left(BE_{X\equiv X} + \frac{1}{2}BE_{Y=Y}\right) - \left(BE_{X=X} + BE_{X=Y}\right)$$

$$= \left(940 + \frac{1}{2}\times 500\right) - (410 + 602)$$

= 178 kJ/mole

$\Delta H_{R.E} = 80 - 178$

$= -98$ kJ/mol

$|\Delta H_{R.E}| = 98$

72. The energy of an electron in first Bohr orbit of H-atom is –13.6 eV. The magnitude of energy value of electron in the first excited state of Be^{3+} is ______ eV. (nearest integer value)

Ans. **(54)**

Sol. $E_T = -13.6\dfrac{z^2}{n^2}$ ev

For energy of H-atom, energy of 1st Bohr orbit $E_1 = -13.6$ eV [z = 1, n = 1]

For Be^{+3} ion, energy of Ist E.S. [z = 4, n = 2]

$$\frac{E_H}{E_{Be^{+3}}} = \frac{z_1^2}{n_1^2}\times\frac{n_2^2}{z_2^2}$$

$$\frac{E_H}{E_{Be^{+3}}} = \frac{1}{1}\times\frac{4}{16}$$

$$E_{Be^{+3}} = -13.6\times 4 = -54.4\,eV$$

$$|E_{Be^{+3}}| = 54.4\,eV$$

73. 20 mL of sodium iodide solution gave 4.74 g silver iodide when treated with excess of silver nitrate solution. The molarity of the sodium iodide solution is ______ M. (Nearest Integer value)

(Given : Na = 23, I = 127, Ag = 108, N = 14, O = 16 g mol^{-1})

Ans. **(1)**

Sol. $NaI_{(aq)} + AgNO_{3(aq)} \rightarrow AgI_{(s)} + NaNO_3(aq)$

M, 20 ml   excess   4.74g

Moles of I^- in NaI = Moles of (I^-) in AgI = $\dfrac{4.74}{235}$

Moles of NaI = $\dfrac{4.74}{235}$

Molarity [NaI] = $\dfrac{4.74}{235\times 0.02}$ = 1.008

74. The equilibrium constant for decomposition of $H_2O(g)$

$$H_2O(g) \rightleftharpoons H_2(g) + \frac{1}{2}O_2(g)\ (\Delta G^\circ = 92.34\text{ kJ mol}^{-1})$$

is 8.0 × 10^{-3} at 2300 K and total pressure at equilibrium is 1 bar. Under this condition, the degree of dissociation (α) of water is _____ × 10^{-2} (nearest integer value).

[Assume α is negligible with respect to 1]

Ans. **(5)**

Sol. $$H_2O(g) \rightleftharpoons H_{2(g)} + \frac{1}{2}O_{2(g)}$$

t = 0   1 mole

$t = t_{eq}$ 1–α   α $\frac{\alpha}{2}$

$n_T = 1 + \frac{\alpha}{2} \simeq 1\ (\alpha << 1)$

$$k_P = \frac{P_{H_2}\cdot P_{O_2}^{1/2}}{P_{H_2O}} = \frac{(\alpha\cdot P)\left(\frac{\alpha}{2}P\right)^{\frac{1}{2}}}{(1-\alpha)P}$$

P = 1

$$8\times 10^{-3} = \frac{\alpha^{3/2}}{\sqrt{2}}$$

$$\alpha^{3/2} = 8\sqrt{2}\times 10^{-3}$$

$\alpha^3 = 128 \times 10^{-6}$

$$\alpha = \sqrt[3]{128}\times 10^{-2}$$

$= 5.03 \times 10^{-2}$

75. Consider the following half cell reaction

$$Cr_2O_7^{2-}(aq) + 6e^- + 14H^+(aq) \rightarrow 2Cr^{3+}(aq) + 7H_2O(l)$$

The reaction was conducted with the ratio of $\frac{[Cr^{3+}]^2}{[Cr_2O_7^{2-}]} = 10^{-6}$. The pH value at which the EMF of the half cell will become zero is ________ .

(nearest integer value)

[Given : standard half cell reduction potential $E^{o}_{Cr_2O_7^{2-}, H^+/Cr^{3+}} = 1.33V, \frac{2.303RT}{F} = 0.059V$]

Ans. **(10)**

Sol. $Cr_2O^{-2}_{7(aq)} + 14H^{+}_{(aq)} + 6e^- \rightarrow 2Cr^{+3}_{(aq)} + 7H_2O_{(\ell)}$

$$E_R = E_R^0 - \frac{0.059}{6}\log\frac{[Cr^{+3}]^2}{[Cr_2O_7^{-2}][H^+]^{14}}$$

$$0 = 1.33 - \frac{0.059}{6}\log\frac{10^{-6}}{[H^+]^{14}}$$

$$\frac{1.33\times 6}{0.059} = \log\frac{10^{-6}}{[H]^{14}}$$

$135.254 = -6 - 14 \log [H^+]$

$141.254 = 14$ pH

$$pH = \frac{141.254}{14} = 10.08$$