Integration

EXERCISE 5.1 [PAGE 119]

Exercise 5.1 | Q 1 | Page 119

Evaluate $\int \frac{-2}{\sqrt{5x-4}-\sqrt{5x-2}}$dx

Solution:

Let I = $\int \frac{-2}{\sqrt{5x-4}-\sqrt{5x-2}}$dx

$= -2\int \frac{1}{\sqrt{5x-4}-\sqrt{5x-2}} \times \frac{\sqrt{5x-4}+\sqrt{5x-2}}{\sqrt{5x-4}+\sqrt{5x-2}}$dx

$= -2\int \frac{\sqrt{5x-4}+\sqrt{5x-2}}{(5x-4)-(5x-2)}$ dx

$= -2\int \frac{\sqrt{5x-4}+\sqrt{5x-2}}{-2}$ dx

$= \int \left[(5x-4)^{\frac{1}{2}} + (5x-2)^{\frac{1}{2}}\right]$dx

$= \int (5x-4)^{\frac{1}{2}}dx + \int (5x-2)^{\frac{1}{2}}$ dx

$= \frac{(5x-4)^{\frac{3}{2}}}{\frac{3}{2}} \times \frac{1}{5} + \frac{(5x-2)^{\frac{3}{2}}}{\frac{3}{2}} \times \frac{1}{5}$ + c

∴ I = $\frac{2}{15}\left[(5x-4)^{\frac{3}{2}} + (5x-2)^{\frac{3}{2}}\right]$ + c

Exercise 5.1 | Q 2 | Page 119

Evaluate $\int \left(1 + x + \frac{x^2}{2!}\right)$dx

Solution:

Let I = $\int \left(1 + x + \frac{x^2}{2!}\right)$ dx

$= \int 1 \cdot dx + \int x \cdot dx + \frac{1}{2} \int x^2 \cdot dx$ .....[∵ 2! = 2]

$= x + x^2 + \frac{1}{2} \cdot \frac{x^3}{3} + c$

∴ I = $x + \frac{x^2}{2} + \frac{x^3}{6} + c$

Exercise 5.1 | Q 3 | Page 119

Evaluate $\int \frac{3x^3 - 2\sqrt{x}}{x}$ dx

Solution:

Let I = $\int \frac{3x^3 - 2\sqrt{x}}{x}$ dx

$= \int \left(\frac{3x^3}{x} - \frac{2x^{\frac{1}{2}}}{x}\right)$ dx

$= \int \left(3x^2 - 2x^{-\frac{1}{2}}\right)$ dx

$= 3\int x^2 \cdot dx - 2\int x^{-\frac{1}{2}} \cdot dx$

$= 3\left(\frac{x^3}{3}\right) - 2\left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right)$ + c

∴ I = $x^3 - 4\sqrt{x} + c$

Exercise 5.1 | Q 4 | Page 119

Evaluate $\int (3x^2 - 5)^2 \, dx$

Solution:

Let I = $\int (3x^2 - 5)^2 \, dx$

$= \int (9x^4 - 30x^2 + 25) \, dx$

$= 9 \int x^4 dx - 30 \int x^2 dx + 25 \int dx$

$= 9\left(\frac{x^5}{5}\right) - 30\left(\frac{x^3}{3}\right) + 25x + c$

∴ I = $\frac{9}{5}x^5 - 10x^3 + 25x + c$

Exercise 5.1 | Q 5 | Page 119

Evaluate $\int \frac{1}{x(x - 1)} \, dx$

Solution:

Let I = $\int \frac{1}{x(x - 1)} \, dx$

$= \int \frac{x - x + 1}{x(x - 1)} \, dx$

$= \int \frac{x - (x - 1)}{x(x - 1)} \, dx$

$= \int \left(\frac{1}{x - 1} - \frac{1}{x}\right) dx$

$$= \int \frac{1}{x - 1} dx - \int \frac{1}{x} dx$$

$$= \log|x - 1| - \log|x| + c$$

$$\therefore I = \log\left|\frac{x - 1}{x}\right| + c$$

Exercise 5.1 | Q 6 | Page 119

If f '(x) = x^2 + 5 and f(0) = - 1, then find the value of f(x).

Solution:

f '(x) = x^2 + 5 ....[Given]

f(x) = ∫f '(x) dx

= ∫(x^2 + 5) dx

= ∫ x^2 dx + 5 ∫ dx

$$\therefore f(x) = \frac{x^3}{3} + 5x + x \quad ....(i)$$

Now f(0) = - 1 ....[Given]

$$\therefore \frac{0^3}{3} + 5(0) + c = -1$$

∴ c = - 1

Substituting c = – 1 in (i), we get

$$f(x) = \frac{x^3}{3} + 5x - 1$$

Exercise 5.1 | Q 7 | Page 119

If f '(x) = $4x^3 - 3x^2 + 2x + k$, f(0) = 1 and f(1) = 4, find f(x).

Solution: f '(x) = $4x^3 - 3x^2 + 2x + k$ ....[Given]

f(x) = ∫ f '(x) dx

$= \int (4x^3 - 3x^2 + 2x + k)\, dx$

$= 4 \int x^3\, dx - 3 \int x^2\, dx + 2 \int x\, dx + k \int dx$

$$= 4\left(\frac{x^4}{4}\right) - 3\left(\frac{x^3}{3}\right) + 2\left(\frac{x^2}{2}\right) kx + c$$

$\therefore f(x) = x^4 - x^3 + x^2 + kx + c$ ....(i)

Now, f(0) = 1 ...[Given]

$\therefore (0)^4 - (0)^3 + (0)^2 + k(0) + c = 1$

$\therefore c = 1$ ....(ii)

Also, f(1) = 4

$$\therefore 1^4 - 1^3 + 1^2 + k(1) + 1 = 4$$

$\therefore 2 + k = 4$

$\therefore k = 2$ ...(iii)

Substituting (ii) and (iii) in (i), we get

$f(x) = x^4 - x^3 + x^2 + 2x + 1$

Exercise 5.1 | Q 8 | Page 119

If $f'(x) = \frac{x^2}{2} - kx + 1$, f(0) = 2 and f(3) = 5, find f(x).

Solution:

$f'(x) = \frac{x^2}{2} - kx + 1$ ...[Given]

$f(x) = \int f'(x)\, dx$

$= \int \left(\frac{x^2}{2} - kx + 1 \right) dx$

$= \frac{1}{2} \int x^2 dx - k \int x dx + \int 1 \cdot dx$

$= \frac{1}{2} \cdot \frac{x^3}{3} - k\left(\frac{x^2}{2}\right) + x + c$

$\therefore$ f(x) = $\frac{x^3}{6} - \frac{k}{2} x^2 + x + c$ ...(i)

Now, f(0) = 2

$\therefore \frac{(0)^3}{6} - \frac{k}{2}(0)^2 + 0 + c = 2$

$\therefore$ c = 2 ...(ii)

Also f(3) = 5 ...[Given]

$\therefore \frac{(3)^3}{6} - \frac{k}{2}(3)^2 + 3 + 2 = 5$

$\therefore \frac{27}{6} - \frac{9k}{2} + 5 = 5$

$\therefore \frac{9}{2} - \frac{9k}{2} = 0$

$\therefore \frac{9k}{2} = \frac{9}{2}$

$\therefore$ k = 1 ....(iii)

Substituting (ii) and (iii) in (i), we get

f(x) = $\frac{x^3}{6} - \frac{x^2}{2} + x + 2$

EXERCISE 5.2 [PAGES 122 - 123]

Exercise 5.2 | Q 1 | Page 122

Evaluate the following.

$$\int x\sqrt{1+x^2}\,dx$$

Solution:

Let I = $\int x\sqrt{1+x^2}\,dx$

Put $1 + x^2 = t$

$\therefore 2x\,.\,dx = dt$

$\therefore x\,.\,dx = \frac{1}{2}\,dt$

$\therefore I = \frac{1}{2}\int \sqrt{t}\cdot dt$

$= \frac{1}{2}\int t^{\frac{1}{2}}\cdot dt$

$= \frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{1}{3}t^{\frac{3}{2}} + c$

$\therefore I = \frac{1}{3}\left(1+x^2\right)^{\frac{3}{2}} + c$

Exercise 5.2 | Q 2 | Page 123

Evaluate the following.

$$\int \frac{x^3}{\sqrt{1+x^4}}\,dx$$

Solution:

Let I = $\int \frac{x^3}{\sqrt{1+x^4}}$ dx

Put 1 + x^4 = t

∴ $4x^3$. dx = dt

∴ x^3 . dx = $\frac{1}{4}$ dt

∴ I = $\frac{1}{4}\int \frac{dt}{\sqrt{t}}$

$= \frac{1}{4}\int t^{\frac{-1}{2}}$ dt

$= \frac{1}{4} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}}$ + c

$= \frac{1}{2}\sqrt{t} + c$

∴ I = $\frac{1}{2}\sqrt{1+x^4}$ + c

Exercise 5.2 | Q 3 | Page 123

Evaluate the following.

$\int \left(e^x + e^{-x}\right)^2 \left(e^x - e^{-x}\right)$dx

Solution:

Let I = $\int \left(e^x + e^{-x}\right)^2 \cdot \left(e^x - e^{-x}\right)$dx

Put $e^x + e^{-x}$ = t

∴ $\left(e^x + e^{-x}\right)$dx = dt

$\therefore$ I = $\int t^2 \cdot dt$

$= \frac{t^3}{3} + c$

$\therefore$ I = $\frac{1}{3}\left(e^x + e^{-x}\right)^3$ + c

Exercise 5.2 | Q 4 | Page 123

Evaluate the following.

$\int \frac{1 + x}{x + e^{-x}}$ dx

Solution:

Let I = $\int \frac{1 + x}{x + e^{-x}}$ dx

$= \int \frac{1 + x}{x + \frac{1}{e^x}}$ dx

$= \int \frac{1 + x}{\frac{x \cdot e^x + 1}{e^x}}$dx

$= \int \frac{e^x(1 + x)}{x \cdot e^x + 1}$ dx

Put $x \cdot e^x + 1 = t$

$\therefore$ $\left[x \cdot (e^x) + e^x(1) + 0\right]$dx = dt

$\therefore$ $e^x(x + 1)$dx = dt

$\therefore$ I = $\int \frac{dt}{t}$

= log |t| + c

$\therefore$ I = log $\left|x \cdot e^x + 1\right|$ + c

Exercise 5.2 | Q 5 | Page 123

Evaluate the following.

$\int (x + 1)(x + 2)^7 (x + 3)dx$

Solution: Let $I = \int (x + 1)(x + 2)^7 (x + 3)dx$

Put x + 2 = t

∴ dx = dt

Also, x = t - 2

∴ x + 1 = t - 2 + 1

= t - 1

and x + 3 = t - 2 + 3

= t + 1

$$\therefore I = \int (t - 1) \cdot t^7 (t + 1) \cdot dt$$

$$= \int (t^2 - 1) \cdot t^7 \cdot dt$$

$$= \int (t^9 - t^7) dt$$

$$= \int t^9 dt - \int t^7 dt$$

$$= \frac{t^{10}}{10} - \frac{t^8}{8} + c$$

$$\therefore I = \frac{(x + 2)^{10}}{10} - \frac{(x + 2)^8}{8} + c$$

Exercise 5.2 | Q 6 | Page 123

Evaluate the following.

$$\int \frac{1}{x \log x} dx$$

Solution:

$$\text{Let I} = \int \frac{1}{x \log x} dx$$

Put log x = t

$$\therefore \frac{1}{x} dx = dt\prime$$

$$\therefore I = \int \frac{1}{t}\, dt = \log|t| + c$$

$$\therefore I = \log|\log x| + c$$

Alternate Method:

$$\text{Let I} = \int \frac{1}{x \cdot \log x} dx$$

$$= \int \frac{1/x\, dx}{\log x}$$

$$\therefore I = \log|\log x| + c \quad ..... \left[\because \int \frac{f\prime(x)}{f(x)} dx = \log|f(x)| + c\right]$$

Exercise 5.2 | Q 7 | Page 123

Evaluate the following.

$$\int \frac{x^5}{x^2 + 1} dx$$

Solution:

Let I = $\int \frac{x^5}{x^2+1}$ dx

$\int \frac{\left(x^2\right)^2 \cdot x}{x^2+1}$ dx

Put x^2 + 1 = t

∴ 2x . dx = dt

∴ x . dx = $\frac{1}{2} \cdot dt$

Also, x^2 = t - 1

∴ I = $\int \frac{(t-1)^2}{t} \cdot \frac{1}{2}$ dt

$= \frac{1}{2} \int \frac{t^2 - 2t + 1}{t}$ dt

$= \frac{1}{2} \int \left(t - 2 + \frac{1}{t}\right)$ dt

$= \frac{1}{2}\left[\frac{t^2}{2} - 2t + \log|t|\right]$ + c

$= \frac{1}{4}t^2 - t + \frac{1}{2}\log|t| + c$

∴ I = $\frac{1}{4}\left(x^2+1\right)^2 - \left(x^2+1\right) + \frac{1}{2}\log\left|x^2+1\right|$ + c

Exercise 5.2 | Q 8 | Page 123

Evaluate the following.

