EXERCISE 13.2 - PROBABILITY

Q No 1 If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $P(A \cap B)$ if A and B are independent events.

Sol. Since A and B are independent events

$$\therefore P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$$

Q No 2. Two cards are drawn at random and without replacement from pack of 52 playing cards. Find the probability that both cards are black.

Soln: Let E_1 = first card drawn is black.

E_2 = 2nd card drawn is black.

$$\therefore \text{Required probability} = P(E_1 \text{ and } E_2) = P(E_1 \cap E_2)$$

$$= P(E_1) \cdot P(E_2 | E_1)$$

$$= \frac{26}{52} \times \frac{25}{51} = \frac{25}{102}.$$

Q No. 3. A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

Sol. No. of good oranges = 12

Total oranges = 15

$\therefore$ Required probability = P(Three good oranges taken out one by one without replacement)

$$= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13} = \frac{44}{91}$$

Q No 4: A fair coin and an unbiased die are tossed. Let 'A' be the event head appears on coin and B be the event 3 appears on die. Check whether A and B are independent or not.

Sol. Here Sample space $S = \left\{ \begin{matrix} (H,1)\ (H,2)\ (H,3)\ (H,4)\ (H,5)\ (H,6) \\ (T,1)\ (T,2)\ (T,3)\ (T,4)\ (T,5)\ (T,6) \end{matrix} \right\}$

A: Head appear on coin

i.e $A = \{(H,1)\ (H,2)\ (H,3)\ (H,4)\ (H,5)\ (H,6)\}$

B: 3 appear on die

$B = \{(H,3)\ (T,3)\} \Rightarrow A \cap B = \{(H,3)\}$

Now $P(A) = \frac{6}{12} = \frac{1}{2}$

$P(B) = \frac{2}{12} = \frac{1}{6}$ and $P(A \cap B) = \frac{1}{12}$

Now $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12} = P(A \cap B)$

$\therefore$ A and B are independent.

Q No 5 A die marked 1, 2, 3 in Red and 4, 5, 6 in green is tossed. Let A be the event No. is even and B is event Number is Red. Are A and B independent?

Sol. Sample Space $S = \{1, 2, 3, 4, 5, 6\}$

A: Number is even $= \{2, 4, 6\}$

B: Number is Red $= \{1, 2, 3\}$

$\therefore A \cap B = \{2\}$

Now $P(A) = \frac{3}{6} = \frac{1}{2}$; $P(B) = \frac{3}{6} = \frac{1}{2}$ $P(A \cap B) = \frac{1}{6}$

Now $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq P(A \cap B)$

$\therefore$ A and B are not independent.

Q No 6. Let E and F be events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent?

Sol. Here $P(E) \times P(F) = \frac{3}{5} \times \frac{3}{10}$

$= \frac{9}{50} \neq \frac{1}{5} = P(E \cap F)$

$\therefore$ E and F are not independent.

Q No. 7 Given that the events E and F are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Find p if they are
(i) Mutually Exclusive (ii) Independent.

Sol. (i) When A and B are mutually Exclusive then

$$A \cap B = \phi \Rightarrow P(A \cap B) = 0$$

Now $\therefore$ $P(A \cup B) = P(A) + P(B)$

$$\therefore \quad \frac{3}{5} = \frac{1}{2} + p$$

$$\Rightarrow \quad p = \frac{3}{5} - \frac{1}{2} = \frac{6-5}{10} = \frac{1}{10}$$

(ii) When A and B are independent.

$$P(A \cap B) = P(A) \times P(B) = \frac{1}{2} \times p = p/2$$

Also $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \quad \frac{3}{5} = \frac{1}{2} + p - \frac{p}{2}$$

$$\Rightarrow \quad 6 = 5 + 10p - 5p$$

$$\Rightarrow \quad 1 = 5p \Rightarrow p = 1/5.$$

Q No. 8 Let A and B be independent events with $P(A) = 0.3$ and $P(B) = 0.4$ find.
(i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(A|B)$ (iv) $P(B|A)$

Sol. Since A and B are independent events

(i) $\therefore$ $P(A \cap B) = P(A) \times P(B)$

$$= 0.3 \times 0.4 = 0.12$$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= 0.3 + 0.4 - 0.12 = 0.7 - 0.12 = 0.58$$

(iii) $$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \times P(B)}{P(B)} = P(A) = 0.3$$

(iv) $$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B) \times P(A)}{P(A)} = P(B) = 0.4.$$

Q No. 9. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, Find $P(\text{not } A \text{ and not } B)$.

Sol. Given $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{8}$

$$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} = \frac{5}{8}$$

Now $P(\text{not } A \text{ and Not } B) = P(A^c \cap B^c) = P((A \cup B)^c)$

$$= 1 - P(A \cup B) = 1 - \frac{5}{8} = \frac{8-5}{8} = \frac{3}{8}$$

QNo. 10 Events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\text{not } A \text{ or nor } B) = \frac{1}{4}$. State whether A and B are independent.

