Simplification

SIMPLE ARITHMETIC OPERATIONS

It is a common need to simplify the expressions formulated according to the statements of the problems relating to the practical life. To do this, it is essential to follow in sequence the mathematical operations given by the term "BODMAS".

BODMAS

Each letter of the word BODMAS stands as follows:

B for bracket : [{(–)}]

There are four brackets, namely, – bar, ( ), { } and [ ]. They are removed, strictly in the order –, ( ), { } and [ ].

O for of: of

D for division : ÷

M for multiplication : ×

A for addition : +

S for subtraction : –

The order of various operations in exercises involving brackets and fractions must be performed strictly according to the order of the letters of the word BODMAS.

Note:

Here, $-\overline{5-8} = -(-3) = 3$.

Illustration 1 Simplify

$$8\frac{1}{2}-\left[3\frac{1}{5}\div 4\frac{1}{2} \text{ of } 5\frac{1}{3}+\left\{11-\left(3-\overline{1\frac{1}{4}-\frac{5}{8}}\right)\right\}\right]$$

Solution: Given expression

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2} \text{ of } \frac{16}{3}+\left\{11-\left(3-\overline{\frac{5}{4}-\frac{5}{8}}\right)\right\}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2} \text{ of } \frac{16}{3}+\left\{11-\left(3-\frac{5}{8}\right)\right\}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2} \text{ of } \frac{16}{3}+\left\{11-\frac{19}{8}\right\}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2} \text{ of } \frac{16}{3}+\frac{69}{8}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2}\times\frac{16}{3}+\frac{69}{8}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{24}{1}+\frac{69}{8}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\times\frac{1}{24}+\frac{69}{8}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{120}+\frac{69}{8}\right]$$

$$=\frac{17}{2}-\left[\frac{16+1035}{120}\right]$$

$$=\frac{17}{2}-\frac{1051}{120}$$

$$=\frac{1020-1051}{120}=-\frac{31}{120}$$

Illustration 2 Simplify

$$5\frac{1}{3}-\left\{4\frac{1}{3}-\left(3\frac{1}{3}-\overline{2\frac{1}{3}-\frac{1}{3}}\right)\right\}$$

Solution: Given expression

$$=\frac{16}{3}-\left\{\frac{13}{3}-\left(\frac{10}{3}-\overline{\frac{7}{3}-\frac{1}{3}}\right)\right\}$$

$$=\frac{16}{3}-\left\{\frac{13}{3}-\left(\frac{10}{3}-\frac{6}{3}\right)\right\}$$

$$=\frac{16}{3}-\left\{\frac{13}{3}-\frac{4}{3}\right\}$$

$$=\frac{16}{3}-\left\{\frac{9}{3}\right\}=\frac{16}{3}-\frac{9}{3}=\frac{7}{3}=2\frac{1}{3}$$

Use of Algebraic Formulae

The following important formulae are sometimes found useful in dealing with the simplifications:

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$
4. $(a+b)^2 - (a-b)^2 = 4ab$
5. $a^2 - b^2 = (a+b)(a-b)$
6. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
   $= a^3 + b^3 + 3ab(a+b)$
7. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
   $= a^3 - b^3 - 3ab(a-b)$
8. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
9. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
10. $\dfrac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca} = (a+b+c)$
11. $a^4 - a^4 = (a^2 + b^2)(a+b)(a-b)$.

Illustration 3 Simplify the following:

(i) $0.32 \times 0.32 + 0.64 \times 0.68 + 0.68 \times 0.68$

Solution: Given expression

$= 0.32 \times 0.32 + 2 \times 0.32 \times 0.68 + 0.68 \times 0.68$

$= (0.32)^2 + 2 \times 0.32 \times 0.68 + (0.68)^2$

$= (0.32 + 0.68)^2 \quad [\because a^2 + 2ab + b^2 = (a+b)^2]$

$= 1^2 = 1$

(ii) $2.45 \times 2.45 - 0.9 \times 2.45 + 0.45 \times 0.45$

Solution: Given expression

$= 2.45 \times 2.45 - 2 \times 2.45 \times 0.45 + 0.45 \times 0.45$

$= (2.45)^2 - 2 \times 2.45 \times 0.45 + (0.45)^2$

$= (2.45 - 0.45)^2 \quad [\because a^2 - 2ab + b^2 = (a-b)^2]$

$= (2)^2 = 4$

(iii) $\dfrac{7 \times \{(146+92)^2 + (146-92)^2\}}{(146)^2 + (92)^2}$

Solution: Given expression

$= \dfrac{7 \times 2\{(146)^2 + (92)^2\}}{(146)^2 + (92)^2}$

$[\because (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)]$

$= 14$

(iv) $\dfrac{(0.345+0.255)^2 - (0.345-0.255)^2}{0.345 \times 1.02}$

Solution: Given expression

$= \dfrac{(0.345+0.255)^2 - (0.345-0.255)^2}{4 \times 0.345 \times 0.255}$

$= \dfrac{4 \times 0.345 \times 0.255}{4 \times 0.345 \times 0.255} \quad [\because (a+b)^2 - (a-b)^2 = 4ab]$

$= 1$

(v) $\dfrac{0.682 \times 0.682 - 0.318 \times 0.318}{0.682 - 0.318}$

Solution: Given expression

$= \dfrac{(0.682)^2 - (0.318)^2}{0.682 - 0.318}$

$= (0.682 + 0.318) \quad \left[\because \dfrac{a^2-b^2}{a-b} = a+b\right]$

$= 1$

(vi) $\dfrac{(3.29)^2 - (0.81)^2}{4}$

Solution: Given expression

$= \dfrac{(3.29)^2 - (0.81)^2}{3.29 + 0.81}$

$= (3.29 - 0.81) \quad \left[\because \dfrac{a^2-b^2}{a+b} = a-b\right]$

$= 2.48$

(vii) $(2.35)^3 + 1.95 \times (2.35)^2 + 7.05 \times (0.65)^2 + (0.65)^3$

Solution: Given expression

$= (2.35)^3 + 3 \times 0.65 \times (2.35)^2$

$+ 3 \times 2.35 \times (0.65)^2 + (0.65)^3$

$= (2.35 + 0.65)^3$

$[\because a^3 + 3a^2b + 3ab^2 + b^3 = (a+b)^3]$

$= (3)^3 = 27$

(viii) $\dfrac{(4.32)^3 - 0.96 \times (4.32)^2 + 12.96 \times (0.32)^2 - (0.32)^3}{4 \times 4 \times 4}$

Solution: Given expression

$= \dfrac{(4.32)^3 - 3 \times 0.32 \times (4.32)^2 + 3 \times 4.32 \times (0.32)^2 - (0.32)^3}{4 \times 4 \times 4}$

$= \dfrac{(4.32 - 0.32)^3}{4^3} \quad [\because a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3]$

$= \left(\dfrac{4}{4}\right)^3 = 1$

(ix) $\dfrac{885 \times 885 \times 885 + 115 \times 115 \times 115}{885 \times 885 + 115 \times 115 - 885 \times 115}$

Solution: Given expression

$= \dfrac{(885)^3 + (115)^3}{(885)^2 + (115)^2 - 885 \times 115}$

$= (885 + 115)$ $\left[\because \dfrac{a^3+b^3}{a^2-ab+b^2} = a+b\right]$

$= 1000$

(x) $\dfrac{0.62\times0.62\times0.62 - 0.41\times0.41\times0.41}{0.62\times0.62 + 0.62\times0.41 + 0.41 + 0.41}$

Solution: Given expression

$= \dfrac{(0.62)^3 - (0.41)^3}{(0.62)^2 + 0.62\times0.41 + (0.41)^2}$

$= (0.62 - 0.41)$ $\left[\because \dfrac{a^3-b^3}{a^2+ab+b^2} = a-b\right]$

$= 0.21$

(xi)

$$\frac{(2.3)^3 + (1.5)^3 + (1.2)^3 - 3\times2.3\times1.5\times1.2}{2.3\times2.3 + 1.5\times1.5 + 1.2\times1.2 - 2.3\times1.5 - 2.3\times1.2 - 1.5\times1.2}$$

Solution: Given expression

$= \dfrac{(2.3)^3 + (1.5)^3 + (1.2)^3 - 3\times2.3\times1.5\times1.2}{(2.3)^2 + (1.5)^2 + (1.2)^2 - 2.3\times1.5 - 2.3\times1.2 - 1.5\times1.2}$

$= (2.3 + 1.5 + 1.2)$

$\left[\because \dfrac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-ac-bc} = a+b+c\right]$

$= 5$

Surds and Indices

a^n is called a **surd** if n is a fraction and a^n is called an **index** if n is an integer. a is called the **base**.

Some Useful Formulae

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = (a^n)^m = a^{mn}$
4. $\left(\dfrac{a}{b}\right)^{-\frac{m}{n}} = \left(\dfrac{b}{a}\right)^{\frac{m}{n}}$
5. $a^m \div b^{-n} = a^m \times b^n$
6. $(\sqrt[n]{a})^n = a$, where 'n' is a +ve integer and 'a' a +ve rational number
7. $\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}$, where '$n$' is a +ve integer and '$a$', '$b$' are rational numbers
8. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$, where '$n$' is a +ve integer and '$a$', '$b$' are rational numbers
9. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$, where '$m$', '$n$' are +ve integers and '$a$' is a +ve rational number
10. $\sqrt[n]{\sqrt[m]{(a^k)^m}} = \sqrt[n]{a^k} = \sqrt[mn]{a^{km}}$, where '$m$', '$n$', '$k$' are +ve integers and '$a$' is a +ve rational number
11. $\sqrt{a}\times\sqrt{a} = a$
12. $\sqrt{a}\times\sqrt{b} = \sqrt{ab}$
13. $(\sqrt{a}+\sqrt{b})^2 = a + b + 2\sqrt{ab}$
14. $(\sqrt{a}-\sqrt{b})^2 = a + b - 2\sqrt{ab}$
15. $a + \sqrt{b} = c + \sqrt{d} \Rightarrow a = c$ and $b = d$
16. $\dfrac{1}{\sqrt{a}-\sqrt{b}} = \dfrac{\sqrt{a}+\sqrt{b}}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})} = \dfrac{\sqrt{a}+\sqrt{b}}{a-b}$
17. $\dfrac{1}{\sqrt{a}+\sqrt{b}} = \dfrac{\sqrt{a}-\sqrt{b}}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})} = \dfrac{\sqrt{a}-\sqrt{b}}{a-b}$
18. If $x = n(n+1)$, then
    (*a*) $\sqrt{x-\sqrt{x-\sqrt{x-\ldots\infty}}} = n$
    (*b*) $\sqrt{x+\sqrt{x+\sqrt{x+\ldots\infty}}} = (n+1)$.

Illustration 4 Find the value of $(243)^{0.8} \div (243)^{0.4}$.

Solution: $(243)^{0.8} \div (243)^{0.4} = (243)^{0.8-0.4}$

$[\because a^m \div a^n = a^{m-n}]$

$= (243)^{0.4}$

$= (3^5)^{\frac{2}{5}} = 3^2 = 9$

Illustration 5 Find the value of $(27)^{2/3} \div (64)^{-4/3}$

Solution: $(27)^{2/3} \div (64)^{-4/3} = (3^3)^{2/3} \times (64)^{4/3}$

$[\because a^m \div b^{-n} = a^m \times b^n]$

$= 3^2 \times (4^3)^{4/3}$

$= 9 \times (4^4) = 9 \times 256 = 2304$

Illustration 6 Find the value of $(-3)^{(-2)^{(-2)^{(-4)}}}$

Solution: $(-3)^{(-2)^{(-2)^{(-4)}}} = \left(-\frac{1}{3}\right)^{(2)^{(-2)^{(-4)}}} = \left(\frac{1}{9}\right)^{(-2)^{(-4)}}$

$= (9)^{(2)^{(-4)}}$

$= (81)^{-4} = \left(\frac{1}{81}\right)^4$

$= \left(\frac{1}{3^4}\right)^4 = \frac{1}{3}$

Illustration 7 Find the value of x if $\sqrt[5]{2x-7} - 3 = 0$

Solution: We have

$\sqrt[5]{2x-7} - 3 = 0$

$\Rightarrow \quad \sqrt[5]{2x-7} = 3$

$\Rightarrow \quad (\sqrt[5]{2x-7})^5 = 3^5$

$\Rightarrow \quad 2x - 7 = 243 \qquad [\because (\sqrt[n]{a})^n = a]$

$\Rightarrow \quad 2x = 250$ or, $x = 125$

Illustration 8 Find the value of $\sqrt[5]{64} \times \sqrt[5]{512}$

Solution: $\sqrt[5]{64} \times \sqrt[5]{512}$

$= \sqrt[5]{64 \times 512} \qquad [\because \sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}]$

$= \sqrt[5]{8^2 \times 8^3} = \sqrt[5]{8^5} = 8 \qquad [\because \sqrt[n]{a^n} = a]$

Illustration 9 Find the value of $\sqrt[3]{\sqrt{729}}$

Solution: $\sqrt[3]{\sqrt{729}} = \sqrt[6]{729} \qquad [\because \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}]$

$= \sqrt[6]{3^6} = 3 \qquad [\because \sqrt[n]{a^n} = a]$

Illustration 10 Find the value of $\frac{\sqrt[7]{\sqrt[5]{(21^7)^5}}}{\sqrt[5]{\sqrt[3]{(7^5)^3}}}$.

Solution: Given expression

$= \frac{\sqrt[7]{(21)^7}}{\sqrt[5]{(7)^5}} \qquad \left[\because \sqrt[n]{\sqrt[m]{(a^p)^m}} = \sqrt[n]{a^p}\right]$

$= \frac{21}{7} = 3 \qquad [\because \sqrt[n]{a^n} = a]$

Illustration 11 Find the value of $\sqrt{5} \times \sqrt{125}$

Solution: $\sqrt{5} \times \sqrt{125} = \sqrt{625} \qquad [\because \sqrt{a} \times \sqrt{b} = \sqrt{ab}]$

$= 25$

Illustration 12 Simplify each of the following by rationalising the denominator.

(i) $\frac{1}{2+\sqrt{3}}$ (ii) $\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

Solution: (i) $\frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$

$= \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$

(ii) $\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{4^2\times 3}+\sqrt{3^2\times 2}}$

$= \frac{7\sqrt{3}-5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} = \frac{7\sqrt{3}-5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} \times \frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}$

$= \frac{(7\sqrt{3}-5\sqrt{2})(4\sqrt{3}-3\sqrt{2})}{(4\sqrt{3}+3\sqrt{2})(4\sqrt{3}-3\sqrt{2})}$

$= \frac{7\sqrt{3}\times 4\sqrt{3} - 7\sqrt{3}\times 3\sqrt{2} - 5\sqrt{2}\times 4\sqrt{3} + 5\sqrt{2}\times 3\sqrt{2}}{(4\sqrt{3})^2-(3\sqrt{2})^2}$

$= \frac{28\sqrt{3\times 3} - 21\sqrt{3\times 2} - 20\sqrt{2\times 3} + 15\sqrt{2\times 2}}{16\times 3 - 9\times 2}$

$= \frac{28\times 3 - 21\times\sqrt{6} - 20\sqrt{6} + 15\times 2}{48-18}$

$= \frac{84-(21\times 20)\sqrt{6}+30}{30} = \frac{114-41\sqrt{6}}{30}$

Illustration 13 If a and b are rational numbers, find the values of a and b in the following equation:

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6}.$$

Solution: $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$

$= \frac{3+2+2\sqrt{3}\times\sqrt{2}}{3-2} = \frac{5+2\sqrt{6}}{1}$

$= 5 + 2\sqrt{6}$

$\therefore \quad \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6}$

$\Rightarrow \quad 5 + 2\sqrt{6} = a + b\sqrt{6}$

On equating rational and irrational parts, we get

$a = 5$ and $b = 2$.

