1. **સંકલિત મેળવો :** $\int \frac{2x-1}{2x+3}\,dx$

➡ $I = \int \frac{2x-1}{2x+3}\,dx$

$= \int \frac{2x+3-4}{2x+3}\,dx$

$= \int \left(\frac{2x+3}{2x+3} - \frac{4}{2x+3}\right) dx$

$= \int 1\,dx - 4\int \frac{1}{2x+3}\,dx$

$= x - 4\left\{\frac{1}{2}\log|2x+3|\right\} + C$

$= x - 2\log|2x+3| + C$

$\therefore I = x - \log|(2x+3)^2| + C$

2. **સંકલિત મેળવો :** $\int \frac{2x+3}{x^2+3x}\,dx$

➡ $I = \int \frac{2x+3}{x^2+3x}\,dx$

હવે $x^2 + 3x = t$ આદેશ લેતાં,

$\therefore (2x+3)dx = dt$

$\therefore I = \int \frac{1}{t}\,dt$

$= \log|t| + C$

$\therefore I = \log|x^2+3x| + C$

3. **સંકલિત મેળવો :** $\int \frac{(x^2+2)}{x+1}\,dx$

➡ $I = \int \frac{x^2+2}{x+1}\,dx$

$= \int \frac{x^2-1+3}{x+1}\,dx$

$= \int \left(\frac{x^2-1}{x+1} + \frac{3}{x+1}\right) dx$

$= \int (x-1)\,dx + 3\int \frac{1}{x+1}\,dx$

$= \int x\,dx - \int 1\,dx + 3\int \frac{1}{x+1}\,dx$

$\therefore I = \frac{x^2}{2} - x + 3\log|x+1| + C$

4. **સંકલિત મેળવો :** $\int \frac{e^{6\log x} - e^{5\log x}}{e^{4\log x} - e^{3\log x}}\, dx$

➡ $I = \int \frac{e^{6\log x} - e^{5\log x}}{e^{4\log x} - e^{3\log x}}\, dx$

$= \int \frac{e^{\log x^6} - e^{\log x^5}}{e^{\log x^4} - e^{\log x^3}}\, dx$

$= \int \frac{x^6 - x^5}{x^4 - x^3}\, dx$

$= \int \frac{x^5(x-1)}{x^3(x-1)}\, dx$

$= \int \frac{x^5}{x^3}\, dx$

$= \int x^2\, dx$

$\therefore I = \int \frac{x^3}{3} + C$

5. **સંકલિત મેળવો :** $\int \frac{1 + \cos x}{x + \sin x}\, dx$

➡ $\therefore I = \int \frac{1 + \cos x}{x + \sin x}\, dx$

અહીં $x + \sin x = t$ આદેશ મૂકો.

