CBSE Class 12 - Mathematics
Sample Paper 03 (2020-21)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
ii. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions
iii. Both Part A and Part B have choices.

Part – A:

i. It consists of two sections- I and II.
ii. Section I comprises of 16 very short answer type questions.
iii. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

i. It consists of three sections- III, IV and V.
ii. Section III comprises of 10 questions of 2 marks each.
iii. Section IV comprises of 7 questions of 3 marks each.
iv. Section V comprises of 3 questions of 5 marks each.
v. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part - A Section - I

1. Let R is the equivalence relation in the set A = {0, 1, 2, 3, 4, 5} given by R = {(a, b): 2 divides (a - b)}. Write the equivalence class [0].

OR

Show that $f: R \rightarrow R$, given by $f(x) = x - [x]$, is neither one-one nor onto.

2. Determine whether the relation is reflexive, symmetric and transitive:

Relation R in the set A of human beings in a town at a particular time given by

R = {(x, y) : x is wife of y}

OR

Let the relation R be defined on the set A = {1, 2, 3, 4, 5} by R = {(a, b) : $|a^2 - b^2| < 8$}. Write R as a set of ordered pairs.

3. Let R ={ (a,b) : a, b $\in N$ and b = a +5, a < 4}. Find the domain and range of R.

4. If [x 1] $\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}$ = O, find x.

5. Find the values of x and y, when $\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$

OR

If A is any $m \times n$ matrix such that AB and BA are both defined show that B is an $n \times m$ matrix.

6. If A is a square matrix of order $n \times n$ such that $|A| = \lambda$, then write the value of $|-A|$.

7. Evaluate $\int_0^{\pi/4} \tan x dx$

OR

Evaluate: $\int \sqrt{1+\sin 2x} dx$

8. Find the area of the region enclosed by the curves y = x , x = e, y = $\frac{1}{x}$ and the positive x-axis

9. Write the order and the degree of the differential equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx}$ + 3y = 0.

OR

Find the order and degree (if any) of the differential equation given below: $\frac{dy}{dx}$ - tan x = 0.

10. Let $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of three distinct points A, B, C. If there exist scalars x, y, z (not all zero) such that $x\vec{a} + y\vec{b} + z\vec{c} = \vec{0}$ and x + y + z = 0, then show that A, B and C lie on a line.

11. Find the sum of the following vectors: $\vec{a} = \hat{i} - 2\hat{j}$, $\vec{b} = 2\hat{i} - 3\hat{j}$, $\vec{c} = 2\hat{i} + 3\hat{k}$.

12. If $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 3$, then find the projection of $\vec{b}$ on $\vec{a}$.

13. Write the formula for the shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b}$ and $\vec{r} = \vec{a_2} + \mu\vec{b}$.

14. Find the distance of the given point (2, 3, -5) from the given plane x + 2y - 2z = 9

15. If P(A) = $\frac{6}{11}$, P(B) = $\frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$, find $P(A \cap B)$

16. Consider the experiment of throwing a die, if a multiple of 3 comes up throw the die again and if any other number comes toss a coin. Find the conditional probability of the event "the coin shows a tail", given that "at least one die shows a 3".

Section - II

17. As we know good planning can save energy, time, and money. A farmer wants to construct a circular well and square garden in his field. He wants to keep the sum of their perimeters 600 m

i. If the radius of the circular garden be r m and the side of the square garden be x m then sum of area S is

a. $S = \pi r^2 + (\frac{600+2\pi r}{4})^2$

b. $S = \pi r^2 + (\frac{300+\pi r}{4})^2$

c. $S = 2\pi r^2 + (\frac{600+2\pi r}{4})^2$

d. $S = \pi r + (\frac{600+\pi r}{4})^2$

ii. Radius of circular well is

a. $r = \frac{600}{\pi+4}$

b. $r = \frac{300}{\pi+4}$

c. $r = \frac{300}{2\pi+4}$

d. $r = \frac{150}{\pi+2}$

iii. For the given condition

a. $\frac{d^2s}{dr^2} = 0$

b. $\frac{d^2s}{dr^2} < 0$

c. $\frac{d^2s}{dr^2} > 0$

d. None of these

iv. The relationship between the side of the square garden and the radius of the circular garden.

a. a = r

b. $2a = r^2$

c. $a = \frac{1}{2}r$

d. a = 2r

v. Find the number which exceeds its square by the greatest possible number.

a. $\frac{1}{2}$

b. 2

c. 1

d. 0

18. In an office, three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03

Based on the above information answer the following:

i. The conditional probability that an error is committed in processing given that Sonia processed the form is:

a. 0.0210

b. 0.04

c. 0.47

d. 0.06

ii. The probability that Sonia processed the form and committed an error is:

a. 0.005

b. 0.006

c. 0.008

d. 0.68

iii. The total probability of committing an error in processing the form is

a. 0

b. 0.047

c. 0.234

d. 1

iv. The manager of the company wants to do a quality check. During the inspection, he selects a form at random from the days' output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Vinay is:

a. 1

b. $\frac{30}{47}$

c. $\frac{20}{47}$

d. $\frac{17}{47}$

v. Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form. The value of $\sum_{i=1}^{3} P(E_i \mid \mathrm{A})$ is

a. 0

b. 0.03

c. 0.06

d. 1

Part - B Section - III

19. Evaluate: $\tan \frac{1}{2}\left(\cos^{-1} \frac{\sqrt{5}}{3}\right)$.

20. Write the minors and cofactors of each element of the first column of the matrix and

hence evaluate the determinant: A = $\begin{bmatrix} -1 & 4 \\ 2 & 3 \end{bmatrix}$

OR

If B = [-7], find det B.

21. Examine that $\sin|x|$ is a continuous function.
22. A circular disc of radius 3cm is being heated. Due to expansion, their radius increase at the rate of 0.05 cm/s. find the rate at which its area is increasing when radius is 3.2cm.
23. Evaluate: $\int e^x \left(\frac{2+\sin 2x}{1+\cos 2x}\right)$ dx

OR

Evaluate: $\int\limits_{-\pi}^{\pi}$ (cos ax - sin bx)^2 dx

24. Find the area of the region bounded by the curve y^2 = x and the lines x = 1, x = 4 and the x - axis.
25. Find the general solution of $\frac{dy}{dx} + y = 1\,(y \neq 1)$
26. If $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}, \quad \vec{a} \neq \overrightarrow{0}$ and $\vec{b} \neq \vec{c}$ show that $\vec{b} = \vec{c} + t\vec{a}$ for some scalar t.
27. Find a vector of magnitude 26 units normal to the plane 12x - 3y + 4z = 1.
28. The probability of a man hitting a target is 1/2. How many times must he fire so that the probability of hitting the target at least once is more than 90%.

