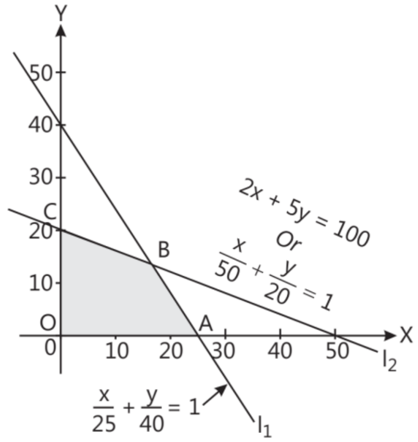
Linear Programming

Case Study Based Questions

Case Study 1

Deepa rides her car at 25 km/hr. She has to spend ₹ 2 per km on diesel and if she rides it at a faster speed of 40 km/hr, the diesel cost increases to ₹ 5 per km. She has ₹ 100 to spend on diesel. Let she travels x kms with speed 25 km/hr and y kms with speed 40 km/hr.

The feasible region for the LPP is shown in the adjacent figure:



Based on the above information, solve the following questions:

Q 1. What is the point of intersection of line l_1 and l_2?

a. $\left(\frac{40}{3}, \frac{50}{3}\right)$

b. $\left(\frac{50}{3}, \frac{40}{3}\right)$

c. $\left(\frac{-50}{3}, \frac{40}{3}\right)$

d. $\left(\frac{-50}{3}, \frac{-40}{3}\right)$

Q2. The corner points of the feasible region shown in above graph are:

a. (0, 25), (20, 0), $\left(\frac{40}{3}, \frac{50}{3}\right)$

b. (0, 0), (25, 0), (0, 20)

c. (0, 0), $\left(\frac{40}{3}, \frac{50}{3}\right)$, (0, 20)

d. (0, 0), (25, 0), $\left(\frac{50}{3}, \frac{40}{3}\right)$, (0, 20)

Q3. If Z = x + y be the objective function and max. Z = 30. The maximum value occurs at point:

a. $\left(\frac{50}{3}, \frac{40}{3}\right)$ b. $(0, 0)$

c. $(25, 0)$ d. $(0, 20)$

Q4. If Z = 6x - 9y be the objective function, then maximum value of Z is:

a. -20 b. 150

c. 180 d. 20

Q5. If Z = 6x + 3y be the objective function, then what is the minimum value of Z?

a. 120 b. 130

c. 0 d. 150

Solutions

1. Let B (x, y) be the point of intersection of the given lines

$$2x + 5y = 100 \quad \ldots(1)$$

and $$\frac{x}{25} + \frac{y}{40} = 1$$

$$\Rightarrow \quad 8x + 5y = 200 \quad \ldots(2)$$

Solving eqs. (1) and (2), we get

$$x = \frac{50}{3}, y = \frac{40}{3}$$

$\therefore$ The point of intersection $B(x, y) = \left(\frac{50}{3}, \frac{40}{3}\right)$.

So, option (b) is correct.

2. The corner points of the feasible region shown in the given graph are

$$O\ (0, 0), A\,(25, 0), B\left(\frac{50}{3}, \frac{40}{3}\right), C\,(0, 20).$$

So, option (d) is correct.

3. Here Z = X + Y

Corner Points	Value of $Z = x + y$
$(0, 0)$	$Z = 0 + 0 = 0$
$(25, 0)$	$Z = 25 + 0 = 25$
$\left(\frac{50}{3}, \frac{40}{3}\right)$	$Z = \frac{50}{3} + \frac{40}{3} = \frac{90}{3} = 30$ (maximum)
$(0, 20)$	$Z = 0 + 20 = 20$

Thus, max $Z = 30$ occurs at point $\left(\frac{50}{3}, \frac{40}{3}\right)$.

So, option (a) is correct.

4.

