Long Answer Type Questions

[4 MARKS]

Que 1. PQRS is a diameter of a circle of radius 6 cm. The length PQ, QR and RS are equal. Semicircles are drawn on PQ and QS as diameters is shown in Fig. 12.36. Find the perimeter and area of the shaded region.

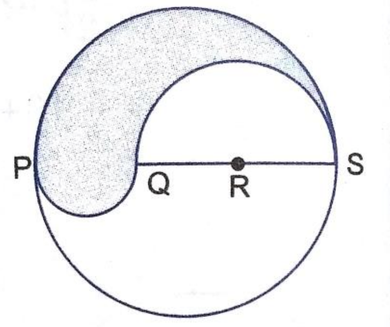

Fig. 12.36

Sol. We have,

PS = Diameter of a circle of radius 6 cm = 12 cm

$\therefore\ PQ = QR = RS = \frac{12}{3} = 4\ cm$

$QS = QR + RS = (4 + 4)\ cm = 8cm$

Hence, required perimeter

= Arc of semicircle of radius 6 cm + Arc of semicircle of radius 4 cm

+ ARC of semi-circle of radius 2 cm

$= (\pi \times 6 + \pi \times 4 + \pi \times 2)\ cm = 12\pi\text{cm} = 12 \times \frac{22}{7} = \frac{264}{7} = 37.71$ cm.

Required area = Area of semicircle with PS as diameter + Area of semicircle with PQ as diameter – Area of semi-circle with QS as diameter

$$= \frac{1}{2} \times \frac{22}{7} \times (6)^2 + \frac{1}{2} \times \frac{22}{7} \times (2)^2 - \frac{1}{2} \times \frac{22}{7} \times (4)^2$$

$$= \frac{1}{2} \times \frac{22}{7}(6^2 + 2^2 - 4^2) = \frac{1}{2} \times \frac{22}{7} \times 24 = \frac{264}{7} cm^2$$

$$= 37.71\ cm^2$$

Que 2. Figure 12.37 depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and white. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.

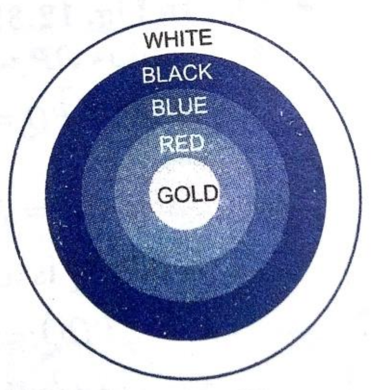

Fig. 12.37

Sol. The area of Gold region = $\pi(10.5)^2 = \frac{22}{7} \times 110.25$

$$= \frac{2425.2}{7} cm^2 = 346.5\ cm^2$$

The area of Red region = $\pi[(21)^2 - (10.5)^2]$

$$= \pi[441 - 110.25]$$

$$= \frac{22}{7} \times 330.75 = \frac{7276.5}{7} cm^2 = 1039.5\ cm^2$$

The area of Blue region = $[(31.5)^2 - (21)^2] = \pi[992.25 - 441]$

$$= \frac{22}{7} \times 551.25 = \frac{12127.5}{7} = 1732.5\ cm^2$$

The area of Black region = $\pi[(42)^2 - (31.5)^2]$

$$= \frac{22}{7}[1764 - 992.25] = \frac{16978.5}{7} = 2425.5\ cm^2$$

And the area of White region = $\pi[(52.5)^2 - (42)^2]$

$$= \frac{22}{7}[2756.25 - 1764]$$

$$= \frac{22}{7} \times 992.25 = \frac{21829.5}{7} = 3118.5\ cm^2$$

Que 3. The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distance travelled by their tips in 2 days.

Sol. In 2 days, the short hand will complete 4 rounds.

$\therefore$ Distance moved by its tip = 4 (Circumference of a circle of radius 4 cm)

$$= 4 \times \left(2 \times \frac{22}{7} \times 4\right) cm = \frac{704}{7} cm$$

In 2 days, the long hand will complete 48 rounds.