OR

Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Section - IV

29. Show that f : R $\rightarrow$ R, defined as f(x) = x^3, is a bijection.
30. If y = $\sin^{-1}$ (cos x) + $\cos^{-1}$ (sin x), prove that $\frac{dy}{dx}$ = -2.
31. Find all points of discontinuity if $f(x) = \begin{cases} |x| + 3, \; if \; x \leqslant -3 \\ -2x, \; if \; -3 < x < 3 \\ 6x + 2, \; if \; x \geqslant 3 \end{cases}$

OR

If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then prove that $\frac{dy}{dx} = \frac{1}{x^3 y}$.

32. Show that the function given by f(x) = sin x is
   i. strictly increasing in $\left(0, \frac{\pi}{2}\right)$
   ii. strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$
   iii. neither increasing nor decreasing in $(0, \pi)$

33. Evaluate: $\int \frac{(2x+1)}{(4-3x-x^2)} dx$.

34. Find the area of the region bounded by the curves $y^2 = 9x$, $y = 3x$.

OR

Find the area of the region bounded by the curve $y=x^2$ and the line $y = 4$.

35. Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that $y = 0$ when $x = 1$.

Section - V

36. For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find numbers a and b such that $A^2 + aA + bI = 0$.

OR

Show that the matrix, $A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$ satisfies the equation, $A^3 - A^2 - 3A - I_3 = O$. Hence, find A^{-1}

37. Find the equation of the plane that contains the point (1,-1,2) and is $\perp$ to each of the plane $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$.

OR

Show that the lines $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$ and $\frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$ intersect and find their point of intersection.

38. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product A requires 3hours on both machines and each unit of product B requires 2 h on the first machine and 1 h on the second machine. Each unit of product A is sold at a profit of ₹7 and B at a profit of ₹4. Find the

production level per day for maximum profit graphically.

OR

A fruit grower can use two types of fertilizer in his garden, brand P and Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brad are given in the table. Testes indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine. If the grower wants to minimize the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

(Kg per bag)		
	Brand P	**Brand Q**
Nitrogen	3	3.5
Phosphoric Acid	1	2
Potash	3	1.5
Chlorine	1.5	2

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Solution

Part - A Section - I

1. Given that R is the equivalence relation in the set A = {0, 1, 2, 3, 4, 5} defined as R = {(a, b): 2 divides (a - b)}.

   Clearly, [0] = {b ∈ A : (0, b) ∈ R}

   = {b ∈ A . 2 divides (0- b)}

   = {b ∈ A : 2 divides (-b)}

   = {0, 2, 4}

   Hence equivalence class of [0] = {0, 2, 4}.

OR

We have, f(x) = x - [x]=$\{x\}$ (fractinal part of x)

Injection test:

f(x) = 0 for all x ∈ Z

So, f is a many-one function.

Surjection test:

Range (f) = [0, 1) ≠ R.

So, f is an into function.

Therefore, f is neither one-one nor onto.

2. It is given that R = {(x, y) : x is wife of y}

   Clearly, (x,x) ∉ R as x cannot be the wife of herself.

   ⇒ R is not reflexive.

   Now, if (x,y) ∈ R, then x is the wife of y.

   ⇒ But y is not wife of x.

   ⇒ (y,x) ∉ R

   ⇒ R is not symmetric.

   Further, let (x,y), (y,z) ∈ R

   ⇒ x is the wife of y and y is the wife of z.

   ⇒ This is not possible.

$\Rightarrow$ (x,z) $\notin$ R

$\Rightarrow$ R is not transitive.

Therefore, R is neither reflexive, nor symmetric, nor transitive.

OR

Given a set A = {1, 2, 3, 4, 5} and relation R = {(a, b) : $|a^2 - b^2| < 8$, a,b $\in A$}

Now according to the questionR= {(a,b): $|a^2 - b^2| < 8$ }

$\Rightarrow$ R = {(1,1),(2,2), (3,3), (4,4), (5,5), (1,2), (2,1), (2,3), (3,2), (3,4), (4,3),}

3. Given R={(a, b):b=a+5,a<4,a, b belongs N}

   Then dom (R) = {1, 2, 3} and range (R) = {6, 7, 8}

4. According to the question, $\begin{bmatrix} x & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = 0$

   Using matrix multiplication,

   $\Rightarrow \begin{bmatrix} x-2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$

   Equating the corresponding elements,

   $\Rightarrow x - 2 = 0$

   $\Rightarrow$ x = 2

5. Given $\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$

   So, x + y = 8 and x - y = 4

   Adding these two gives 2x = 1

   $\Rightarrow$ x = 6

   and, y = 2

   Conclusion : x = 6 and y = 2

OR

Since we are given A is an $m \times n$ matrix such that AB, exists.

Therefore, the number of rows in B, should be equal to the number of columns in A.

Thus, B has n rows.

Further, BA exists, therefore the number of columns in B, should be equal to the number of rows in A.

So, B has m columns.

Hence, B is an $n \times m$ matrix.

6. Since $|kA| = k^m|A|$

Given that, k = -1, m = n and $|A| = \lambda$, we get

$|-A| = (-1)^n \times \lambda$

Hence, $|-A| = \lambda$ if n is even

and $|-A| = -\lambda$ if n is odd.