$\int \frac{2x+6}{\sqrt{x^2+6x+3}}$ dx

Solution:

Let I = $\int \frac{2x+6}{\sqrt{x^2+6x+3}}$ dx

Put x^2 + 6x + 3 = t

∴ (2x + 6) dx = dt

∴ I = $\int \frac{dt}{\sqrt{t}}$

$= \int t^{\frac{-1}{2}}$ dt

$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}}$ + c

$= 2\sqrt{t}$ + c

∴ I = $2\sqrt{x^2+6x+3}$ + c

Alternate Method:

Let I = $\int \frac{2x+6}{\sqrt{x^2+6x+3}}$ dx

$\frac{d}{dx}\left(x^2+6x+3\right)$ = 2x + 6

∴ I = $\int \frac{\frac{d}{dx}\left(x^2+6x+3\right)}{\sqrt{x^2+6x+3}}$ dx

∴ I = $2\sqrt{x^2+6x+3}$ + c .... $\left[\because \int \frac{f\prime(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c\right]$

Exercise 5.2 | Q 9 | Page 123

Evaluate the following.

$$\int \frac{1}{\sqrt{x}+x}\,dx$$

Solution:

Let I = $\int \frac{1}{\sqrt{x}+x}\,dx$

$= \int \frac{1}{\sqrt{x}(1+\sqrt{x})}dx$

Put $1+\sqrt{x}=t$

$\therefore \frac{1}{2\sqrt{x}}dx = dt$

$\therefore \frac{1}{\sqrt{x}}dx = 2\,dt$

$\therefore I = \int \frac{2\cdot dt}{t}$

$= 2\int \frac{1}{t}\cdot dt$

$= 2\log|t| + c$

$\therefore I = 2\log\left|1+\sqrt{x}\right| + c$

Exercise 5.2 | Q 10 | Page 123

Evaluate the following.

$$\int \frac{1}{x(x^6+1)}\,dx$$

Solution:

Let I = $\int \frac{1}{x(x^6+1)}$ dx

$= \frac{x^5}{x^6(x^6+1)}$dx

Put x^6 = t

$\therefore$ $6x^5$ dx = dt

$\therefore x^5 \cdot dx = \frac{1}{6} \cdot dt$

$\therefore$ I = $\frac{1}{6}\int \frac{dt}{t(t+1)}$

$= \frac{1}{6}\int \frac{(t+1)-t}{t(t+1)}$ dt

$= \frac{1}{6}\int \left(\frac{1}{t} - \frac{1}{t+1}\right)$ dt

= $\frac{1}{6}$ [log | t | - log |t + 1|] + c

$= \frac{1}{6}\log\left|\frac{t}{t+1}\right|$ + c

$\therefore$ I = $\frac{1}{6}\log\left|\frac{x^6}{x^6+1}\right|$ + c

EXERCISE 5.3 [PAGE 123]

Exercise 5.3 | Q 1 | Page 123

Evaluate the following.

$$\int \frac{(3e)^{2t}+5}{4e^{2t}-5}\,dt$$

Solution:

Let I = $\int \frac{(3e)^{2t}+5}{4e^{2t}-5}\,dt$

Let $(3e)^{2t}+5 = A\left(4e^{2t}-5\right)+B\frac{d}{dt}\left(4e^{2t}-5\right)$

$= 4Ae^{2t}-5A+B\left(8e^{2t}\right)$

$\therefore (3e)^{2t}+5 = (4A+8B)e^{2t}-5A$

Comparing the coefficients of e^{2t} and constant term on both sides, we get

4A + 8B = 3 and - 5A = 5

Solving these equations, we get

A = - 1 and B = $\frac{7}{8}$

$\therefore$ I = $\int \frac{-1\left(4e^{2t}-5\right)+\frac{7}{8}\left(8e^{2t}\right)}{4e^{2t}-5}\,dt$

$= -\int dt+\frac{7}{8}\int \frac{8e^{2t}}{4e^{2t}-5}\,dt$

$\therefore$ I = $-t+\frac{7}{8}\log\left|4e^{2t}-5\right|$ + c ..... $\left[\int \frac{f\prime(x)}{f(x)}dx = \log|f(x)|+c\right]$

Exercise 5.3 | Q 2 | Page 123

Evaluate the following.

$$\int \frac{20 - 12e^x}{3e^x - 4} dx$$

Solution:

Let I = $\int \frac{20 - 12e^x}{3e^x - 4} dx$

Let $20 - 12e^x = A(3e^x - 4) + B \frac{d}{dx}(3e^x - 4)$

$= 3Ae^x - 4A + 3Be^x$

$\therefore 20 - 12e^x = (3A + 3B)e^x - 4A$

Comparing the coefficients of e^x and constant term on both sides, we get

$-4A = 20$ and $3A + 3B = -12$

Solving these equations, we get

$A = -5$ and $B = 1$

$$\therefore I = \int \frac{-5(3e^x - 4) + 3e^x}{3e^x - 4} dx$$

$$= -5\int dx + \int \frac{3e^x}{3e^x - 4} dx$$

$\therefore I = -5x + \log|(3e^x - 4)| + c$ .... $\left[\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c\right]$

Exercise 5.3 | Q 3 | Page 123

Evaluate the following.

$$\int \frac{3e^x + 4}{2e^x - 8} dx$$

Solution:

Let I = $\int \frac{3e^x + 4}{2e^x - 8}$dx

Let $3e^x + 4 = A(2e^x - 8) + B\frac{d}{dx}(2e^x - 8)$

$= 2Ae^x - 8A + B(2e^x)$

$\therefore 3e^x + 4 = (2A + 2B)e^x - 8A$

Comparing the coefficients of e^x and constant term on both sides, we get

2A + 2B = 3 and - 8A = 4

Solving these equations, we get

$A = -\frac{1}{2}$ and B = 2

$\therefore I = \int \frac{-\frac{1}{2}(2e^x - 8) + 2(2e^x)}{2e^x - 8}dx$

$= -\frac{1}{2}\int dx + 2\int \frac{2e^x}{2e^x - 8}dx$

$\therefore I = -\frac{1}{2}x + 2\log|2e^x - 8| + c$ ..... $\left[\int \frac{f\prime(x)}{f(x)}dx = \log|f(x)| + c\right]$

Exercise 5.3 | Q 4 | Page 123

Evaluate the following.

$\int \frac{2e^x + 5}{2e^x + 1}dx$

Solution:

Let I = $\int \frac{2e^x + 5}{2e^x + 1}dx$

Let $2e^x + 5 = A(2e^x + 1) + B \frac{d}{dx}(2e^x + 1)$

$= 2 Ae^x + A + B(2e^x)$

$\therefore 2e^x + 5 = (2A + 2B)e^x + A$

Comparing the coefficients of e^x and constant term on both sides, we get

2A + 2B = 2 and A = 5

Solving these equations, we get

B = - 4

$$\therefore I = \int \frac{5(2e^x + 1) - 4(2e^x)}{2e^x + 1} dx$$

$$= 5 \int dx - 4 \int \frac{2e^x}{2e^x + 1} dx$$

$$\therefore I = 5x - 4 \log |2e^x + 1| + c \quad .... \left[\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c\right]$$

EXERCISE 5.4 [PAGES 128 - 129]

Exercise 5.4 | Q 1 | Page 129

Evaluate the following.

$$\int \frac{1}{4x^2 - 1} dx$$

Solution:

$$\text{Let } I = \int \frac{dx}{4x^2 - 1}$$

$$= \frac{1}{4} \int \frac{dx}{x^2 - \frac{1}{4}}$$

$$= \frac{1}{4} \int \frac{dx}{x^2 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{4} \times \frac{1}{2 \times \frac{1}{2}} \log\left|\frac{x - \frac{1}{2}}{x + \frac{1}{2}}\right| + c$$

$$\therefore I = \frac{1}{4} \log\left|\frac{2x - 1}{2x + 1}\right| + c$$

Alternate Method:

$$\text{Let } I = \int \frac{dx}{4x^2 - 1} = \int \frac{dx}{(2x^2) - (1)^2}$$

$$= \frac{1}{2 \times 1} \times \frac{1}{2} \log\left|\frac{2x - 1}{2x + 1}\right| + c$$

$$\therefore I = \frac{1}{4} \log\left|\frac{2x - 1}{2x + 1}\right| + c$$

Exercise 5.4 | Q 2 | Page 129

Evaluate the following.

$$\int \frac{1}{x^2 + 4x - 5} dx$$

Solution:

$$\text{Let } I = \int \frac{1}{x^2 + 4x - 5} dx$$

$$= \int \frac{1}{x^2 + 4x + 4 - 4 - 5} dx$$

$$= \int \frac{1}{(x + 2)^2 - 9} dx$$

$$= \int \frac{1}{(x + 2)^2 - 3^2} dx$$

$$= \frac{1}{2 \times 3} \log \left| \frac{(x+2)-3}{(x+2)+3} \right| + c$$

$$\therefore I = \frac{1}{6} \log \left| \frac{x-1}{x+5} \right| + c$$

Exercise 5.4 | Q 3 | Page 129

Evaluate the following.

$$\int \frac{1}{4x^2 - 20x + 17} \, dx$$

Solution:

$$\text{Let } I = \int \frac{1}{4x^2 - 20x + 17} \, dx$$

$$= \frac{1}{4} \int \frac{1}{x^2 - 5x + \frac{17}{4}} \, dx$$

$$= \frac{1}{4} \int \frac{1}{x^2 - 2 \cdot \frac{5}{2} x + \frac{25}{4} - \frac{25}{4} + \frac{17}{4}} \, dx$$

$$= \frac{1}{4} \int \frac{1}{\left(x - \frac{5}{2}\right)^2 - \frac{8}{4}} \, dx$$

$$= \frac{1}{4} \int \frac{1}{\left(x - \frac{5}{2}\right)^2 - \left(\sqrt{2}\right)^2} + c$$

$$= \frac{1}{4} \times \frac{1}{2\sqrt{2}} \log \left| \frac{x - \frac{5}{2} - \sqrt{2}}{x - \frac{5}{2} + \sqrt{2}} \right| + c$$

$$\therefore I = \frac{1}{8\sqrt{2}} \log \left| \frac{2x - 5 - 2\sqrt{2}}{2x - 5 + 2\sqrt{2}} \right| + c$$

Exercise 5.4 | Q 4 | Page 129

Evaluate the following.

$$\int \frac{x}{4x^2 - 2x^2 - 3}\, dx$$

Solution:

Let I = $\displaystyle\int \frac{x}{4x^2 - 2x^2 - 3}\, dx$

Put $x^2 = t$

$\therefore$ 2x dx = dt

$\therefore$ x dx = $\dfrac{dt}{2}$

$\therefore$ I = $\displaystyle\frac{1}{2}\int \frac{dt}{4t^2 - 2t - 3}$

$$= \frac{1}{2 \times 4}\int \frac{dt}{t^2 - \frac{1}{2}t - \frac{3}{4}}$$

$$= \frac{1}{8}\int \frac{dt}{t^2 - \frac{1}{2}t - \frac{3}{4}}$$

$$= \frac{1}{8}\int \frac{dt}{t^2 - 2 \cdot \frac{1}{4} \cdot t + \frac{1}{16} - \frac{1}{16} - \frac{3}{4}}$$

$$= \frac{1}{8}\int \frac{dt}{t^2 - 2 \cdot \frac{1}{4} \cdot t + \frac{1}{16} - \frac{1}{16} - \frac{3}{4}}$$

$$= \frac{1}{8}\int \frac{dt}{\left(t - \frac{1}{4}\right)^2 - \left(\frac{1+12}{16}\right)}$$

$$= \frac{1}{8}\int \frac{dt}{\left(t - \frac{1}{4}\right)^2 - \frac{13}{16}}$$

$$= \frac{1}{8} \int \frac{dt}{\left(t - \frac{1}{4}\right)^2 - \left(\frac{\sqrt{13}}{4}\right)^2}$$

$$= \frac{1}{8} \cdot \frac{1}{2 \times \frac{\sqrt{13}}{4}} \log\left|\frac{t - \frac{1}{4} - \frac{\sqrt{13}}{4}}{t - \frac{1}{4} + \frac{\sqrt{13}}{4}}\right| + c$$

$$= \frac{1}{4\sqrt{13}} \log\left|\frac{4t - 1 - \sqrt{13}}{4t - 1 - \sqrt{13}}\right| + c$$

$$\therefore I = \frac{1}{4\sqrt{13}} \log\left|\frac{4x^2 - 1 - \sqrt{13}}{4x^2 - 1 + \sqrt{13}}\right| + c$$