Corner Points	Value of $Z = 6x - 9y$
$(0, 0)$	$Z = 6 \times 0 - 9 \times 0 = 0$
$(25, 0)$	$Z = 6 \times 25 - 9 \times 0 = 150$ (maximum)
$(0, 20)$	$Z = 6 \times 0 - 9 \times 20 = -180$
$\left(\frac{50}{3}, \frac{40}{3}\right)$	$Z = 6 \times \frac{50}{3} - 9 \times \frac{40}{3} = -20$

Thus, maximum value of Z is 150.

So, option (b) is correct.

5.

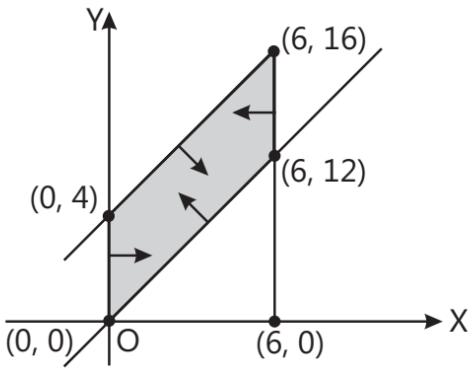
Corner Points	Value of $Z = 6x + 3y$
$(0, 0)$	$Z = 6 \times 0 + 3 \times 0 = 0$ (minimum)
$(25, 0)$	$Z = 6 \times 25 + 3 \times 0 = 150$
$(0, 20)$	$Z = 6 \times 0 + 3 \times 20 = 60$
$\left(\frac{50}{3}, \frac{40}{3}\right)$	$Z = 6 \times \frac{50}{3} + 3 \times \frac{40}{3} = 140$

Thus, minimum value of Z is 0.

So, option (c) is correct.

Case Study 2

The feasible region for an LPP is shown shaded in the figure. Let $F = 3x - 4y$ be objective function. (NCERT EXEMPLAR)



Based on the above information, solve the following questions:

Q1. Find the maximum value of F.

Q2. Find (Maximum of F + Minimum of F).

Solutions

1. Construct the following table of values of the objective function F:

Corner Points	Value of $F = 3x - 4y$
(0, 0)	$3 \times 0 - 4 \times 0 = 0$ (maximum)
(6, 12)	$3 \times 6 - 4 \times 12 = -30$
(6,16)	$3 \times 6 - 4 \times 16 = -46$ (minimum)
(0, 4)	$3 \times 0 - 4 \times 4 = -16$

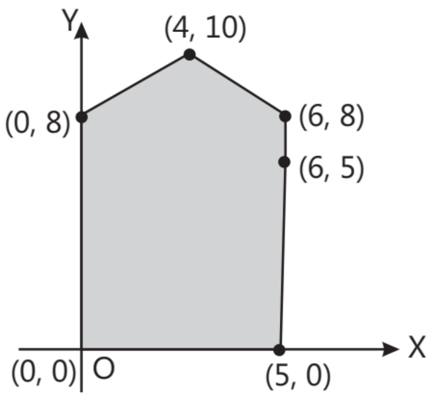
Hence, maximum value of F is 0

2. Minimum of $F = -46$.

$\therefore$ Maximum of F + Minimum of $F = 0 + (-46) = -46$.

Case Study 3

The feasible region for a LPP is shown shaded in the figure. Let Z = 3x - 4y be the objective function. (NCERT EXEMPLAR)



Based on the above information, solve the following questions:

Q 1. Find the points at which maximum and minimum of Z occurs.

Q 2. Find (Maximum Value of Z – Minimum Value of Z).

Solutions

1. Construct the following table of values of the objective function:

Corner Points	Value of $Z = 3x - 4y$
(0, 0)	$3 \times 0 - 4 \times 0 = 0$
(5, 0)	$3 \times 5 - 4 \times 0 = 15$ (Maximum)
(6, 5)	$3 \times 6 - 4 \times 5 = -2$
(6, 8)	$3 \times 6 - 4 \times 8 = -14$
(4, 10)	$3 \times 4 - 4 \times 10 = -28$
(0, 8)	$3 \times 0 - 4 \times 8 = -32$ (Minimum)

$\therefore$ Minimum of $Z = -32$ at (0, 8)
and maximum of $Z = 15$ at (5, 0).