Illustration 14 Find the value of

$$(\sqrt{72+\sqrt{72+\sqrt{72+\ldots\infty}}}) \div (\sqrt{12-\sqrt{12-\sqrt{12-\ldots\infty}}})$$

Solution: Since $72 = 9 \times 8$

therefore, $72 + \sqrt{72+\sqrt{72+\ldots\infty}} = 9$

Also, since $12 = 4 \times 3$

therefore, $\sqrt{12-\sqrt{12-\sqrt{12-\ldots\infty}}} = 3$

Thus, the given expression $= \frac{9}{3} = 3$

Fractions

1. Continued Fraction

Fractions of the form $7+\cfrac{2}{5+\cfrac{3}{4+\cfrac{2}{3+\cfrac{1}{4}}}}$

are called **continued fractions.**

To simplify a continued fraction, we start from the bottom and work upwards

Illustration 15 Simplify $3-\cfrac{1}{4+\cfrac{7}{9-\cfrac{5}{6+\cfrac{2}{3}}}}$.

Solution: $3-\cfrac{1}{4+\cfrac{7}{9-\cfrac{5}{6+\cfrac{2}{3}}}} = 3-\cfrac{1}{4+\cfrac{7}{9-\cfrac{15}{20}}}$

[Multiply the numerator and denominator of the lowest term $\cfrac{5}{\left(6+\frac{2}{5}\right)}$ by 3 to get $\frac{15}{20}$.]

$= 3-\cfrac{1}{4+\cfrac{7}{9-\frac{3}{4}}} = 3-\cfrac{1}{4+\frac{28}{33}}$

[Multiply the numerator and denominator of the lowest term $\cfrac{7}{\left(9-\frac{3}{4}\right)}$ by 4 to get $\frac{28}{33}$]

$= 3-\frac{33}{160}$

[Multiply the numerator and denominator of the term $\cfrac{1}{\left(4+\frac{28}{33}\right)}$ by 33 to get $\frac{33}{160}$]

$= \frac{480-33}{160} = \frac{447}{160} = 2\frac{127}{160}$

2. Comparison of Fractions

The following points are found useful while comparing two or more fractions:

(a) If the denominators of the fractions are same, the largest is one whose numerator is the largest.

Illustration 16 Which is the largest fraction among

$\frac{3}{8}, \frac{7}{8}$ and $\frac{5}{8}$?

Solution: $\frac{7}{8}$

(b) If the numerators of the fractions are same, the largest is one whose denominator is the smallest.

Illustration 17 Which is the largest fraction among

$\frac{5}{2}, \frac{5}{7}$ and $\frac{5}{9}$?

Solution: $\frac{5}{2}$

(c) If neither the numerators nor denominators of the fractions are same then they are converted into equivalent fractions of the same denominator by taking the L.C.M. of the denominators of the given fractions. Then, the fractions are compared according to (1).

Illustration 18 Which is the largest fraction among

$\frac{1}{2}, \frac{2}{3}, \frac{4}{5}$ and $\frac{5}{8}$?

Solution: L.C.M. of 2, 3, 5 and 8 = 120

Then, $\frac{1}{2} = \frac{1\times60}{2\times60} = \frac{60}{120}$

$\frac{2}{3} = \frac{2\times40}{3\times40} = \frac{80}{120}$

$\frac{4}{5} = \frac{4\times24}{5\times24} = \frac{96}{120}$

and, $\frac{5}{8} = \frac{5\times15}{8\times15} = \frac{75}{120}$

Now, the denominator of these fractions are same and the largest numerator is 96. Hence, the largest fraction is $\frac{96}{120}$, that is, $\frac{4}{5}$

(d) Two fractions can also be compared by cross multiplication method.

Illustration 19 Which is greater $\frac{6}{13}$ or $\frac{5}{7}$?

Solution: Step 1. Cross multiply the two given fractions

$$\frac{6}{13} \times \frac{5}{7},$$

we get $6 \times 7 = 42$ and $13 \times 5 = 65$

Step 2. Since 65 is greater than 42 and in 65, the numerator of $\frac{5}{7}$ is included, $\therefore$ $\frac{5}{7}$ is greater than $\frac{6}{13}$

(e) If the difference of the numerator and denominator of each of the given fractions be same then the fraction of the largest numerator is the smallest.

Illustration 20 Which one of the following fractions is the largest?

$$\frac{2}{3}, \frac{3}{4}, \frac{5}{6} \text{ and } \frac{9}{10}$$

Solution: Since in each of the given fractions the difference between the numerator and denominator is same and the largest numerator is 9, therefore, the largest fraction is $\frac{9}{10}$

(f) In the given fractions, $\frac{x}{y}, \frac{x+a}{y+b}, \frac{x+2a}{y+2b}, \ldots, \frac{x+na}{y+nb}$, where $a < b$

(a) If $\frac{\text{Increase in numerator}}{\text{Increase in denominator}}$ > first fraction, the last value is the greatest

(b) If $\frac{\text{Increase in numerator}}{\text{Increase in denominator}}$ < first fraction, the last value is the least.

(c) If $\frac{\text{Increase in numerator}}{\text{Increase in denominator}}$ = first fraction, all values are equal.

Illustration 21 Which one the following fractions is the greatest?

$$\frac{3}{8}, \frac{4}{11}, \frac{5}{14}, \frac{6}{17}, \frac{7}{20}.$$

Solutoin: Since, $\frac{\text{Increase in numerator}}{\text{Increase in denominator}} = \frac{1}{3}$ is less than the first fraction $\frac{3}{8}$, therefore, the first fraction $\frac{3}{8}$ is the greatest

Illustration 22 Which of the following fractions is the least?

$$\frac{2}{5}, \frac{4}{11}, \frac{6}{17}, \frac{8}{23}$$

Solution: Since $\frac{\text{Increase in numerator}}{\text{Increase in denominator}} = \frac{2}{6} = \frac{1}{3}$ is less than the first fraction $\frac{2}{5}$, therefore, the last fraction $\frac{8}{23}$ is the least

3. Inserting a fraction between two given fractions

To insert a fraction between two given fractions $\frac{a_1}{b_1}$ and $\frac{a_2}{b_2}$, the following steps may be useful:

Step 1 The numerators of the two given fractions are added to get the numerator of the resulting fraction, that is, $a_1 + a_2$.

Step 2 The denominators of the two given fractions are added to get the denominator of the resulting fraction, that is, $b_1 + b_2$.

Step 3 Resulting fraction = $\frac{a_1 + a_2}{b_1 + b_2}$.

Illustration 23 Insert one fraction between $\frac{2}{5}$ and $\frac{4}{7}$.

Solution: Using the above method,

$$\frac{2}{5}, \frac{2+4}{5+7}, \frac{4}{7} = \frac{2}{5}, \frac{6}{12}, \frac{4}{7} \text{ or, } \frac{2}{5}, \frac{1}{2}, \frac{4}{7}$$

Illustration 24 Insert three fractions between $\frac{5}{7}$ and $\frac{9}{11}$.

Solution: Using the above method,

$$\frac{5}{7}, \frac{5+9}{7+11}, \frac{9}{11} = \frac{5}{7}, \frac{14}{18}, \frac{9}{11} \text{ or, } \frac{5}{7}, \frac{7}{9}, \frac{9}{11}$$

Further,

$$\frac{5}{7}, \frac{5+7}{7+9}, \frac{7}{9}, \frac{7+9}{9+11}, \frac{9}{11} = \frac{5}{7}, \frac{12}{16}, \frac{7}{9}, \frac{16}{20}, \frac{9}{11}$$

or $\frac{5}{7}, \frac{3}{4}, \frac{7}{9}, \frac{4}{5}, \frac{9}{11}$

Thus, the three fractions inserted between $\frac{5}{7}$ and $\frac{9}{11}$ are $\frac{3}{4}, \frac{7}{9}$ and $\frac{4}{5}$

Practice Exercises

Difficulty Level-1
(Based on Memory)

1. If $\left(a+\frac{1}{a}\right)^2 = 3$, then what is the value of $a^3+\frac{1}{a^3}$?

(*a*) $\frac{10\sqrt{3}}{3}$ (*b*) 0

(*c*) $3\sqrt{3}$ (*d*) $6\sqrt{3}$

[Based on MAT, 2003]

2. If $a = \frac{}{x \quad y}$ and $b = \frac{y}{x-y}$ then $\frac{ab}{a+b}$ is equal to:

(*a*) $\frac{xy}{x^2+y^2}$ (*b*) $\frac{x^2+y^2}{xy}$

(*c*) $\frac{x}{x+y}$ (*d*) $\left(\frac{y}{x+y}\right)^2$

[Based on MAT, 2003]

3. If $x = \frac{4ab}{a+b}$, then the value of $\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}$ is equal to:

(*a*) 0 (*b*) $1\frac{1}{9}$

(*c*) $2\frac{1}{9}$ (*d*) None of these

[Based on MAT, 2003]

4. Given $\frac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}} = \frac{5}{2}$. The value of x is:

(*a*) 1 (*b*) $\frac{331}{5}$

(*c*) $\frac{263}{20}$ (*d*) $\frac{17}{21}$

[Based on MAT, 2002]

5. The value of $\frac{1.073\times1.073-0.927}{1.073-0.927}+\frac{(3^4)^4\times9^6}{(27)^7\times(3)^9}$ is:

(*a*) $2\frac{1}{3}$ (*b*) $2\frac{1}{5}$

(*c*) $2\frac{1}{9}$ (*d*) 3

6. 9^6+7, when divided by 8, would have a remainder of:

(*a*) 0 (*b*) 6

(*c*) 5 (*d*) None of these

[Based on MAT, 2001]

7. H.C.F. of 3240, 3600 and a third number is 36 and their L.C.M. is $2^4\times3^5\times5^2\times7^2$. The third number is:

(*a*) $2^2\times5^3\times7^2$ (*b*) $2^2\times3^5\times7^2$

(*c*) $2^3\times3^5\times7^2$ (*d*) $2^5\times5^2\times7^2$

[Based on MAT, 2001]

8. The value of $\frac{2^{1/2}3^{1/3}4^{1/4}}{10^{-1/5}5^{3/5}}\div\frac{3^{4/3}5^{-7/5}}{4^{-3/5}6}$ is:

(*a*) 5 (*b*) 6

(*c*) 10 (*d*) 15

9. The value of $\left(\frac{2.75\times2.75\times2.75-2.25\times2.25\times2.25}{2.75\times2.75+2.75\times2.25+2.25\times2.25}\right)$ is:

(*a*) 0.30 (*b*) 0.50

(*c*) 3 (*d*) 5

[Based on MAT, 2005]

10. If $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\left(1-\frac{1}{70}\right) = \frac{x}{70}$, then what is the value of x?

(*a*) 69 (*b*) 35

(*c*) 20 (*d*) 1

11. $\frac{(0.6)^0-(0.1)^{-1}}{\left(\frac{3}{2^3}\right)^{-1}\left(\frac{3}{2}\right)^3-\left(\frac{1}{3}\right)^{-1}}$ is equal to:

(*a*) $-\frac{3}{2}$ (*b*) $-\frac{1}{2}$

(*c*) $\frac{2}{3}$ (*d*) $\frac{3}{2}$

[Based on MAT, 2005]

12. What is the missing figure in the expression given below?

$$\frac{16}{7}\times\frac{16}{7}-\frac{*}{7}\times\frac{9}{7}+\frac{9}{7}\times\frac{9}{7}=1$$

(*a*) 1 (*b*) 7

(*c*) 4.57 (*d*) 32

[Based on MAT, 2000]

13. If the numerator and the denominator of a proper fraction are increased by the same quantity, then the resulting fraction is:

(*a*) Always greater than the original fraction.

(*b*) Always less than the original fraction.

(*c*) Always equal to the original fraction.

(*d*) None of these.

[Based on MAT, 2001]

14. A man completes $\frac{2}{15}$ th of his journey by aeroplane, $\frac{2}{5}$ th by train and the rest by taxi. What part of his journey does he complete by taxi?

(*a*) $\frac{8}{15}$ (*b*) $\frac{7}{15}$

(*c*) $\frac{9}{15}$ (*d*) None of these

15. The unit digit of $3^6 \times 4^7 \times 6^3 \times 7^4 \times 8^2 \times 9^5$ is:

(*a*) 6 (*b*) 5

(*c*) 4 (*d*) 2

16. If a man spends five-sixths part of money and then again earns part of the remaining money, what half part of his money is with him now?

(*a*) 1/2 (*b*) 1/4

(*c*) 2/3 (*d*) 3/4

17. The difference between the squares of two consecutive odd integers is always divisible by:

(*a*) 8 (*b*) 7

(*c*) 6 (*d*) 3

[Based on Narsee Manjee Inst. of Man. Studies, 2003]

18. What is the difference between the largest and the smallest fractions $\frac{5}{8}, \frac{21}{35}, \frac{9}{16}$ and $\frac{6}{7}$?

(*a*) $\frac{33}{112}$ (*b*) $\frac{11}{37}$

(*c*) $\frac{13}{41}$ (*d*) $\frac{9}{35}$

19. When one-fourth of a number is subtracted from one-third of the same number, the remainder obtained is 12. The number is:

(*a*) 144 (*b*) 72

(*c*) 120 (*d*) 63

20. Simplify $\left(\frac{1}{64}\right)^0 + (64)^{-1/2} + (-32)^{4/5}$:

(*a*) $17\frac{1}{8}$ (*b*) $17\frac{3}{8}$

(*c*) $11\frac{7}{8}$ (*d*) $17\frac{7}{8}$

21. $(1.06 + 0.04)^2 - ? = 4 \times 1.06 \times 0.04$:

(*a*) 1.04 (*b*) 1.4

(*c*) 1.5 (*d*) Cannot be determined

22. If $\frac{a^2+b^2}{c^2+d^2} = \frac{ab}{cd}$, then find the value of $\frac{a+b}{a-b}$ in terms of c and d only.

(*a*) $\frac{c+d}{cd}$ (*b*) $\frac{cd}{c+d}$

(*c*) $\frac{c-d}{c+d}$ (*d*) $\frac{c+d}{c-d}$

23. The value of is $\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)\ldots\left(2-\frac{999}{1001}\right)$:

(*a*) $\frac{1003}{3}$ (*b*) $\frac{1003}{1001}$

(*c*) $\frac{1}{1001}$ (*d*) None of these

24. The sum of the two digits of a number is 13 and the difference between the number (x) and that formed by reversing the digits (y) is 27, i.e., $(x - y)$ is 27. Find the number.

(*a*) 72 (*b*) 58

(*c*) 27 (*d*) 85

[Bases on IMT Ghaziabad, 2002]

25. Find a positive number which when increased by 17 is equal to 60 times the reciprocal of the number.

(*a*) 20 (*b*) 10

(*c*) 3 (*d*) 17

[Based on IMT Ghaziabad, 2002]

26. Four of the following five parts numbered (1), (2), (3), (4) and (5) are exactly equal. The number of the part which is not equal to the remaining four parts will be your answer.