$\therefore (1 + \cos x)\, dx = dt$

$I = \int \frac{1}{t}\, dt$

$= \log |t| + C$

$I = \log |x + \sin x| + C$

6. **સંકલિત મેળવો :** $\int \frac{1}{1 + \cos x}\, dx$

➡ $I = \int \frac{1}{1 + \cos x}\, dx$

$= \int \frac{1}{2\cos^2\left(\frac{x}{2}\right)}\, dx$

$= \int \frac{1}{2} \sec^2\left(\frac{x}{2}\right) dx$

હવે $\frac{x}{2} = t$ આદેશ લેતાં,

$\therefore \frac{1}{2}\, dx = dt$

$\therefore I = \int \sec^2 t\ \mathrm{dt}$

$= \tan(t) + C$

$\therefore I = \tan\left(\frac{x}{2}\right) + C$

7. **સંકલિત મેળવો :** $\int \tan^2 x \sec^4 x\, dx$

➡ $I = \int \tan^2 x \sec^4 x\, dx$

$= \int \tan^2 x \cdot \sec^2 x \cdot \sec^2 x\, dx$

$= \int \tan^2 x\, (1 + \tan^2 x) \sec^2 x\, dx$

હવે $\tan x = t$ આદેશ લેતાં,

$\therefore \sec^2 x\, dx = dt$

$\therefore I = \int t^2(1 + t^2)dt$

$= \int(t^4 + t^2)dt$

$= \frac{t^5}{5} + \frac{t^3}{3} + C$

$\therefore I = \frac{\tan^5 x}{5} + \frac{\tan^3 x}{3} + C$

8. સંકલિત મેળવો : $\int \frac{\sin x + \cos x}{\sqrt{1 + \sin(2x)}} dx$

➡ અહીં $1 + \sin(2x)$

$= \sin^2 x + \cos^2 x + 2\sin x \cos x$

$= (\sin x + \cos x)^2$

$\therefore \sqrt{1 + \sin 2x} = \sqrt{(\sin x + \cos x)^2}$

$= \sin x + \cos x$

$\therefore I = \int \frac{\sin x + \cos x}{\sqrt{1 + \sin(2x)}} dx$

$= \int \frac{\sin x + \cos x}{\sin x + \cos x} dx$

$= \int 1\, dx$

$\therefore I = x + C$

9. સંકલિત મેળવો : $\int \sqrt{1 + \sin x}\, dx$

➡ અહીં $1 + \sin x$

$= \sin^2\left(\frac{x}{2}\right) + \cos^2\left(\frac{x}{2}\right) + 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$

$= \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2$

$\therefore \sqrt{1 + \sin x} = \sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}$

$= \sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)$

$I = \int \sqrt{1 + \sin x}$

$= \int\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) dx$

$= \frac{-\cos\left(\frac{x}{2}\right)}{\frac{1}{2}} + \frac{\sin\left(\frac{x}{2}\right)}{\frac{1}{2}} + C$

$\therefore I = -2\cos\left(\frac{x}{2}\right) + 2\sin\left(\frac{x}{2}\right) + C$

10. સંકલિત મેળવો : $\int \frac{1}{\sqrt{16 - 9x^2}} dx$

➡ $I = \int \frac{1}{\sqrt{16 - 9x^2}} dx$

$$= \frac{1}{\sqrt{9\left(\frac{16}{9} - x^2\right)}}\, dx$$

$$= \frac{1}{3}\int \frac{1}{\sqrt{\left(\frac{4}{3}\right)^2 - x^2}}\, dx = \frac{1}{3}\sin^{-1}\left(\frac{x}{\frac{4}{3}}\right) + C$$

$$\therefore I = \frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right) + C$$

11. સંકલિત મેળવો : $\int \frac{x}{x^4 - 1}\, dx$

➡ $I = \int \frac{x}{x^4 - 1}\, dx$

$x^2 = t$ આદેશ લેતાં,

$\therefore 2x\, dx = dt$

$\therefore x\, dx = \frac{1}{2}\, dt$

$$\therefore I = \frac{1}{2}\int \frac{1}{t^2 - 1}\, dt$$

$$= \frac{1}{2}\left\{\frac{1}{2}\log\left|\frac{t - 1}{t + 1}\right|\right\} + C$$

$$= \frac{1}{4}\log\left|\frac{x^2 - 1}{x^2 + 1}\right| + C$$

$$\therefore I = \frac{1}{4}\left\{\log|x^2 - 1| - \log|x^2 + 1|\right\} + C$$

12. સંકલિત મેળવો : $\int \frac{\cos x - \cos 2x}{1 - \cos x}\, dx$

➡ $I = \int \frac{\cos x - \cos 2x}{1 - \cos x}\, dx$

$$= \int \frac{-2\sin\left(\frac{3x}{2}\right)\sin\left(\frac{-x}{2}\right)}{2\sin^2\left(\frac{x}{2}\right)}\, dx$$

$\left(\because \cos C - \cos D = -2\sin\left(\frac{C + D}{2}\right)\sin\left(\frac{C - D}{2}\right)\right.$
અને $\left. 1 - \cos x = 2\sin^2\left(\frac{x}{2}\right)\right)$

$$= \int \frac{\sin\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right)}{\sin^2\left(\frac{x}{2}\right)}\, dx \qquad (\because \sin(-\theta) = -\sin\theta)$$

$$= \int \frac{\sin\left(\frac{3x}{2}\right)}{\sin\left(\frac{x}{2}\right)}\, dx$$

$$= \int \frac{3\sin\left(\frac{x}{2}\right) - 4\sin^3\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)}\, dx$$