Sol. Given $P(\text{not } A \text{ or not } B) = \frac{1}{4}$

$$\therefore \quad P(A^c \cup B^c) = \frac{1}{4}$$

$$P((A \cap B)^c) = \frac{1}{4} \Rightarrow 1 - P(A \cap B) = \frac{1}{4}.$$

$$\Rightarrow P(A \cap B) = 1 - \frac{1}{4} = \frac{3}{4}$$

Now $P(A) \times P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24} \neq \frac{3}{4} = P(A \cap B)$

$\Rightarrow$ A and B are not independent.

QNo. 11. Given two independent events A and B such that $P(A) = 0.3$, $P(B) = 0.6$. Find (i) $P(A \text{ and } B)$ (ii) $P(A \text{ and not } B)$ (iii) $P(A \text{ or } B)$ (iv) $P(\text{Neither } A \text{ nor } B)$

Sol.

(i) $P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B) = 0.3 \times 0.6 = 0.18$

(ii) $P(A \text{ and not } B) = P(A \cap B^c) = P(A) \times P(B^c) = P(A) \times (1 - P(B))$

$$= 0.3 \times (1 - 0.6) = 0.3 \times 0.4 = 0.12$$

(Note that if A and B are independent then A and B^c are also independent)

(iii) $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= 0.3 + 0.6 - 0.18$$

$$= 0.9 - 0.18 = 0.72$$

(iv) $P(\text{Neither } A \text{ nor } B) = P(A^c \cap B^c) = P((A \cup B)^c)$

$$= 1 - P(A \cup B) = 1 - 0.72 = 0.28$$

QNo 12. A die is tossed thrice. Find the probability of getting an odd no. at least once.

Sol. When a die is tossed thrice, sample space

$S = \{(x, y, z);\ x, y, z \in \{1, 2, 3, 4, 5, 6\}\}$ which contains $6 \times 6 \times 6 = 216$ equally likely sample points.

Let E: An odd no. at least once.

E^c: Not an odd number any time.

i.e. $E^c = \{(x, y, z);\ x, y, z \in \{2, 4, 6\}\}$

$\Rightarrow E^c$ contains $3\times3\times3 = 27$ simple events.

$\therefore$ Required probability $= P(E) = 1 - P(E^c)$

$$= 1 - \frac{27}{216} = 1 - \frac{1}{8} = \frac{7}{8}$$

Q.No.13. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that (i) both balls are red. (ii) first ball is black and second is red. (iii) One of them is black and other is red.

Sol:

Number of black balls $= 10$

No. of Red balls $= 8$

Total balls $= 18$

(i) P(Both balls are red) = P(R and R)

$$= P(R) \times P(R) = \frac{8}{18} \times \frac{8}{18} = \frac{16}{81}$$

(ii) P(first black Second is red) $= P(B) \times P(R) = \frac{10}{18} \times \frac{8}{18} = \frac{20}{81}$

(iii) P(one is black and other is Red) = P(BR or RB)

$$= P(B) \times P(R) + P(R) \times P(B)$$

$$= \frac{10}{18} \times \frac{8}{18} + \frac{8}{18} \times \frac{10}{18} = \frac{20}{81} + \frac{20}{81} = \frac{40}{81}$$

Q.No14. Probability of solving specific problem independently by A and B are $1/2$ and $1/3$ respectively. If both try to solve the problem independently, find the probability that

(i) The problem is solved (ii) Exactly one of them solves the problem

Sol:

Let E_1: A Solves the problem

E_2: B solves the problem.

Then $P(E_1) = \frac{1}{2}$; $P(E_2) = \frac{1}{3}$

(i) Required probability = P(Problem is solved)

$= 1 - P(\text{Problem is not solved})$

$= 1 - P(E_1^c \cap E_2^c)$

$= 1 - P(E_1^c) \times P(E_2^c)$

$= 1 - \left(1-\frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) = 1 - \frac{1}{2} \times \frac{2}{3} = 1 - \frac{1}{3} = \frac{2}{3}$

(ii) Required probability = P(Exactly one solves problem)

$= P((E_1 \text{ and not } E_2) \text{ or } (E_2 \text{ and not } E_1))$

$= P(E_1 \cap E_2^c) + P(E_2 \cap E_1^c)$

$= P(E_1) \times P(E_2^c) + P(E_2) \times P(E_1^c)$

$$= P(E_1)(1-P(E_2)) + P(E_2)(1-P(E_1))$$
$$= \frac{1}{2}\left(1-\frac{1}{3}\right) + \frac{1}{3}\left(1-\frac{1}{2}\right) = \frac{1}{2}\times\frac{2}{3} + \frac{1}{3}\times\frac{1}{2} = \frac{1}{3}+\frac{1}{6} = \frac{2+1}{6} = \frac{3}{6} = \frac{1}{2}$$

Q No. 15. One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are events E and F independent.