(*a*) $36 \times 15 + 27 \times 13$

(*b*) $53 \times 4 + 64 \div 16 \times 7$

(*c*) $328 \div 41 \times 21 + 9 \times 2^3$

(*d*) $\sqrt{1024} \times 11 - 16 \times 7$

[Based on IRMA, 2002]

27. What approximate value should come in place of question mark (?) in the following equation?

$$95.975^{3.5} \div 16.001^{3.5} \times 6.002^{1.5} \div 35.99^2 = ?$$

(*a*) 36 (2) 16

(*c*) 96 (*d*) 6

[Based on IRMA, 2002]

28. If $x * y = (x+2)^2(y-2)$ then $7 * 5 = ?$

(*a*) 234 (*b*) 243

(*c*) 343 (*d*) 423

29. If m and n are whole numbers such that $m^n = 121$, then $(m-1)^{n+1} = ?$

(*a*) 10 (*b*) 102

(*c*) 1000 (*d*) 104

30. If we multiply a fraction by itself and divide the product by the square of its reciprocal, the fraction so obtained is $3\frac{13}{81}$. The original fraction is:

(*a*) $\frac{16}{9}$ (*b*) $\frac{8}{9}$

(*c*) $\frac{4}{3}$ (*d*) $\frac{1}{3}$

31. Suppose $a = 2/3\ b$, $b = 2/3\ c$, and $c = 2/3\ d$. What would be the value of b as a fraction of d?

(*a*) 2/3 (*b*) 4/3

(*c*) 4/9 (*d*) 8/27

[Based on I.P. Univ., 2002]

32. Find x and y:

$$x^2 - xy = 4 \text{ and } y^2 - xy = -3$$

(*a*) (4, 3) or (–4, –3)

(*b*) (4, 1) or (–4, –1)

(*c*) (3, 4) or (–3, –4)

(*d*) $x = y = 0$

[Based on SCMHRD, 2002]

33. If $a * b = \frac{ab}{a+b}$, find $3 * (3 * -1)$:

(*a*) –3 (*b*) –1.5

(*c*) 2/3 (*d*) 3

[Based on SCMHRD, 2002]

34. Solve $\left(\frac{9}{4}\right)^x \left(\frac{8}{27}\right)^{x-1} = \frac{2}{3}$:

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

[Based on SCMHRD, 2002]

35. $\left(2\frac{3}{x}\right) \times \left(y\frac{1}{2}\right) = 7\frac{3}{4}$, find the values of x and y.

(*a*) (3, 19) (*b*) (3, 14)

(*c*) (14, 3) (*d*) (24, 6)

36. Which of the given numbers is the greatest?

(*a*) $6\sqrt[3]{5}$ (*b*) $8\sqrt[3]{2}$

(*c*) $2\sqrt[3]{130}$ (*d*) $\sqrt[3]{900}$

[Based on REC Tiruchirapalli, 2002]

37. If $x = 2 + 2^{2/3} + 2^{1/3}$, then the value of $x^3 - 6x^2 + 6x$ is:

(*a*) 3 (*b*) 2

(*c*) 1 (*d*) None of these

[Based on REC Tiruchirapalli, 2002]

38. What fraction must be subtracted from the sum of $\frac{1}{4}$ and $\frac{1}{6}$ to have an average of $\frac{1}{12}$ of all the three fractions?

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$

(*c*) $\frac{1}{4}$ (*d*) $\frac{1}{6}$

39. If we multiply a fraction by itself and divide the product by its reciprocal, the fraction thus obtained is $18\frac{26}{27}$. The fraction is:

(*a*) $\frac{8}{27}$ (*b*) $2\frac{2}{3}$

(*c*) $1\frac{1}{3}$ (*d*) None of these

40. Simplify: $\dfrac{2\frac{1}{7} - 2\frac{1}{2}}{2\frac{1}{4} + 1\frac{1}{7}} \div \dfrac{1}{2 + \dfrac{1}{2 + \dfrac{1}{2 - \frac{1}{2}}}}$.

(*a*) $\frac{-1}{2}$ (*b*) $-\frac{1}{8}$

(*c*) $-\frac{1}{6}$ (*d*) $-\frac{1}{4}$

41. Which of the following fractions is less than $\frac{7}{8}$ and greater than $\frac{1}{3}$?

(*a*) $\frac{1}{4}$ (*b*) $\frac{23}{24}$

(*c*) $\frac{11}{12}$ (*d*) $\frac{17}{24}$

42. In a college, one-fifths of the girls and one-eighths of the boys took part in a social camp. What of the total number of students in the college took part in the camp?

(*a*) $\frac{13}{40}$ (*b*) $\frac{13}{80}$

(*c*) $\frac{2}{13}$ (*d*) Data inadequate

43. Which of the following is true?

A. $99/101 < 97/99 < 95/97$

B. $95/97 < 97/99 < 99/101$

C. $(95/97)^2 > (97/99)^2 > (99/101)^2$

D. $(99/101)^2 > (97/99)^2 > (95/97)^2$

(*a*) Only A (*b*) Only B

(*c*) B and C (*d*) B and D

[Based on MAT (Dec), 2006]

44. If $G = H + \sqrt{\frac{4}{L}}$, then L equals:

(*a*) $4/(G-H)^2$ (*b*) $4(G-H)^2$

(*c*) $4/(G^2-H^2)$ (*d*) $4(G^2-H^2)$

[Based on MAT, 1997]

45. If $\frac{1}{x} = \frac{1}{y} + \frac{1}{z}$, then z equals:

(*a*) $xy/(x-y)$ (*b*) $x-y$

(*c*) $xy/(y-x)$ (*d*) $(x-y)/xy$

[Based on MAT, 1997]

46. If $(x-3)(2x+1)=0$, then possible values of $2x+1$ are:

(*a*) 0 only (*b*) 0 and 3

(*c*) $-\frac{1}{2}$ and 3 (*d*) 0 and 7

[Based on MAT, 1997]

47. Simplify $2 \div [2 + 2 \div \{2 + 2 \div (2 + 2 \div 3)\}]$:

(*a*) 13/15 (*b*) 17/15

(*c*) 11/15 (*d*) None of these

48. Simplify $\frac{3}{10} \div \frac{3}{7}$ of $\left(2\frac{3}{10} + 2\frac{3}{5}\right) + \frac{1}{5} \div 1\frac{2}{5} - \frac{2}{7}$

(*a*) 1 (*b*) 2

(*c*) 0 (*d*) 3

49. $1+1 \div \left\{1+1 \div \left(1-\frac{1}{3}\right)\right\} = ?$

(*a*) $\frac{7}{5}$ (*b*) $\frac{2}{3}$

(*c*) $\frac{4}{5}$ (*d*) None of these

50. $48 \div 12 \times \left(\frac{9}{8} \text{ of } \frac{4}{3} + \frac{3}{4} \text{ of } \frac{2}{3}\right) = ?$

(*a*) 9 (*b*) 12

(*c*) 15 (*d*) None of these

51. If $a^x = b$, $b^y = c$ and $c^z = a$, then the value of $xyz = \sqrt{?}$

(*a*) 0 (*b*) 1

(*c*) –1 (*d*) 2

[Based on MAT, 1997]

52. If $ax = b$, $by = c$ and $cz = a$, then xyz is equal to:

(*a*) 0 (*b*) –1

(*c*) 1 (*d*) $a+b+c$

[Based on MAT, 1998]

53. If $x = \frac{2}{3}y$ and $y = \frac{2}{3}z$ and $z = \frac{2}{3}w$, what fraction of w is y?

(*a*) $\frac{8}{27}$ (*b*) $\frac{4}{9}$

(*c*) $\frac{2}{3}$ (*d*) $\frac{4}{3}$

[Based on MAT, 1998]

54. How many multiples of 9 can be found which are less than 9999 and are perfect cubes?

(*a*) 5 (*b*) 6

(*c*) 7 (*d*) 8

55. If $\sqrt{6} = 2.45$, then the value of $\frac{(3\sqrt{2}-\sqrt{3})}{(3\sqrt{2}+\sqrt{2})}$ is:

(*a*) 0.40 (*b*) 0.41

(*c*) 0.42 (*d*) 0.44

[Based on MAT, 1998]

56. Three-fourths of 68 is less than two-thirds of 114 by:

(*a*) 12 (*b*) 25

(*c*) 35 (*d*) 48

[Based on MAT, 1998]

57. The remainder when 8^{7185} is divided by 7 is:

(*a*) 5 (*b*) 1

(*c*) 6 (*d*) 4

58. The value of $(1/x^2) + (1/y^2)$, where $x = 2 + \sqrt{3}$ and $y = 2 - \sqrt{3}$, is:

(*a*) 14 (*b*) 12

(*c*) 10 (*d*) 16

[Based on MAT, 1999]

59. If $9^x = \frac{9}{3^x}$ then x is:

(*a*) 1/3 (*b*) 2/3

(*c*) 3 (*d*) 4/3

[Based on MAT, 1999]

60. If $\frac{5a+3b}{2a-3b} = \frac{23}{5}$, then the value of $a:b$ is:

(*a*) 2:1 (*b*) 1:4

(*c*) 1:2 (*d*) 4:1

[Based on MAT, 1999]

61. Find the value of $x^4 + \frac{1}{x^4}$, if $x = 3 + 2\sqrt{2}$:

(*a*) 1154 (*b*) 1024

(*c*) 1734 (*d*) None of these

[Based on NMAT, 2006]

62. The value of expression:

$$\left(\frac{x^b}{x^c}\right)^{b+c-a}\left(\frac{x^c}{x^a}\right)^{c+a-b}\left(\frac{x^a}{x^b}\right)^{a+b-c}$$

(*a*) $x^{ab+bc+ca}$ (*b*) 1

(*c*) x^{abc} (*d*) x^{a+b+c}

[Based on NMAT, 2005]

63. When simplified, the product:

$$\left\{2-\frac{1}{3}\right\}\left\{2-\frac{3}{5}\right\}\left\{2-\frac{5}{7}\right\}\ldots\left\{2-\frac{999}{1001}\right\}$$

is equal to:

(*a*) $\frac{1003}{13}$ (*b*) $\frac{1001}{13}$

(*c*) $\frac{991}{1001}$ (*d*) None of these

[Based on NMAT, 2005]

Difficulty Level-2 (Based on Memory)

1. If $\frac{a^3-b^3}{a^3+b^3}=\frac{13}{14}$, then find $\frac{a+b}{a-b}$:

(*a*) $\frac{3}{2}$ (*b*) 1

(*c*) 2 (*d*) None of these

[Based on FMS (Delhi), 2004]

2. Arrange the following in ascending order of values:

$$\frac{87}{83},\left(\frac{87}{83}\right)^2,\frac{87}{89},\left(\frac{87}{89}\right)^2$$

(*a*) $\frac{87}{89},\frac{87}{89},\left(\frac{87}{89}\right)^2,\left(\frac{87}{83}\right)^2$

(*b*) $\frac{87}{83},\frac{87}{89},\left(\frac{87}{83}\right)^2,\left(\frac{87}{89}\right)^2$

(*c*) $\left(\frac{87}{89}\right)^2,\frac{87}{89},\frac{87}{83},\left(\frac{87}{83}\right)^2$

(*d*) $\left(\frac{87}{83}\right)^2,\frac{87}{83},\frac{87}{89},\left(\frac{87}{89}\right)^2$

[Based on FMS (Delhi), 2004]

3. If two-thirds part of a number is 96, what is the value of three-fourths part of the same number?

(*a*) 48 (*b*) 192

(*c*) 108 (*d*) 72

4. If the difference between four-fifths part and three-fourths part of a number is 4, what is the number?

(*a*) 60 (*b*) 100

(*c*) 80 (*d*) 40

5. The unit digit of the product of all the prime numbers between 1 and $(11)^{11}$ is:

(*a*) 6 (*b*) 5

(*c*) 4 (*d*) 0

6. Find the remainder when $7^{21}+7^{22}+7^{23}+7^{24}$ is divided by 25.

(*a*) 0 (*b*) 2

(*c*) 4 (*d*) 6

[Based on FMS (Delhi), 2004]

7. A four-digit number is formed, using digits 1, 2, 3 and 4, without repeating any one of them. What is the sum of all such possible numbers?

(*a*) 66600 (*b*) 66660

(*c*) 66666 (*d*) 60000

[Based on FMS (Delhi), 2004]

8. How many multiples of 9 can be found which are less than 9999 and are perfect cubes?

(*a*) 5 (*b*) 6

(*c*) 7 (*d*) 8

[Based on FMS (Delhi), 2004]

9. If $\sqrt{a^b}=5b+a^2$ then (a, b) could be:

(*a*) (3, 4) (*b*) (2, 12)

(*c*) (4, 18) (*d*) (6, 4)

[Based on IIT Joint Man. Ent. Test, 2004]

10. Manmohan spends one-fifth part of his money as pocket money and four-fifths of the remainder in other affairs. If he is left with ₹48 per month, what is the monthly income?

(*a*) ₹360 (*b*) ₹400

(*c*) ₹320 (*d*) ₹300

11. What is the smallest number with which 1800 must be multiplied to make it a perfect cube?

(*a*) 12 (*b*) 5

(*c*) 18 (*d*) 15

[Based on IIT Joint Man. Ent. Test, 2004]

12. The number $3^9 + 3^{12} + 3^{15} + 3^n$ is a perfect cube of an integer for natural number n equalling:

(*a*) 12 (*b*) 13

(*c*) 14 (*d*) 15

[Based on IITTM, Gwalior, 2003]

13. The number of ways, in which 8064 can be resolved as the product of two factors, is:

(*a*) 10 (*b*) 16

(*c*) 24 (*d*) 48

[Based on IITTM, Gwalior, 2003]

14. Find the positive integer, which when added to the numerator and denominator of $\frac{2}{3}$, will result in a fraction nearest to $\frac{13}{15}$.

(*a*) 6 (*b*) 5

(*c*) 4 (*d*) 3

[Based on IITTM, Gwalior, 2003]

15. If $x = 5 + 2\sqrt{6}$, then $\left(\frac{\sqrt{x}-1}{\sqrt{x}}\right)$ is equal to:

(*a*) $2\sqrt{3}$ (*b*) $\sqrt{3}$

(*c*) $2\sqrt{2}$ (*d*) None of these

[Based on IITTM, Gwalior, 2003]

16. The largest number in the sequence $1, 2^{\frac{1}{2}}, 3^{\frac{1}{3}}, 4^{\frac{1}{4}}$ is:

(*a*) 1 (*b*) $2^{\frac{1}{2}}$

(*c*) $3^{\frac{1}{3}}$ (*c*) $4^{\frac{1}{4}}$

[Based on IITTM, Gwalior, 2003]

17. The term $(r^2 + s)^{1/2}$ is approximately equal to $r + \frac{s}{2r}$. Which of the following is the closest approximation to $(85)^{1/2}$?

(*a*) 9.06 (*b*) 9.34

(*c*) 9.22 (*d*) 9.28

[Based on REC Tiruchiraplli, 2003]

18. The highest score in an inning was two-ninths of the total score and the next highest was two-ninths of the remainder. These scores differ by 8 runs. What was the total score in the innings?