$\therefore$ Distance moved by its tip = 48 (Circumference of a circle of radius 6 cm)

$$= 48 \times \left(2 \times \frac{22}{7} \times 6\right) cm = \frac{12672}{7} cm$$

Hence, Sum of the distance moved by the tips of two hands of the clock

$= \frac{704}{7} + \frac{12672}{7} = \frac{13376}{7}$

= 1910.86 cm

Que 4. Fig. 12.38, depicts a racing track whose left and right ends are semicircular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:
(i) the distance around the track along its inner edge.
(ii) the area of the track.

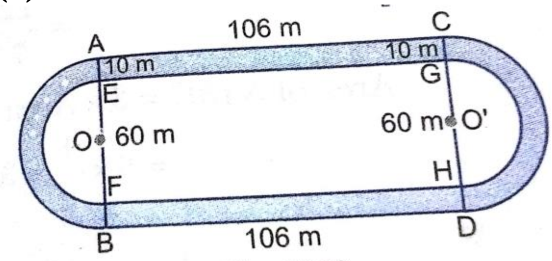

Fig. 12.38

Sol. Here, we have

OE = O'G = 30 m

AE = CG = 10 m

OA = O'C = (30 + 10) m = 40 m

AC = EG = FH = BD = 106 m

(i) The distance around the track along its inner edge

= EG + FH + 2 × (circumference of the semicircle of radius OE = 30 cm)

$= 106 + 106 + 2\left(\frac{1}{2} \times 2\pi \times 30\right) = 212 + 60\pi$

$= 212 + 60 \times \frac{22}{7} = \left(212 + \frac{1320}{7}\right) m = \left(\frac{1484+1320}{7}\right) m = \frac{2804}{7} m = 400\frac{4}{7} m$

(ii) Area of the track = Area of the shaded region

= Area of rectangle AEGC + Area of rectangle BFHD + 2 (Area of the semicircle of radius 40 m – Area of the semicircle with radius 30 m)

$= [(10 \times 106) + (10 \times 106)] + 2\left\{\frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2\right\}$

$= 1060 + 1060 + \frac{22}{7}[(40)^2 - (30)^2]$

$= 2120 + \frac{22}{7} \times 700 = 2120 + 2200 = 4320\, m^2$

Que 5. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig. 12.39). Find the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$)

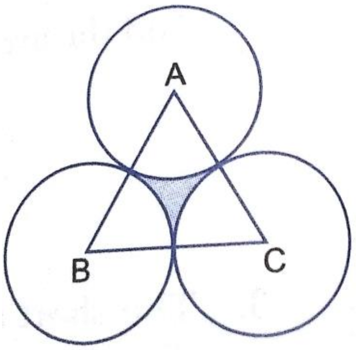

Fig. 12.39

Sol. Let each side of the equilateral triangle be x cm. Then,

Area of equilateral triangle ABC = 17320.5 cm^2 (Given)

$\Rightarrow \quad \frac{\sqrt{3}}{4}x^2 = 17320.5 \qquad \Rightarrow \quad \frac{1.73205}{4}x^2 = 17320.5$

$\Rightarrow \quad x^2 = \frac{4 \times 17320.5}{1.73205} \qquad \Rightarrow \quad x^2 = 40000$

$\therefore \quad x = 200\ cm$

Thus, radius of each circle = $\frac{200}{2} = 100\ cm$

Now, area of shaded region = Area of $\Delta ABC - 3 \times$ Area of a sector of angle 60° and radius 100 cm

$= 17320.5 - 3 \times \frac{60}{360} \times \pi \times (100)^2$

$= 17320.5 - \frac{1}{2} \times \pi \times 100 \times 100 = 17320.5 - 3.14 \times 5000$

= 17320.5 – 15700 = 1620.5 cm^2

Que 6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. 12.40. Find the area of the design.

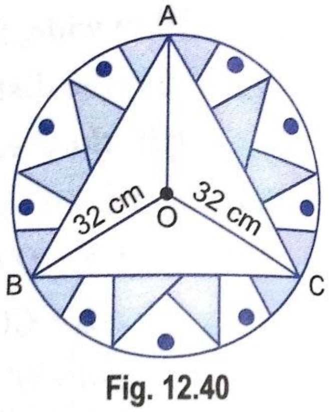

Fig. 12.40

Sol. Here, ΔABC is an equilateral triangle. Let O be the circumcenter of circumcircle.