($\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta$ સૂત્રમાં $\theta = \frac{x}{2}$ મૂકતાં)

$= \int\left(3 - 4\sin^2\left(\frac{x}{2}\right)\right) dx$

$= 3\int 1\, dx - 4\int \left(\frac{1-\cos x}{2}\right) dx$

$= 3\int 1\, dx - 2\int 1\, dx + 2\int \cos x\, dx$

$= 3x - 2x + 2\sin x + C$

$\therefore\ I = x + 2\sin x + C$

13. **નિયત સંકલિત મેળવો :** $\displaystyle\int_1^2 \frac{1}{\sqrt{(x-1)(2-x)}}\, dx$

➡ અહીં $(x - 1)(2 - x) = 2x - x^2 - 2 + x$

$= -x^2 + 3x - 2$

$= -x^2 + 2\left(\frac{3}{2}\right)x - \frac{9}{4} + \frac{9}{4} - 2$

$= \frac{1}{4} - \left(x^2 - 2\left(\frac{3}{2}\right)x + \frac{9}{4}\right)$

$= \frac{1}{4} - \left(x - \frac{3}{2}\right)^2$

$= \left(\frac{1}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2$

$$\therefore\ I = \int_1^2 \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2}}\, dx$$

$$= \left[\sin^{-1}\left(\frac{x - \frac{3}{2}}{\frac{1}{2}}\right)\right]_1^2$$

$$\left(\because \int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + C\right)$$

$$= \left[\sin^{-1}(2x - 3)\right]_1^2$$

$= \sin^{-1}(4 - 3) - \sin^{-1}(2 - 3)$

$= \sin^{-1}(1) - \sin^{-1}(-1)$

$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right)$

$= \frac{\pi}{2} + \frac{\pi}{2}$

$\therefore\ I = \pi$

14. **સંકલિત મેળવો :** $\displaystyle\int \frac{x}{\sqrt{x} + 1}\, dx$

➡ $I = \displaystyle\int \frac{x}{\sqrt{x} + 1}\, dx$

હવે $\sqrt{x} = t$ આદેશ લેતાં,

$\therefore\ x = t^2$

$\therefore\ dx = 2t\ dt$

$$\therefore \text{I} = \int \frac{t^2}{t+1} \cdot 2t\ dt$$

$$= 2\int \frac{t^3}{t+1}\ dt$$

$$\text{I} = 2\int \frac{(t^3+1)-1}{t+1}\ dt$$

$$= 2\int \left(\frac{t^3+1}{t+1} - \frac{1}{t+1}\right) dt$$

$$= 2\left\{\int \frac{(t+1)\ (t^2-t+1)}{t+1}\ dt - \int \frac{1}{t+1}\ dt\right\} + 1$$

$$= 2\left\{\int (t^2 - t + 1)\, dt - \log|t+1|\right\} + \text{C}$$

$$\text{I} = 2\left\{\frac{t^3}{3} - \frac{t^2}{2} + t - \log|t+1|\right\} + \text{C} \qquad ....(1)$$

હવે $\sqrt{x} = t$

$\therefore\ x^{\frac{1}{2}} = t$

$$\therefore \left(x^{\frac{1}{2}}\right)^3 = t^3$$

$\therefore\ t^3 = x^{\frac{3}{2}}$ અને $x^{\frac{1}{2}} = t \Rightarrow x = t^2$

$$\therefore \text{I} = 2\left\{\frac{x^{\frac{3}{2}}}{3} - \frac{x}{2} + \sqrt{x} - \log\left|\sqrt{x}+1\right|\right\} + \text{C}$$