Exercise 5.4 | Q 5 | Page 128

Evaluate the following.

$$\int \frac{x^3}{16x^8 - 25}\, dx$$

Solution:

Let $I = \int \frac{x^3}{16x^8 - 25}\, dx$

Put $x^4 = t$

$\therefore 4x^3\, dx = dt$

$\therefore x^3\, dx = \frac{1}{4}\, dt$

$$\therefore I = \frac{1}{4} \int \frac{dt}{16t^2 - 25}$$

$$= \frac{1}{4 \times 16} \int \frac{dt}{t^2 - \frac{25}{16}}$$

$$= \frac{1}{64} \int \frac{dt}{t^2 - \left(\frac{5}{4}\right)^2}$$

$$= \frac{1}{64} \times \frac{1}{2 \times \frac{5}{4}} \log\left|\frac{t - \frac{5}{4}}{t + \frac{5}{4}}\right| + c$$

$$= \frac{1}{160} \log\left|\frac{4t - 5}{4t + 5}\right| + xc$$

$$\therefore I = = \frac{1}{160} \log\left|\frac{4x^4 - 5}{4x^4 + 5}\right| + c$$

Exercise 5.4 | Q 6 | Page 129

Evaluate the following.

$$\int \frac{1}{a^2 - b^2x^2} dx$$

Solution:

Let $I = \int \frac{1}{a^2 - b^2x^2} dx$

$$= \frac{1}{b^2} \int \frac{1}{\frac{a^2}{b^2} - x^2} dx$$

$$= \frac{1}{b^2} \int \frac{1}{\left(\frac{a}{b}\right)^2 - x^2} dx$$

$$= \frac{1}{b^2} \times \frac{1}{2\left(\frac{a}{b}\right)} \log\left|\frac{\frac{a}{b} + x}{\frac{a}{b} - x}\right| + c$$

$$\therefore I = \frac{1}{2ab} \log\left|\frac{a + bx}{a - bx}\right| + c$$

Alternate Method:

Let $I = \int \frac{dx}{a^2 - b^2x^2} = \int \frac{dx}{a^2 - (bx)^2}$

$$= \frac{1}{2 \times a} \times \frac{1}{b} \log \left| \frac{a + bx}{a - bx} \right| + c$$

$$\therefore I = \frac{1}{2ab} \log \left| \frac{a + bx}{a - bx} \right| + c$$

Exercise 5.4 | Q 7 | Page 129

Evaluate the following.

$$\int \frac{1}{7 + 6x - x^2} \, dx$$

Solution:

$$\text{Let } I = \int \frac{1}{7 + 6x - x^2} \, dx$$

$$= \int \frac{1}{7 + 9 - 9 + 6x - x^2} \, dx$$

$$= \int \frac{1}{16 - (x^2 - 6x + 9)} \, dx$$

$$= \int \frac{1}{(4)^2 - (x - 3)^2} \, dx$$

$$= \frac{1}{2 \times 4} \log \left| \frac{4 + x - 3}{4 - (x - 3)} \right| + c$$

$$\therefore I = \frac{1}{8} \log \left| \frac{1 + x}{7 - x} \right| + c$$

Exercise 5.4 | Q 8 | Page 129

Evaluate the following.

$$\int \frac{1}{\sqrt{3x^2 + 8}} \, dx$$

Solution:

$$\text{Let } I = \int \frac{1}{\sqrt{3x^2 + 8}} \, dx$$

$$\int \frac{1}{\sqrt{\left(\sqrt{3}x\right)^2 + \left(\sqrt{8}\right)^2}}\,dx$$

$$= \frac{\log\left|\sqrt{3}x + \sqrt{\left(\sqrt{3}x\right)^2 + \left(\sqrt{8}\right)^2}\right|}{\sqrt{3}} + c$$

$$\therefore I = \frac{1}{\sqrt{3}}\log\left|\sqrt{3}x + \sqrt{3x^2 + 8}\right| + c$$

Alternate method:

$$\text{Let } I = I = \int \frac{1}{\sqrt{3x^2 + 8}}dx = \frac{1}{\sqrt{3}}\int \frac{1}{\sqrt{x^2 + \frac{8}{3}}}\,dx$$

$$= \frac{1}{\sqrt{3}}\int \frac{1}{\sqrt{x^2 + \left(\frac{2\sqrt{2}}{\sqrt{3}}\right)^2}}\,dx$$

$$= \frac{1}{\sqrt{3}}\log\left|x + \sqrt{x^2 + \left(\frac{2\sqrt{2}}{\sqrt{3}}\right)^2}\right| + c_1$$

$$= \frac{1}{\sqrt{3}}\log\left|x + \sqrt{x^2 + \frac{8}{3}}\right| + c_1$$

$$= \frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{3}x + \sqrt{3x^2 + 8}}{\sqrt{3}}\right| + c_1$$

$$= \frac{1}{\sqrt{3}}\log\left|\sqrt{3}x + \sqrt{3x^2 + 8}\right| - \frac{1}{\sqrt{3}}\log\sqrt{3} + c_1$$

$\therefore$ I = $\frac{1}{\sqrt{3}}\log\left|\sqrt{3}x + \sqrt{3x^2 + 8}\right|$ + c

where c = $c_1 - \frac{1}{\sqrt{3}}\log\sqrt{3}$

Exercise 5.4 | Q 9 | Page 129

Evaluate the following.

$$\int \frac{1}{\sqrt{x^2 + 4x + 29}}\ dx$$

Solution:

Let I = $\int \frac{1}{\sqrt{x^2 + 4x + 29}}$ dx

$= \int \frac{1}{\sqrt{x^2 + 2 \cdot 2x + 4 - 4 + 29}}$ dx

$= \int \frac{1}{\sqrt{(x + 2)^2 + 25}}$ dx

$= \int \frac{dx}{\sqrt{(x + 2)^2 + 5^2}}$

$= \log\left|(x + 2) + \sqrt{(x + 2)^2 + 5^2}\right|$ + c

$\therefore$ I = $= \log\left|(x + 2) + \sqrt{x^2 + 4x + 29}\right|$ + c

Exercise 5.4 | Q 10 | Page 129

Evaluate the following.

$$\int \frac{1}{\sqrt{3x^2 - 5}}\ dx$$

Solution:

Let I = $\int \frac{1}{\sqrt{3x^2-5}}\,dx$

$= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2-\frac{5}{3}}}\,dx$

$= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2-\left(\frac{\sqrt{5}}{\sqrt{3}}\right)^2}}\,dx$

$= \frac{1}{\sqrt{3}} \log\left|x+\sqrt{x^2-\left(\frac{\sqrt{5}}{\sqrt{3}}\right)^2}\right| + c_1$

$= \frac{1}{\sqrt{3}} \log\left|x+\sqrt{x^2-\frac{5}{3}}\right| + c_1$

$= \frac{1}{\sqrt{3}} \log\left|\frac{\sqrt{3}x+\sqrt{3x^2-5}}{\sqrt{3}}\right| + c_1$

$= \frac{1}{\sqrt{3}} \log\left|\sqrt{3}x+\sqrt{3x^2-5}\right| - \frac{1}{\sqrt{3}}\log\sqrt{3} + c_1$

∴ I = $\frac{1}{\sqrt{3}} \log\left|\sqrt{3}x+\sqrt{3x^2-5}\right| + c,$

where c = $c_1 - \frac{1}{\sqrt{3}}\log\sqrt{3}$

Exercise 5.4 | Q 11 | Page 129

Evaluate the following.

$\int \frac{1}{\sqrt{x^2-8x-20}}\,dx$

Solution:

Let I = $\int \frac{1}{\sqrt{x^2 - 8x - 20}}$ dx

$$= \int \frac{1}{\sqrt{x^2 - 2 \cdot 4x + 16 - 16 - 20}} \text{ dx}$$

$$= \int \frac{dx}{\sqrt{(x - 4)^2 - 36}} \text{ dx}$$

$$= \int \frac{dx}{\sqrt{(x - 4)^2 - 6^2}} \text{ dx}$$

$$= \log\left|(x - 4) + \sqrt{(x - 4)^2 - 6^2}\right| + c$$

$$\therefore I = \log\left|(x - 4) + \sqrt{x^2 - 8x - 20}\right| + c$$

EXERCISE 5.5 [PAGE 133]

Exercise 5.5 | Q 1 | Page 133

Evaluate the following.

∫ x log x dx

Solution: Let I = ∫ x log x dx

$$= \log x \int x\, dx - \int\left[\frac{d}{dx}(\log x) \int x \ dx\right] dx$$

$$= \log x \cdot \frac{x^2}{2} - \int\left[\frac{1}{x} \times \frac{x^2}{2}\right] \text{ dx}$$

$$= \frac{x^2}{2} \log x - \frac{1}{2} \int x \ dx$$

$$= \frac{x^2}{2}\log x - \frac{1}{2} \cdot \frac{x^2}{2} + c$$

$$\therefore I = \frac{x^2}{2}\log x - \frac{x^2}{4} + c$$

Exercise 5.5 | Q 2 | Page 133

Evaluate the following.

$$\int x^2 e^{4x} dx$$

Solution:

Let $I = \int x^2 e^{4x} dx$

$$= x^2 \int e^{4x}\, dx - \int \left[\frac{d}{dx}(x^2) \int e^{4x} dx\right] dx$$

$$= x^2 \cdot \frac{e^{4x}}{4} - \int 2x \cdot \frac{e^{4x}}{4}\, dx$$

$$= \frac{x^2 \cdot e^{4x}}{4} - \frac{1}{2} \int x \cdot e^{4x}\, dx$$

$$= \frac{x^2 \cdot e^{4x}}{4} - \frac{1}{2}\left[x \int e^{4x}\, dx - \int \left(\frac{d}{dx}(x) \int e^{4x} dx\right) dx\right]$$

$$= \frac{x^2 \cdot e^{4x}}{4} - \frac{1}{2}\left[x \cdot \frac{e^{4x}}{4} - \int 1 \cdot \frac{e^{4x}}{4}\, dx\right]$$

$$= \frac{x^2 e^{4x}}{4} - \frac{1}{2}\left[\frac{x \cdot e^{4x}}{4} - \frac{1}{4} \int e^{4x} dx\right]$$

$$= \frac{x^2 e^{4x}}{4} - \frac{1}{2}\left[\frac{x \cdot e^{4x}}{4} - \frac{1}{4} \cdot \frac{e^{4x}}{4}\right] + c$$

$$= \frac{x^2 e^{4x}}{4} - \frac{xe^{4x}}{8} + \frac{e^{4x}}{32} + c$$

$$\therefore I = \frac{e^{4x}}{4}\left[x^2 - \frac{x}{2} + \frac{1}{8}\right] + c$$

Exercise 5.5 | Q 3 | Page 133

Evaluate the following.

$$\int x^2 e^{3x} dx$$

Solution:

$$\text{Let } I = \int x^2 e^{3x} dx$$

$$= x^2 \int e^{3x}\, dx - \int \left[\frac{d}{dx}(x^2) \int e^{3x} dx\right] dx$$

$$= x^2 \cdot \left(\frac{e^{3x}}{3}\right) - \int 2x \cdot \frac{e^{3x}}{3}\, dx$$

$$= \frac{x^2}{3} e^{3x} - \frac{2}{3} \int x \cdot e^{3x}\, dx$$

$$= \frac{x^2}{3} e^{3x} - \frac{2}{3}\left[x \int e^{3x}\, dx - \int \left(\frac{d}{dx}(x) \int e^{3x} dx\right) dx\right]$$

$$= \frac{x^2 \cdot e^{3x}}{3} - \frac{2}{3}\left[x \cdot \frac{e^{3x}}{3} - \int 1 \cdot \frac{e^{3x}}{3}\, dx\right]$$

$$= \frac{x^2 \cdot e^{3x}}{3} - \frac{2}{3}\left[\frac{1}{3} xe^{3x} - \frac{1}{3} \int e^{3x}\, dx\right]$$

$$= \frac{x^2 \cdot e^{3x}}{3} - \frac{2}{3}\left[\frac{1}{3} xe^{3x} - \frac{1}{3} \cdot \frac{e^{3x}}{3}\right] + c$$

$$\therefore I = \frac{1}{3} x^2 \cdot e^{3x} - \frac{2}{9} xe^{3x} + \frac{2}{27} e^{3x} + c$$

Exercise 5.5 | Q 4 | Page 133

Evaluate the following.

$$\int x^3 e^{x^2} dx$$

Solution:

Let I = $\int x^3 e^{x^2} dx$

$$= \int x^2 \cdot x \cdot e^{x^2}\ dx$$

Put $x^2 = t$

$$\therefore 2x \cdot dx = dt$$

$\therefore$ x dx = $\frac{dt}{2}$

$\therefore$ I = $\frac{1}{2}\int te^t\ dt$

$$= \frac{1}{2}\left[t\int e^t\ dt - \int\left[\frac{d}{dt}(t)\int e^t dt\right]dt\right]$$

$$= \frac{1}{2}\left[te^t - \int 1 \cdot e^t dt\right]$$

$$= \frac{1}{2}\left(te^t - e^t\right) + c = \frac{1}{2}e^t(t - 1) + c$$

$\therefore$ I = $\frac{1}{2}e^{x^2}\left(x^2 - 1\right) + c$

Exercise 5.5 | Q 5 | Page 133

Evaluate the following.

$$\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx$$

Solution:

Let I = $\int e^{x}\left(\frac{1}{x}-\frac{1}{x^2}\right)$dx

Put f(x) = $\frac{1}{x}$

∴ f '(x) = $\frac{1}{x}$

∴ I = $\int e^{x}[f(x)+f\prime(x)]$ dx

$= e^{x}\cdot f(x)+c$

∴ I = $e^{x}\cdot\frac{1}{x}+c$

Exercise 5.5 | Q 6 | Page 133

Evaluate the following.

$$\int e^{x}\frac{x}{(x+1)^2}\ dx$$

Solution:

Let I = $\int\left(\frac{x}{(x+1)^2}\right)e^{x}$ dx

$= \int e^{x}\left(\frac{(x+1)-1}{(x+1)^2}\right)$ dx

$= \int e^{x}\left(\frac{x+1}{(x+1)^2}-\frac{1}{(x+1)^2}\right)$ dx

$= \int e^{x}\left(\frac{1}{x+1}-\frac{1}{(x+1)^2}\right)$ dx

Put f(x) = $\frac{1}{x+1}$

$\therefore$ f '(x) = $\dfrac{-1}{(x+1)^2}$

$\therefore$ I = $\int e^x[f(x) + f\prime(x)]$ dx

$= e^x \cdot f(x) + c$

$\therefore$ I = $e^x\left(\dfrac{1}{x+1}\right)$ + c

Exercise 5.5 | Q 7 | Page 133

Evaluate the following.

$\int e^x \dfrac{x - 1}{(x+1)^3}$ dx

Solution:

Let I = $\int e^x \dfrac{(x - 1)}{(x+1)^3} dx = \int e^x \dfrac{(x+1-1-1)}{(x+1)^3}$ dx

$= \int e^x \left[\dfrac{x+1}{(x+1)^3} - \dfrac{2}{(x+1)^3}\right]$ dx

$= \int e^x \left[\dfrac{1}{(x+1)^2} - \dfrac{2}{(x+1)^3}\right]$ dx

Put f(x) = $\dfrac{1}{(x+1)^2}$

$\therefore$ f '(x) = $\dfrac{-2}{(x+1)^3}$

$\therefore$ I = $\int e^x[f(x) \ + \ f\prime(x)]$ dx

$$= e^x f(x) + c$$

$$= e^x \times \frac{1}{(x+1)^2} + c$$

$$\therefore I = \frac{e^x}{(x+1)^2} + c$$

Exercise 5.5 | Q 8 | Page 133

Evaluate the following.

$$\int e^x \left[(\log x)^2 + \frac{2\log x}{x}\right] dx$$

Solution:

$$\text{Let } I = \int e^x \left[(\log x)^2 + \frac{2\log x}{x}\right] dx$$

Put $f(x) = (\log x)^2$

$$\therefore f'(x) = \frac{2\log x}{x}$$

$$\therefore I = \int e^x [f(x) + f'(x)] + dx$$

$$= e^x f(x) + c$$

$$\therefore I = e^x (\log x)^2 + c$$

Exercise 5.5 | Q 9 | Page 133

Evaluate the following.

$$\int \left[\frac{1}{\log x} - \frac{1}{(\log x)^2}\right] dx$$

Solution:

Let I = $\int\left[\frac{1}{\log x}-\frac{1}{(\log x)^2}\right]$ dx

Put log x = t

∴ x = e^t

∴ dx = e^t dt

∴ I = $\int e^t\left[\frac{1}{t}-\frac{1}{t^2}\right]$ dt

Put f(t) = $\frac{1}{t}$

∴ f '(x) = $\frac{-1}{t^2}$

∴ I = $\int e^t[f(t)+f\prime(x)]$ dt

$= e^t \ f(t)$ + c

∴ I = $e^t\left(\frac{1}{t}\right)+c=\frac{x}{\log x}$ + c

Exercise 5.5 | Q 10 | Page 133

Evaluate the following.

$\int\frac{\log x}{(1+\log x)^2}$ dx

Solution:

Let I = $\int\frac{\log x}{(1+\log x)^2}$ dx

Put log x = t

$\therefore x = e^t$

$\therefore dx = e^t \, dt$

$\therefore I = \int \frac{t}{(1+t)^2} e^t \, dt$

$$= \int e^t \left[\frac{(t+1)-1}{(1+t)^2} \right] dt$$

$$= \int e^t \left[\frac{t+1}{(1+t)^2} - \frac{1}{(1+t)^2} \right] dt$$

$$= \int e^t \left[\frac{1}{(1+t)^2} - \frac{1}{(1+t)^2} \right] dt$$

Put $f(t) = \frac{1}{1+t}$

$\therefore f'(t) = \frac{-1}{(1+t)^2}$

$$\therefore \int e^t [f(t) + f\prime(t)] \, dt$$

$$= e^t \, f(t) + c$$

$$= e^t \cdot \frac{1}{1+t} + c$$

$\therefore I = \frac{x}{1+\log x} + c$

EXERCISE 5.6 [PAGE 135]

Exercise 5.6 | Q 1 | Page 135

Evaluate: $\int \frac{2x + 1}{(x + 1)(x - 2)} dx$

Solution:

Let I = $\int \frac{2x + 1}{(x + 1)(x - 2)} dx$

Let $\frac{2x + 1}{(x + 1)(x - 2)} = \frac{A}{x + 1} + \frac{B}{x - 2}$

$\therefore$ 2x + 1 = A(x - 2) + B(x + 1) ....(i)

Putting x = - 1 in (i), we get

2(-1) + 1 = A(- 3) + B(0)

$\therefore$ - 1 = -3A

$\therefore$ A = $\frac{1}{3}$

Putting x = 2 in (i), we get

2(2) + 1 = A(0) + B(3)

$\therefore$ 5 = 3B

$\therefore$ B = $\frac{5}{3}$

$\therefore \frac{2x + 1}{(x + 1)(x - 2)} = \frac{\frac{1}{3}}{x + 1} + \frac{\frac{5}{3}}{x - 2}$

$\therefore$ I = $\int \left(\frac{\left(\frac{1}{3}\right)}{x + 1} + \frac{\left(\frac{5}{3}\right)}{x - 2} \right) dx$

$\therefore \frac{1}{3} \int \frac{1}{x+1} dx + \frac{5}{3} \int \frac{1}{x-2}$ dx

$\therefore$ I = $\frac{1}{3} \log|x+1| + \frac{5}{3} \log|x-2| + c$

Exercise 5.6 | Q 2 | Page 135

Evaluate: $\int \frac{2x+1}{x(x-1)(x-4)}$ dx

Solution:

Let I = $\int \frac{2x+1}{x(x-1)(x-4)}$ dx

Let $\frac{2x+1}{x(x-1)(x-4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-4}$

$\therefore$ 2x + 1 = A(x - 1)(x - 4) + Bx(x - 4) + Cx(x - 1) ....(i)

Putting x = 0 in (i), we get

0 + 1 = A(0 - 1)(0 - 4) + B(0)(- 4) + C(0)(- 1)

$\therefore$ 1 = 4A

$\therefore$ A = $\frac{1}{4}$

Putting x = 1 in (i), we get

2(1) + 1 = A(0)(-3) + B(1)(1 - 4) + C(1)(0)

$\therefore$ 3 = - 3B

$\therefore$ B = - 1

Putting x = 4 in (i), we get

2(4) + 1 = A(3)(0) + B(4)(0) + C(4)(4 - 1)

$\therefore$ 9 = C(4)(3)

$\therefore$ C = $\frac{3}{4}$

$$\therefore \frac{2x+1}{x(x-1)(x-4)} = \frac{\frac{1}{4}}{x} + \frac{-1}{x-1} + \frac{\frac{3}{4}}{x-4}$$

$$\therefore I = \int\left(\frac{\frac{1}{4}}{x} + \frac{-1}{x-1} + \frac{\frac{3}{4}}{x-4}\right) dx$$

$$= \frac{1}{4}\int \frac{1}{x} dx - \int \frac{1}{x-1} dx + \frac{3}{4}\int \frac{1}{x-4} dx$$

$$\therefore I = \frac{1}{4}\log|x| - \log|x-1| + \frac{3}{4}\log|x-4| + c$$

Exercise 5.6 | Q 3 | Page 135

Evaluate: $\int \frac{x^2+x-1}{x^2+x-6} dx$

Solution:

Let I = $\int \frac{x^2+x-1}{x^2+x-6} dx$

$$= \int \frac{(x^2+x-6)+5}{x^2+x-6} dx$$

$$= \int\left[\frac{x^2+x-6}{x^2+x-6} + \frac{5}{x^2+x-6}\right] dx$$

$$= \int\left[1 + \frac{5}{x^2+x-6}\right] dx$$

$$\int\left[1 + \frac{5}{(x+3)(x-2)}\right] dx$$

Let $\frac{5}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$

$\therefore$ 5 = A(x - 2) + B(x + 2) ....(i)

Putting x = 2 in (i), we get

5 = A (0) + B (5)

∴ 5 = 5B

∴ B = 1

Putting x = -3 in (i), we get

5 = A(- 5) + B (0)

∴ 6 = - 5A

∴ A = - 1

$$\therefore \frac{5}{(x+3)(x-2)} = \frac{-1}{x+3} + \frac{1}{x-2}$$

$$\therefore I = \int \left[1 + \frac{-1}{x+3} + \frac{1}{x-2}\right] dx$$

$$= \int dx - \int \frac{1}{x+3} dx + \int \frac{1}{x-2} dx$$

∴ I = x - log |x + 3| + log |x - 2| + c

Exercise 5.6 | Q 4 | Page 135

Evaluate: $\int \frac{x}{(x-1)^2(x+2)} dx$

Solution:

Let $I = \int \frac{x}{(x-1)^2(x+2)} dx$

Let $\frac{x}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$

∴ $x = A (x - 1) (x + 2) + B (x + 2) + C (x - 1)^2$ ....(i)

Putting x = 1 in (i), we get

$1 = A (0) (3) + B (3) + C (0)^2$

∴ 1 = 3B

∴ B = 1/3

Putting x = -2 in (i), we get

- 2 = A(- 3) (0) + B (0) + C (9)

∴ - 2 = 9C

∴ C = - 2/9

Putting x = - 1 in (i), we get

- 1 = A(- 2) (1) + B (1) + C (4)

$$\therefore -1 = -2A + \frac{1}{3} - \frac{8}{9}$$

$$\therefore -1 = -2A - \frac{5}{9}$$

$$\therefore 2A = -\frac{5}{9} + 1 = \frac{4}{9}$$

$$\therefore A = \frac{2}{9}$$

$$\therefore \frac{x}{(x-1)^2(x+2)} = \frac{\frac{2}{9}}{x-1} + \frac{\frac{1}{3}}{(x-1)^2} + \frac{-\frac{2}{9}}{x+2}$$

$$\therefore I = \int \left[\frac{\frac{2}{9}}{x-1} + \frac{\frac{1}{3}}{(x-1)^2} + \frac{-\frac{2}{9}}{x+2}\right] dx$$

$$= \frac{2}{9}\int \frac{1}{x-1}dx + \frac{1}{3}\int (x-1)^{-2}dx - \frac{2}{9}\int \frac{1}{x+2}dx$$

$$= \frac{2}{9}\log|x-1| + \frac{1}{3}\cdot\frac{(x-1)^{-1}}{-1} - \frac{2}{9}\log|x+2| + c$$

$$= \frac{2}{9}\log|x-1| - \frac{2}{9}\log|x+2| - \frac{1}{3}\times\frac{1}{x-1} + c$$

$$\therefore I = \frac{2}{9}\log\left|\frac{x-1}{x+2}\right| - \frac{1}{3(x-1)} + c$$

Exercise 5.6 | Q 5 | Page 135

Evaluate: $\int \frac{3x - 2}{(x+1)^2(x+3)}\, dx$

Solution:

Let I = $\int \frac{3x - 2}{(x+1)^2(x+3)}\, dx$

Let $\frac{3x - 2}{(x+1)^2(x+3)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+3}$

$\therefore$ 3x - 2 = (x + 3) [A(x + 1) + B] + C$(x + 1)^2$ ....(i)

Putting x = - 1 in (i), we get

3(- 1) - 2 = (–1 + 3)[A(0) + B] + C(0)