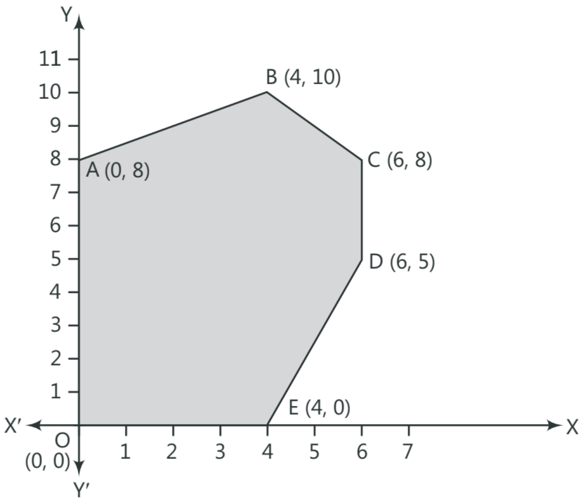
2. Here, Max. $Z = 15$ and Min. $Z = -32$

$\therefore$ Maximum value of Z – Minimum value of Z

$$= 15 - (-32)$$
$$= 15 + 32 = 47.$$

Case Study 4

The corner points of the feasible region determined by the system of linear constraints are as shown below:



Based on the above information, solve the following questions: (NCERT EXEMPLAR)

Q1. Let Z = 3x - 4y be the objective function. Find the maximum and minimum value of Z and also the corresponding points at which the maximum and minimum value occurs.

Q2. Let Z = px + qy, where p,q > 0 be the objective function. Find the condition on p and q so that the maximum value of Z occurs at B (4, 10) and C (6, 8). Also mention the number of optimal solutions in this case.

Solutions

1. The values of Z at corner points are as follows:

Corner Points	$\boldsymbol{Z = 3x - 4y}$
$O\,(0, 0)$	$3 \times 0 - 4 \times 0 = 0$
$A\,(0, 8)$	$3 \times 0 - 4 \times 8 = -32$ (minimum)
$B\,(4, 10)$	$3 \times 4 - 4 \times 10 = -28$
$C\,(6, 8)$	$3 \times 6 - 4 \times 8 = -14$
$D\,(6, 5)$	$3 \times 6 - 4 \times 5 = -2$
$E\,(4, 0)$	$3 \times 4 - 4 \times 0 = 12$ (maximum)

So, the maximum value of Z is 12 at (4, 0) and the minimum value of Z is -32 at (0, 8).

2. Since the maximum value of Z = px + qy occurs at B (4, 10) and C (6, 8).

$$\therefore \quad 4p + 10q = 6p + 8q$$
$$\Rightarrow \quad 6p - 4p = 10q - 8q$$
$$\Rightarrow \quad 2p = 2q$$
$$\Rightarrow \quad p = q$$

which is the required condition.

The number of optimal solution are infinite.

Solutions for Questions 5 to 9 are Given Below

Case Study 5

Linear programming is a method for finding the optimal values (maximum or minimum) of quantities subject to the constraints when relationship is expressed as linear equations or inequations.

Based on the above information, answer the following questions.

(i) The optimal value of the objective function is attained at the points

(a) on X-axis

(b) on Y-axis

(c) which are corner points of the feasible region

(d) none of these

(ii) The graph of the inequality $3x + 4y < 12$ is

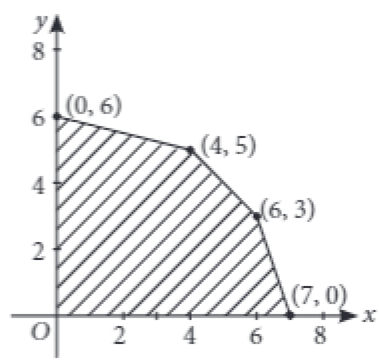
(a) half plane that contains the origin

(b) half plane that neither contains the origin nor the points of the line $3x + 4y = 12$.