Radius, r = 32 cm.

Now, area of circle = πr^2

$$= \frac{22}{7} \times 32 \times 32 = \frac{22528}{7}\ cm^2$$

Area of ΔABC = 3 × Area of ΔBOC

$$= 3 \times \frac{1}{2} \times 32 \times 32 \times \sin 120°$$

$$[\angle BOC = 2\angle BAC = 2 \times 60° = 120°]$$

$$= 3 \times 16 \times 32 \times \frac{\sqrt{3}}{2} \quad \left(\because \sin 120° = \sin(180° - 60°) = \sin 60° = \frac{\sqrt{3}}{2}\right)$$

$$= 3 \times 16 \times 16 \times \sqrt{3} = 768\sqrt{3}\ cm^2$$

∴ Area of the design = Area of the circle – Area of ΔABC

$$= \left(\frac{22528}{7} - 768\sqrt{3}\right)\ cm^2$$

$$= (3218.28 - 1330.176)\ cm^2 = 1888.7\ cm^2$$

Que 7. In Fig. 12.41, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

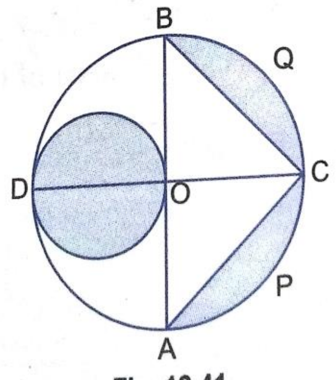

Fig. 12.41

Sol. Here, area of sector OBQC = $\frac{90°}{360°} \times \pi \times (7)^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{2} cm^2$$

And, Area of $\Delta OBC = \frac{1}{2} \times OC \times OB = \frac{1}{2} \times 7 \times 7 = \frac{49}{2}\ cm^2$

∴ Area of the segment BQC = Area of sector OBQC – Area of ΔOBC

$$= \frac{77}{2} - \frac{49}{2} = \frac{28}{2} = 14\ cm^2$$

Similarly, area of the segment APC = 14 cm^2

Now, the area of the circle with OD as diameter = $\pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} cm^2$

Hence, the total area of the shaded region

$$= \left(14 + 14 + \frac{77}{2}\right)\ cm^2 = \left(28 + \frac{77}{2}\right)\ cm^2$$

$$= \left(\frac{56+77}{2}\right)\ cm^2 = \frac{133}{2} cm^2 = 66.5\ cm^2$$

Que 8. In Fig. 1242 ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.

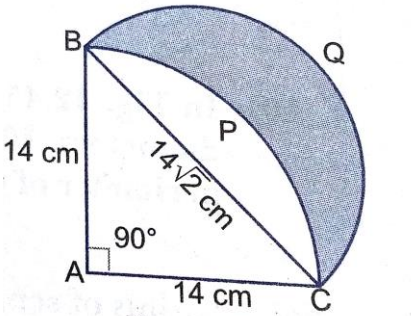

Fig. 12.42

Sol. In ΔABC, we have

$BC = \sqrt{(AC)^2 + (AB)^2}$ (By Pythagoras Theorem)

$= \sqrt{(14)^2 + (14)^2} = \sqrt{196 + 196} = \sqrt{392} = 14\sqrt{2}$

Now, area of sector ABPC = $\frac{90°}{360°} \times \pi(14)^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154\ cm^2$$

And, Area of segment BPC = Area of sector ABPC – Area of ΔABC

= (154 – 98) cm^2 = 56 cm^2

Now, we have radius of semi-circle BQC = $\frac{14\sqrt{2}}{2}\ cm = 7\sqrt{2}\ cm$

∴ Area of semi-circle $= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} = 154\ \text{cm}^2$

Hence, area of the shaded region

= Area of the semi-circle BQC – Area of the segment BPC

= (154 – 56) cm^2 = 98 cm^2