(*a*) 162 (*b*) 152

(*c*) 142 (*d*) 132

19. $\frac{885\times885\times885+115\times115\times115}{885\times885+115\times115-885\times115} = ?$

(*a*) 1000 (*b*) 770

(*c*) 885 (*d*) 115

[Based on FMS (Delhi), 2003]

20. If $\left(x+\frac{1}{x}\right) = 3$, then the value of $\left(x^6+\frac{1}{x^6}\right)$ is:

(*a*) 927 (*b*) 414

(*c*) 364 (*d*) 322

[Based on FMS (Delhi), 2003]

21. The value of $\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}$ is:

(*a*) 2 (*c*) 3

(*d*) 2.327 (*d*) 2.273

[Based on FMS (Delhi), 2003]

22. The value of $\left(1+\frac{1}{x+1}\right)\left(1+\frac{1}{x+2}\right)$:

$\times\left(1+\frac{1}{x+3}\right)\left(1+\frac{1}{x+4}\right)$ is:

(*a*) $1+\frac{1}{x+5}$ (*b*) $\frac{1}{x+5}$

(*c*) $x+\frac{1}{x+5}$ (*d*) $\frac{x+5}{x+1}$

[Based on FMS (Delhi), 2003]

23. The value of $\frac{1-x^4}{1+x} \div \frac{1+x^2}{x} \times \frac{1}{x(1-x)} = ?$

(*a*) $\frac{1}{x}$ (*b*) $1+x$

(*c*) $1-x^2$ (*d*) 1

[Based on FMS (Delhi), 2003]

24. If $(a/b)^{x-1} = (b/a)^{x-3}$, then x is equal to:

(*a*) 1 (*b*) 1/2

(*c*) 7/2 (*d*) 2

[Based on FMS (Delhi), 2003]

25. The expression $\frac{1}{1+x^{(b-a)}+x^{(c-a)}} + \frac{1}{1+x^{(a-b)}+x^{(c-b)}}$

$+ \frac{1}{1+x^{(b-c)}+x^{(a-c)}}$:

(a) x^{a-b-c} (b) 1

(c) 0 (d) None of these

[Based on FMS (Delhi), 2003]

26. $\frac{1}{1+a^{n-m}}+\frac{1}{1+a^{m-n}}$ is equal to:

(a) 0 (b) 1

(c) 1/2 (d) a^{m+n}

[Based on FMS (Delhi), 2003]

27. Consider the following statements:

A. If $a^x = b$, $b^y = c$, $c^z = a$, then $xyz = 0$.

B. If $p = a^x$, $q = a^y$, $(p^y q^x)^z = a^2$, then $xyz = 1$

C. $x^a = y^b = z^c$, $ab + bc + ca = 0$, then $xyz = 1$.

Of these statements:

(a) A and B are correct

(b) B and C are correct

(c) Only A is correct

(d) A and C are correct

[Based on FMS (Delhi), 2003]

28. Simplify:

$$\frac{a^{1/2}+a^{-1/2}}{1-a}+\frac{1-a^{-1/2}}{1+\sqrt{a}}$$

(a) $\frac{a}{a-1}$ (b) $\frac{a-1}{2}$

(c) $\frac{2}{a-1}$ (d) $\frac{2}{1-a}$

29. Let $(a/b) - (b/a) = x:y$ and $(x - y) = (a/b) + (b/a)$, then x is equal to:

(a) $(a + b)/a$ (b) $(a + b)/b$

(c) $(a - b)/a$ (d) None of these

[Based on IIFT, 2003]

30. If $(3)^{(x+y)} = 81$ and $81^{(x-y)} = 3$, then the values of x and y are:

(a) 17/8, 9/8 (b) 17/8, 15/8

(c) 17/8, 11/8 (d) 15/8, 11/8

[Based on IIFT, 2003]

31. Given that x and y are real numbers, let $P(x, y) = x^2 - y^2$. Then, $P(3, P(3,4)) = ?$

(a) –40 (b) –7

(c) 40 (d) 7

[Based on IIFT, 2003]

32. Of the two-digit numbers (those from 10 to 99, both inclusive), how many have a second digit greater than the first digit?

(a) 38 (b) 40

(c) 36 (d) 41

[Based on SCMHRD Ent. Exam., 2003]

33. How much more is 1/2 of 2/3 than 3/4 of 1/3?

(a) 1/4 (b) 1/3

(c) 1/12 (d) 7/12

[Based on SCMHRD Ent. Exam., 2003]

34. Assuming that in Harappan era, rocks, stones and pebbles were used for money. The following used to be the currency valuations: 1 rock = 7 stones, 1 stone = 7 pebbles. If a person used 6 rocks to purchase a cave that costs 5 rocks, 2 stones and 3 pebbles, then how much would the change be?

(a) 4 stones, 4 pebbles

(b) 5 stones, 4 pebbles

(c) 1 rock, 5 stones, 4 pebbles

(d) 5 stones, 5 pebbles

[Based on SCMHRD Ent. Exam., 2003]

35. In an examination, a student was asked to find three-fourths of a certain number. By mistake he found three-fourths of it. His answer was 150 more than the correct answer. The given number is:

(a) 180 (b) 240

(c) 280 (d) 290

36. If $\frac{a}{a+b}=\frac{17}{23}$, what is $\frac{a+b}{a-b}$ equal to?

(a) $\frac{11}{23}$ (b) $\frac{17}{32}$

(c) $\frac{23}{11}$ (d) $\frac{23}{17}$

37. If we multiply a fraction by itself and divide the product by its reciprocal, the fraction thus obtained is $18\frac{26}{27}$. The original fraction is:

(a) $\frac{8}{27}$ (b) $2\frac{2}{3}$

(c) $1\frac{1}{3}$ (d) None of these

38. A boy was asked to multiply a given number by $\frac{8}{17}$. Instead, he divided the given number by $\frac{8}{17}$ and got the result 225 more than what he should have got if he had multiplied the number by $\frac{8}{17}$. The given number was:

(a) 8 (b) 17

(c) 64 (d) 136

39. If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, then the value of $\left[\frac{a^2+ab+b^2}{a^2-ab+b^2}\right]$ would be:

(*a*) 3/4 (*b*) 4/3

(*c*) 3/5 (*d*) 5/3

[Based on FMS (Delhi), 2002]

40. If $(x+y):(x-y) = 4:1$, then $(x^2+y^2):(x^2-y^2) = ?$

(*a*) 25:9 (*b*) 16:1

(*c*) 8:17 (*d*) 17:8

[Based on FMS (Delhi), 2002]

41. A number consists of two digits whose sum is 7. If the digits are reversed, then the number is increased by 27. The number is:

(*a*) 25 (*b*) 34

(*c*) 16 (*d*) 52

[Based on FMS (Delhi), 2002]

42. The expression:

$$(2a-3b)(4a+0.5b) - a(8a-11b)$$

(*a*) Is always positive for all a and b

(*b*) Is always negative for all a and b

(*c*) Is zero

(*d*) Depends on the values of a and b

[Based on SCMHRD, 2002]

43. The difference between the sum of $1\frac{3}{4}, 2\frac{1}{3}, 3\frac{5}{12}, 5\frac{1}{5}$ and $2\frac{1}{6}$ and the nearest whole number is:

(*a*) $\frac{2}{15}$ (*b*) $\frac{13}{15}$

(*c*) $\frac{11}{60}$ (*d*) None of these

44. At the first stop on his route, a driver unloaded two-fifths of the packages in his van. After he unloaded another three packages at his next stop, half of the original number of packages in the van remained. How many packages were in the van before the first delivery?

(*a*) 10 (*b*) 25

(*c*) 30 (*d*) 36

[Based on REC Tiruchirapalli, 2002]

45. $\left[1+\frac{1}{1+\frac{1}{1+\frac{1}{3}}}\right] + 1\frac{4}{7}$ is equal to:

(a) $1\frac{1}{3}$ (b) $1\frac{1}{4}$

(*c*) $1\frac{1}{7}$ (*d*) None of these

[Based on FMS (Delhi), 2006]

46. The value of $\frac{1}{2}+\frac{1}{2.3}+\frac{1}{2.3.4}+\frac{1}{2.3.4.5}$ correct to three places of decimal is:

(*a*) 0.713 (*b*) 0.715

(*c*) 0.717 (*d*) 0.718

47. A person was to multiply a fraction by $\frac{6}{7}$. Instead, he divided and got an answer which exceeds the correct answer by $\frac{1}{7}$. The correct answer was:

(*a*) $\frac{6}{13}$ (*b*) $\frac{36}{91}$

(*c*) $\frac{7}{13}$ (*d*) None of these

48. In a certain college, the number of girls is twice the number of boys. One-fifth of the girls and one-eighth of the boys took part in a social camp. What part of the total number of students took part in the camp?

(*a*) $\frac{7}{40}$ (*b*) $\frac{7}{80}$

(*c*) $\frac{2}{12}$ (*d*) $\frac{1}{24}$

49. How many $\frac{1}{8}$s are there in $37\frac{1}{2}$?

(*a*) 300 (*b*) 400

(*c*) 500 (*d*) Cannot be determined

50. $\left\{7\frac{1}{2}+\frac{1}{2}\div\frac{1}{2} \text{ of } \frac{1}{4}-\frac{2}{5}\times 2\frac{1}{3}\div 1\frac{7}{8} \text{ of } \left(1\frac{2}{5}-1\frac{1}{3}\right)\right\} = ?$

(*a*) $3\frac{1}{5}$ (*b*) $2\frac{1}{24}$

(*c*) $4\frac{1}{30}$ (*d*) None of these

51. The value of $1+\frac{1}{4\times 3}+\frac{1}{4\times 3^2}+\frac{1}{4\times 3^3}$ up to four places of decimals is:

(*a*) 1.1202 (*b*) 1.1203

(*c*) 1.1204 (*d*) None of these

52. $7\frac{1}{2}-\left[2\frac{1}{4}\div\left\{1\frac{1}{4}-?\left(1\frac{1}{2}-\frac{1}{3}-\frac{1}{6}\right)\right\}\right]=3$:

(*a*) $\frac{1}{4}$ (*b*) $\frac{3}{4}$

(*c*) $\frac{4}{3}$ (*d*) None of these

53. Find the remainder when $7^{21}+7^{22}+7^{23}+7^{24}$ is divided by 25.

(*a*) 0 (*b*) 2

(*c*) 4 (*d*) 6

54. A four-digit number is formed, using digits 1, 2, 3 and 4, without repeating any one of them. What is the sum of all such possible numbers?

(*a*) 66600 (*b*) 66660

(*c*) 66666 (*d*) 60000

55. What is the smallest number with which 1800 must be multiplied to make it a perfect cube?

(*a*) 12 (*b*) 5

(*c*) 18 (*d*) 15

56. The number $3^9+3^{12}+3^{15}+3n$ is a perfect cube of an integer for natural number n equalling:

(*a*) 12 (*b*) 13

(*c*) 14 (*d*) 15

57. The rank of 2/9 in the following fraction when expressed in ascending order is –2/3, 1/7, 0, 4/9, 2/9, 14/15, 9/11.

(*a*) 4 (*b*) 5

(*c*) 6 (*d*) 9

58. The sum of the squares of two numbers is 3341 and the difference of their squares is 891. The numbers are:

(*a*) 25, 46 (*b*) 35, 46

(*c*) 25, 36 (*d*) None of these

[Based on FMS (MS), 2006]

59. $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)...\left(1-\frac{1}{n}\right)$ is equal to:

(*a*) $\frac{1}{n}$ (*b*) $\frac{2}{n}$

(*c*) $\frac{2}{n(n+1)}$ (*d*) $\frac{2(n-1)}{n}$

[Based on FMS (MS), 2006]

60. If $\left[x^4+\frac{1}{x^4}\right]=322$, the value of $\left[x-\frac{1}{x}\right]$ is:

(*a*) 4 (*b*) 6

(*c*) 8 (*d*) $3\sqrt{2}$

[Based on FMS, 2005]

61. The value of:

$(x^a/x^b)^{(a+b)}\cdot(x^b/x^c)^{(b+c)}\cdot(x^c/x^a)^{(c+a)}$ is:

(*a*) 0 (*b*) x^{abc}

(*c*) x^{a+b+c} (*d*) 1

[Based on FMS, 2006]

62. The value of [1/(216) – 213 + 1/(256) – 314 + 1/(243) – 1/5] is:

(*a*) 107 (*b*) 105

(*c*) 103 (*d*) None of these

[Based on FMS, 2006]

63. If x is an integer such that $x+\frac{1}{x}=\frac{17}{4}$, then the value of $x-\frac{1}{x}$ is:

(*a*) 4 (*b*) $\frac{13}{4}$

(*c*) $\frac{15}{4}$ (*d*) $\frac{1}{4}$

[Based on FMS, 2006]

64. If $\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}=2\frac{1}{6}$, then the value of x is:

(*a*) $\frac{6}{13}$ or $\frac{4}{13}$ (*b*) $\frac{3}{2}$ or $\frac{2}{3}$

(*c*) $\frac{5}{2}$ or $\frac{2}{3}$ (*d*) $\frac{9}{13}$ or $\frac{4}{13}$

[Based on FMS, 2009]

65. If $\frac{a+b}{b+c}=\frac{c+d}{d+a}$, then:

(*a*) a must equal c

(*b*) $a+b+c+d$ must equal zero

(*c*) either $a=c$ or $a+b+c+d=0$, or both

(*d*) $a(b+c+d)=c(a+b+d)$

[Based on FMS, 2009]

66. Let r be the result of doubling both the base and the exponent of a^b, $b\neq 0$. If r equals the product of a^b by x^b, then x equals:

(*a*) $2a$ (*b*) $4a$

(*c*) 2 (*d*) 4

[Based on FMS, 2010]

67. If $\frac{m}{n}=\frac{4}{3}$ and $\frac{r}{t}=\frac{9}{14}$, the value of $\frac{3mr-nt}{4nt-7mr}$ is:

(*a*) $-5\frac{1}{2}$ (*b*) $-\frac{11}{14}$

(*c*) $-1\frac{1}{4}$ (*d*) $\frac{11}{14}$

[Based on FMS, 2011]

68. Simplify $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4$; the result is:

(a) a^{16} (b) a^{12}

(c) a^8 (d) a^4

[Based on FMS, 2011]

69. The expression $1-\frac{1}{1+\sqrt{3}}+\frac{1}{1-\sqrt{3}}$ equals:

(a) $1-\sqrt{3}$ (b) 1

(c) $-\sqrt{3}$ (d) $\sqrt{3}$

[Based on FMS, 2011]

70. The expression $2+\sqrt{2}+\frac{1}{2+\sqrt{2}}+\frac{1}{\sqrt{2}-2}$ equals:

(a) 2 (b) $2-\sqrt{2}$

(c) $2+\sqrt{2}$ (d) $2\sqrt{2}$

[Based on FMS, 2011]

71. $\frac{2^{n+4}-2(2^n)}{2(2^{n+3})}$ when simplified is:

(a) $2^{n+1}-\frac{1}{8}$ (b) -2^{n+1}

(c) $1-2^n$ (d) $\frac{7}{8}$

[Based on FMS, 2011]

72. If $u_1=\sqrt{3}, u_2=\sqrt{3\sqrt{3}}, u_3=\sqrt{3\sqrt{3\sqrt{3}}}$ etc, $u_{10}:u_9$ is:

(a) $\sqrt{3}$ (b) $3^{\frac{1}{10}}$

(c) $3^{\frac{1}{20}}$ (d) None of these

[Based on IIFT, 2007]