(c) whole XOY-plane excluding the points on line $3x + 4y = 12$

(d) None of these

(iii) The feasible region for an LPP is shown in the figure. Let $Z = 2x + 5y$ be the objective function. Maximum of Z occurs at



(a) (7, 0)

(b) (6, 3)

(c) (0, 6)

(d) (4, 5)

(iv) The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is

(a) $p = q$

(b) $p = 2q$

(c) $q = 2p$

(d) $q = 3p$

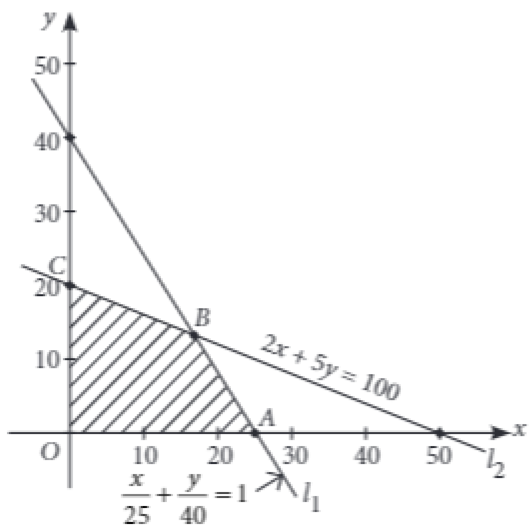
(v) The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40), (60, 20), (60, 0). The objective function is $Z = 4x + 3y$.
Compare the quantity in Column A and Column B

Column A	Column B
Maximum of Z	325

(a) The quantity in column A is greater
(b) The quantity in column B is greater
(c) The two quantities are equal
(d) The relationship cannot be determined on the basis of the information supplied.

Case Study 6

Deepa rides her car at 25 km/hr. She has to spend ₹ 2 per km on diesel and if she rides it at a faster speed of 40 km/hr, the diesel cost increases to ₹ 5 per km. She has ₹ 100 to spend on diesel. Let she travels x kms with speed 25 km/hr and y kms with speed 40 km/hr. The feasible region for the LPP is shown below :



Based on the above information, answer the following questions.

(i) What is the point of intersection of line l_1 and l_2.

(a) $\left(\frac{40}{3}, \frac{50}{3}\right)$ (b) $\left(\frac{50}{3}, \frac{40}{3}\right)$ (c) $\left(\frac{-50}{3}, \frac{40}{3}\right)$ (d) $\left(\frac{-50}{3}, \frac{-40}{3}\right)$

(ii) The corner points of the feasible region shown in above graph are

(a) (0, 25), (20, 0), $\left(\frac{40}{3}, \frac{50}{3}\right)$ (b) (0, 0), (25, 0), (0, 20)

(c) (0, 0), $\left(\frac{40}{3}, \frac{50}{3}\right)$, (0, 20) (d) (0, 0), (25, 0), $\left(\frac{50}{3}, \frac{40}{3}\right)$, (0, 20)

(iii) If $Z = x + y$ be the objective function and max $Z = 30$. The maximum value occurs at point

(a) $\left(\frac{50}{3}, \frac{40}{3}\right)$ (b) (0, 0) (c) (25, 0) (d) (0, 20)

(iv) If $Z = 6x - 9y$ be the objective function, then maximum value of Z is

(a) –20 (b) 150 (c) 180 (d) 20

(v) If $Z = 6x + 3y$ be the objective function, then what is the minimum value of Z?

(a) 120 (b) 130 (c) 0 (d) 150

Case Study 7

Corner points of the feasible region for an LPP are (0, 3), (5, 0), (6, 8), (0, 8). Let $Z = 4x - 6y$ be the objective function.
Based on the above information, answer the following questions.