15. સંકલિત મેળવો : $\int \sqrt{\frac{a+x}{a-x}}\ dx$

➡ $\text{I} = \int \sqrt{\frac{a+x}{a-x}}\ dx$

અહીં $x = a\cos(2\theta)$ આદેશ લેતાં,

$dx = -2a\ \sin(2\theta)\ d\theta$

તથા $\sqrt{\frac{a+x}{a-x}} = \sqrt{\frac{a + a\cos(2\theta)}{a - a\cos(2\theta)}}$

$$= \sqrt{\frac{1+\cos(2\theta)}{1-\cos 2\theta}}$$

$$= \sqrt{\frac{2\cos^2\theta}{2\sin^2\theta}}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$\text{I} = \int \frac{\cos\theta}{\sin\theta}\ (-2a\ \sin 2\theta)\ d\theta$$

$$= -2a \int \frac{\cos\theta}{\sin\theta} \times 2\sin\theta\ \cos\theta\ d\theta$$

$= -2a \int 2\cos^2\theta \, d\theta$

$= -2a \int (1 + \cos 2\theta) \, d\theta$

$= -2a \left\{\theta + \frac{\sin 2\theta}{2}\right\} + C$

$\therefore I = -2a \left\{\frac{2\theta + \sin 2\theta}{2}\right\} + C$

$= -a \{2\theta + \sin 2\theta\} + C$

હવે $x = a \cos 2\theta$

$\therefore \frac{x}{a} = \cos(2\theta)$

$\therefore 2\theta = \cos^{-1}\left(\frac{x}{a}\right)$ અને $\sin 2\theta = \sqrt{1 - \cos^2(2\theta)}$

$= \sqrt{1 - \frac{x^2}{a^2}}$

$\therefore I = -a \left\{\cos^{-1}\left(\frac{x}{a}\right) + \sqrt{1 - \frac{x^2}{a^2}}\right\} + C$

અન્ય રીત :

➡ $I = \int \sqrt{\frac{a + x}{a - x}} \, dx$

અહીં $x = a \cos(2\theta)$ આદેશ લેતાં,

$dx = -2a \sin(2\theta) \, d\theta$

તથા $\sqrt{\frac{a + x}{a - x}} = \sqrt{\frac{a + a\cos(2\theta)}{a - a\cos(2\theta)}}$

$= \sqrt{\frac{1 + \cos(2\theta)}{1 - \cos 2\theta}}$

$= \sqrt{\frac{2\cos^2\theta}{2\sin^2\theta}}$

$= \frac{\cos\theta}{\sin\theta}$

$I = \int \frac{\cos\theta}{\sin\theta} (-2a \sin 2\theta) \, d\theta$

$= -2a \int \frac{\cos\theta}{\sin\theta} \times 2\sin\theta \cos\theta \, d\theta$

$= -2a \int 2\cos^2\theta \, d\theta$

$= -2a \int (1 + \cos 2\theta) \, d\theta$

$= -2a \left\{\theta + \frac{\sin 2\theta}{2}\right\} + C$

$\therefore I = -2a \left\{\frac{2\theta + \sin 2\theta}{2}\right\} + C$

$= -a \{2\theta + \sin 2\theta\} + C$

હવે $x = a \cos 2\theta$

$\therefore \frac{x}{a} = \cos(2\theta)$

$\therefore 2\theta = \cos^{-1}\left(\frac{x}{a}\right)$ અને $\sin 2\theta = \sqrt{1 - \cos^2(2\theta)}$

$$= \sqrt{1 - \frac{x^2}{a^2}}$$

$$\therefore I = -a \left\{ \cos^{-1}\left(\frac{x}{a}\right) + \sqrt{1 - \frac{x^2}{a^2}} \right\} + C$$

અન્ય રીત :

