Pair of Linear Equations in Two Variables

Algebraic Methods

By Substitution

Solve: $7x - 15y = 2$ (i)

$x + 2y = 3$ (ii)

Solution: From equation (ii), $x = 3 - 2y$ substitute value of x in eq. (i)

$$7(3 - 2y) - 15y = 2$$

$$-29y = -19$$

$$\Rightarrow \quad y = \frac{19}{29}$$

Now, from $x = 3 - 2y$

$$x = 3 - 2\left(\frac{19}{29}\right) = \frac{49}{29}$$

By Elimination

Solve: $x + 3y = 6$ (i)

$2x + 3y = 12$ (ii)

Solution: Now, from (ii) – (i)

$$x = 6$$

and from (i) × 2 – (ii)

$$3y = 0$$

$$y = 0$$

General Form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

where $a_1, b_1, c_1, a_2, b_2, c_2$ are Real Numbers

Solution through graph

Each solution (x, y) corresponds to a point on the line representing the equation and vice-versa

Graphical Representation

Pair of Lines: $= x + -2y = 0$

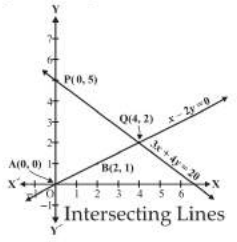
$3x + 4y - 20 = 0$

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-2}{4}, \frac{c_1}{c_2} = \frac{0}{-20}$$

Compare the Ratios: $= \frac{a_1}{a_2} = \frac{b_1}{b_2}$

Algebraic Interpretation : unique solution

Graphical Representation



Intersecting Lines

Pair of Lines: $= x + 2y - 4 = 0$

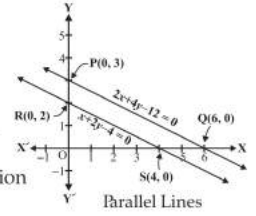
$2x + 4y - 12 = 0$

$$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4}, \frac{c_1}{c_2} = \frac{-4}{-12}$$

Compare the Ratios $= \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Algebraic Interpretation : No solution

Graphical Representation



Parallel Lines

Pair of Lines: $= 2x + 3y - 9 = 0$

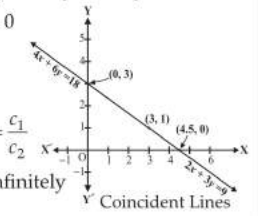
$4x + 6y - 18 = 0$

$$\frac{a_1}{a_2} = \frac{2}{4}, \frac{b_1}{b_2} = \frac{3}{6}, \frac{c_1}{c_2} = \frac{-9}{-18}$$

Compare the Ratios $= \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Algebraic Interpretation = Infinitely many solutions

Graphical Representation



Coincident Lines

Trace the Mind Map

- First Level
- Second Level
- Third Level