7. $I = \int_0^{\pi/4} \tan x dx$

$= [log \sec x]_0^{\frac{\pi}{4}}$

$=[log \sec \frac{\pi}{4} - log \sec 0]$

$=[log\sqrt{2} - log 1]$

$= [log 2^{\frac{1}{2}} - 0]$

$= \frac{1}{2} \log 2$

OR

Let $I = \int \sqrt{1 + \sin 2x} dx$. Then,

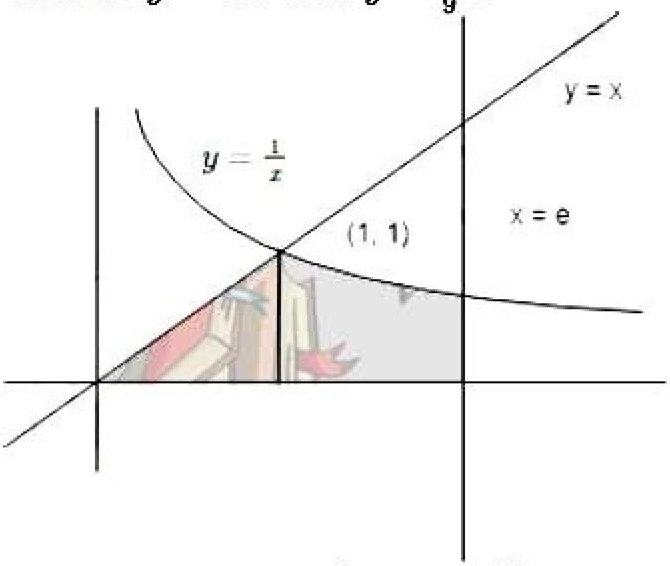
$\Rightarrow \quad I = \int \sqrt{\sin^2 x + \cos^2 x + 2 \sin x \cos x} dx$

$\Rightarrow \quad I = \int(\sin x + \cos x) dx$

$= \int \sin x dx + \int \cos x dx$

$= -\cos x + \sin x + C$

8. We have $y = 4x^2$ and $y = \frac{1}{9}x^2$



Required area $=2\int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right) dy$

$= 2\left(\frac{5y}{2} \frac{\sqrt{y}}{3/2}\right)_0^2$

$= 2.\frac{5}{3}2\sqrt{2} = \frac{20\sqrt{2}}{3}$

9. In the given equation, the highest-order derivative is $\frac{d^2y}{dx^2}$ and its power is 1

∴ its order = 2 and degree = 1.

OR

The given equation is $\frac{dy}{dx}$ - tan x = 0

In this equation, the highest-order derivative is $\frac{dy}{dx}$ whose power is 1

∴ its order = 1 and degree = 1.

10. $x\vec{a} + y\vec{b} + z\vec{c} = \vec{0}$

It is given that x, y, z are not all zero. So, let z be non-zero. Then,

$\Rightarrow z\vec{c} = -(x\vec{a} + y\vec{b})$

$\Rightarrow \vec{c} = -\frac{(x\vec{a}+y\vec{b})}{z} \Rightarrow \vec{c} = \frac{x\vec{a}+y\vec{b}}{x+y}$ [∵ x + y + z = 0 ∴ z = -(x + y)]

This shows that the point C divides the line joining the points A and B in the ratio y : x.

Hence, A, B and C lie on the same line.

11. Given,

$\vec{a} = \hat{i} - 2\hat{j}, \vec{b} = 2\hat{i} - 3\hat{j}, \vec{c} = 2\hat{i} + 3\hat{k}.$

So, Sum of the above three vectors = $\vec{a} + \vec{b} + \vec{c} = \hat{i} - 2\hat{j} + 2\hat{i} - 3\hat{j} + 2\hat{i} + 3\hat{k}$

$= 5\hat{i} - 5\hat{j} + 3\hat{k}$

12. If $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 3$, then we have to find the projection of $\vec{b}$ *on* $\vec{a}$.

Projection of $\vec{b}$ *on* $\vec{a} = \frac{\vec{a}.\vec{b}}{|\vec{a}|} = \frac{3}{2}$

13. The shortest distance d between the parallel lines $\vec{r} = \overrightarrow{a_1} + \lambda\vec{b}$ and $\vec{r} = \overrightarrow{a_2} + \mu\vec{b}$ is given by

d = $\frac{|(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b}|}{|\vec{b}|}$

14. Given point is (2, 3, -5) and the plane is x + 2y - 2z = 9

$d = \left|\frac{2+6+10-9}{\sqrt{1+4+4}}\right|$

$= \left|\frac{9}{\sqrt{9}}\right|$

$= \frac{9}{3}$

= 3

15. Given that P(A) = $\frac{6}{11}$, P(B) = $\frac{5}{11}$, $P(A \cup B) = \frac{7}{11}$

we know that $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$\Rightarrow P(A\cap B)=\frac{6}{11}+\frac{5}{11}-\frac{7}{11}=\frac{11-7}{11}$

$\Rightarrow P(A\cap B)=\frac{4}{11}$

16. S = {(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

(1, H), (2, H), (3, H), (4, H), (5, H), (1, T), (2, T), (3, T), (4, T), (5, T)}

$\therefore$ n(S) = 20

P (first die shows a multiple of 3) = $\frac{12}{36}=\frac{1}{3}$

P (first die shows a number which is not a multiple of 3) = $\frac{4}{6}\times\frac{1}{2}+\frac{4}{6}\times\frac{1}{2}=\frac{8}{12}=\frac{2}{3}$

Let A = the coin shows a tail = {(1, T), (2, T), (4, T), (5, T)}

B = at least one die shows a 3 = {(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)}

$A\cap B=\phi$

$n(A)=4, n(B)=6, n(A\cap B)=0$

P (B) = $\frac{6}{36}=\frac{1}{6}$ and $P(A\cap B)$ = 0

$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{0}{\frac{6}{36}}=0$

Section - II

17. i. (a) $S=\pi r^2+(\frac{600+2\pi r}{4})^2$
    ii. (b) $r=\frac{300}{\pi+4}$
    iii. (c) $\frac{d^2s}{dr^2}$ >0
    iv. (d) a = 2r
    v. (a) $\frac{1}{2}$
18. i. (b) 0.04
    ii. (c) 0.008
    iii. (b) 0.047
    iv. (d) 17/47
    v. (d) 1

Part - B Section - III

19. Suppose $\cos^{-1}\frac{\sqrt{5}}{3}=\theta$. Then, $\cos\theta=\frac{\sqrt{5}}{3}$, where $\theta\in[0,\pi]$

$\therefore\ \tan\frac{1}{2}\left(\cos^{-1}\frac{\sqrt{5}}{3}\right)=\tan\frac{1}{2}\theta$ = $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\sqrt{\frac{(1-\sqrt{5}/3)}{(1+\sqrt{5}/3)}}$

= $\sqrt{\frac{(3-\sqrt{5})}{(3+\sqrt{5})}\times\frac{(3-\sqrt{5})}{(3-\sqrt{5})}}=\frac{(3-\sqrt{5})}{2}$

20. Let M_{ij} and C_{ij} represents the minor and co-factor of an element, where i and j represent the row and column.

The minor of matrix can be obtained for particular element by removing the row and column where the element is present. Then finding the absolute value of the matrix newly formed.