(i) The minimum value of Z occurs at

(a) (6, 8) (b) (5, 0) (c) (0, 3) (d) (0, 8)

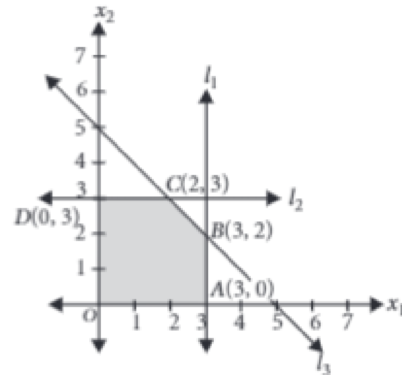
(ii) Maximum value of Z occurs at

(a) (5, 0) (b) (0, 8) (c) (0, 3) (d) (6, 8)

(iii) Maximum of Z – Minimum of Z =

(a) 58 (b) 68 (c) 78 (d) 88

(iv) The corner points of the feasible region determined by the system of linear inequalities are



(a) (0, 0), (–3, 0), (3, 2), (2, 3)

(b) (3, 0), (3, 2), (2, 3), (0, –3)

(c) (0, 0), (3, 0), (3, 2), (2, 3), (0, 3)

(d) None of these

(v) The feasible solution of LPP belongs to

(a) first and second quadrant (b) first and third quadrant

(c) only second quadrant (d) only first quadrant

Case Study 8

Suppose a dealer in rural area wishes to purpose a number of sewing machines. He has only ₹ 5760 to invest and has space for at most 20 items for storage. An electronic sewing machine costs him ₹ 360 and a manually operated sewing machine ₹ 240. He can sell an electronic sewing machine at a profit of ₹ 22 and a manually operated sewing machine at a profit of ₹ 18.
Based on the above information, answer the following questions.

(i) Let x and y denotes the number of electronic sewing machines and manually operated sewing machines purchased by the dealer. If it is assume that the dealer purchased atleast one of the the given machines, then

(a) $x + y \geq 0$ (b) $x + y < 0$ (c) $x + y > 0$ (d) $x + y \leq 0$

(ii) Let the constraints in the given problem is represented by the following inequalities.

$x + y \leq 20$

$360x + 240y \leq 5760$

$x, y \geq 0$

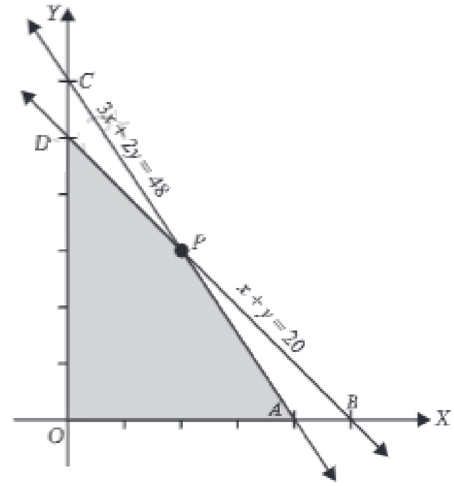
Then which of the following point lie in its feasible region.

(a) (0, 24) (b) (8, 12) (c) (20, 2) (d) None of these

(iii) If the objective function of the given problem is maximise $z = 22x + 18y$, then its optimal value occur at

(a) (0, 0) (b) (16, 0) (c) (8, 12) (d) (0, 20)

(iv) Suppose the following shaded region $APDO$, represent the feasible region corresponding to mathematical formulation of given problem.
Then which of the following represent the coordinates of one of its corner points.



(a) (0, 24)

(b) (12, 8)

(c) (8, 12)

(d) (6, 14)

(v) If an LPP admits optimal solution at two consecutive vertices of a feasible region, then

(a) the required optimal solution is at the midpoint of the line joining two points.

(b) the optimal solution occurs at every point on the line joining these two points.

(c) the LPP under consideration is not solvable.

(d) the LPP under consideration must be reconstructed.

Case Study 9

Let R be the feasible region (convex polygon) for a linear programming problem and let $Z = ax + by$ be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.
Based on the above information, answer the following questions.

(i) Objective function of a L.P.P. is

(a) a constant

(b) a function to be optimised

(c) a relation between the variables

(d) none of these

(ii) Which of the following statement is correct?

(a) Every LPP has at least one optimal solution.

(b) Every LPP has a unique optimal solution.

(c) If an LPP has two optimal solutions, then it has infinitely many solutions.