(i) E: The card drawn is spade
F: The card drawn is ace.

(ii) E: The card draw is black
F: The card drawn is king

(iii) E: The card drawn is a king or Queen
F: The card drawn is queen or Jack

Sol: (i) Here $P(E) = P(\text{Card drawn is spade})$
$$= \frac{13}{52} = \frac{1}{4}$$
and $P(F) = P(\text{Card draw is ace}) = \frac{4}{52} = \frac{1}{13}$.

Also $P(E\cap F) = P(\text{Card drawn is an ace of spade})$
$$= \frac{1}{52}$$

Now $P(E)\times P(F) = \frac{1}{4}\times\frac{1}{13} = \frac{1}{52} = P(E\cap F)$

$\Rightarrow$ E and F are independent events.

(ii) Here $P(E) = P(\text{Card drawn is black}) = \frac{26}{52} = \frac{1}{2}$

$P(F) = P(\text{Card drawn is king}) = \frac{4}{52} = \frac{1}{13}$

Also $P(E\cap F) = P(\text{Card drawn is black king}) = \frac{2}{52} = \frac{1}{26}$

Now $P(E)\times P(F) = \frac{1}{2}\times\frac{1}{13} = \frac{1}{26} = P(E\cap F)$

$\therefore$ E, F are Independent events.

(iii) Here $P(E) = P(\text{Card drawn is king or queen}) = \frac{8}{52} = \frac{2}{13}$

$P(F) = P(\text{Card drawn is queen or jack}) = \frac{8}{52} = \frac{2}{13}$

Also. $P(\text{Card drawn is queen}) = 4/52 = \frac{1}{13}$

Since $P(E)\times P(F) = \frac{2}{13}\times\frac{2}{13} \neq \frac{1}{13} = P(E\cap F)$

$\therefore$ E and F are not Independent.

Q No. 16. In a hostel 60% of students read Hindi News paper, 40% read Eng. news paper and 20% read both Hindi and Eng. News paper. A student is selected at random.

(a) find the probability she read neither Hindi nor Eng.

(b) If she reads Hindi News paper, find the probability that she reads English News paper.

(c) If she reads Eng. News paper, find the probability that she reads Hindi News paper.

Sol.. Let A : A student reads Hindi News paper

B : A students reads English News paper.

Then $P(A) = \frac{60}{100} = \frac{3}{5}$ $\quad P(B) = \frac{40}{100} = \frac{2}{5}$

and $P(A \cap B) = \frac{20}{100} = \frac{1}{5}$.

(i) Required probability = P(Neither Hindi Nor Eng)

$= P((A \cup B)^c) = 1 - P(A \cup B)$

$= 1 - (P(A) + P(B) - P(A \cap B))$

$= 1 - \left(\frac{3}{5} + \frac{2}{5} - \frac{1}{5}\right) = 1 - \frac{4}{5} = \frac{1}{5}$.

(ii) Required probability = P(Reads Eng. when reads Hindi)

$= P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{1/5}{3/5} = \frac{1}{3}$.

(iii) Required Probability = P(Reads Hindi when reads Eng.)

$= P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/5}{2/5} = 1/2$.

QNo. 17. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

(A) 0 $\quad$ (B) $\frac{1}{3}$ $\quad$ (C) $\frac{1}{12}$ $\quad$ (D) $\frac{1}{36}$.

Sol. Here samples space $S = \{(x, y) ; x, y \in \{1, 2, 3, 4, 5, 6\}\}$ has $6 \times 6 = 36$ sample points.

P(Even prime on each dice) $= P((2, 2)) = \frac{1}{36}$

$\therefore$ (D) is correct answer.

Q.No. 18. Two Events A and B will be independent, if

(A) A and B are mutually exclusive.

(B) $P(A'B') = [1 - P(A)][1 - P(B)]$

(C) $P(A) = P(B)$

(D) $P(A) + P(B) = 1$

Soln. A and B are independent if

$$P(A \cap B) = P(A)\,P(B)$$

ie If $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$$= 1 - P(A \cup B)$$

$$= 1 - (P(A) + P(B) - P(A \cap B))$$

$$= 1 - P(A) - P(B) + P(A \cap B)$$

$$= 1 - P(A) - P(B) + P(A)\,P(B)$$

$$= (1 - P(A)) - (P(B) - P(A)P(B))$$

$$= (1 - P(A)) - P(B)(1 - P(A))$$

$$= (1 - P(A))(1 - P(B))$$

$\therefore$ B is correct option.

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