Also, $C_{ij} = (-1)^{i+j} \times M_{ij}$

A = $\begin{bmatrix} -1 & 4 \\ 2 & 3 \end{bmatrix}$

$M_{11} = 3$

$M_{21} = 4$

$C_{11} = (-1)^{1+1} \times M_{11} = 1 \times 3 = 3$

$C_{21} = (-1)^{2+1} \times 4 = -1 \times 4 = -4$

Now expanding along the first column we get

$|A| = a_{11} \times C_{11} + a_{21} \times C_{21}$

$= -1 \times 3 + 2 \times (-4) = -11.$

OR

$|B| = 7$ [since $|a| = a$, for some constant a]

21. Let $f(x) = |x|$ and $g(x) = \sin|x|$, then

$(gof)x = g\{f(x)\} = g(|x|) = \sin|x|$

Now, f and g being continuous, it follows that their composite, (gof) is continuous.

Therefore, $\sin|x|$ is continuous.

22. Given , $\frac{dr}{dt} = 0.05cm/sec$

$A = \pi r^2$

Now $\frac{dA}{dt} = 2\pi r.\frac{dr}{dt}$

$= 2\pi(3.2) \times 0.05$(given r=3.2cm)

$= 0.320\pi cm^2/s$

23. Let I = $\int e^x \left(\frac{2+\sin 2x}{1+\cos 2x}\right)$ dx, then we have

$I = \int e^x \left(\frac{2+2\sin x\cos x}{2\cos^2 x}\right)$dx

$\Rightarrow I = \int e^x \left(\underset{f'}{\sec^2 x} + \underset{f}{\tan x}\right) dx = \int \underset{II}{e^x} \underset{I}{\tan x}$ dx + $\int e^x \sec^2 x\, dx$

$\Rightarrow I = (\tan x) e^x - \int e^x \sec^2 x\, dx + \int e^x \sec^2 x\, dx + C$

$= e^x \tan x + C$

OR

Let I = $\int_{-\pi}^{\pi}$ (cos ax - sin bx)^2 dx, then we have

I = $\int_{-\pi}^{\pi}$ ($\cos^2$ax + $\sin^2$bx - 2 cos ax sin bx) dx

= $\int_{-\pi}^{\pi}$ $\cos^2$ ax dx + $\int_{-\pi}^{\pi}$ $\sin^2$ bx dx - $\int_{-\pi}^{\pi}$ 2 cos ax sin bx dx

= 2 $\int_0^{\pi}$ $\cos^2$ ax dx + 2 $\int_0^{\pi}$ $\sin^2$ bx dx - 0 [since $\cos^2$ ax and $\sin^2$ bx are even functions and cos ax sin bx is an odd function.]

$\therefore\ I = 2\int_0^{\pi} \frac{1+\cos 2ax}{2} dx + 2\int_0^{\pi} \frac{1-\cos 2bx}{2} dx$

= $\int_0^{\pi}$ (1 + cos 2 ax) dx + $\int_0^{\pi}$ (1 - cos 2 bx) dx

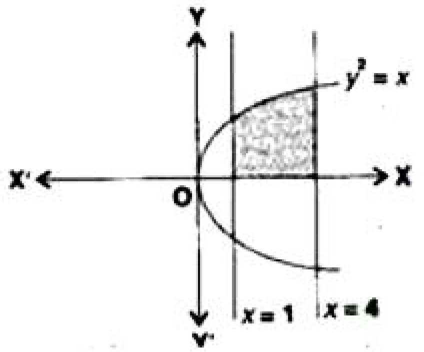
= $\int_0^{\pi}$ (1 + cos 2 ax + 1 - cos 2 bx) dx

= $\int_0^{\pi}$ (2 + cos 2 ax - cos 2 bx) dx

$= 2[x]_0^{\pi} + \left[\frac{\sin 2ax}{2a}\right]_0^{\pi} - \left[\frac{\sin 2bx}{2b}\right]_0^{\pi}$

$= 2\pi + \frac{\sin 2a\pi}{2a} - \frac{\sin 2b\pi}{2b}$

24. Equation of the curve (rightward parabola) is $y^2 = x$



$\therefore y = \sqrt{x}$ ...(i)

$\therefore$ Required area (shaded region)

$= \left|\int_1^4 y dx\right| = \left|\int_1^4 \sqrt{x}\, dx\right|$ [From eq. (i)]

$= \left|\int_1^4 x^{\frac{1}{2}} dx\right| = \left|\frac{\left(x^{\frac{3}{2}}\right)_1^4}{\frac{3}{2}}\right| = \left|\frac{2}{3}\left(4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right)\right|$

$= \left|\frac{2}{3}\left(4^{\frac{1}{2}\times 3} - 1^{\frac{1}{2}\times 3}\right)\right| = \left|\frac{2}{3}(8-1)\right| = \frac{2}{3} \times 7 = \frac{14}{3}$sq. units

25. Given: Differential equation $\frac{dy}{dx} + y = 1$

$\Rightarrow \frac{dy}{dx} = 1 - y$

$\Rightarrow dy = (1-y)\,dx$

$\Rightarrow dy = -(y-1)\,dx$

$\Rightarrow \frac{dy}{y-1} = -dx$

Integrating both sides,

$\Rightarrow \int \frac{dy}{y-1} dx = -\int 1 dx$

$\Rightarrow \log|y-1| = -x + c$

$\Rightarrow |y-1| = e^{-x+c}$ [∵ if log x = t, then x = e^x]

$\Rightarrow y - 1 = \pm e^{-x+c}$

$\Rightarrow y = 1 \pm e^{-x} e^{c}$

$\Rightarrow y = 1 \pm e^{c} e^{-x}$

$\Rightarrow y = 1 + Ae^{-x}$, where $A = \pm e^{c}$

26. We have,

$\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$

$\Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = \overrightarrow{0}$

$\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = \overrightarrow{0}$

$\Rightarrow \vec{a} = \overrightarrow{0}$ or, $\vec{b} - \vec{c} = \overrightarrow{0}$ or, $\vec{a} \| (\vec{b} - \vec{c})$

$\Rightarrow \vec{a} = \overrightarrow{0}$ or, $\vec{b} = \vec{c}$ or,, $\vec{a} \| (\vec{b} - \vec{c})$

$\Rightarrow \vec{a} \| (\vec{b} - \vec{c})$ $[\because \vec{a} \neq \overrightarrow{0}$ and $\vec{b} \neq \vec{c}]$

$\Rightarrow \vec{b} - \vec{c} = t\vec{a}$ for some scalar t

$\Rightarrow \vec{b} = \vec{c} + t\vec{a}$

27. The given equation of the plane is 12x - 3y + 4z = 1

$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (12\hat{i} - 3\hat{j} + 4\hat{k}) = 1$

$\Rightarrow \vec{r} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 1$

it is the vector equation of the plane.