16. સંકલિત મેળવો : $\int \frac{x^{\frac{1}{2}}}{1 + x^{\frac{3}{4}}} \, dx$

➥ $I = \int \frac{x^{\frac{1}{2}}}{1 + x^{\frac{3}{4}}} \, dx$

હવે $x^{\frac{1}{4}} = t$ આદેશ લેતાં,

$\therefore x = t^4$

$\therefore dx = 4t^3 \, dt$

તથા $x = t^4 \Rightarrow x^{\frac{1}{2}} = (t^4)^{\frac{1}{2}}$

$\therefore x^{\frac{1}{2}} = t^2$

$$\therefore I = \int \frac{t^2 \cdot (4t^3) \, dt}{1 + t^3}$$

$$= 4 \int \frac{t^2 \cdot t^3}{1 + t^3} \, dt$$

$$= 4 \int \frac{t^2 \cdot t^3 + t^2 - t^2}{1 + t^3} \, dt$$

$$= 4 \int \left(\frac{t^2(t^3 + 1)}{t^3 + 1} - \frac{t^2}{1 + t^3} \right) dt$$

$$= 4 \int t^2 \, dt - 4 \int \frac{t^2}{1 + t^3} \, dt$$

$$= \frac{4t^3}{3} - \frac{4}{3} \int \frac{3t^2}{1 + t^3} \, dt$$

$$= \frac{4t^3}{3} - \frac{4}{3} \int \frac{f'(t)}{f(t)} \, dt$$

જ્યાં $f(t) = 1 + t^3$

$\therefore f'(t) = 3t^2$

$$\therefore I = \frac{4}{3} t^3 - \frac{4}{3} \log |f(t)| + C$$

$$= \frac{4}{3} t^3 - \frac{4}{3} \log |1 + t^3| + C$$

$$\therefore I = \frac{4}{3} \left(x^{\frac{3}{4}} \right) - \frac{4}{3} \log |1 + x^{\frac{3}{4}}| + C$$

17. સંકલિત મેળવો : $\int \frac{\sqrt{1 + x^2}}{x^4} \, dx$

➥ $I = \int \frac{\sqrt{1 + x^2}}{x} \times \frac{1}{x^3} \, dx$

$$= \int \sqrt{\frac{1 + x^2}{x^2}} \cdot \frac{1}{x^3}\, dx$$

$$= \int \left(\sqrt{1 + \frac{1}{x^2}}\right) \cdot \left(\frac{1}{x^3}\right) dx$$

અહીં $1 + \frac{1}{x^2} = t^2$ આદેશ લેતાં,

$\therefore\ 1 + x^{-2} = t^2$

$\therefore\ (0 + -2x^{-3})dx = 2t\, dt$

$\therefore\ x^{-3}\, dx = -t\, dt$

$\therefore\ \frac{1}{x^3}\, dx = -t\, dt$

$$\therefore \text{I} = -\int t^2 \cdot dt = -\frac{t^3}{3} + \text{C}$$

$$= -\frac{1}{3}\left(1 + \frac{1}{x^2}\right)^{\frac{3}{2}} + \text{C}$$

18. **સંકલિત મેળવો :** $\int \frac{1}{\sqrt{3t - 2t^2}}\, dt$

➡ અહીં $3t - 2t^2$

$$= -2\left(t^2 - \frac{3}{2}t\right)$$

$$= -2\left\{t^2 - 2\left(\frac{1}{2}\right) \cdot \frac{3}{2}\, t\right\}$$

$$= -2\left\{t^2 - 2\left(\frac{3}{4}\right) t + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right\}$$

$$= -2\left\{\left(t - \frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right\}$$

$$= 2\left\{\left(\frac{3}{4}\right)^2 - \left(t - \frac{3}{4}\right)^2\right\}$$

$$\therefore \text{I} = \int \frac{1}{\sqrt{2\left[\left(\frac{3}{4}\right)^2 - \left(t - \frac{3}{4}\right)^2\right]}}\, dt$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(\frac{3}{4}\right)^2 - \left(t - \frac{3}{4}\right)^2}}\, dt$$

$$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{t - \frac{3}{4}}{\frac{3}{4}}\right) + \text{C}$$

$$\left(\because \int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + \text{C}\right)$$

$$\therefore \text{I} = \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{4t - 3}{3}\right) + \text{C}$$

19. **સંકલિત મેળવો :** $\int \frac{3x-1}{\sqrt{x^2+9}}\,dx$

➡ $I = \int \frac{3x-1}{\sqrt{x^2+9}}\,dx$

$= \int \left(\frac{3x}{\sqrt{x^2+9}} - \frac{1}{\sqrt{x^2+9}}\right) dx$

$= 3\int \frac{3x}{\sqrt{x^2+9}}\,dx - \int \frac{3x}{\sqrt{x^2+9}}\,dx$

$= 3\,I_1 - I_2$ ....(1)