(d) None of these

(iii) In solving the LPP : "minimize $f = 6x + 10y$ subject to constraints $x \geq 6$, $y \geq 2$, $2x + y \geq 10$, $x \geq 0$, $y \geq 0$" redundant constraints are

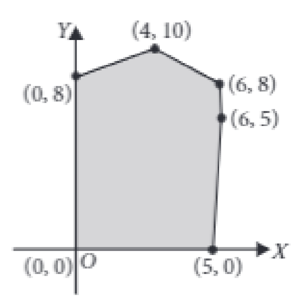
(a) $x \geq 6, y \geq 2$

(b) $2x + y \geq 10, x \geq 0, y \geq 0$

(c) $x \geq 6$

(d) none of these

(iv) The feasible region for a LPP is shown shaded in the figure.
Let $Z = 3x - 4y$ be the objective function. Minimum of Z occurs at



(a) (0, 0)

(b) (0, 8)

(c) (5, 0)

(d) (4, 10)

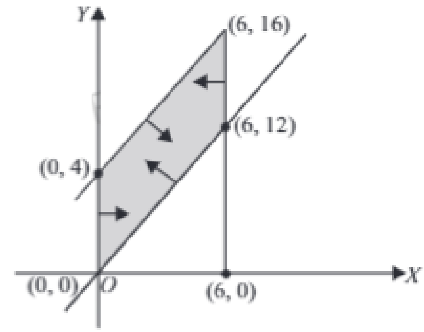
(v) The feasible region for a LPP is shown shaded in the figure. Let $F = 3x - 4y$ be the objective function. Maximum value of F is

(a) 0

(b) 8

(c) 12

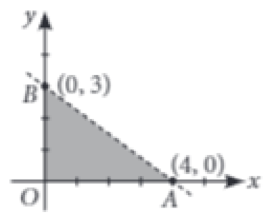
(d) –18



HINTS & EXPLANATIONS

5. (i) (c): When we solve an L.P.P. graphically, the optimal (or optimum) value of the objective function is attained at corner points of the feasible region.

(ii) (d): From the graph of $3x + 4y < 12$ it is clear that it contains the origin but not the points on the line $3x + 4y = 12$.



(iii) (d): Maximum of objective function occurs at corner points.

Corner Points	Value of $Z = 2x + 5y$
(0, 0)	0
(7, 0)	14
(6, 3)	27
(4, 5)	33 ← Maximum
(0, 6)	30

(iv) (d): Value of $Z = px + qy$ at $(15, 15) = 15p + 15q$ and that at $(0, 20) = 20q$. According to given condition, we have

$15p + 15q = 20q \Rightarrow 15p = 5q \Rightarrow q = 3p$

(v) (b): Construct the following table of values of the objective function :

Corner Point	Value of $Z = 4x + 3y$
(0, 0)	$4 \times 0 + 3 \times 0 = 0$
(0, 40)	$4 \times 0 + 3 \times 40 = 120$
(20, 40)	$4 \times 20 + 3 \times 40 = 200$
(60, 20)	$4 \times 60 + 3 \times 20 = 300$ ← Maximum
(60, 0)	$4 \times 60 + 3 \times 0 = 240$

6. (i) (b): Let $B(x, y)$ be the point of intersection of the given lines

$$2x + 5y = 100 \quad \text{...(i)}$$

and $$\frac{x}{25} + \frac{y}{40} = 1 \Rightarrow 8x + 5y = 20 \quad \text{...(ii)}$$

Solving (i) and (ii), we get

$$x = \frac{50}{3}, y = \frac{40}{3}$$

$\therefore$ The point of intersection $B(x, y) = \left(\frac{50}{3}, \frac{40}{3}\right)$.

(ii) (d): The corner points of the feasible region shown in the given graph are

$$(0, 0), A(25, 0), B\left(\frac{50}{3}, \frac{40}{3}\right), C(0, 20).$$

(iii) (a): Here $Z = x + y$

Corner Points	Value of $Z = x + y$
(0, 0)	0
(25, 0)	25
$\left(\frac{50}{3}, \frac{40}{3}\right)$	30 ← Maximum
(0, 20)	20

Thus, max $Z = 30$ occurs at point $\left(\frac{50}{3}, \frac{40}{3}\right)$.