$\therefore$ - 5 = 2B

$\therefore$ B = $-\frac{5}{2}$

Putting x = - 3 in (i), we get

3(- 3)-2 = 0[A(–3 + 1) + B] + C$(–2)^2$

$\therefore$ - 11 = 4C

$\therefore$ C = $-\frac{11}{4}$

Putting x = 0 in (i), we get

3(0)- 2 = 3[A(0 + 1) + B] + C$(0 + 1)^2$

$\therefore$ - 2 = 3A + 3B + C

$$\therefore -2 = 3A + 3\left(-\frac{5}{2}\right) - \frac{11}{4}$$

$$\therefore 3A = -2 + \frac{15}{2} + \frac{11}{4} = \frac{-8+30\ 11}{4} = \frac{33}{4}$$

$$\therefore A = \frac{33}{4} \times \frac{1}{3} = \frac{11}{4}$$

$$\therefore \frac{3x - 2}{(x+1)^2(x+3)} = \frac{\frac{11}{4}}{x+1} + \frac{-\frac{5}{2}}{(x+1)^2} + \frac{-\frac{11}{4}}{x+3}$$

$$\therefore I = \int\left(\frac{\frac{11}{4}}{x+1} - \frac{\frac{5}{2}}{(x+1)^2} - \frac{\frac{11}{4}}{x+3}\right) dx$$

$$= \frac{11}{4}\int\frac{dx}{x+1} - \frac{5}{2}\int(x+1)^{-2}dx - \frac{11}{4}\int\frac{dx}{x+3}$$

$$= \frac{11}{4}\log|x+1| - \frac{5}{2}\left(-\frac{1}{x+1}\right) - \frac{11}{4}\log|x+3| + c$$

$$= \frac{11}{4}[\log|x+1| - \log|x+3|] + \frac{5}{2(x+1)} + c$$

$$\therefore I = \frac{11}{4}\log\left|\frac{x+1}{x+3}\right| + \frac{5}{2(x+1)} + c$$

Exercise 5.6 | Q 6 | Page 135

Evaluate: $\int \frac{1}{x(x^5+1)}\ dx$

Solution:

Let I = $\int \frac{1}{x(x^5 + 1)}$ dx

∴ I = $\int \frac{x^4}{x^5(x^5 + 1)}$ dx

Put x^5 = t

$\therefore 5x^4 \, dx = dt$

$\therefore x^4 \, dx = \frac{dt}{5}$

∴ I = $\int \frac{1}{t(t + 1)} \cdot \frac{dt}{5}$

Let $\frac{1}{t(t + 1)} = \frac{A}{t} + \frac{B}{t + 1}$

∴ 1 = A(t + 1) + Bt ....(i)

Putting t = –1 in (i), we get

1 = A(0) + B(- 1)

∴ 1 = - B

∴ B = - 1

Putting t = 0 in (i), we get

1 = A(1) + B(0)

∴ A = 1

$\therefore \frac{1}{t(t + 1)} = \frac{1}{t} + \frac{-1}{t + 1}$

∴ I = $\frac{1}{5} \int \left(\frac{1}{t} + \frac{-1}{t + 1}\right)$ dt

$$= \frac{1}{5}\left[\int \frac{1}{t}dt - \int \frac{1}{t+1}dt\right]$$

$$= \frac{1}{5}[\log|t| - \log|t+1|] + c$$

$$= \frac{1}{5}\log\left|\frac{t}{t+1}\right| + c$$

$$\therefore I = \frac{1}{5}\log\left|\frac{x^5}{x^5+1}\right| + c$$

Exercise 5.6 | Q 7 | Page 135

Evaluate: $\int \frac{1}{x(x^n + 1)}\, dx$

Solution:

Let $I = \int \frac{1}{x(x^n+1)}\, dx$

$$\therefore I = \int \frac{x^{n-1}}{x^{n-1} \times x(x^n+1)}\, dx$$

$$\therefore I = \int \frac{x^{n-1}}{x^n(x^n+1)}\, dx$$

Put $x^n = t$

$$\therefore nx^{n-1}\, dx = dt$$

$$\therefore x^{n-1}\, dx = \frac{dt}{n}$$

$$\therefore I = \int \frac{1}{t(t+1)} \cdot \frac{dt}{n}$$

Let $\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1}$

$\therefore$ 1 = A(t + 1) + Bt ....(i)

Putting t = –1 in (i), we get

1 = A(0) + B(- 1)

$\therefore$ 1 = - B

$\therefore$ B = - 1

Putting t = 0 in (i), we get

1 = A(1) + B(0)

$\therefore$ A = 1

$$\therefore \frac{1}{t(t+1)} = \frac{1}{t} + \frac{-1}{t+1}$$

$$\therefore I = \frac{1}{n}\int\left(\frac{1}{t} + \frac{-1}{t+1}\right) dt$$

$$= \frac{1}{n}\left[\int \frac{1}{t}dt - \int \frac{1}{t+1}dt\right]$$

$$= \frac{1}{n}[\log|t| - \log|t+1|] + c$$

$$= \frac{1}{n}\log\left|\frac{t}{t+1}\right| + c$$

$$\therefore I = \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + c$$

Exercise 5.6 | Q 8 | Page 135

Evaluate: $\int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x}\, dx$

Solution:

Let I = $\int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x}\, dx$

$$= \int \frac{5x^2 + 20x + 6}{x(x^2 + 2x + 1)}\, dx$$

$$= \int \frac{5x^2 + 20x + 6}{x(x + 1)^2}\, dx$$

Let $\frac{5x^2 + 20x + 6}{x(x + 1)^2} = \frac{A}{x} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2}$

∴ $5x^2 + 20x + 6 = A(x + 1)^2 + B(x + 1)x + Cx$ ...(i)

Putting x = 0 in (i), we get

$5(0) + 20(0) + 6 = A(1)^2 + B(1)(0) + C(0)$

∴ A = 6

Putting x = - 1 in (i), we get

5 (1) + 20(- 1) + 6 = A (0)+ B (0) (- 1) + C (-1)

∴ - 9 = - C

∴ C = 9

Putting x = 1 in (i), we get

$5 (1) + 20 (1) + 6 = A (2)^2 + B (2) (1) + C (1)$

∴ 31 = 4A + 2B + C

∴ 31 = 4(6) + 2B + 9

∴ B = - 1

$$\therefore \frac{5x^2 + 20x + 6}{x(x + 1)^2} = \frac{6}{x} + \frac{-1}{x + 1} + \frac{9}{(x + 1)^2}$$

$$\therefore I = \int \left[\frac{6}{x} + \frac{-1}{x + 1} + \frac{9}{(x + 1)^2}\right] dx$$

$$= 6\int \frac{1}{x}dx - \int \frac{1}{x+1}dx + 9\int (x+1)^{-2}\,dx$$

$$= 6\log|x| - \log|x+1| + 9\frac{(x+1)^{-1}}{-1} + c$$

$$\therefore I = 6\log|x| - \log|x+1| - \frac{9}{x+1} + c$$

MISCELLANEOUS EXERCISE 5 [PAGES 137 - 139]

Miscellaneous Exercise 5 | Q 1.01 | Page 137

Choose the correct alternative from the following.

The value of $\int \frac{dx}{\sqrt{1 - x}}$ is

Options

$2\sqrt{1-x} + c$

$- 2\sqrt{1-x} + c$

$\sqrt{x} + c$

$x + c$

Solution:

$- 2\sqrt{1-x} + c$

Miscellaneous Exercise 5 | Q 1.02 | Page 137

Choose the correct alternative from the following.

$\int \sqrt{1-x^2}dx =$

Options

$$\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left(x+\sqrt{1+x^2}\right)+c$$

$$\frac{2}{3}\left(1+x^2\right)^{\frac{3}{2}}+c$$

$$\frac{1}{3}\left(1+x^2\right)+c$$

$$\frac{x}{\sqrt{1+x^2}}+c$$

Solution:

$$\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left(x+\sqrt{1+x^2}\right)+c$$

Miscellaneous Exercise 5 | Q 1.03 | Page 137

Choose the correct alternative from the following.

$$\int x^2(3)^{x^3}dx =$$

Options

$$(3)^{x^3}+c$$

$$\frac{(3)^{x^3}}{3\cdot\log 3}+c$$

$$\log 3(3)^{x^3}+c$$

$$x^2(3)^{x^3}+c$$

Solution:

$$\frac{(3)^{x^3}}{3\cdot\log 3}+c$$

Explanation:

Let I = $\int x^2 \cdot (3)^{x^3} dx$

Put $x^3 = t$

$\therefore 3x^2 dx = dt$

$\therefore x^2 dx = \frac{1}{3} dt$

$\therefore I = \frac{1}{3} \int 3^t \cdot dt$

$= \frac{1}{3} \cdot \frac{3^t}{\log 3} + c$

$= \frac{(3)^{x^3}}{3 \log 3} + c$

Miscellaneous Exercise 5 | Q 1.04 | Page 137

Choose the correct alternative from the following.

$\int \frac{x+2}{2x^2+6x+5} dx = p \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{2} \int \frac{dx}{2x^2+6x+5}$, then p = ?

1. 1/3
2. 1/2
3. **1/4**
4. 2

Solution: 1/4

Explanation:

Let x + 2 = p $\frac{d}{dx}\left(2x^2 + 6x + 5\right) + q$

= p(4x + 6) + q

∴ x + 2 = 4px + 6p + q

∴ 4p = 1 and 6p + q = 2

∴ p = 1/4

Miscellaneous Exercise 5 | Q 1.05 | Page 137

Choose the correct alternative from the following.

$$\int \frac{dx}{(x - x^2)} =$$

1. **log x – log (1 – x) + c**
2. log (1 - x^2) + c
3. - log x + log(1 - x) + c
4. log (x - x^2) + c

Solution: log x – log (1 – x) + c

Explanation:

$$\text{Let I} = \int \frac{dx}{(x - x^2)}$$

$$= \int \frac{1}{x(1 - x)} \, dx$$

$$= \int \frac{(1 - x) + x}{x(1 - x)} \, dx$$

$$= \int \left(\frac{1}{x} + \frac{1}{1 - x} \right) dx$$

$$= \log|x| + \frac{\log|1 - x|}{-1} + c$$

$$= \log|x| - \log|1 - x| + c$$

Miscellaneous Exercise 5 | Q 1.06 | Page 137

Choose the correct alternative from the following.

$$\int \frac{dx}{(x - 8)(x + 7)} =$$

Options

$$\frac{1}{15}\log\left|\frac{x+2}{x-1}\right| + c$$

$$\frac{1}{15}\log\left|\frac{x+8}{x+7}\right| + c$$

$$\frac{1}{15}\log\left|\frac{x-8}{x+7}\right| + c$$

(x - 8)(x - 7) + c

Solution:

$$\frac{1}{15}\log\left|\frac{x-8}{x+7}\right| + c$$

Explanation:

Let I = $\int \frac{dx}{(x-8)(x+7)}$

$$= \frac{1}{15}\int \frac{15 \cdot dx}{(x-8)(x+7)}$$

$$= \frac{1}{15}\int \frac{(x+7)-(x-8)}{(x-8)(x+7)}\ dx$$

$$= \frac{1}{15}\left(\int \frac{1}{x-8} - \int \frac{1}{x+7}\right) dx$$

$$= \frac{1}{15}[\log|x-8| - \log|x+7|] + c$$

$$= \frac{1}{15}\log\left|\frac{x-8}{x+7}\right| + c$$

Miscellaneous Exercise 5 | Q 1.07 | Page 137

Choose the correct alternative from the following.