હવે $I_1 = 3\int \frac{x}{\sqrt{x^2+9}}\,dx$

$\therefore \sqrt{x^2+9} = t$ આદેશ લેતાં,

$\therefore x^2 + 9 = t^2$

$\therefore 2x\,dx = 2t\,dt$

$\therefore x\,dx = t\,dt$

$\therefore I_1 = 3\int \frac{t\,dt}{t} = 3\int 1\,dt$

$= 3t + C$

$\therefore I_1 = 3\left(\sqrt{x^2+9}\right) + C'$ ....(i)

તથા $I_2 = \int \frac{1}{\sqrt{x^2+9}}\,dx$

$= \log|x + \sqrt{x^2+9}| + C''$ ....(ii)

$\therefore I = 3\left(\sqrt{x^2+9}\right) - \log|x + \sqrt{x^2+9}| + C$

(પરિણામ (i) અને (ii) પરથી) જ્યાં $C = C' - C''$

20. **સંકલિત મેળવો :** $\int \sqrt{5-2x+x^2}\,dx$

➡ $I = \int \sqrt{x^2-2x+5}\,dx$

$= \int \sqrt{x^2-2x+1+4}\,dx$

$= \int \sqrt{(x-1)^2+(2)^2}\,dx$

હવે $x - 1 = t$ આદેશ લેતાં,

$\therefore dx = dt$

$= \int \sqrt{(t^2)+(2)^2}\,dt$

$= \frac{t\cdot\sqrt{t^2+(2)^2}}{2} + \frac{(2)^2}{2}\log|t + \sqrt{t+4}|$

($\because \int \sqrt{a^2+x^2}$ ના સૂત્ર મુજબ)

$= \left\{\frac{(x-1)}{2}\sqrt{(x-1)^2+4} + 2\log|(x-1)+\sqrt{(x-1)^2+4}|\right\}$

$= \frac{x-1}{2}\cdot\sqrt{x^2-2x+5} + 2\log|(x-1)+\sqrt{x^2-2x+5}| + C$

21. **સંકલિત મેળવો :** $\int \frac{x^2}{1-x^4}\,dx$

➡ $I = \int \frac{x^2}{1-x^4}\,dx$

$= \frac{1}{2}\int \frac{2x^2}{1-x^4}\,dx$

$= \frac{1}{2}\int \frac{1+x^2-1+x^2}{1-x^4}\,dx$

$= \frac{1}{2}\int \frac{(1+x^2)-(1-x^2)}{1-x^4}\,dx$

$= \frac{1}{2}\left\{\int \frac{1+x^2}{1-x^4}\,dx - \int \frac{1-x^2}{1-x^4}\,dx\right\}$

$= \frac{1}{2}\left\{\int \frac{1}{1-x^2}\,dx - \int \frac{1}{1+x^2}\,dx\right\}$

$\therefore I = \frac{1}{2}\left\{\frac{1}{2}\log\left|\frac{1+x}{1-x}\right| - \tan^{-1}x\right\} + C$

$\therefore I = \frac{1}{4}\log\left|\frac{1+x}{1-x}\right| - \frac{1}{2}\tan^{-1}x + C$

22. **સંકલિત મેળવો :** $\int \sqrt{2ax - x^2}\,dx$

➡ $I = \int \sqrt{2ax - x^2}\,dx$

$= \int \sqrt{a^2 - a^2 + 2ax - x^2}\,dx$

$= \int \sqrt{a^2 - (x-a)^2}\,dx$

હવે $x - a = t$ આદેશ મૂકો.

$\therefore\ dx = dt$

$I = \int \sqrt{a^2 - t^2}\,dt$

$= \frac{t \cdot \sqrt{a^2 - t^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{t}{a}\right) + C$

$= \frac{t \cdot \sqrt{a^2 - (x-a)^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C$

$\therefore I = \frac{(x-a)\sqrt{2ax - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C$

23. **સંકલિત મેળવો :** $\int \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}\,dx$

➡ $I = \int \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}\,dx$

$= \int \frac{\sin^{-1}x}{(1-x^2)\cdot\sqrt{1-x^2}}\,dx$

હવે $\sin^{-1}x = t$ મૂકો.