73. What is the value of x that would satisfy $(\sqrt{3}+\sqrt{2})^x+(\sqrt{3}-\sqrt{2})^x=10$:

(a) ± 2 (b) $-\frac{1}{3}$

(c) 4 (d) -4

[Based on ATMA, 2008]

74. Which of the following is/are true?

I. $(x^{b-c})^a (x^{c-a})^b (x^{a-b})^c = 1$

II. Fractions $\frac{3}{18}, \frac{19}{20}$ of $\frac{16}{19}$ are in descending order.

III. If $\log(x+1)-\log(x-1)=\log 2$, then $x=3$

(a) I and II (b) I Only

(c) I and III (d) II and III

[Based on ATMA, 2005]

75. If $\frac{a^2-ab+b^2}{a^2+ab+b^2}=\frac{1}{3}$, then find $\frac{a}{b}$.

(a) 1 (b) 2

(c) 3 (d) 4

[Based on CAT, 2009]

76. Which one of the terms $2^{1/3}, 3^{1/4}, 4^{1/6}, 6^{1/8}$ and $10^{1/12}$ is the largest?

(a) $2^{1/3}$ (b) $3^{1/4}$

(c) $4^{1/6}$ (d) $10^{1/12}$

[Based on CAT, 2012]

77. If $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$, then $xy+yz+zx$ is equal to:

(a) $\frac{(a+b+c)^2}{x^2+y^2+z^2}$

(b) $\frac{x^2(a+b+c)^2-a^2(x^2+y^2+z^2)}{2a^2}$

(c) $\frac{ax+by+cz}{(a+b+c)^2}$

(d) $\frac{ax^2+by^2+cz^2}{(a+b+c)^2}$

[Based on CAT, 2013]

78. If $x+\frac{1}{x}=1$ and $p=x^{4000}+\frac{1}{x^{4000}}$ and q be the digit at units place in the number $2^{2^n}+1$, n being a natural number greater than 1, then $(p+q)$ is equal to:

(a) 8 (b) 6

(c) 4 (d) 2

[Based on CAT, 2013]

79. Find the value of $\frac{1}{1+\cfrac{1}{3-\cfrac{4}{2+\cfrac{1}{3-\frac{1}{2}}}}}+\frac{3}{3-\cfrac{4}{3+\cfrac{1}{2-\frac{1}{2}}}}$:

(a) $\frac{13}{7}$ (b) $\frac{15}{7}$

(c) $\frac{11}{21}$ (d) $\frac{17}{28}$

[Based on CAT, 1996]

80. 5^6-1 is divisible by:

(a) 13 (b) 31

(c) 5 (d) None of these

[Based on CAT, 1995]

81. For the product $n(n+1)(2n+1)$, $n \in N$, which one of the following is not necessarily true?

(*a*) It is even

(*b*) Divisible by 3

(*c*) Divisible by the sum of the squares of first n natural numbers

(*d*) Never divisible by 237

[Based on CAT, 1995]

82. Prof. Suman takes number of quizzes for a course. All the quizzes are out of 100. A student can get an A grade in the course if the average of her scores is more than or equal to 90. Grade B is awarded to a student if the average of her score is between 87 and 89 (both included). If the average is below 87, the student gets a C grade. Ramesh is preparing for the last quiz and he realizes that he must score a minimum of 97 to get an A grade. After the quiz, he realizes that he will score70, and he will just manage a B. How many quizzes did Prof. Suman take?

(*a*) 6 (*b*) 7

(*c*) 8 (*d*) 9

[Based on XAT, 2014]

Answer Keys

Difficulty Level-1

1. (*b*)	**2.** (*a*)	**3.** (*a*)	**4.** (*c*)	**5.** (*c*)	**6.** (*a*)	**7.** (*b*)	**8.** (*c*)	**9.** (*b*)	**10.** (*d*)	**11.** (*a*)	**12.** (*d*)	**13.** (*a*)
14. (*b*)	**15.** (*a*)	**16.** (*b*)	**17.** (*a*)	**18.** (*a*)	**19.** (*a*)	**20.** (*a*)	**21.** (*a*)	**22.** (*d*)	**23.** (*a*)	**24.** (*d*)	**25.** (*c*)	**26.** (*a*)
27. (*d*)	**28.** (*b*)	**29.** (*c*)	**30.** (*c*)	**31.** (*c*)	**32.** (*a*)	**33.** (*a*)	**34.** (*d*)	**35.** (*c*)	**36.** (*a*)	**37.** (*b*)	**38.** (*d*)	**39.** (*b*)
40. (*d*)	**41.** (*d*)	**42.** (*c*)	**43.** (*d*)	**44.** (*a*)	**45.** (*c*)	**46.** (*d*)	**47.** (*c*)	**48.** (*c*)	**49.** (*a*)	**50.** (*b*)	**51.** (*b*)	**52.** (*c*)
53. (*b*)	**54.** (*c*)	**55.** (*d*)	**56.** (*b*)	**57.** (*b*)	**58.** (*a*)	**59.** (*b*)	**60.** (*d*)	**61.** (*a*)	**62.** (*b*)	**63.** (*d*)		

Difficulty Level-2

1. (*c*)	**2.** (*c*)	**3.** (*c*)	**4.** (*c*)	**5.** (*d*)	**6.** (*a*)	**7.** (*b*)	**8.** (*b*)	**9.** (*b*)	**10.** (*d*)	**11.** (*d*)	**12.** (*c*)	**13.** (*c*)
14. (*b*)	**15.** (*d*)	**16.** (*c*)	**17.** (*c*)	**18.** (*a*)	**19.** (*a*)	**20.** (*d*)	**21.** (*a*)	**22.** (*d*)	**23.** (*d*)	**24.** (*d*)	**25.** (*b*)	**26.** (*b*)
27. (*b*)	**28.** (*d*)	**29.** (*d*)	**30.** (*b*)	**31.** (*a*)	**32.** (*c*)	**33.** (*c*)	**34.** (*a*)	**35.** (*c*)	**36.** (*c*)	**37.** (*b*)	**38.** (*d*)	**39.** (*b*)
40. (*d*)	**41.** (*a*)	**42.** (*b*)	**43.** (*a*)	**44.** (*c*)	**45.** (*d*)	**46.** (*c*)	**47.** (*b*)	**48.** (*a*)	**49.** (*a*)	**50.** (*c*)	**51.** (*b*)	**52.** (*b*)
53. (*a*)	**54.** (*b*)	**55.** (*d*)	**56.** (*c*)	**57.** (*a*)	**58.** (*b*)	**59.** (*b*)	**60.** (*a*)	**61.** (*d*)	**62.** (*c*)	**63.** (*c*)	**64.** (*d*)	**65.** (*c*)
66. (*b*)	**67.** (*b*)	**68.** (*d*)	**69.** (*a*)	**70.** (*a*)	**71.** (*d*)	**72.** (*d*)	**73.** (*a*)	**74.** (*c*)	**75.** (*a*)	**76.** (*d*)	**77.** (*b*)	**78.** (*b*)
79. (*d*)	**80.** (*b*)	**81.** (*d*)	**82.** (*d*)									

Explanatory Answers

Difficulty Level-1

1. (*b*) $\left(a+\frac{1}{a}\right)^2 = 3 \Rightarrow a+\frac{1}{a} = \sqrt{3}$

$\therefore \left(a+\frac{1}{a}\right)^3 = a^3+\frac{1}{a^3}+3\left(a+\frac{1}{a}\right)$

$\Rightarrow 3\sqrt{3} = a^3+\frac{1}{a^3}+3\sqrt{3}$

$\Rightarrow a^3+\frac{1}{a^3} = 0.$

2. (*a*) $a = \frac{x}{x+y}, b = \frac{y}{x-y}$

$\therefore ab = \frac{xy}{x^2-y^2}, a+b = \frac{x^2+y^2}{x^2-y^2}$

$\therefore \frac{ab}{a+b} = \frac{xy}{x^2+y^2}.$

3. (a) $\dfrac{x+2a}{x-2a}+\dfrac{x+2b}{x-2b}=\dfrac{\dfrac{6ab+2a^2}{a+b}}{\dfrac{2ab-2a^2}{a+b}}+\dfrac{\dfrac{6b^2+2ab}{a+b}}{\dfrac{2ab-2b^2}{a+b}}$

$$=\frac{2a(3b+a)}{2a(b-a)}+\frac{2b(3b+a)}{2b(a-b)}$$

$$=\frac{3b+a}{b-a}-\frac{3b+a}{b-a}=0.$$

4. (c) Given expression

$$\Rightarrow \frac{(\sqrt{x+4}+\sqrt{x-10})^2}{(x+4)-(x-10)}=\frac{25}{4}$$

$$\Rightarrow \frac{x+4+x-10+2\sqrt{x^2-6x-40}}{14}=\frac{5}{2}$$

$$\Rightarrow 2x+2\sqrt{x^2-6x-40}=41$$

$$\Rightarrow (2x-41)^2=[-2\sqrt{x^2-6x-40}]^2$$

$$\Rightarrow 4x^2+1681-164x=4(x^2-6x-40)$$

$$=4x^2-24x-160$$

$$\Rightarrow 140x=1841$$

$$\Rightarrow x=\frac{263}{20}.$$

5. (c) $\dfrac{1.073\times1.073-0.927\times0.927}{1.073-0.927}+\dfrac{(3^4)^4\times(9)^6}{(27)^7\times(3)^9}$

$$=\frac{(1.073)^2-(0.927)^2}{1.073-0.927}+\frac{(3^4)^4\times(3^2)^6}{(3^3)^7\times(3)^9}$$

$$=\frac{(1.073+0.927)(1.073-0.927)}{1.073-0.927}+\frac{3^{28}}{3^{30}}$$

$$=2+\frac{1}{3^2}=2+\frac{1}{9}=2\frac{1}{9}.$$

6. (a) Consider $9, 9^2, 9^3, 9^4, 9^6$. Each of the these numbers, when divided by 8, will leave a remainder of 1.

$\therefore$ 9^6+7, when divided by 8, will leave a remainder of 0.

7. (b) The product of the numbers = H.C.F × L.C.M.

Let the third number be x.

$\therefore$ $3240\times3600\times x=36\times2^4\times3^5\times5^2\times7^2$

$$\Rightarrow x=\frac{36\times2^4\times3^5\times5^2\times7^2}{(2^3\times3^4\times5)\times(2^4\times3^2\times5^2)}$$

$$\left[\because \begin{matrix}3240=2^3\times3^4\times5\\ 3600=2^4\times3^2\times5^2\end{matrix}\right]$$

$$\Rightarrow x=\frac{(2^2\times3^2)\times2^4\times3^5\times5^2\times7^2}{2^3\times3^4\times5\times2^4\times3^2\times5^2}$$

$$=\frac{2^6\times3^7\times5^2\times7^2}{2^7\times3^6\times5^3}$$

$$=2^2\times3^5\times7^2.$$

8. (c) $\dfrac{2^{1/2}\times3^{1/3}\times4^{1/4}}{10^{-1/5}\times5^{3/5}}\div\dfrac{3^{4/3}\times5^{-7/5}}{4^{-3/5}\times6}$

$$=\frac{2^{1/2}\times3^{1/3}\times(2^2)^{1/4}\times10^{1/5}}{5^{3/5}}\div\frac{3^{4/3}\times4^{3/5}}{5^{7/5}\times6}$$

$$=\frac{2^{1/2}\times3^{1/3}\times2^{1/2}\times2^{1/5}\times5^{1/5}}{5^{3/5}}\times\frac{5^{7/5}\times2\times3}{3^{4/3}\times2^{6/5}}$$

$$=2^{\frac{1}{2}+\frac{1}{2}+\frac{1}{5}-\frac{6}{5}+1}\times3^{\frac{1}{3}-\frac{4}{3}+1}\times5^{\frac{1}{5}-\frac{3}{5}+\frac{7}{5}}$$

$$=2^1\times3^0\times5^1=2\times5=10$$

9. (b) By using $\dfrac{a^3-b^3}{a^2+ab+b^2}=a-b$

$$\frac{(2.75)^3-(2.25)^3}{(2.75)^2+2.75\times2.25+(2.25)^2}=2.75-2.25=0.50.$$

10. (d) $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\left(1-\frac{1}{70}\right)=\frac{x}{70}$

$$\therefore \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\ldots\frac{69}{70}=\frac{x}{70}$$

$$\therefore \frac{1}{70}=\frac{x}{70}$$

$$\therefore x=1.$$

11. (a) $\dfrac{1-\left(\frac{1}{10}\right)^{-1}}{\left(\frac{2^3}{3}\right)\left(\frac{3}{2}\right)^3+(-3)}=\dfrac{1-10}{\frac{2^3}{3}\cdot\frac{3^3}{2^3}-3}$

$$=\frac{-9}{9-3}=-\frac{3}{2}.$$

12. (d) $-\dfrac{*}{7}\times\dfrac{9}{7}=-\dfrac{256}{49}-\dfrac{81}{49}+1$

$$\Rightarrow -*\times9=-256-81+49$$

$$\Rightarrow *=32.$$

13. (a) $\dfrac{1}{2}<\dfrac{1+1}{2+1}\Rightarrow\dfrac{1}{2}<\dfrac{2}{3}$

$$\frac{2}{3}<\frac{2+1}{3+1}\Rightarrow\frac{2}{3}<\frac{3}{4}$$

$$\frac{2}{5}<\frac{2+1}{5+1}\Rightarrow\frac{2}{5}<\frac{1}{2}.$$

14. (*b*) Journey completed by aeroplane and train

$$= \frac{2}{15} + \frac{2}{5} = \frac{2+6}{15} = \frac{8}{15}$$

$\therefore$ Remaining journey $= 1 - \frac{8}{15} = \frac{7}{15}$

$\therefore$ He completed $\frac{7}{15}$ th part of his journey by taxi.

15. (*a*) The unit digit of 3^6 is 9
The unit digit of 4^7 is 4
The unit digit of 6^3 is 6
The unit digit of 7^4 is 1
The unit digit of 8^2 is 4
The unit digit of 1^5 is 9
Therefore the unit digit of the given expression is 6
(Since $9 \times 4 \times 6 \times 1 \times 4 \times 9 = 7776$).

16. (*b*) Let the money with the man at first be ₹1

$\therefore$ Money spent $= \frac{5}{6}$ of $1 = ₹\frac{5}{6}$

$\therefore$ Remaining money $= 1 - \frac{5}{6} = ₹\frac{1}{6}$

and money earned $= \frac{1}{2}$ of $₹\frac{1}{6} = ₹\frac{1}{12}$

$\therefore$ Total money with him now

$$= \frac{1}{6} + \frac{1}{12} = ₹\frac{3}{12} = ₹\frac{1}{4}$$

$\therefore$ $\frac{1}{4}$ th part of his money is with him now.

17. (*a*) $(2x+3)^2 - (2x+1)^2 = 4x^2 + 12x + 9 - (4x^2 + 4x + 1)$
$= 8x + 8 = 8(x+1)$.