$\int \left(x + \frac{1}{x}\right)^3 dx$ =

Options

$\frac{1}{4}\left(x + \frac{1}{x}\right)^4$ + c

$\frac{x^4}{4} + \frac{3x^2}{2} + 3\log x - \frac{1}{2x^2} + c$

$\frac{x^4}{4} + \frac{3x^2}{2} + 3\log x + \frac{1}{x^2} + c$

$\left(x - x^{-1}\right)^3$ + c

Solution:

$\frac{x^4}{4} + \frac{3x^2}{2} + 3\log x - \frac{1}{2x^2} + c$

Explanation:

Let I = $\int \left(x + \frac{1}{x}\right)^3 dx$

$\int \left(x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}\right)$ dx

$= \frac{x^4}{4} + 3\frac{x^2}{2} + 3\log|x| - \frac{1}{2x^2}$ + c

Miscellaneous Exercise 5 | Q 1.08 | Page 137

Choose the correct alternative from the following.

$$\int\left(\frac{e^{2x} + e^{-2x}}{e^{x}}\right)dx =$$

Options

$e^{x} - \frac{1}{3e^{3x}}$ **+ c**

$e^{x} + \frac{1}{3e^{3x}}$ + c

$e^{-x} + \frac{1}{3e^{3x}}$ + c

$e^{-x} + \frac{1}{3e^{3x}} + c$

Solution:

$e^{x} - \frac{1}{3e^{3x}}$ + c

Explanation:

$$\int\left(\frac{e^{2x} + e^{-2x}}{e^{x}}\right)dx = \int\left(e^{x} + e^{-3x}\right) dx$$

$$= e^{x} - \frac{1}{3}e^{-3x} + c$$

Miscellaneous Exercise 5 | Q 1.09 | Page 137

Choose the correct alternative from the following.

$$\int (1 - x)^{-2} dx =$$

1. $(1+x)^{-1}$ + c
2. **$(1-x)^{-1}$ + c**

3. $(1-x)^{-1}-1 + c$
4. $(1-x)^{-1}+1 + c$

Solution:

$(1-x)^{-1}$ + c

Explanation:

$$\int (1-x)^{-2}dx = \frac{(1-x)^{-1}}{-1 \times -1} + c$$

$$= (1-x)^{-1} + c$$

Miscellaneous Exercise 5 | Q 1.1 | Page 138

Choose the correct alternative from the following.

$$\int \frac{\left(x^3+3x^2+3x+1\right)}{(x+1)^5}\,dx =$$

Options

$\frac{-1}{x+1}$ **+ c**

$\left(\frac{-1}{x+1}\right)^5$ + c

log(x + 1) + c

log $|x + 1|^5$ + c

Solution:

$\frac{-1}{x+1}$ + c

Explanation:

$$\int \frac{\left(x^3+3x^2+3x+1\right)}{(x+1)^5}\,dx = \int \frac{(x+1)^3}{(x+1)^5}\,dx$$

$$= \int \frac{1}{(x+1)} dx$$

$$= \frac{-1}{x+1} + c$$

Miscellaneous Exercise 5 | Q 2.1 | Page 138

Fill in the Blank.

$$\int \frac{5\left(x^6+1\right)}{x^2+1} dx = x^5 + \_\_\_\_\_ x^3 + 5x + c$$

Solution:

$$\int \frac{5\left(x^6+1\right)}{x^2+1} dx = x^5 + \underline{\frac{-5}{3}} x^3 + 5x + c$$

Explanation:

$$\int \frac{5\left(x^6+1\right)}{x^2+1} dx = 5 \int \frac{\left(x^2+1\right)\left(x^4-x^2+1\right)}{x^2+1} dx$$

$$= 5 \int \left(x^4 - x^2 + 1\right) dx$$

$$= 5\left(\frac{x^5}{5} - \frac{x^3}{3} + x\right) + c$$

$$= x^2 - \frac{5}{3}x^3 + 5x + c$$

Miscellaneous Exercise 5 | Q 2.2 | Page 138

Fill in the Blank.

$$\int \frac{x^2+x-6}{(x-2)(x-1)} dx = x + \_\_\_\_\_ + c$$

Solution:

$$\int \frac{x^2+x-6}{(x-2)(x-1)} dx = x + \underline{4 \log |x-1|} + c$$

Explanation:

$$\int \frac{x^2 + x - 6}{(x - 2)(x - 1)} dx = \int \frac{(x + 3)(x - 2)}{(x - 2)(x - 1)} dx$$

$$= \int \frac{x + 3}{x - 1} dx$$

$$= \int \frac{(x - 1) + 4}{x - 1} dx$$

$$= \int \left(1 + \frac{4}{x - 1}\right) dx$$

$= x + 4 \log |x - 1| + c$

Miscellaneous Exercise 5 | Q 2.3 | Page 138

Fill in the Blank.

If $f'(x) = \frac{1}{x} + x$ and $f(1) = \frac{5}{2}$, then $f(x) = \log x + \frac{x^2}{2}$ + ______

Solution:

If $f'(x) = \frac{1}{x} + x$ and $f(1) = \frac{5}{2}$, then $f(x) = \log x + \frac{x^2}{2}$ + **2**

Explanation:

$f(x) = \int f'(x)\, dx$

$$= \int \left(\frac{1}{x} + x\right) dx$$

$f(x) = \log |x| + \frac{x^2}{2} + c$ ...(i)

$f(1) = \frac{5}{2}$

$f(1) = \log 1 + \frac{1^2}{2} + c$

$\therefore \frac{5}{2} = 0 + \frac{1}{2} + c$

$\therefore c = 2$

$\therefore f(x) = \log |x| + \frac{x^2}{2} + 2$

Miscellaneous Exercise 5 | Q 2.4 | Page 138

Fill in the Blank.

To find the value of $\int \frac{(1 + \log x)dx}{x}$ the proper substitution is _______

Solution:

To find the value of $\int \frac{(1 + \log x)dx}{x}$ the proper substitution is **1 + log x = t.**

Miscellaneous Exercise 5 | Q 2.5 | Page 138

Fill in the Blank.

$\int \frac{1}{x^3}[\log x^x]^2 dx = P(\log x)^3$ + c, then P = ______

Solution:

$\int \frac{1}{x^3}[\log x^x]^2 dx = P(\log x)^3$ + c, then P = $\frac{1}{3}$

Explanation:

Let I = $\int \frac{1}{x^3}[\log x^x]^2 dx = P \cdot (\log x)^3 + c$

I = $\int \frac{1}{x^3}[\log x^x]^2 dx = \int \frac{1}{x^3}(x \log x)^2 \cdot dx$

$$= \int \frac{1}{x^3} \cdot x^2 \cdot (\log x)^2 dx = \int \frac{1}{x} (\log x)^2 \cdot dx$$

∴ Put log x = t

$$\therefore \frac{1}{x} dx = dt$$

$$\therefore I = \int t^2 \cdot dt$$

$$= \frac{t^3}{3} + c$$

$$= \frac{1}{3} (\log x)^3 + c$$

$$\therefore P = \frac{1}{3}$$

Miscellaneous Exercise 5 | Q 3.1 | Page 138

State whether the following statement is True or False.

The proper substitution for $\int x (x^x)^x (2 \log x + 1) dx$ is $(x^x)^x = t$

1. **True**
2. False

Solution: True

Explanation:

Let **I** = $\int x (x^x)^x (2 \log x + 1) dx$

Put $(x^x)^x = t$

Taking logarithm of both sides, we get

$\log (x^x)^x = \log t$

$$\therefore x^2 \cdot \log x = \log t$$

Differentiating w.r.t. x, we get

$$x^2 \cdot \frac{1}{x} + (\log x) \cdot 2x = \frac{1}{t} \cdot \frac{dt}{dx}$$

$$\therefore (x + 2x \log x)dx = \frac{1}{t} \cdot dt$$

$\therefore$ x(1 + 2 log x) dx = $\frac{1}{t} \cdot dt$

$\therefore$ I = $\int t \cdot \frac{1}{t} \cdot dt = \int 1 \cdot dt = t + c = (x^x)^x$ + c

Miscellaneous Exercise 5 | Q 3.2 | Page 138

State whether the following statement is True or False.

If $\int x\, e^{2x}$ dx is equal to e^{2x} f(x) + c, where c is constant of integration, then f(x) is $\frac{2x-1}{2}$.

1. True
2. **False**

Solution: False

Explanation:

Let I = $\int x \cdot e^{2x}$ dx

$= x \int e^{2x} \cdot dx - \int \left[\frac{d}{dx}(x) \int e^{2x} \cdot dx\right]$ dx

$$= x \cdot \frac{e^{2x}}{2} - \int 1 \cdot \frac{e^{2x}}{2} \cdot dx$$

$= \frac{x}{2}e^{2x} - \frac{1}{2} \int e^{2x} +$ c

$= \frac{x}{2}e^{2x} - \frac{1}{2} \cdot \frac{e^{2x}}{2}$ + c

$$= e^{2x}\left(\frac{x}{2} - \frac{1}{4}\right) + c$$

$$= e^{2x}\left(\frac{2x - 1}{4}\right) + c$$

$$\therefore f(x) = \frac{2x - 1}{4}$$

Miscellaneous Exercise 5 | Q 3.3 | Page 138

State whether the following statement is True or False.

If $\int x\, f(x)\, dx = \frac{f(x)}{2}$, then find $f(x) = e^{x^2}$

1. **True**
2. False

Solution: True

Explanation:

If $f(x) = e^{x^2}$, then

$$\int x \cdot f(x) dx = \int x \cdot e^{x^2} \cdot dx$$

Put $x^2 = t$

$\therefore 2x\ dx = dt$

$\therefore x\ dx = \frac{1}{2}\, dt$

$$\therefore \int x \cdot f(x) dx = \frac{1}{2} \int e^t \cdot dt$$

$$= \frac{1}{2} e^t + c$$

$$= \frac{1}{2} e^{x^2} + c$$

$$= \frac{1}{2} f(x) + c$$

Miscellaneous Exercise 5 | Q 3.4 | Page 138

State whether the following statement is True or False.

If $\int \frac{(x - 1)dx}{(x+1)(x-2)}$ = A log |x + 1| + B log |x - 2| + c, then A + B = 1.

1. **True**
2. False

Solution: True

Explanation:

Let I = $\frac{(x - 1)}{(x+1)(x-2)}$ dx

Let $\frac{(x - 1)}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$

∴ x - 1 = A(x - 2) + B(x + 1) ....(i)

Putting x = –1 in (i), we get

- 1 - 1 = A(- 1 - 2)

∴ - 2 = - 3A

∴ A = $\frac{2}{3}$

Putting x = 2 in (i), we get

2 - 1 = B(2 + 1)

∴ 1 = 3B

∴ I = $\int \left(\frac{\frac{2}{3}}{x+1} + \frac{\frac{1}{3}}{x-2}\right)$ dx

$= \frac{2}{3}\int \frac{1}{x+1}dx + \frac{1}{3}\int \frac{1}{x-2}$ dx

Comparing the above with

A log |x + 1| + B log |x - 2| + c, we get

$\therefore$ A = $\frac{2}{3}, B = \frac{1}{3}$

$\therefore$ A + B = $\frac{2}{3} + \frac{1}{3} = 1$

Miscellaneous Exercise 5 | Q 3.5 | Page 138

State whether the following statement is True or False.

For $\int \frac{x - 1}{(x+1)^3} e^x dx = e^x$ f(x) + c, f(x) = $(x + 1)^2$.

1. True
2. **False**

Solution: False

Explanation:

Let I = $\frac{(x - 1)}{(x+1)^3} \cdot e^x$ dx

$= \int e^x \left[\frac{(x+1) - 2}{(x+1)^3}\right]$ dx

$= \int e^x \left[\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}\right]$ dx

$= \int e^x \left[(x+1)^{-2} - 2(x+1)^{-3}\right]$ dx

Put f(x) = $(x + 1)^{-2}$

$\therefore$ f '(x) = - 2 $(x + 1)^{-3}$

$\therefore$ I = $e^x[f(x) + f'(x)]$ dx

$= e^x \cdot f(x)$ + c

$= e^x \cdot (x + 1)^{-2}$ + c

$\therefore$ f(x) = $(x + 1)^{-2}$

Miscellaneous Exercise 5 | Q 4.1 | Page 138

Evaluate $\int \frac{5x^2 - 6x + 3}{2x - 3}$ dx

Solution:

Let I = $\int \frac{5x^2 - 6x + 3}{2x - 3}$ dx

We perform actual division and express the result as:

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$\frac{5}{2}x + \frac{3}{4}$$

$$2x - 2 \overline{)\, 5x^2 - 6x + 3}$$

$$5x^2 - \frac{15}{2}x$$

$$(-) \quad (\pm)$$

$$\frac{3x}{2} + 3$$

$$\frac{3x}{2} - \frac{9}{4}$$

$$(-) \quad (\pm)$$

$$\frac{21}{4}$$

$\therefore$ I = $\int\left(\frac{5}{2}x + \frac{3}{4} + \frac{\frac{21}{4}}{2x - 2}\right)$ dx

$= \frac{5}{2}\int x\,dx + \frac{3}{4}\int dx + \frac{21}{4}\int\frac{1}{2x-3}$ dx

$= \frac{5}{2}\cdot\frac{x^2}{2} + \frac{3}{4}x + \frac{21}{4}\cdot\frac{\log|2x-3|}{2}$ + c

$\therefore$ I = $\frac{5x^2}{4} + \frac{3}{4}x + \frac{21\log|2x-3|}{8}$ + c

Miscellaneous Exercise 5 | Q 4.1 | Page 138

Evaluate $\int(5x+1)^{\frac{4}{9}}$ dx

Solution:

Let I = $\int(5x+1)^{\frac{4}{9}}$ dx

$= \frac{(5x+1)^{\frac{4}{9}+1}}{\left(\frac{4}{9}+1\right)\times 5}$ + c

$= \frac{(5x+1)^{\frac{13}{9}}}{\frac{13}{9}\times 5}$ + c

$\therefore$ I = $\frac{9}{65}(5x+1)^{\frac{13}{9}}$ + c

Miscellaneous Exercise 5 | Q 4.1 | Page 138

Evaluate $\int\frac{1}{(2x+3)}$ dx

Solution:

Let I = $\int \frac{1}{2x + 3}$ dx

∴ I = $\frac{\log|2x + 3|}{2}$ + c

Miscellaneous Exercise 5 | Q 4.1 | Page 138

Evaluate $\int \frac{x - 1}{\sqrt{x + 4}}$ dx

Solution:

Let I = $\int \frac{x - 1}{\sqrt{x + 4}}$ dx

= $\int \frac{(x + 4) - 5}{\sqrt{x + 4}}$ dx

= $\int \left(\sqrt{x + 4} - \frac{5}{\sqrt{x + 4}}\right)$ dx

$= \int \left[(x + 4)^{\frac{1}{2}} - 5(x + 4)^{-\frac{1}{2}}\right]$ dx

$= \frac{(x + 4)^{\frac{3}{2}}}{\frac{3}{2}} - 5\frac{(x + 4)^{\frac{1}{2}}}{\frac{1}{2}}$ + c

∴ I = $\frac{2}{3}(x + 4)^{\frac{3}{2}} - 10\sqrt{x + 4}$ + c

Miscellaneous Exercise 5 | Q 4.1 | Page 138

Evaluate: If f '(x) = $\sqrt{x}$ and f(1) = 2, then find the value of f(x).