(iv) (b):

Corner Points	Value of $Z = 6x - 9y$
(0, 0)	0
(25, 0)	150 ← Maximum
$\left(\frac{50}{3}, \frac{40}{3}\right)$	–20
(0, 20)	–180

(v) (c):

Corner Points	Value of $Z = 6x + 3y$
(0, 0)	0 ← Minimum
(25, 0)	150
$\left(\frac{50}{3}, \frac{40}{3}\right)$	140
(0, 20)	60

7. Construct the following table of values of objective function

Corner Points	Value of $Z = 4x - 6y$
(0, 3)	$4 \times 0 - 6 \times 3 = -18$
(5, 0)	$4 \times 5 - 6 \times 0 = 20$
(6,8)	$4 \times 6 - 6 \times 8 = -24$
(0, 8)	$4 \times 0 - 6 \times 8 = -48$

(i) **(d):** Minimum value of Z is -48 which occurs at (0, 8).

(ii) **(a):** Maximum value of Z is 20, which occurs at (5, 0).

(iii) **(b):** Maximum of Z – Minimum of Z
$= 20 - (-48) = 20 + 48 = 68$

(iv) **(c):** The corner points of the feasible region are $O(0, 0)$, $A(3, 0)$, $B(3, 2)$, $C(2, 3)$, $D(0, 3)$.

(v) **(d)**

8. (i) (c)

(ii) **(b):** Since (8, 12) satisfy all the inequalities therefore (8, 12) is the point in its feasible region.

(iii) **(c):** At (0, 0), $z = 0$
At (16, 0), $z = 352$
At (8, 12), $z = 392$
At (0, 20), $z = 360$
It can be observed that max z occur at (8, 12). Thus, z will attain its optimal value at (8, 12).

(iv) **(c):** We have, $x + y = 20$ ...(i)
and $3x + 2y = 48$ ...(ii)
On solving (i) and (ii), we get
$x = 8, y = 12$.
Thus, the coordinates of P are (8, 12) and hence (8, 12) is one of its corner points.

(v) **(b):** The optimal solution occurs at every point on the line joining these two points.

9. **(i)** **(b):** Objective function is a linear function (involve variable) whose maximum or minimum value is to be found.

(ii) **(c):** If optimal solution is obtained at two distinct points A and B (corners of the feasible region), then optimal solution is obtained at every point of segment $[AB]$.

(iii) **(b):** When $x \geq 6$ and $y \geq 2$, then
$2x + y \geq 2 \times 6 + 2$, *i.e.*, $2x + y \geq 14$
Hence, $x \geq 0$, $y \geq 0$ and $2x + y \geq 10$ are automatically satisfied by every point of the region
$\{(x, y) : x \geq 6\} \cap \{(x, y) : y \geq 2\}$

(iv) **(b):** Construct the following table of values of the objective function :

Corner Point	Value of $Z = 3x - 4y$
(0, 0)	$3 \times 0 - 4 \times 0 = 0$
(5, 0)	$3 \times 5 - 4 \times 0 = 15$
(6, 5)	$3 \times 6 - 4 \times 5 = -2$
(6, 8)	$3 \times 6 - 4 \times 8 = -14$
(4,10)	$3 \times 4 - 4 \times 10 = -28$
(0, 8)	$3 \times 0 - 4 \times 8 = -32$ ← Minimum

Minimum of $Z = -32$ at (0, 8)

(v) **(a):** Construct the following table of values of the objective function F:

Corner Point	Value of $F = 3x - 4y$
(0, 0)	$3 \times 0 - 4 \times 0 = 0$ ← Maximum
(6, 12)	$3 \times 6 - 4 \times 12 = -30$
(6, 16)	$3 \times 6 - 4 \times 16 = -46$
(0, 4)	$3 \times 0 - 4 \times 4 = -16$

Hence, maximum of $F = 0$