18. (*a*) L.C.M. of 7, 8, 16 and 35 = 560

$\therefore$ $\frac{5}{8} = \frac{5\times70}{8\times70} = \frac{350}{560}$

$\frac{21}{35} = \frac{21\times16}{35\times16} = \frac{336}{560}$

$\frac{9}{16} = \frac{9\times35}{16\times35} = \frac{315}{560}$

and, $\frac{6}{7} = \frac{6\times80}{7\times80} = \frac{480}{560}$

$\therefore$ Difference between the largest and the smallest fractions

$$= \frac{6}{7} - \frac{9}{16} = \frac{480}{560} - \frac{315}{560}$$

$$= \frac{165}{560} = \frac{33}{112}.$$

19. (*a*) Let the number be 1

$\therefore$ $\frac{1}{3}$ of $1 = \frac{1}{3}$ and, $\frac{1}{4}$ of $1 = \frac{1}{4}$

$\therefore$ $\frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$

$\therefore$ Number $12 \div \frac{1}{12} = 144$.

20. (*a*) $\left(\frac{1}{64}\right)^0 + (64)^{-1/2} + (-32)^{4/5}$

$= 1 + (8^2)^{-1/2} + (-1\times32)^{4/5}$
$= 1 + 8^{-1} + [(-1)^{4/5} \times (32)^{4/5}]$
$= 1 + 8^{-1} + [((-1)^2)^{2/5} \times (2^5)^{4/5}]$
$= 1 + \frac{1}{8} + [1 \times 16] = 17\frac{1}{8}$.

21. (*a*) Putting x for ? and solving

$(1.06 + 0.04)^2 - x = 4 \times 1.06 \times 0.04$

Here, $1.06 = a$ and $0.04 = b$

$\therefore$ $(a+b)^2 - x = 4ab$

$\therefore$ $x = (a+b)^2 - 4ab = (a-b)^2$
$= (1.06 - 0.04)^2$
$= (1.02)^2 = 1.0404$.

22. (*d*) $\frac{a^2+b^2}{c^2+d^2} = \frac{ab}{cd}$ or, $\frac{a^2+b^2}{c^2+d^2} = \frac{2ab}{2cd}$

or, $\frac{a^2+b^2+2ab}{a^2+b^2-2ab} = \frac{c^2+d^2+2cd}{c^2+d^2-2cd}$

[by componendo and dividendo]

or, $\left(\frac{a+b}{a-b}\right)^2 = \left(\frac{c+d}{c-d}\right)^2$

$\therefore$ $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.

23. (*a*) Given product

$$= \frac{5}{3} \times \frac{7}{5} \times \frac{9}{7} \times \ldots \times \frac{1003}{1001} = \frac{1003}{3}$$

24. (*d*) Let the number be $10p + q$

$\therefore$ $p + q = 13$ and $(10p+q) - (10q+p) = 27$
$(x = 10p+q, y = 10q+p)$

i.e., $9p - 9q = 27$, i.e., $p - q = 3$

$\therefore$ $p = 8, q = 5$

Hence the required number = 85.

25. (*c*) Let the number be x

$\therefore$ $x + 17 = \frac{60}{x} \Rightarrow x^2 + 17x - 60 = 0$

$\Rightarrow (x+20)(x-3)=0$

$\Rightarrow \quad x=3\ (x>0).$

26. (*a*) (1) $=36\times15\div27\times13$

$=36\times\frac{15}{27}\times13=260$

(2) $=53\times5+64\div16\times7$

$=53\times4+\frac{64}{16}\times7$

$=212+28=240$

(3) $=\frac{328}{41}\times21+9\times2^3$

$=168+9\times8$

$=168+72=240$

(4) $=\sqrt{1024}\times11-16\times7$

$=32\times11-112$

$=352-112=240$

(5) $=17\times18-\sqrt{121}\times6$

$=17\times18-11\times6$

$=306-66=240.$

27. (*d*) Given expression $=\frac{96^{3.5}\times6^{1.5}}{16^{3.5}}\times\frac{1}{36^2}$

$=\frac{6^5}{6^4}=6.$

28. (*b*) Substituting $x=7$ and $y=5$, we get

$7*5=(7+2)^2(5-2)=(9)^2\times3=243.$

29. (*c*) Given that $m^n=121\Rightarrow m^n=11^2$

Hence, $m=11$, $n=2$. Substituting these values

$(m-1)^{n+1}=(11-1)^{2+1}=10^3=1000.$

30. (*c*) Let x be the fraction

$x\times x\div\left(\frac{1}{x}\right)^2=3\frac{13}{81}\Rightarrow x^4=\frac{256}{81}=\left(\frac{4}{3}\right)^4$

$\therefore \quad x=\frac{4}{3}.$

31. (*c*) $b=\frac{2}{3}c,\ c=\frac{2}{3}d$

$\Rightarrow \quad b=\frac{2}{3}\times\frac{2}{3}d=\frac{4}{9}d.$

32. (*a*) $x^2-xy=4\Rightarrow x(x-y)=4 \quad (1)$

$y^2-xy=-3\Rightarrow y(y-x)=-3 \quad (2)$

Equations (1) and (2)

$\Rightarrow \quad \frac{4}{x}=\frac{3}{y}\Rightarrow\frac{x}{y}=\frac{4}{3}.$

33. (*a*) Given expression

$=3*\left(\frac{3\times(-1)}{3+(-1)}\right)=3*\left(\frac{-3}{2}\right)$

$=\frac{3\times\left(\frac{-3}{2}\right)}{3+\left(\frac{-3}{2}\right)}=\frac{\frac{-9}{2}}{\frac{3}{2}}=-3.$

34. (*d*) $\left(\frac{9}{4}\right)^x\times\left(\frac{8}{27}\right)^{x-1}=\frac{2}{3}$

$\Rightarrow \quad \left(\frac{3}{2}\right)^{2x}\times\left(\frac{2}{3}\right)^{3(x-1)}=\frac{2}{3}$

$\Rightarrow \quad \left(\frac{2}{3}\right)^{-2x}\times\left(\frac{2}{3}\right)^{3x-3}=\left(\frac{2}{3}\right)^1$

$\Rightarrow \quad -2x+(3x-3)=1\Rightarrow x=4.$

35. (*c*) Taking the quotient 2, y and 7, we get $2y=7$, which gives the quotient as 3

$\therefore\ y=3$. Substituting the value of y, we get

$2\frac{3}{x}\times3\frac{1}{2}=7\frac{3}{4}$

Now, $\frac{7\frac{3}{4}}{3\frac{1}{2}}=2\frac{3}{x}\Rightarrow2\frac{3}{14}=2\frac{3}{x}.$

$\therefore \quad x=14,\ y=3.$

36. (*a*) $(6\times\sqrt[3]{5})^3=216\times5=1080$

$(8\times\sqrt[3]{2})^3=512\times2=1024$

$(2\times\sqrt[3]{130})^3=8\times130=1040$

$(\sqrt[3]{900})^3=900.$

37. (*b*) $x=2+2^{2/3}+2^{1/3}$

$\Rightarrow \quad (x-2)=2^{2/3}+2^{1/3}$

$\Rightarrow \quad (x-2)^3=(2^{2/3}+2^{1/3})^3$

$=4+2+3\times2^{2/3}\times2^{1/3}$

$[2^{2/3}+2^{1/3}]$

$=6+3\times2(x-2)$

$\Rightarrow \quad (x-2)^3=6+6x-12=6x-6$

$\Rightarrow \quad x^3-8-6x(x-2)=6x-6$

$\Rightarrow \quad x^3-6x^2+6x=2.$

38. (*d*) Let $\frac{1}{4}+\frac{1}{6}-x=3\times\frac{1}{12}$

Then, $\frac{1}{4}+\frac{1}{6}-x=\frac{1}{4}$ or, $x=\frac{1}{6}.$

39. (*b*) Let the fraction be $\frac{a}{b}$, then

$$\left(\frac{a}{b}\times\frac{a}{b}\right)\div\frac{b}{a}=18\frac{26}{27}=\frac{512}{27}$$

or, $$\left(\frac{a}{b}\right)^3=\left(\frac{8}{3}\right)^3$$

$\therefore$ $$\frac{a}{b}=\frac{8}{3}=2\frac{2}{3}.$$

40. (*d*) Given expression

$$=\frac{\frac{15}{7}-\frac{5}{2}}{\frac{9}{4}+\frac{8}{7}}\div\frac{1}{2+\frac{1}{2+\frac{2}{3}}}$$

$$=\frac{-5}{14}\times\frac{28}{95}\div\frac{1}{2+\frac{3}{8}}$$

$$=\frac{-2}{19}\div\frac{8}{19}=\frac{-2}{19}\times\frac{19}{8}=\frac{-1}{4}.$$

41. (*d*) $\frac{1}{3}=0.33$ and $\frac{7}{8}=0.875$

$\frac{1}{4}=0.25$ does not lie between 0.33 and 0.875

$\frac{23}{24}=0.96$ which exceeds 0.875

$\frac{11}{12}=0.92$ which exceeds 0.875

$\frac{17}{24}=0.708$ which lies between 0.33 and 0.875.

42. (*c*) Out of the 5 girls, 1 took part in the camp.

Out of the 8 boys, 1 took part in the camp.

Out of the 13 students, 2 took part in the camp.

$\therefore$ $\frac{2}{13}$ th of total number of students took part in the camp.

43. (*d*) $\frac{99}{101}=0.9802,\ \left(\frac{99}{101}\right)^2=0.9607$

$\frac{97}{99}=0.9797,\ \left(\frac{97}{99}\right)^2=0.9600$

$\frac{95}{97}=0.9793,\ \left(\frac{95}{97}\right)^2=0.9592$

Hence, only *B* and *D* are correct.

44. (*a*) $$G=H+\sqrt{\frac{4}{L}}$$

$\Rightarrow$ $$(G-H)^2=\frac{4}{L}$$

$\Rightarrow$ $$L=\frac{4}{(G-H)^2}.$$

45. (*c*) $$\frac{1}{z}=\frac{1}{x}-\frac{1}{y}=\frac{y-x}{xy}$$

$\Rightarrow$ $$z=\frac{xy}{y-x}.$$

46. (*d*) $$(x-3)(2x+1)=0$$

$\Rightarrow$ $x-3=0$ or, $2x+1=0$

If $x=3,\ 2x+1=7$

Hence, possible values of $2x+1$ are 0 and 7.

47. (*c*) Given expression

$$=2\div\left[2+2\div\left\{2+2\div\left(2+\frac{2}{3}\right)\right\}\right]$$

$$=2\div\left[2+2\div\left\{2+2\times\frac{3}{8}\right\}\right]$$

$$=2\div\left[2+2\div\frac{11}{4}\right]=2\div\left[2+2\times\frac{4}{11}\right]$$

$$=2\div\frac{30}{11}=2\times\frac{11}{30}=\frac{11}{15}.$$

48. (*c*) Given expression

$$=\frac{3}{10}\div\frac{3}{7}\text{ of }\left(\frac{23}{10}+\frac{13}{5}\right)+\frac{1}{5}\times\frac{5}{7}-\frac{2}{7}$$

$$=\frac{3}{10}\div\frac{3}{7}\text{ of }\frac{49}{10}+\frac{1}{7}-\frac{2}{7}=\frac{3}{10}\div\frac{21}{10}-\frac{1}{7}$$

$$=\frac{3}{10}\times\frac{10}{21}-\frac{1}{7}=\frac{1}{7}-\frac{1}{7}=0.$$

49. (*a*) Given expression

$$=1+1\div\left\{1+1\div\left(\frac{2}{3}\right)\right\}$$

$$=1+1\div\left\{1+1\times\frac{3}{2}\right\}$$

$$=1+1\div\left\{1+\frac{3}{2}\right\}=1+1\div\frac{5}{2}$$

$$=1+1\times\ -\ =1+\frac{2}{5}=\frac{7}{5}.$$

50. (*b*) Given expression

$$=48\div12\times\left(\frac{9}{8}\text{ of }\frac{4}{3}\div\frac{3}{4}\text{ of }\frac{2}{3}\right)$$

$$=\frac{48}{12}\times\left\{\left(\frac{9}{8}\times\frac{4}{3}\right)\div\left(\frac{3}{4}\times\frac{2}{3}\right)\right\}$$

$$=\frac{48}{12}\times\left(\frac{3}{2}\times2\right)=4\times3=12.$$

51. (*b*) $a^x = b,\ b^y = c,\ c^z = a$

On multiplying, we get

$a^x \times b^y \times c^z = a \times b \times c$

$\Rightarrow \quad (abc)^{xyz} = (abc)^1$

$\Rightarrow \quad xyz = 1 = \sqrt{1}$

52. (*c*) Here, $a = c^z = (b^y)^z = b^{yz} = (a^x)^{yz} = a^{xyz}$

$\therefore \quad a^1 = a^{xyz}$

$\therefore \quad xyz = 1.$

53. (*b*) Given $x = \frac{2}{3}y,\ y = \frac{2}{3}z,\ z = \frac{2}{3}w$

$\therefore \quad \frac{3}{2}y = z = \frac{2}{3}w$

or, $\quad y = \frac{2}{3} \times \frac{2}{3}w = \frac{4}{9}w$

54. (*c*) 27, 216, 729, 1728, 3375, 5832, 9261

55. (*d*) Given expression $= \frac{3\sqrt{2} - \sqrt{3}}{3\sqrt{2} + \sqrt{2}} = \frac{3\sqrt{2} - \sqrt{3}}{4\sqrt{2}}$

$= \frac{3}{4} - \frac{1}{4}\frac{\sqrt{3}}{\sqrt{2}}$

$= \frac{3}{4} - \frac{1}{4}\frac{\sqrt{6}}{2} = \frac{3}{4} - \frac{1}{4} \times \frac{2.45}{2}$

$= \frac{3}{4} - 0.31 = 0.75 - 0.31 = 0.44.$

56. (*b*) $\frac{2}{3}$ of $114 - \frac{3}{4}$ of $68 = 76 - 51 = 25.$

57. (*b*) Since $\frac{(a+1)^n}{a}$ leaves always remainder 1.

$\therefore \quad \frac{8^{1785}}{7} = \frac{(7+1)^{1785}}{7}$ gives the remainder 1.

58. (*a*) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{(2+\sqrt{3})^2} + \frac{1}{(2-\sqrt{3})^2}$

$= \frac{1}{7+4\sqrt{3}} + \frac{1}{7-4\sqrt{3}} = 14.$

59. (*b*) $9^x = \frac{9}{3^x} \Rightarrow 3^{2x} = \frac{3^2}{3x}$

$\Rightarrow \quad 3^{2x} = 3^{2-x} = 2x = 2 - x$

$\Rightarrow \quad x = 2/3.$

60. (*d*) $\frac{5a+3b}{2a-3b} = \frac{23}{5} \Rightarrow \frac{5\frac{a}{b}+3}{2\frac{a}{b}-3} = \frac{23}{5}$

$\Rightarrow \quad 25\frac{a}{b} + 15 = 46\frac{a}{b} - 69$

$\Rightarrow \quad 21\frac{a}{b} = 84 \Rightarrow \frac{a}{b} = \frac{4}{1}.$

61. (*a*) $x = 3 + 2\sqrt{2}$

$= 3 + 2 \times 1.414$

$= 5.828$, i.e., less than 6

Number must be close and less than $6^4 = 1296$

$\therefore$ Required number = 1154.