Solution:

f '(x) = $\sqrt{x}$ ....[Given]

f(x) = ∫ f '(x)

$$= \int \sqrt{x}\, dx$$

$$= \int x^{\frac{1}{2}}\, dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

∴ f(x) = $\frac{2}{3}x^{\frac{3}{2}} + c$ ...(i)

Now, f(1) = 2 ....[Given]

$$\therefore \frac{2}{3}(1)^{\frac{3}{2}} + c = 2$$

∴ c = $2 - \frac{2}{3}$

∴ c = $\frac{4}{3}$

∴ f(x) = $\frac{2}{3}x^{\frac{3}{2}} + \frac{4}{3}$

Miscellaneous Exercise 5 | Q 4.1 | Page 138

Evaluate: ∫ |x| dx if x < 0

Solution: |x| = x; x ≥ 0

= x; x < 0

Let **I** = ∫ |x| dx, if x < 0

= ∫ - x dx

∴ I = $\frac{-x^2}{2}$ + c

Miscellaneous Exercise 5 | Q 4.2 | Page 138

Evaluate: Find the primitive of $\frac{1}{1+e^x}$

Solution:

Let I = $\int \frac{1}{1+e^x} dx$

Dividing Nr. and Dr. by e^x, we get

I = $\int \frac{e^{-x}}{e^{-x}+1} dx$

Put $e^{-x}+1$ = t

$\therefore -e^{-x} dx = dt$

$\therefore e^{-x} dx = -dt$

$\therefore$ I = $\int \frac{-dt}{t} = -\log|t| + c$

$\therefore$ I = - log $|e^{-x}+1|$ + c

Miscellaneous Exercise 5 | Q 4.2 | Page 138

Evaluate: $\int \frac{ae^x + be^{-x}}{(ae^x - be^{-x})} dx$

Solution:

Let I = $\int \frac{ae^x + be^{-x}}{(ae^x - be^{-x})} dx$

Put ae^x - be^{-x} = t

$\therefore (ae^x + be^{-x}) dx = dt$

$\therefore$ I = $\int \frac{dt}{t} = \log|t|$ + c

$\therefore$ I = log $\left|ae^{x} + be^{-x}\right|$ + c

Miscellaneous Exercise 5 | Q 4.2 | Page 138

Evaluate: $\int \frac{1}{2x + 3x \log x}$ dx

Solution:

Let I = $\int \frac{1}{2x + 3x \cdot \log x}$ dx

$= \int \frac{1}{x(2 + 3 \log x)}$ dx

Put 2 + 3 log x = t

$\therefore 3 \cdot \frac{1}{x} dx$ = dt

$\therefore \frac{1}{x} dx = \frac{1}{3} dt$

$\therefore$ I = $\frac{1}{3} \int \frac{1}{t} \cdot dt$

$= \frac{1}{3}$ log |t| + c

$\therefore$ I = $\frac{1}{3}$ log |2 + 3 log x| + c

Miscellaneous Exercise 5 | Q 4.2 | Page 138

Evaluate: $\int \frac{1}{\sqrt{x} + x}$ dx

Solution:

Let I = $\int \frac{1}{\sqrt{x} + x}$ dx

$$= \int \frac{1}{\sqrt{x}(1+\sqrt{x})}\ dx$$

Put $1 + \sqrt{x}$ = t

$$\therefore \frac{1}{2\sqrt{x}} dx = dt$$

$$\therefore \frac{1}{\sqrt{x}} dx = 2\ dt$$

$$\therefore I = \int \frac{2 \cdot dt}{t}$$

$$= 2\int \frac{1}{t}\ dt$$

= 2 log |t| + c

$$\therefore I = 2 \log \left|1+\sqrt{x}\right| + c$$

Miscellaneous Exercise 5 | Q 4.2 | Page 138

Evaluate: $\int \frac{2e^x - 3}{4e^x + 1}\ dx$

Solution:

Let I = $\int \frac{2e^x - 3}{4e^x + 1}\ dx$

Let $2e^x - 3 = A(4e^x + 1) + B\frac{d}{dx}(4e^x + 1)$

$$\therefore 2e^x - 3 = (4A + 4B)e^x + A$$

Comparing the coefficients of e^x and constant term on both sides, we get

4A + 4B = 2 and A = - 3

Solving these equations, we get

$B = \frac{7}{2}$

$\therefore I = \frac{-3(4e^x + 1) + \frac{7}{2}(4e^x)}{4e^x + 1}\ dx$

$= -3\int dx + \frac{7}{2}\int \frac{4e^x}{4e^x + 1} dx$

$\therefore I = -3x + \frac{7}{2}\log|4e^x + 1| + c \quad ...\left[\because \int \frac{f\prime(x)}{f(x)} dx = \log|f(x)| + c\right]$

Miscellaneous Exercise 5 | Q 4.3 | Page 138

Evaluate: $\int \frac{dx}{\sqrt{4x^2 - 5}}$

Solution:

Let $I = \int \frac{dx}{\sqrt{4x^2 - 5}}$

$= \int \frac{1}{\sqrt{4\left(x^2 - \frac{5}{4}\right)}} dx$

$= \frac{1}{2}\int \frac{1}{\sqrt{x^2 - \left(\frac{\sqrt{5}}{2}\right)^2}}\ dx$

$= \frac{1}{2}\log\left|x + \sqrt{x^2 - \left(\frac{\sqrt{5}}{2}\right)^2}\right| + c$

$\therefore I = \frac{1}{2}\log\left|x + \sqrt{x^2 - \frac{5}{4}}\right| + c$

Miscellaneous Exercise 5 | Q 4.3 | Page 138

Evaluate: $\int \frac{dx}{3 - 2x - x^2}$

Solution:

Let I = $\int \frac{dx}{3 - 2x - x^2}$

$3 - 2x - x^2 = -x^2 - 2x + 3$

$= -(x^2 + 2x - 3)$

$= -(x^2 + 2x + 1 - 4)$

$= -[(x + 1)^2 - 4]$

$= (2)^2 - (x + 1)^2$

$\therefore I = \int \frac{dx}{(2)^2 - (x+1)^2}$

$= \frac{1}{2(2)} \log\left|\frac{2 + x + 1}{2 - (x + 1)}\right| + c$

$\therefore I = \frac{1}{4} \log\left|\frac{3 + x}{1 - x}\right| + c$

Miscellaneous Exercise 5 | Q 4.3 | Page 138

Evaluate: $\int \frac{dx}{9x^2 - 25}$

Solution:

Let I = $\int \frac{dx}{9x^2 - 25}$

$= \int \frac{1}{9\left(x^2 - \frac{25}{9}\right)} dx$

$$= \frac{1}{9} \int \frac{1}{x^2 - \left(\frac{5}{3}\right)^2} dx$$

$$= \frac{1}{9} \cdot \frac{1}{2 \cdot \frac{5}{3}} \log \left| \frac{x - \frac{5}{3}}{x + \frac{5}{3}} \right| + c$$

$$\therefore I = \frac{1}{30} \log \left| \frac{3x - 5}{3x + 5} \right| + c$$

Miscellaneous Exercise 5 | Q 4.3 | Page 139

Evaluate: $\int \frac{e^x}{\sqrt{e^{2x} + 4e^x + 13}} dx$

Solution:

Let $I = \int \frac{e^x}{\sqrt{e^{2x} + 4e^x + 13}} dx$

$$= \int \frac{e^x}{\sqrt{(e^x)^2 + 4e^x + 13}} dx$$

Put $e^x = t$

$\therefore e^x \, dx = dt$

$$\therefore I = \frac{dt}{\sqrt{t^2 + 4t + 13}}$$

$$= \int \frac{1}{\sqrt{t^2 + 4t + 4 - 4 + 13}} dt$$

$$= \int \frac{1}{\sqrt{(t + 2)^2 + 9}} dt$$

$$= \int \frac{1}{\sqrt{(t + 2)^2 + (3)^2}} dt$$

$$= \log\left|t + 2 + \sqrt{(t+2)^2 + (3)^2}\right| + c$$

$$= \log\left|t + 2 + \sqrt{t^2 + 4t + 13}\right|$$

$$\therefore I = \log\left|e^x + 2 + \sqrt{e^{2x} + 4e^x + 13}\right|$$

Miscellaneous Exercise 5 | Q 4.3 | Page 139

Evaluate: $\int \frac{dx}{x\left[(\log x)^2 + 4\log x - 1\right]}$

Solution:

Let $I = \int \frac{dx}{x\left[(\log x)^2 + 4\log x - 1\right]}$

Put log x = t

$$\therefore \frac{1}{x}dx = dt$$

$$\therefore I = \int \frac{dt}{t^2 + 4t - 1}$$

$$= \int \frac{1}{t^2 + 4t + 4 - 4 - 1}dt$$

$$= \int \frac{1}{(t+2)^2 - 5}\ dt$$

$$= \int \frac{1}{(t+2)^2 - \left(\sqrt{5}\right)^2}\ dt$$

$$= \frac{1}{2\sqrt{5}}\log\left|\frac{t + 2 - \sqrt{5}}{t + 2 + \sqrt{5}}\right| + c$$

$$\therefore I = \frac{1}{2\sqrt{5}} \log \left| \frac{\log x + 2 - \sqrt{5}}{\log x + 2 + \sqrt{5}} \right| + c$$

Miscellaneous Exercise 5 | Q 4.3 | Page 139

Evaluate: $\int \frac{dx}{5 - 16x^2}$

Solution:

$$\text{Let } I = \int \frac{dx}{5 - 16x^2}$$

$$= \int \frac{1}{16\left(\frac{5}{16} - x^2\right)} dx$$

$$= \frac{1}{16} \int \frac{1}{\left(\frac{\sqrt{5}}{4}\right)^2 - x^2} dx$$

$$= \frac{1}{16} \cdot \frac{1}{2\frac{\sqrt{5}}{4}} \log \left| \frac{\frac{\sqrt{5}}{4} + x}{\frac{\sqrt{5}}{4} - x} \right| + c$$

$$\therefore I = \frac{1}{8\sqrt{5}} \log \left| \frac{\sqrt{5} + 4x}{\sqrt{5} - 4x} \right| + c$$

Miscellaneous Exercise 5 | Q 4.3 | Page 139

Evaluate: $\int \frac{dx}{25x - x(\log x)^2}$

Solution:

$$\text{Let } I = \int \frac{dx}{25x - x(\log x)^2}$$

$$= \int \frac{1}{x\left[25 - (\log x)^2\right]} dx$$

Put log x = t

$\therefore \frac{1}{x}$ dx = dt

$\therefore$ I = $\int \frac{dt}{25 - t^2}$

$= \int \frac{1}{(5)^2 - t^2}$ dt

$= \frac{1}{2(5)} \cdot \log\left|\frac{5 + t}{5 - t}\right|$ + c

$\therefore$ I = $\frac{1}{10}\log\left|\frac{5 + \log x}{5 - \log x}\right|$ + c

Miscellaneous Exercise 5 | Q 4.3 | Page 139

Evaluate: $\int \frac{e^x}{4e^{2x} - 1}$ dx

Solution:

Let I = $\int \frac{e^x}{4e^{2x} - 1}$ dx

$= \int \frac{e^x}{4(e^x)^2 - 1}$ dx

Put e^X = t

$\therefore$ e^X dx = dt

$\therefore$ I = $\int \frac{dt}{4t^2 - 1}$

$= \frac{1}{4} \int \frac{1}{t^2 - \frac{1}{4}}$ dx

$= \frac{1}{4} \int \frac{1}{t^2 - \left(\frac{1}{2}\right)^2}$ dt

$$= \frac{1}{4} \cdot \frac{1}{2\left(\frac{1}{2}\right)} \log \left| \frac{t - \frac{1}{2}}{t + \frac{1}{2}} \right| + c$$

$$= \frac{1}{4} \log \left| \frac{2t - 1}{2t + 1} \right| + c$$

Miscellaneous Exercise 5 | Q 4.4 | Page 139

Evaluate: $\int (\log x)^2 \, dx$

Solution:

Let I = $\int (\log x)^2 \, dx$

$$= \int (\log x)^2 \cdot 1 dx$$

$$= (\log x)^2 \int 1 \cdot dx - \int \left[\frac{d}{dx} (\log x)^2 \int 1 \cdot dx \right] dx$$

$$= x(\log x)^2 \cdot x - \int 2 \log x \cdot \frac{1}{x} \cdot x \cdot dx$$

$$= x(\log x)^2 - 2 \int (\log x) \cdot 1 \cdot dx$$

$$= x(\log x)^2 - 2 \left[\log x \int 1 \cdot dx - \int \left\{ \frac{d}{dx} (\log x) \int 1 \cdot dx \right\} \right] dx$$

$$= x(\log x)^2 - 2 \left[(\log x)x - \int \frac{1}{x} \cdot x \cdot dx \right]$$

$$= x(\log x)^2 - 2 \left[x \log x - \int 1 \cdot dx \right]$$

$$= x(\log x)^2 - 2(x \log x - x) + c$$

∴ I = $x(\log x)^2 - 2x \log x - 2x + c$

Miscellaneous Exercise 5 | Q 4.4 | Page 139

Evaluate: $\int e^x \frac{1+x}{(2+x)^2}\, dx$

Solution:

Let I = $\int e^x \frac{1+x}{(2+x)^2}\, dx$

$= \int e^x \left[\frac{(2+x)-1}{(2+x)^2}\right] dx$

$= \int e^x \left[\frac{1}{2+x} - \frac{1}{(2+x)^2}\right] dx$

Let f(x) = $\frac{1}{2+x}$

∴ f '(x) = $\frac{-1}{(2+x)^2}$

∴ I = $\int e^x [f(x) + f'(x)]\, dx$

$= e^x \cdot f(x) + c$

$= e^x \cdot \frac{1}{2+x}$ + c

∴ I = $\frac{e^x}{2+x}$ + c

Miscellaneous Exercise 5 | Q 4.4 | Page 139

Evaluate: $\int x \cdot e^{2x}\, dx$

Solution:

Let I = $\int x \cdot e^{2x}$ dx

$= x \int e^{2x} dx - \int \left[\frac{d}{dx}(x) \int e^{2x} \cdot dx \right]$ dx

$= x \cdot \frac{e^{2x}}{2} - \int 1 \cdot \frac{e^{2x}}{2}$ dx

$= \frac{1}{2} xe^{2x} - \frac{1}{2} \int e^{2x}$ dx

$= \frac{1}{2} x\, e^{2x} - \frac{1}{2} \cdot \frac{e^{2x}}{2}$ + c

∴ I = $\frac{1}{4} e^{2x}(2x - 1)$ + c

Miscellaneous Exercise 5 | Q 4.4 | Page 139

Evaluate: $\int \log(x^2 + x)$ dx

Solution:

Let I = $\int \log(x^2 + x)$ dx

$= \int \log(x^2 + x) \cdot 1 \cdot dx$

$= \log(x^2 + x) \int 1 \cdot dx - \int \left\{ \frac{d}{dx} \log(x^2 + x) \int 1 \cdot dx \right\}$dx

$= \log(x^2 + x) \cdot x - \int \frac{1}{x^2 + x} \cdot (2x + 1) \cdot x \cdot dx$

$= x \cdot \log(x^2 + x) - \int \frac{1}{x(x + 1)} \cdot (2x + 1) \cdot x \cdot dx$

$= x \cdot \log(x^2 + x) - \int \frac{2x + 1}{x + 1}$dx

$$= x \cdot \log\left(x^2 + x\right) - \int \frac{(2x+2)-1}{x+1}\, dx$$

$$= x \cdot \log\left(x^2 + x\right) - \int \left[\frac{2(x+1)}{x+1} - \frac{1}{x+1}\right] dx$$

$$= x \cdot \log\left(x^2 + x\right) - \int \left[2 - \frac{1}{x+1}\right] dx$$

$$= x \cdot \left[\log\left(x^2 + x\right)\right] - (2x - \log|x+1|) + c$$

$$\therefore I = x \cdot \left[\log\left(x^2 + x\right)\right] - 2x + \log|x+1| + c$$

Miscellaneous Exercise 5 | Q 4.4 | Page 139

Evaluate: $\int e^{\sqrt{x}}\, dx$

Solution:

Let $I = \int e^{\sqrt{x}}\, dx$

Put $\sqrt{x} = t$

$\therefore x = t^2$

$\therefore dx = 2t\ dt$

$\therefore I = \int e^t \cdot 2t dt$

$$= 2\int t \cdot e^t \cdot dt$$

$$= 2\left[t \int e^t dt - \int \left\{\frac{d}{dx}(t) \int e^t \cdot dt\right\} dt\right]$$

$$= 2\left[t \cdot e^t - \int 1 \cdot e^t dt\right]$$

$= 2\left(te^t - e^t\right)$ + c

$= 2e^t(t - 1)$ + c

$\therefore$ I = $2e^{\sqrt{x}}\left(\sqrt{x} - 1\right)$ + c

Miscellaneous Exercise 5 | Q 4.4 | Page 139

Evaluate: $\int \sqrt{x^2 + 2x + 5}\, dx$

Solution:

Let I = $\int \sqrt{x^2 + 2x + 5}\, dx$

$= \int \sqrt{x^2 + 2x + 1 + 4}\, dx$

$= \int \sqrt{(x + 1)^2 + (2)^2}\, dx$

$= \frac{x + 1}{2}\sqrt{(x + 1)^2 + (2)^2} + \frac{(2)^2}{2}\log\left|(x + 1) + \sqrt{(x + 1)^2 + (2)^2}\right|$ + c

$\therefore$ I = $\frac{x + 1}{2}\sqrt{x^2 + 2x + 5} + 2\log\left|(x + 1) + \sqrt{x^2 + 2x + 5}\right|$ + c

Miscellaneous Exercise 5 | Q 4.4 | Page 139

Evaluate: $\int \sqrt{x^2 - 8x + 7}\, dx$

Solution:

Let I = $\int \sqrt{x^2 - 8x + 7}\, dx$

$= \int \sqrt{x^2 - 8x + 16 - 9}\, dx$

$= \int \sqrt{(x + 4)^2 + (3)^2}\, dx$

$$= \frac{x-4}{2}\sqrt{(x-4)^2-(3)^2} - \frac{(3)^2}{2}\log\left|(x - 4) + \sqrt{(x - 4)^2-(3)^2}\right| + c$$

$$\therefore I = \frac{x-4}{2}\sqrt{x^2-8x+7} + \frac{9}{2}\log\left|(x - 4) + \sqrt{x^2-8x+7}\right| + c$$

Miscellaneous Exercise 5 | Q 4.5 | Page 139

Evaluate: $\int \frac{3x-1}{2x^2-x-1}\,dx$

Solution:

Let $I = \int \frac{3x-1}{2x^2-x-1}\,dx$

$$= \int \frac{3x-1}{(x - 1)(2x+1)}\,dx$$

Let $\frac{3x-1}{(x - 1)(2x+1)} = \frac{A}{x - 1} + \frac{B}{2x+1}$

$\therefore 3x - 1 = A(2x + 1) + B(x - 1)$ ...(i)

Putting $x = 1$ in (i), we get

$3(1) - 1 = A(2 + 1) + B(0)$

$\therefore 2 = 3A$

$\therefore A = \frac{2}{3}$

Putting $x = -\frac{1}{2}$ in (i), we get

$$3\left(-\frac{1}{2}\right) - 1 = A(0) + B\left[-\frac{1}{2} - 1\right]$$

$$\therefore -\frac{5}{2} = B\left(-\frac{3}{2}\right)$$

$\therefore B = \frac{5}{3}$

$$\therefore \frac{3x - 1}{(x - 1)(2x + 1)} = \frac{\frac{2}{3}}{x - 1} + \frac{\frac{5}{3}}{2x + 1}$$

$$\therefore I = \int \left(\frac{\frac{2}{3}}{x - 1} + \frac{\frac{5}{3}}{2x + 1} \right) dx$$

$$= \frac{2}{3} \int \frac{1}{x - 1} dx + \frac{5}{3} \int \frac{1}{2x + 1} dx$$

$$\therefore I = \frac{2}{3} \log|x - 1| + \frac{5}{3} \frac{\log|(2x + 1)|}{2} + c$$

Miscellaneous Exercise 5 | Q 4.5 | Page 139

Evaluate: $\int \frac{2x^3 - 3x^2 - 9x + 1}{2x^2 - x - 10} dx$

Solution:

We perform actual division and express the result as:

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$\begin{array}{r|l} & x - 1 \\ 2x^2 - x - 10 & 2x^3 - 3x^2 - 9x + 1 \\ & 2x^3 - x^2 - 10x \\ & \underline{(-)\quad(+)\quad(+)} \\ & -2x^2 + x + 1 \\ & -2x^2 + x + 10 \\ & \underline{(+)\quad(-)\quad(-)} \\ & -9 \end{array}$$

$\therefore$ I = $\int\left(x - 1 + \frac{-9}{2x^2 - x - 10}\right)$ dx

$$= \int x \cdot dx - \int 1 \cdot dx - 9\int \frac{1}{2x^2 - x - 10}dx$$

Here $2x^2 - x - 10$

$$= 2\left(x^2 + \frac{1}{2}x + \frac{1}{16} - 5 - \frac{1}{16}\right)$$

$$= 2\left[\left(x - \frac{1}{4}\right)^2 - \frac{81}{16}\right]$$

$\therefore$ I = $\int x \cdot dx - \int 1 \cdot dx - \frac{9}{2}\int \frac{1}{\left(x - \frac{1}{4}\right)^2 - \left(\frac{9}{4}\right)^2}$ dx

$$= \frac{x^2}{2} - x - \frac{9}{2} \cdot \frac{1}{2\left(\frac{9}{4}\right)}\log\left|\frac{x - \frac{1}{4} - \frac{9}{4}}{x - \frac{1}{4} + \frac{9}{4}}\right| + c_1$$

$$= \frac{x^2}{2} - x - \log\left|\frac{x - \frac{5}{2}}{x + 2}\right| + c_1$$

$$= \frac{x^2}{2} - x - \log\left|\frac{2x - 5}{2(x + 2)}\right| + c_1$$

$$= \frac{x^2}{2} - x + \log\left|\frac{2(x + 2)}{2x - 5}\right| + c_1$$

$$= \frac{x^2}{2} - x + \log\left|\frac{x + 2}{2x - 5}\right| + \log 2 + c_1$$

$\therefore$ I = $\frac{x^2}{2} - x + \log\left|\frac{x + 2}{2x - 5}\right| + c \text{ where } c = c_1 + \log 2$

Miscellaneous Exercise 5 | Q 4.5 | Page 139

Evaluate: $\int \frac{1 + \log x}{x(3 + \log x)(2 + 3\log x)}$ dx

Solution:

Let I = $\int \frac{1 + \log x}{x(3 + \log x)(2 + 3\log x)}$ dx

Put log x = t

∴ $\frac{1}{x}$ dx = dt

∴ I = $\int \frac{1 + t}{(3 + t)(2 + 3t)}$ dt

Let $\frac{1 + t}{(3 + t)(2 + 3t)} = \frac{A}{3 + t} + \frac{B}{2 + 3t}$

∴ 1 + t = A(2 + 3t) + B(3 + t) ...(i)

Putting t = – 3 in (i), we get

1 -3 = A(2 - 9) + B(0)

∴ - 2 = A (- 7)

∴ A = $\frac{2}{7}$

Putting t = $-\frac{2}{3}$ in (i), we get

$$1 - \frac{2}{3} = A(0) + B\left(3 - \frac{2}{3}\right)$$

$$\therefore \frac{1}{3} = B\left(\frac{7}{3}\right)$$

$\therefore$ B = $\dfrac{1}{7}$

$$\therefore \frac{1+t}{(3+t)(2+3t)} = \frac{\frac{2}{7}}{3+t} + \frac{\frac{1}{7}}{2+3t}$$

$$\therefore I = \int\left(\frac{\frac{2}{7}}{3+t} + \frac{\frac{1}{7}}{2+3t}\right) dt$$

$$= \frac{2}{7}\int\frac{1}{3+t}dt + \frac{1}{7}\int\frac{1}{2+3t}\, dt$$

$$= \frac{2}{7}\log|3+t| + \frac{1}{7}\cdot\frac{\log|2+3t|}{3} + c$$

$$\therefore I = \frac{2}{7}\log|3+\log x| + \frac{1}{21}\log|2+3\log x| + c$$