Because the vector equation of the plane is $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

So, the normal vector, $\vec{n} = 12 \quad \hat{i} - 3 \quad \hat{j} + 4\hat{k}$

$|\vec{n}| = \sqrt{144 + 9 + 16} = 13$

Unit vector parallel to $\vec{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{12 \quad \hat{i} - 3\hat{j} + 4\hat{k}}{13}$

So, the vector of magnitude 26 units normal to the plane

$= 26 \times \frac{12\ \hat{i} - 3\hat{j} + 4\hat{k}}{13}$

$= 2\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right)$

$= 24\hat{i} - 6\hat{i} + 8\hat{k}$

Which is the required Vector.

28. Suppose he fires n times. Let X be a random variable denoting the number of times he hits the target in n trials Then, we have,

P (X =r) = ${}^nC_r\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{n-r}$ = ${}^nC_r\left(\frac{1}{2}\right)^n$, where, r= 0, 1, 2,...,n

Now, P (X ≥ 1) > $\frac{90}{100}$

$\Rightarrow 1 - P(X=0) > \frac{90}{100}$

$\Rightarrow P(X=0) < 1 - \frac{90}{100}$

$\Rightarrow {}^nC_0\left(\frac{1}{2}\right)^n < \frac{1}{10}$

$\Rightarrow \left(\frac{1}{2}\right)^n < \frac{1}{10}$

Clearly, $\frac{1}{2} > \frac{1}{10}, \left(\frac{1}{2}\right)^2 > \frac{1}{10}, \left(\frac{1}{2}\right)^3 > \frac{1}{10}$, but $\left(\frac{1}{2}\right)^4 < \frac{1}{10}$

$\because \left(\frac{1}{2}\right)^n < \frac{1}{10} \Rightarrow$ n = 4,5, 6,...

Thus, man must fire at least 4 times

OR

Let us define events,

A_1 : Red ball is transferred from bag I to II.

A_2 : Black ball is transferred from bag I to II.

$\therefore P(A_1) = \frac{3}{7}$ and $P(A_2) = \frac{4}{7}$

Let X be the event that the drawn ball is red

$\therefore$ when red ball is transferred from bag I to II, $P(X|A_1) = \frac{5}{10}$ = $\frac{1}{2}$

And, when black ball is transferred from bag I to II, $P(X|A_2) = \frac{4}{10}$ = $\frac{2}{5}$

Hence, $P(A_2|X) = \frac{P(A_2)P(X|A_2)}{P(A_1)P(X|A_1)+P(A_2)P(X|A_2)}$

= $\frac{\frac{4}{7}\times\frac{2}{5}}{\frac{3}{7}\times\frac{1}{2}+\frac{4}{7}\times\frac{2}{5}}$

= $\frac{16}{31}$

Section - IV

29. We observe the following properties of f.

Injectivity: Let x, y $\in$ R such that f(x) = f(y). Then,

f(x) = f(y)

$\Rightarrow x^3 = y^3$

$\Rightarrow$ x = y

So, f : R$\rightarrow$ R is one-one.

Surjectivity: Let y $\in$ R (co-domain). Then,

$f(x) = y \Rightarrow x^3 = y \Rightarrow x = y^{1/3}$

Clearly, $y^{1/3} \in$ R (domain) for all y $\in$ R (co-domain).

Thus, for each y $\in$ R (co-domain) there exists $x = y^{1/3} \in$ R (domain) such that $f(x) = x^3 = y$.

So, f : R $\rightarrow$ R is onto.

Hence, f : R $\rightarrow$ R is a bijection.

30. Given equation,

$y = \sin^{-1}(\cos x) + \cos^{-1}(\sin x)$

Let $s = \sin^{-1}(\cos x)$ and $t = \cos^{-1}(\sin x)$

Therefore, y = s + t ....(i)

For $\sin^{-1}(\cos x)$

Let u = cos x

Therefore, $s = \sin^{-1} u$

Differentiating above equation w.r.t. x,

$\therefore \frac{ds}{dx} = \frac{ds}{du} \cdot \frac{du}{dx}$ (By chain rule)

$\therefore \frac{ds}{dx} = \frac{d}{du}\left(\sin^{-1} u\right) \cdot \frac{d}{dx}(\cos x)$

$= \frac{1}{\sqrt{1-u^2}} \cdot (-\sin x)$

$= \frac{1}{\sqrt{1-(\cos x)^2}} \cdot (-\sin x)$

$= \frac{1}{\sqrt{\sin^2 x}} \cdot (-\sin x)$ ($\because \sin^2 x + \cos^2 x = 1$)

$= \frac{1}{\sin x} \cdot (-\sin x)$

= -1

$\therefore \frac{ds}{dx}$ = -1 .....(i)

For $\cos^{-1}(\sin x)$

Let u = sin x

Therefore, $t = \cos^{-1} u$

Differentiating above equation w.r.t.x,

$\therefore \frac{dt}{dx} = \frac{dt}{du} \cdot \frac{du}{dx}$ (By chain rule)

$\therefore \frac{dt}{dx} = \frac{d}{du}\left(\cos^{-1} u\right) \cdot \frac{d}{dx}(\sin x)$

$= \frac{-1}{\sqrt{1-u^2}} \cdot (\cos x)$

$= \frac{-1}{\sqrt{1-(\sin x)^2}} \cdot (\cos x)$

$= \frac{-1}{\sqrt{\cos^2 x}} \cdot (\cos x)$ ($\because \sin^2 x + \cos^2 x = 1$)

$= \frac{-1}{\cos x} \cdot (\cos x)$

= -1

$\therefore \frac{dt}{dx}$ = -1 .....(i)

Differentiating eq(i) w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{d}{dx}(s+t)$

$= \frac{ds}{dx} + \frac{dt}{dx}$

= -1 - 1 ......[From (ii) and (iii)

= -2

$\therefore \frac{dy}{dx}$ = -2

Hence proved.