62. (*b*) $x^{(b-c)(b+c-a)} \times x^{(c-a)(c+a-b)} \times x^{(a-b)(a+b-c)}$

$= x^0 = 1.$

63. (*d*) When simplified

$\left\{2 - \frac{1}{3}\right\}\left\{2 - \frac{3}{5}\right\}\left\{2 - \frac{5}{7}\right\} \ldots \left\{2 - \frac{999}{1001}\right\}$

$= \frac{5}{3} \times \frac{7}{5} \times \frac{9}{7} \times \frac{11}{9} \times \ldots \times \frac{1003}{1001} = \frac{1003}{3}.$

DIFFICULTY LEVEL-2

1. (*c*) $\frac{a^3 - b^3}{a^3 + b^3} = \frac{13}{14}$

$\Rightarrow 14a^3 - 14b^3 = 13a^3 + 13b^3$

$\Rightarrow \quad a^3 = 27b^3 \Rightarrow \frac{a}{b} = 3$

$\therefore \quad \frac{a+b}{a-b} = \frac{\frac{a}{b}+1}{\frac{a}{b}-1} = \frac{4}{2} = 2.$

2. (*c*) $\left(\frac{87}{89}\right)^2 = 0.9555$

$\frac{87}{89} = 0.9775$

$\frac{87}{83} = 1.0481$

$\left(\frac{87}{83}\right)^2 = 1.0985$

If $\quad x < 1 \Rightarrow x^2 < x$

If $\quad x > 1 \Rightarrow x^2 > x.$

3. (*c*) $\because \quad \frac{2}{3}$ part $= 96$

$\therefore \quad \frac{3}{4}$ part $= 96 \times \frac{3}{2} \times \frac{3}{4} = 108.$

4. (*c*) Let the number be 1

$\therefore \quad \frac{4}{5}$ of $1 = \frac{4}{5}$ and, $\frac{3}{4}$ of $1 = \frac{3}{4}$

$\therefore$ Difference $= \frac{4}{5} - \frac{3}{4} = \frac{1}{20}$

$\therefore \quad$ Number $= 4 \div \frac{1}{20} = 80.$

5. (*d*) The set of prime numbers.

$S = \{2, 3, 5, 7, 11, 13, \ldots\}.$

Since there is one 5 and one 2 which gives 10 after multiplying mutually, it means the unit digit will be zero.

6. (*a*) Given expression $= 7^{21}(1 + 7 + 7^2 + 7^3)$
$= 7^{21} \times 400,$
which is completely divisible by 25
Hence, remainder is zero.

7. (*b*) Sum of the digits in the thousand's place
$= 6000 + 12000 + 18000 + 24000 = 60000$
Sum of the digits in the hundred's place
$= 600 + 1200 + 1800 + 2400 = 6000$
Sum of the digits in the ten's place
$= 60 + 120 + 180 + 240 = 600$
Sum of the digits in the unit's place
$= 6 + 12 + 18 + 24 = 60.$

8. (*b*) 27, 216, 729, 1728, 3375, 5832, 9261.

9. (*b*) Put $a = 2, b = 12$ in $\sqrt{a^b} = 5b + a^2$

$\therefore \quad \sqrt{2^{12}} = 5 \times 12 + 4 = 64$

$\Rightarrow \quad 2^6 = 64$, which is true.

10. (*d*) Let the monthly income of Manmohan be ₹1

$\therefore$ Pocket money $= \frac{1}{5}$ of ₹1 $= ₹\frac{1}{5}$

and remainder $= 1 - \frac{1}{5} = ₹\frac{4}{5}$

$\therefore$ Other expenses $= \frac{4}{5}$ of ₹$\frac{4}{5}$ = ₹$\frac{16}{25}$

$\therefore$ Saving $= \frac{4}{5} - \frac{16}{25} = ₹\frac{4}{25}$

$\therefore$ Monthly income $= 48, \frac{4}{25} = ₹300.$

11. (*d*) $1800 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

$\therefore$ 1800 must be multiplied by $3 \times 5 = 15$ so that the resulting number becomes a perfect cube.

12. (*c*) For, $n = 14, 3^9(1 + 3^3 + 3^6 + 3^5)$
$= 3^9(1 + 27 + 729 + 243)$
$= 3^9 \times 10^3.$

13. (*c*) 8064 is divisible by 1, 2, 3, 4, 6, 7, 8, 9, 12, 14, 16, 18, 21, 24, 28, 32, 36, 42, 48, 56, 63, 64, 72, 84.

14. (*b*) Let $\frac{2+K}{3+K} = \frac{13}{15} \Rightarrow 2K = 9 \Rightarrow K = 4.5 \approx 5.$

15. (*d*) $x = 5 + 2\sqrt{6}$

$\Rightarrow \quad \sqrt{x} = \sqrt{5 + 2\sqrt{6}} = \sqrt{A} + \sqrt{B}$, say

$\therefore \quad x = 5 + 2\sqrt{6} = A + B + 2\sqrt{AB}$

$\Rightarrow \quad A + B = 5, AB = 6$

$\Rightarrow \quad A = 3, B = 2$

$\Rightarrow \quad \sqrt{x} = \sqrt{3} + \sqrt{2}$

$\therefore \quad \frac{\sqrt{x}-1}{\sqrt{x}} = \frac{\sqrt{3}+\sqrt{2}-1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= 1 - \sqrt{3} + \sqrt{2} = 1 - (\sqrt{3} - \sqrt{2}).$

16. (*c*) $1^{1/2} = 1, (2^{1/2})^{12} = 2^6 = 64$

$(3^{1/3})^{12} = 3^4 = 81, (4^{1/4})^{12} = 4^3 = 64.$

17. (*c*) $r = 9, s = 4$

$\therefore \; 85^{1/2} = r + \frac{s}{2r} = 9 + \frac{4}{18} = 9\frac{2}{9} = 9.22.$

18. (*a*) Let the total score be x runs, such that

$\frac{2}{9}x - \frac{2}{9} \times \left(x - \frac{2}{9}x\right) = 8$ or, $\frac{2}{9}x - \frac{2}{9} \times \frac{7}{9}x = 8$

or $\frac{2}{9}x \times \frac{2}{9} = 8$ or, $x = 162.$

19. (*a*) $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

Put, $a = 885, b = 115$, we have

$? = \frac{a^3 + b^3}{a^2 + b^2 - ab} = a + b = 1000.$

20. (*d*) $x + \frac{1}{x} = 3 \Rightarrow \left(\quad - \right) = 27$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 27$

$\Rightarrow x^3 + \frac{1}{x^3} = 18$

$\Rightarrow \left(x^3 + \frac{1}{x^3}\right)^2 = 324$

$\Rightarrow x^6 + \frac{1}{x^6} + 2 = 324$

$\Rightarrow \quad x^6 + \frac{1}{x^6} = 322.$

21. (*a*) $\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}$

$= \frac{(2.3)^2 - (.3)^3}{(2.3)^2 + 2.3 \times .3 + (.3)^2}$

$= 2.3 - .3 = 2$

$[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)].$

22. (*d*) Given expression

$= \frac{x+2}{x+1} \times \frac{x+3}{x+2} \times \frac{x+4}{x+3} \times \frac{x+5}{x+4}$

$= \frac{x+5}{x+1}.$

23. (*d*) $? = \frac{(1+x^2)(1+x)(1-x)}{(1+x)} \times \frac{x}{1+x^2} \times \frac{1}{x(1-x)} = 1.$

24. (d) $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3} = \left(\frac{a}{b}\right)^{-x+3}$

$\therefore \quad x-1 = -x+3 \Rightarrow x = 2.$

25. (*b*) Given expression

$$= \frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} + \frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}} = 1.$$

26. (*b*) $\frac{1}{1+\frac{a^n}{a^m}} + \frac{1}{1+\frac{a^m}{a^n}} = \frac{a^m}{a^m+a^n} + \frac{a^n}{a^m+a^n} = 1.$

27. (*b*) (B) is correct.

$$(p^y q^x)^z = a^2 \Rightarrow (a^{xy} \times a^{yx})^z = a^2$$

$$\Rightarrow \quad (a^{2xy})^z = a^2$$

$$\Rightarrow \quad a^{2xyz} = a^2$$

$$\Rightarrow \quad 2xyz = 2 \Rightarrow xyz = 1$$

(C) is correct.

$$x^a = y^b = z^c$$

$$\Rightarrow \quad a \log x = b \log y = c \log z$$

$$\therefore \quad ab + bc + ca = 0$$

$$\Rightarrow \quad c\frac{\log z}{\log x} \times c\frac{\log z}{\log y} + c\frac{\log z}{\log y} \times c + c \times c\frac{\log z}{\log x} = 0.$$

$$\Rightarrow \quad \log z \log z + \log x \log z + \log y \log z = 0$$

$$\Rightarrow \quad \log z + \log x + \log y = 0$$

$$\Rightarrow \quad \log (xyz) = 0 = \log 1 \Rightarrow xyz = 1.$$

28. (*d*) $\frac{a^{1/2}+a^{-1/2}}{1-a} + \frac{1-a^{-1/2}}{1+\sqrt{a}}$

$$= \frac{a^{1/2}+a^{-1/2}}{(1+a^{1/2})(1-a^{1/2})} + \frac{1-a^{-1/2}}{1+a^{1/2}}$$

$$= \frac{a^{1/2}+a^{-1/2}+(1-a^{-1/2})(1-a^{1/2})}{(1+a^{1/2})(1-a^{1/2})}$$

$$= \frac{a^{1/2}+a^{-1/2}+1-a^{-1/2}-a^{1/2}+1}{1-a} = \frac{2}{1-a}.$$

29. (*d*) $\frac{x}{y} = \frac{a}{b} - \frac{b}{a} = \frac{a^2-b^2}{ab}$

$$x - y = \frac{a}{b} + \frac{b}{a} = \frac{a^2+b^2}{ab}$$

$$\Rightarrow x = \frac{a^2+b^2}{ab} + y = \frac{a^2+b^2}{ab} + \frac{abx}{a^2-b^2}$$

$$\Rightarrow x\left[1 - \frac{ab}{a^2-b^2}\right] = \frac{a^2+b^2}{ab}$$

$$\Rightarrow \quad x = \frac{a^2+b^2}{ab} \times \frac{(a^2-b^2)}{a^2-b^2-ab}.$$

30. (*b*) $3^{x+y} = 81 = 34,\ 81^{x-y} = 3$

$$\Rightarrow \quad x+y = 4,\ (3^4)^{(x-y)} = 3^1$$

$$\Rightarrow \quad x+y = 4,\ 4x-4y = 1$$

$$x = \frac{17}{8},\ y = \frac{15}{8}.$$

31. (*a*) $P(x, y) = x^2 - y^2$

$\therefore \quad P(3, 4) = 3^2 - 4^2 = 9 - 16 = -7$

$\Rightarrow P(3, P(3, 4)) = P(3, -7)$

$= (3)^2 - (-7)^2 = 9 - 49$

$= -40.$

32. (*c*) 12 to 19:8, 23 to 29 : 7,
34 to 39:6, 45 to 49 : 5,
56 to 59:4, 67 to 69 : 3,
78 to 79:2, 89 : 1
Total : 36.

33. (*c*) $\frac{1}{2}$ of $\frac{2}{3} - \frac{3}{4}$ of $\frac{1}{3} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}.$

34. (*a*) Cost of Cave

= 5 rocks + 2 stones + 3 pebbles

= 35 stones + 14 pebbles + 3 pebbles

= 245 pebbles + 14 pebbles + 3 pebbles

= 262 pebbles

1 rock = 49 pebbles

$\therefore$ To use 6 rocks, it requires 294 pebbles

$\therefore$ Change required = 294 – 262 = 32 pebbles

= 4 stones and 4 pebbles.

35. (*c*) $\frac{3}{4}x - \frac{3}{14}x = 150$ or, $\frac{15}{28}x = 150$

$\therefore \quad x = \frac{150 \times 28}{15} = 280.$

36. (*c*) Given that $\frac{a}{a+b} = \frac{17}{23}$

i.e., if $a = 17$, then $a + b = 23$ or, $b = 6$

$$a - b = 17 - 6 = 11$$

$$\therefore \quad \frac{a+b}{a-b} = \frac{23}{11}.$$

37. (*b*) $x \times x \div \frac{1}{x} = 18\frac{26}{27}$ or, $x^3 = \frac{512}{27}$

$$\therefore \quad x^3 = \left(\frac{8}{3}\right)^3 \text{ and so } x = \frac{8}{3} = 2\frac{2}{3}.$$

38. (*d*) $x \times \frac{17}{8} - x \times \frac{8}{17} = 225$ or, $\frac{225}{136}x = 225$

$\therefore \quad x = 136.$

39. (*b*) $a = \frac{\sqrt{5}+1}{\sqrt{5}-1} = \frac{(\sqrt{5}+1)^2}{4} = \frac{6+2\sqrt{5}}{4} = \frac{3+\sqrt{5}}{2}$

$b = \frac{\sqrt{5}-1}{\sqrt{5}+1} = \frac{(\sqrt{5}-1)^2}{4} = \frac{6-2\sqrt{5}}{4} = \frac{3-\sqrt{5}}{2}$

$a^2 = \left(\frac{3+\sqrt{5}}{2}\right)^2 = \frac{14+6\sqrt{5}}{4}$

$b^2 = \left(\frac{3-\sqrt{5}}{2}\right)^2 = \frac{14-6\sqrt{5}}{4}$

$ab = \left(\frac{\sqrt{5}+1}{\sqrt{5}-1}\right)\left(\frac{\sqrt{5}-1}{\sqrt{5}+1}\right) = \frac{4}{1} = 1$

$\therefore \quad \frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{32}{24} = \frac{4}{3}.$

40. (*d*) $\frac{x+y}{x-y} = \frac{4}{1} \Rightarrow x = \frac{5}{3}y$

$\therefore \quad x^2+y^2 = \frac{25}{9}y^2 + y^2 = \frac{34y^2}{9}$

$x^2-y^2 = \frac{25}{9}y^2 - y^2 = \frac{16y^2}{9}$

$\therefore \quad \frac{x^2+y^2}{x^2-y^2} = \frac{34}{16} = \frac{17}{8}.$

41. (*a*) Let the number be $10x + y$.

$\therefore \quad x + y = 7$

and, $10y + x = 10x + y + 27$

$\Rightarrow \quad y = 5, x = 2$

$\therefore$ The number $= 10x + y = 25.$

42. (*b*) Given expression $= -\frac{3}{2}b^2$. Since b^2 is always positive, therefore, $-\frac{3}{2}b^2$ is always negative.

43. (*a*) $\text{Sum} = \frac{7}{4} + \frac{7}{3} + \frac{41}{12} + \frac{26}{5} + \frac{13}{6}$

$= \frac{105+140+205+312+130}{60}$

$= \frac{892}{60} = 14\frac{13}{15}$

which is nearer to 15 than 14

$\text{Difference} = 15 - 14\frac{13}{15} = \frac{2}{15}.$

44. (*c*) Suppose there were x packages in the van before delivery.

$\therefore$ After first delivery, the number of packages in the van

$= x - \frac{2}{5}x = \frac{3}{5}x$

After second delivery, the number of packages in the van

$= \frac{3}{5}x - 3 = \frac{3x-15}{5}.$

$\therefore \quad \frac{3x-15}{5} = \frac{x}{2}$ (Given)

$\Rightarrow \quad x = 30.$

45. (*d*) $\left[1 + \frac{1}{1+\frac{1}{1+\frac{1}{3}}}\right] + 1\frac{4}{7}$

$= \left[1 + \frac{1}{1+\frac{3}{4}}\right] + \frac{11}{7} = \left[1 + \frac{4}{7}\right] + \frac{11}{7}$

$= \frac{11}{7} + \frac{11}{7} = \frac{22}{7}.$

46. (*c*) $\frac{1}{2} = 0.50000$ (1)

$\frac{}{2.3} = 0.16667$ (2) (divide (1) by 3)

$\frac{1}{2.3.4} = 0.04167$ (3) (divide (2) by 4)

$\frac{1}{2.3.4.5} = 0.00833$ (4) (divide (3) by 5)

Adding, we have 0.71667 or 0.717 up to three places.

47. (*b*) Let x be the fraction

$\frac{7}{6}x - \frac{6}{7}x = \frac{1}{7} \Rightarrow x = \frac{6}{13}$

Correct answer $= \frac{6}{7}x = \frac{6}{7} \times \frac{6}{13} = \frac{36}{91}.$

48. (*a*) Let total number of students $= x$

Number of girl students $= \frac{2x}{3}$

Number of boy students $= \frac{x}{3}$

Number of girls who took part in camp

$= \frac{1}{5}\left(\frac{2x}{3}\right) = \frac{2}{15}x$

Number of boys who took part in camp

$= \frac{1}{8}\left(\frac{x}{3}\right) = \frac{x}{24}$

Total number of students who took part in camp

$$= \frac{2}{15}x + \frac{x}{24} = \left(\frac{16+5}{120}\right)x$$

$$= \frac{7}{40}x.$$

49. (*a*) Number of $\frac{1}{8}$'s $= \frac{75}{2} \div \frac{1}{8} = \frac{75}{2} \times 8 = 300.$

50. (*c*) Given expression

$$= \frac{15}{2} + \frac{1}{2} \div \frac{1}{8} - \frac{2}{5} \times \frac{7}{3} \div \frac{15}{8} \text{ of } \left(\frac{7}{5} - \frac{4}{3}\right)$$

$$= \frac{15}{2} + 4 - \frac{2}{5} \times \frac{7}{3} \div \frac{15}{8} \text{ of } \frac{1}{15}$$

$$= \frac{15}{2} + 4 - \frac{2}{5} \times \frac{7}{3} \div \frac{1}{8}$$

$$= \frac{15}{2} + 4 - \frac{2}{5} \times \frac{7}{3} \times \frac{8}{1}$$

$$= \frac{15}{2} + 4 - \frac{112}{15} = \frac{23}{2} - \frac{112}{15}$$

$$= \frac{121}{30} = 4\frac{1}{30}.$$

51. (*b*) Given expression

$$= \frac{108+9+3+1}{108} = \frac{121}{108} = 1.1203.$$

52. (*b*) Let missing figure $= x$

$$\frac{15}{2} - \left[\frac{9}{4} \div \left\{\frac{5}{4} - x\left(\frac{3}{2} - \frac{1}{3} - \frac{1}{6}\right)\right\}\right] = 3$$

$$\frac{15}{2} - \left[\frac{9}{4} \div \left\{\frac{5}{4} - x\right\}\right] = 3$$

$$\frac{15}{2} - 3 = \frac{9/4}{5/4 - x}$$

$$\frac{9}{2} = \frac{9}{5 - 4x}$$

$$5 - 4x = 2$$

or, $$x = \frac{3}{4}$$

53. (*a*) Given expression $= 7^{21}(1 + 7 + 7^2 + 7^3)$

$= 7^{21} \times 400$

which is completely divisible by 25

Hence, remainder is zero.

54. (*b*) Sum of the digits in the thousand's place

$= 6000 + 12000 + 18000 + 24000 = 60000$

Sum of the digits in the hundred's place

$= 600 + 1200 + 1800 + 2400 = 6000$

Sum of the digits in the ten's place

$= 60 + 120 + 180 + 240 = 600$

Sum of the digits in the unit's place

$= 6 + 12 + 18 + 24 = 60.$

55. (*d*) $1800 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

$\therefore$ 1800 must be multiplied by $3 \times 5 = 15$ so that the resulting number becomes a perfect cube.

56. (*c*) For $n = 14$, $3^9(1 + 3^3 + 3^6 + 3^5)$

$= 3^9(1 + 27 + 729 + 243)$

$= 3^9 \times 10^3.$

57. (*a*) $-\frac{2}{3}, 0, \frac{1}{7}, \frac{2}{9}, \frac{4}{9}, \frac{9}{11}, \frac{14}{15}.$

58. (*b*)

$$x^2 + y^2 = 3341$$

$$x^2 - y^2 = 891$$

$$2x^2 = 4232$$

$\Rightarrow x = 46, y = 35.$

59. (*b*) $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \ldots \times \frac{n-1}{n} = \frac{2}{n}.$

60. (*a*) Given $x^4 + \frac{1}{x^4} = 322$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 322$$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 324$$

$$\therefore x^2 + \frac{1}{x^2} = 18$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 + 2 = 18$$

$$\Rightarrow x - \frac{1}{x} = \sqrt{16} = 4.$$

61. (*d*) $(x)^{(a-b)(a+b)+(b-c)(b+c)+(c-a)(c+a)}$

$= x^0 = 1.$

62. (*c*) The given expression is

$$6^2 + 4^3 + 3 = 36 + 64 + 3 = 103.$$

63. (*c*) $x + \frac{1}{x} = \frac{17}{4}$, $x - \frac{1}{x} = ?$

Now, $\left(x - \frac{1}{x}\right)^2 = \left(x + \frac{1}{x}\right)^2 - 4 \cdot x \cdot \frac{1}{x}$

Putting the value, we get

$$x - \frac{1}{x} = \frac{15}{4}.$$

64. (*d*) Given $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = \frac{13}{6}$

$$\Rightarrow \frac{x}{1-x} + \frac{1-x}{x} + 2\sqrt{\frac{x(1-x)}{x(1-x)}} = \frac{169}{36}$$

$$\Rightarrow \frac{x^2 + (1-x)^2}{x(1-x)} + 2 = \frac{169}{36}$$

$$\Rightarrow \quad \frac{x^2+1+x^2-2x+2x-2x^2}{x(1-x)} = \frac{169}{36}$$

$$\Rightarrow \quad \frac{1}{x(1-x)} = \frac{169}{36}$$

$$\Rightarrow \quad x = \frac{9}{13} \text{ or } \frac{4}{13}.$$

65. (*c*) $\quad \frac{a+b}{b+c} = \frac{c+d}{a+d}$

$$\Rightarrow \quad a^2+ad+ab+bd = bc+bd+c^2+cd$$
$$\Rightarrow \quad a^2+ad+ab = c^2+bc+cd$$
$$\Rightarrow \quad ad+ab-bc-cd = c^2-a^2$$
$$\Rightarrow \quad a(b+d)-c(b+d) = (c-a)(a+c)$$
$$\Rightarrow \quad (a-c)(b+d) = -(a-c)(a+c)$$
$$\Rightarrow \quad (a-c)(b+d)+(a-c)(a+c) = 0$$
$$\Rightarrow \quad (b+d+a+c)(a-c) = 0$$

So, either $a = c$ or $a+b+c+d = 0$ or both.

66. (*b*) According to the question,

$$r = (2a)^{2b} = 2^{2b} \times a^{2b} = (4)^b \times (a^b)^2$$

Also, $\quad r = a^b \times x^b$

$$\Rightarrow \quad a^b \times x^b = 4^b \times (a^b)^2$$
$$\Rightarrow \quad x^b = 4^b \times a^b$$
$$\Rightarrow \quad x^b = (4a)^b$$
$$\therefore \quad x = 4a.$$

67. (*b*) Given, $\frac{m}{n} = \frac{4}{3}$ and $\frac{r}{t} = \frac{9}{14}$

Put the values and calculate

$$\frac{3mr-nt}{4nt-7mr} = \frac{3\times4\times9-3\times14}{4\times3\times14-7\times4\times9}$$
$$= \frac{108-42}{168-252} = \frac{66}{-84} = -\frac{11}{14}.$$

68. (*d*) $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4 = (((a^9)^{1/6})^{1/3})^4 \, (((a^9)^{1/3})^{1/6})^4$

$$= (a^2)(a^2) = a^4.$$

69. (*a*) $1 - \frac{1}{1+\sqrt{3}} + \frac{1}{1-\sqrt{3}} = \frac{1+\sqrt{3}-1}{1+\sqrt{3}} + \frac{1}{1-\sqrt{3}}$

$$= \frac{\sqrt{3}}{1+\sqrt{3}} + \frac{1}{1-\sqrt{3}}$$
$$= \frac{\sqrt{3}(1-\sqrt{3})+1+\sqrt{3}}{1-(\sqrt{3})^2}$$
$$= \frac{\sqrt{3}-3+1+\sqrt{3}}{-2} = \frac{2\sqrt{3}-2}{-2}$$
$$= 1-\sqrt{3}.$$

70. (*a*) $2+\sqrt{2}+\frac{1}{2+\sqrt{2}}+\frac{1}{\sqrt{2}-2}$

$$= 2+\sqrt{2}+\frac{1}{2+\sqrt{2}}-\frac{1}{2-\sqrt{2}}$$
$$= 2+\sqrt{2}+\frac{2-\sqrt{2}-2-\sqrt{2}}{(2+\sqrt{2})(2-\sqrt{2})}$$
$$= 2+\sqrt{2}-\frac{2\sqrt{2}}{2}$$

$$2+\sqrt{2}-\sqrt{2} = 2.$$

71. (*d*) $\quad \frac{2^{n+4}-2(2^n)}{2(2^{n+3})} = \frac{2^{n+4}-2^{n+1}}{2^{n+4}}$

$$\frac{2^{n+4}}{2^{n+4}} - \frac{2^{n+1}}{2^{n+4}} = 1-\frac{1}{8} = \frac{7}{8}.$$

72. (*d*) $u_1 = 3^{1/2}, u_2 = \sqrt{3\sqrt{3}} = 3^{3/4}, u_3 = \sqrt{3\sqrt{3\sqrt{3}}} = 3^{7/8}$

$$\Rightarrow \quad u_n = 3^{\frac{2^n-1}{2^n}}$$

$$\Rightarrow \quad u_{10} = 3^{\frac{2^{10}-1}{2^{10}}}, u_9 = 3^{\frac{2^9-1}{2^9}}$$

$$\therefore \quad \frac{u_{10}}{u_9} = 3^{\frac{2^{10}-1}{2^{10}} - \frac{2^9-1}{2^9}}$$
$$= 3^{\frac{2^{10}-1-2(2^9-1)}{2^{10}}}$$
$$= 3^{\frac{2^{10}-1-2^{10}+2}{2^{10}}} = 2^{\frac{1}{2^{10}}}.$$

73. (*a*) By putting $x = 2$, we have

$$(\sqrt{3}+\sqrt{2})^2 + (\sqrt{3}-\sqrt{2})^2$$
$$= 3+2\sqrt{6}+2+3-2\sqrt{6}+2 = 10$$

Again putting $x = -2$ we get

$$(\sqrt{3}+\sqrt{2})^{-2} + (\sqrt{3}-\sqrt{2})^{-2}$$
$$= \frac{1}{(\sqrt{3}+\sqrt{2})^2} + \frac{1}{(\sqrt{3}-\sqrt{2})^2}$$
$$= \frac{(\sqrt{3}-\sqrt{2})^2+(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3}+\sqrt{2})^2(\sqrt{3}-\sqrt{2})^2}$$
$$= \frac{10}{\{(\sqrt{3})^2-(\sqrt{2})^2\}^2} = \frac{10}{(3-2)^2}$$
$$= 10/1 = 10.$$

So, that $x = \pm 2$ will satisfy the equation.

74. (*c*) $\quad x^{ab-ac+bc-ab+ac-cb} = 1$ (Statement I)

According to statement III,

$$\log(4) - \log(2) = \log 2$$
$$\log\left(\frac{4}{2}\right) = \log 2$$
$$2 = 2.$$

75. (*a*) Given,

$$\frac{a^2-ab+b^2}{a^2+ab+b^2} = \frac{1}{3}$$

By componendo–dividendo,

$$\frac{2(a^2+b^2)}{-2(ab)}=\frac{4}{-2}$$

Therefore,

$$\frac{a^2+b^2}{ab}=2$$

So,

$a^2+b^2=2ab$

$\Rightarrow a^2-2ab+b^2=0$

$\Rightarrow (a-b)^2=0$

$\Rightarrow a=b$

$\Rightarrow \frac{a}{b}=1$

76. (*d*) We have

$2^{\frac{1}{3}}=(2^8)^{\frac{1}{24}}$

$3^{\frac{1}{4}}=(3^6)^{\frac{1}{24}}$

$4^{\frac{1}{6}}=2^{\frac{1}{3}}=(2^8)^{\frac{1}{24}}$

$6^{\frac{1}{8}}=(6^3)^{\frac{1}{24}}$

$10^{\frac{1}{12}}=(10^2)^{\frac{1}{24}}$

Clearly, $10^{\frac{1}{12}}$ is the largest term.

77. (*b*) Let, $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}=k$

Then, $x= ak$, $y = bk$ and $z = ck$

$(x + y + z) = k(a + b + c)$

On squaring both sides, we get

$(x+y+z)^2=k^2(a+b+c)^2$

$\Rightarrow 2(xy+yz+zx)=k^2(a+b+c)^2-(x^2+y^2+z^2)$

$\therefore\ xy+yz+zx=\frac{k^2(a+b+c)^2-(x^2+y^2+z^2)}{2}$

Also, $k=\frac{x}{a}$

$\Rightarrow k^2=\frac{x^2}{a^2}$

$\therefore\ xy+yz+zx=\frac{x^2(a+b+c)^2-(x^2+y^2+z^2)a^2}{2a^2}$

78. (*b*) We have

$x+\frac{1}{x}=1$

$\Rightarrow\ x^2-x+1=0$

Since $x \neq -1$, therefore

$(x + 1)(x^2 - x + 1) = 0$

$\Rightarrow x^3+1=0$

$\Rightarrow x^3=-1$

$\Rightarrow (x^3)^{1333}x=-x$

$\Rightarrow x^{4000}=-x$

Now, $p=x^{4000}+\frac{1}{x^{4000}}=-x+\frac{1}{-x}=-(x+\frac{1}{x})=-1$

Let, $n = 2$

Then, $q = 2^{2^2}+1=16+1=17$

Units place digit = 7

$\therefore\ p+q=-1+7=6.$

79. (*d*) Given expression is equal to $\frac{4}{7}+\frac{11}{7}=\frac{15}{7}$.

80. (*b*) $(5^6-1)=(5^3)^2-(1)^2=(125)^2-(1)^2$

$=(125+1)(125-1)=126\times124=31\times4\times126$

It is therefore clear that the expression is divisible by 31.

81. (*d*) It is clear that for $n = 237$, the expression $n(n + 1)(2n + 1)$ is divisible by 237

Hence, option (d) is not necessarily true.

82. (*d*) Let the total number of quizzes = x:

And score of previous quizzes = y

$\therefore \frac{y+97}{x}=90, \frac{y+70}{x}=87$

$\Rightarrow\ \frac{y}{x}+\frac{97}{x}=90, \frac{y}{x}+\frac{70}{x}=87$

$\Rightarrow\ \frac{y}{x}=90-\frac{97}{x}$ (1)

and $\frac{y}{x}=87-\frac{70}{x}$ (2)

From (1) and (2), we get

$90-\frac{97}{x}=87-\frac{70}{x}$

$\Rightarrow 3=\frac{-70}{x}+\frac{97}{x}=\frac{27}{x}$

$\therefore x=9.$