31. At x = - 3

f(-3) = |-3| + 3 = 3 + 3 = 6

$\lim_{x \to -3^+} f(x) = \lim_{x \to -3} (-2x)$

$= -2 \times -3 = 6$

$\lim_{x \to -3^-} f(x) = \lim_{x \to -3} |x| + 3 = |-3| + 3 = 3 + 3 = 6$

Hence, f(x) is continuous at x = -3

At x = 3

$f(x) = 6 \times 3 + 2 = 20$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (-2x)$

$= \lim_{h \to 0} -2(3-h)$ = - 6

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (6x+2)$

$= \lim_{h \to 0} [6(3+h)+2]$ = 20

$\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x)$

Hence f(x) is not continuous at x =3

OR

We have,

$x^2 + y^2 = t - \frac{1}{t}$

$\Rightarrow \quad \left(x^2+y^2\right)^2 = \left(t-\frac{1}{t}\right)^2$

$\Rightarrow \quad x^4+y^4+2x^2y^2 = t^2+\frac{1}{t^2}-2$

$\Rightarrow \quad x^4+y^4+2x^2y^2 = x^4+y^4-2 \quad \left[\because x^4+y^4 = t^2+\frac{1}{t^2}\right]$

$\Rightarrow 2x^2y^2 = -2$

$\Rightarrow x^2y^2 = -1 \Rightarrow y^2 - \frac{1}{x^2} \Rightarrow y^2 = -x^{-2}$

Differentiating with respect to x, we get

$2y\frac{dy}{dx} = -(-2)x^{-3} \Rightarrow y\frac{dy}{dx} = \frac{1}{x^3} \Rightarrow \frac{dy}{dx} = \frac{1}{x^3y}$

LHS=RHS

Hence Proved..

32. i. The function is f(x) = sin x

Then, f'(x) = cos x

Since for each $x \in \left(0, \frac{\pi}{2}\right)$, cos x > 0, we have f'(x) > 0

Therefore, function f(x) is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

ii. The function is f(x) = sin x

Then, f'(x) = cos x

Since for each $x \in \left(\frac{\pi}{2}, \pi\right)$, cos x < 0, we have f'(x) < 0

Therefore, the function f(x) is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

iii. The function is f(x) = sin x

Then, f'(x) = cos x

Since for each $x \in \left(0, \frac{\pi}{2}\right)$, cos x > 0, we have f'(x) > 0

Therefore, f(x) is strictly increasing in $\left(0, \frac{\pi}{2}\right)$ ...(i)

Now, the function is f(x) = sin x

Then, f'(x) = cos x

Since, for each $x \in \left(\frac{\pi}{2}, \pi\right)$, cos x < 0, we have f'(x) < 0

Therefore, f(x) is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$ ...(ii)

From (i) and (ii)

It is clear that the function f(x) is neither increasing nor decreasing in $(0, \pi)$.

33. Let the given integral be,

$I = \int \frac{2x+1}{(4-3x-x^2)}dx$

$= \int \frac{2x+1}{(1-x)(4+x)}dx$

Now using partial fractions by Putting $\frac{2x+1}{(1-x)(4+x)} = \frac{A}{1-x} + \frac{B}{4+x}$ ... (1)

A(4 + x) + B(1 - x) = 2x + 1

Now put 1 - x = 0

Therefore, x = 1

A(5) + B(0) = 3

$A=\frac{3}{5}$

Now put 4 + x = 0

Therefore, x = -4

A(0) + B(5) = -8 + 1 = -7

$B=\frac{-7}{5}$

Now From equation (1) we get,

$\frac{2x+1}{(1-x)(4+x)}=\frac{3}{5}\times\frac{1}{1-x}+\frac{-7}{5}\times\frac{1}{4+x}$

$\int\frac{2x+1}{(1-x)(4+x)}dx=\frac{3}{5}\int\frac{1}{1-x}dx+\frac{-7}{5}\int\frac{1}{4+x}dx$

$=\frac{-3}{5}\log|1-x|-\frac{7}{5}\log|4+x|+c$

$=-\frac{1}{5}[3\log|1-x|+7\log|4+x|]+c$

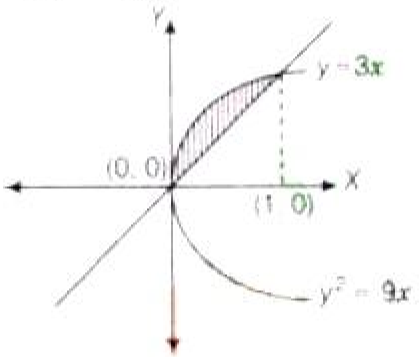
34. We have y^2 = 9x and y = 3x

$\Rightarrow(3x)^2=9x$

$\Rightarrow 9x^2-9x=0$

$\Rightarrow 9x(x-1)=0$

$\Rightarrow x=1,0$



$\therefore$ Required area, $A=\int_0^1\sqrt{9x}\,dx-\int_0^1 3x\,dx$

$=3\int_0^1 x^{1/2}dx-3\int_0^1 x\,dx$

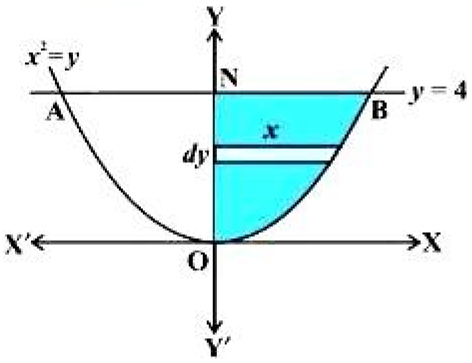
$=3\left[\frac{x^{3/2}}{3/2}\right]_0^1-3\left[\frac{x^2}{2}\right]_0^1$

$=3\left(\frac{2}{3}-0\right)-3\left(\frac{1}{2}-0\right)$

$=2-\frac{3}{2}=\frac{1}{2}$ sq units

OR

The required area is shown in fig below by shaded region;



The points of intersection of two curves can be calculated and are (-1,1) and (2,4) as shown in fig.

The required area is given as;

$2\int_0^4 x dy = 2$ (Area of the region BONB bounded by curve, y - axis and the lines y = 0 and y = 4)

$= 2\int_0^4 \sqrt{y} dy = 2 \times \frac{2}{3}\left[y^{\frac{3}{2}}\right]_0^4 = \frac{4}{3} \times 8 = \frac{32}{3}$

Which is the required area.

35. According to the question,

Given differential equation is

$\frac{dy}{dx} = 1 + x + y + xy$

$\Rightarrow \frac{dy}{dx} = 1(1 + x) + y(1 + x)$

$\Rightarrow \quad \frac{dy}{dx} = (1 + x)(1 + y)$

$\frac{1}{(1+y)} dy = (1 + x)dx$

On integrating both sides, we get

$\int \frac{1}{1+y} dy = \int (1 + x)dx$

$\Rightarrow \log|1 + y| = x + \frac{x^2}{2} + C$ ...(i)

Also, given that y = 0, when x = 1.

On substituting x = 1, y = 0 in Eq. (i), we get

$\log|1 + 0| = 1 + \frac{1}{2} + C \Rightarrow C = -\frac{3}{2} [\because \log 1 = 0]$

Now, on substituting the value of C in Eq. (i), we get

$\log|1 + y| = x + \frac{x^2}{2} - \frac{3}{2}$

which is the required particular solution of given differential equation.

Section - V

36. Given: $A=\begin{bmatrix}3 & 2\\1 & 1\end{bmatrix}$

$\therefore A^2=A.A=\begin{bmatrix}3 & 2\\1 & 1\end{bmatrix}\begin{bmatrix}3 & 2\\1 & 1\end{bmatrix}$

$=\begin{bmatrix}9+2 & 6+2\\3+1 & 2+1\end{bmatrix}=\begin{bmatrix}11 & 8\\4 & 3\end{bmatrix}$

$\therefore A^2+aA+bI_2=0$

$\Rightarrow \begin{bmatrix}11 & 8\\4 & 3\end{bmatrix}+a\begin{bmatrix}3 & 2\\1 & 1\end{bmatrix}+b\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=0$

$\Rightarrow \begin{bmatrix}11 & 8\\4 & 3\end{bmatrix}+\begin{bmatrix}3a & 2a\\a & a\end{bmatrix}+\begin{bmatrix}b & 0\\0 & b\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$\Rightarrow \begin{bmatrix}11+3a+b & 8+2a+0\\4+a+0 & 3+a+b\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$\therefore$ We have $11+3a+b=0$....(i)

$8+2a+0=0$ ...(ii)

$\Rightarrow 2a=-8$

$\Rightarrow a=-4$

Here a = -4 satisfies 4 + a + 0 = 0 also, therefore a = -4

Putting a = -4 in eq. (i), 11 - 12 + b = 0 $\Rightarrow b-1=0 \Rightarrow b=1$

Here also b = 1 satisfies 3 + a + b = 0, therefore b = 1

Therefore, a = - 4 and b = 1

OR

Here, we have:

$A=\begin{bmatrix}1 & 0 & -2\\-2 & -1 & 2\\3 & 4 & 1\end{bmatrix}$

$A^3 = A^2.A$

$A^2=\begin{bmatrix}1 & 0 & -2\\-2 & -1 & 2\\3 & 4 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & -2\\-2 & -1 & 2\\3 & 4 & 1\end{bmatrix}$

$=\begin{bmatrix}1+0-6 & 0+0-8 & -2+0-2\\-2+2+6 & 0+1+8 & 4-2+2\\3-8+3 & 0-4+4 & -6+8+1\end{bmatrix}=\begin{bmatrix}-5 & -8 & -4\\6 & 9 & 4\\-2 & 0 & 3\end{bmatrix}$

$A^2.A = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$= \begin{bmatrix} -5^2 + 16 - 12 & 0 - 8 + 16 & 10 - 16 - 4 \\ 6 - 18 + 12 & 0 - 9 + 16 & -12 + 18 + 4 \\ -2 - 0 + 9 & 0 - 0 - 12 & 4 + 0 + 3 \end{bmatrix}$

$= \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix}$

Now, $A^3 - A^2 - 3A - I$

$= \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix} - \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1 + 5 & -8 + 8 & -10 + 4 \\ 0 - 6 & 7 - 9 & 10 - 4 \\ 7 + 2 & 12 - 0 & 7 - 3 \end{bmatrix} + \begin{bmatrix} -3 - 1 & -0 - 0 & 6 - 0 \\ 6 - 0 & +3 - 1 & -6 - 0 \\ -9 - 0 & -12 + 0 & -3 - 1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 & -6 \\ -6 & -2 & 6 \\ 9 & 12 & 4 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 6 \\ 6 & 2 & -6 \\ -9 & -12 & -4 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Thus, $A^3 - A^2 - 3A - I = 0$

Multiply both sides by A^{-1}, we get

$A^{-1}A^3 - A^{-1}A^2 - 3A^{-1}A - IA^{-1} = 0$

$A^2 - A - 3I = A^{-1}$ ...(since $A^{-1}A = I$)

$\Rightarrow A^{-1} = (A^2 - A - 3I)$

$= \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$= \begin{bmatrix} -5-1-3 & -8-0-0 & -4+2-0 \\ 6+2-0 & 7+1-3 & 4-2-0 \\ -2-3-0 & 0-4-0 & 3-1-3 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

Hence, $A^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

37. The equation of the plane containing the given point is

$A(x - 1) + B(y + 1) + C(z - 2) = 0$....[i]

Condition of $\perp$ to the plane given in (i) with the planes

$2x + 3y - 2z = 5$, $x + 2y - 3z = 8$ implies,

$2A + 3B - 2C = 0$

$A + 2B - 3C = 0$

On solving these equations we get

$A = -5, B = 4, C = 1$

Required equation of the plane is:-

$5x - 4y - z = 7$

OR

Given Cartesian equations of lines

$L_1 = \frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$

Line L1 is passing through point (1, -1, 1) and has direction ratios (3, 2, 5)

Thus, vector equation of line L1 is

$\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 5\hat{k})$

And

$L_2 : \frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$

Line L_2 is passing through point (2, -1, 1) and has direction ratios (2, 3, -2)

Thus, vector equation of line L2 is

$\vec{\mathbf{r}} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) + \mu(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

Now, to calculate distance between the lines,

$\vec{r} = (\hat{\imath} - \hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 5\hat{k})$

$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(2\hat{i} + 3\hat{j} - 2\hat{k})$

Here, we have

$\overrightarrow{a_1} = 1 - j + \hat{k}$

$\overrightarrow{b_1} = 3\hat{i} + 2\hat{j} + 5\hat{k}$

$\overrightarrow{a_2} = 2\hat{i} + \hat{j} - \hat{k}$

$\overrightarrow{b_2} = 2\hat{i} + 3\hat{j} - 2\hat{k}$

Thus,

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 5 \\ 2 & 3 & -2 \end{vmatrix}$$

$= \hat{i}(-4 - 15) - \hat{j}(-6 - 10) + \hat{k}(9 - 4)$

$\Rightarrow \overrightarrow{b_1} \times \overrightarrow{b_2} = -19\hat{i} + 16\hat{j} + 5\hat{k}$

$\Rightarrow |\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-19)^2 + 16^2 + 5^2}$

$= \sqrt{361 + 256 + 25}$

$= \sqrt{642}$

$\overrightarrow{a_2} - \overrightarrow{a_1} = (2 - 1)\hat{i} + (1 + 1)\hat{j} + (-1 - 1)\hat{k}$

$\therefore \overrightarrow{a_2} - \overrightarrow{a_1} = 1 + 2\hat{j} - 2\hat{k}$

Now,

$(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1}) = (-19\hat{i} + 16\hat{j} + 5\hat{k}) \cdot (\hat{i} + 2\hat{j} - 2\hat{k})$

$= ((-19) \times 1) + (16 \times 2) + (5 \times (-2))$

= -19 + 32 - 10

= 3

Thus, the shortest distance between the given lines is

$$d = \left| \frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|} \right|$$

$\Rightarrow d = \left| \frac{3}{\sqrt{642}} \right|$

$\therefore d = \frac{3}{\sqrt{642}}$ units

As $d \neq 0$

Hence, given lines do not intersect each other.

38. Let the manufacturer produces x units per day of product A and y units per day of

product B respectively. Using the given information, construct the following table to make the required in equations and the objective functions.

Products	Produce(in units)	Machine I(in hours)	Machine II (in hours)	Profit (in Rs)
A	x units	3x hours	3x hours	Rs. 7x
B	y units	2y hours	1y hours	Rs. 4y
Total	(x + y) units	(3x + 2y) hours	(3x + y) hours	Rs. (7x + 4y)
Availability		maximum 12 hours	maximum 9 hours	to be maximised

To get the maximum profit, we need to take the objective function , as 7x+ 4y, which the equation for the profit function of the given data.

i.e. Let Z = 7x + 4y be the objective function to be maximised subject to the following constraints,

3x + 2y $\leq$ 12 (time constraint for machine I)

3x + y $\leq$ 9 (time constraint or machine II)

and x $\geq$ 0, y $\geq$0 (non negative constraints which represents the first quadrant)

Now, consider the equations of the given in equations and make a table to sketch the lines in the graph.

3x + 2y = 12........(i)

3x + y = 9 ....(ii)

Table for line 3x + 2y = 12 or $y = \frac{12-3x}{2}$ is

x	0	4
y	6	0

From the above table , we get the information that the line (i) passes though the points (0, 6) and (4, 0).To get the feasible region of the inequality (i),

On replacing the point O(0, 0) in the inequality 3x + 2y $\leq$ 12, we get

0 + 0 $\leq$ 12

$\Rightarrow$0 $\leq$ 12 [true]

So, the half plane includes the origin and represents the region below the line.

Table for line 3x + y = 9 or y = 9- 3x is given as follows.

x	0	3

y	9	0

From the table we can get the points (0, 9) and (3, 0) through which the line (ii) passes. To get the feasible region of the given line (ii) ,

On replacing the point O (0, 0) in the inequality $3x + y \leq 9$, we get $0+0 \leq 9$, which is true. So, the half plane includes the origin and below the line. Also, $x \geq 0$ and $y \geq 0$, is region representing only the Ist quadrant. Hence the feasible region has the corner points OABCO, which is the bounded feasible region.

Now, to find the point of intersection of the given lines (i) and (ii) , we subtract Eq. (ii) from Eq. (i). we get

$(3x+ 2y) - (3x+ y) = 12 - 9$, $3x + 2y - 3x - y = 12$

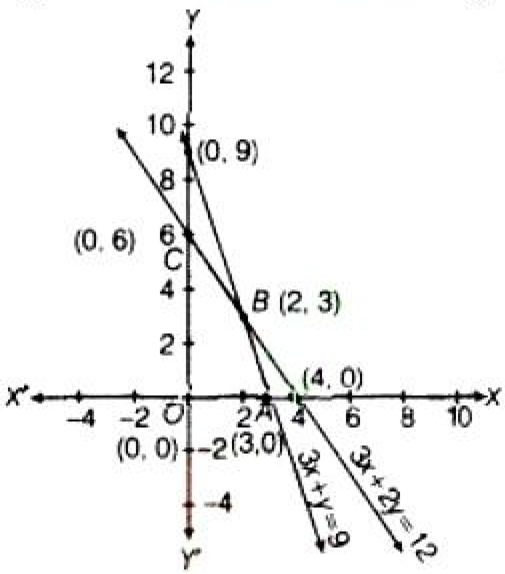
$\Rightarrow y = 3$. Replacing the value of y = 3 in the given equation, we get ,

$\therefore 3x = 12 - 2y$

$= 12 - (2 \times 3) = 6$, when $3x = 6$, we get x = 3.

$\Rightarrow x = 2$

Thus, the point of intersection of the two given lines is at B(2, 3). Hence, all the triple regions meet in the bounded region represented by the corner points OABCO.



Here, we see that OABCO is the required bounded feasible region, whose comer points are O(0, 0), A(3, 0), B(2,3) and C(0, 6) which is the bounded area in the first quadrant only. The values of Z at these comer points are as follows

Corner points	Z = 7x + 4y
O(0, 0)	Z = 0 + 0 = 0
A(3, 0)	$Z = (7 \times 3) + 0 = 21$

B(2, 3)	Z = (7$\times$ 2)+(4$\times$ 3) = 14+ 12=26
C(0, 6)	Z = (7 $\times$ 0)+ (4 $\times$ 6)= 0+ 24 = 24

Hence, to obtain the maximum profit of Rs.26, the manufacturer has to produce 2 units per day of product A and 3 units per day of product B.

OR

Let the fruit grower mix x bags of brand P and Y bags of brand Q

$z = 3x + \frac{7}{2}y$

$x + 2y \geqslant 240$

$3x + \frac{3}{2}y \geqslant 270$

$\frac{3}{2}x + 2y \leqslant 310$

$x \geqslant 0, y \geqslant 0$

Solving these equations, we get

x = 40, y = 100

Then, Z = 3(40) + $\frac{7}{2}$(100) = 470

Hence minimum = 470 Kg

P = 40

Q = 100

Amount of Nitrogen $= 3 \times 40 + 3 \times 100$ = 120 + 150 = 270