$\therefore \frac{1}{\sqrt{1-x^2}}\, dx = dt$

તથા $x = \sin t$ થાય.

$\therefore I = \frac{t}{1-\sin^2 t}\, dt$

$= \int \frac{t}{\cos^2 t}\, dt$

$= \int t \sec^2 t\, dt$

$u = t$ અને $v = \sec^2 t$ લઈ ખંડશઃ સંકલન કરતાં,

$\frac{du}{dt} = 1$ અને $\int v\, dt = \int \sec^2 t\, dt = \tan t$

$I = u \int v\, dt - \int (u' \int v dt)\, dt$

$= t(\tan t) - \int \tan t\, dt$

$= t(\tan t) - \log|\sec t| + C$

$= t(\tan t) + \log|\cos t| + C$

$= \frac{t \cdot \sin t}{\cos t} + \log|\cos t| + C$

$\therefore I = \frac{(\sin^{-1}x)x}{\sqrt{1-x^2}} + \log|\sqrt{1-x^2}| + C$

24. સંકલિત મેળવો : $\int \frac{\cos(5x) + \cos(4x)}{1 - 2\cos(3x)}\, dx$

➡ $I = \int \frac{2\cos\left(\frac{9x}{2}\right)\cos\left(\frac{x}{2}\right)}{1 - 2\left(2\cos^2\left(\frac{3x}{2}\right) - 1\right)}\, dx$

$= \int \frac{2\cos\left(\frac{9x}{2}\right)\cos\left(\frac{x}{2}\right)}{3 - 4\cos^2\left(\frac{3x}{2}\right)}\, dx$

$= -\int \frac{2\cos\left(\frac{9x}{2}\right)\cos\left(\frac{x}{2}\right)}{4\cos^2\left(\frac{3x}{2}\right) - 3}\, dx$

$= -\int \frac{2\cos\left(\frac{9x}{2}\right)\cos\left(\frac{x}{2}\right)\cos\left(\frac{3x}{2}\right)}{4\cos^3\left(\frac{3x}{2}\right) - 3\cos\left(\frac{3x}{2}\right)}\, dx$

$= -\int \frac{2\cos\left(\frac{9x}{2}\right)\cos\left(\frac{x}{2}\right)\cos\left(\frac{3x}{2}\right)}{\cos\left(\frac{3(3x)}{2}\right)}\, dx$

($\because \cos(3\theta) = 4\cos^3\theta - 3\cos\theta$ ના સૂત્ર મુજબ)

$I = \int -2\cos\left(\frac{3x}{2}\right)\cos\left(\frac{x}{2}\right)\, dx$

$= -\int [\cos(2x) + \cos(x)]\, dx$

$\therefore I = -\left\{\frac{\sin(2x)}{2} + \sin x\right\} + C$

25. **સંકલિત મેળવો :** $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x}\, dx$

➡ અહીં $\sin^6 x + \cos^6 x$

$= (\sin^2 x)^3 + (\cos^2 x)^3$

$= (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x \cos^2 x\, (\sin^2 x + \cos^2 x)$

$(\because a^3 + b^3 = (a + b)^3 - 3ab(a + b))$

$= 1 - 3\sin^2 x \cos^2 x$

$\therefore I = \int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cdot \cos^2 x}\, dx$

$= \int \frac{1 - 3\sin^2 x \cos^2 x}{\sin^2 x \cos^2 x}\, dx$

$= \int \left(\frac{1}{\sin^2 x \cos^2 x} - 3\right) dx$

$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\, dx - 3\int 1\, dx$

$= \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right) dx - 3x$

$= \int \sec^2 x\, dx + \int \operatorname{cosec}^2 x\, dx - 3x$

$\therefore I = \tan x - \cot x - 3x + C$

અન્ય રીત :

$= \int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x}\, dx$

$= \int \frac{(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x)}{\sin^2 x \cos^2 x}\, dx$

$= \int \frac{\sin^4 x}{\sin^2 x \cos^2 x}\, dx - \int \frac{\sin^2 x \cos^2 x}{\sin^2 x \cos^2 x}\, dx + \int \frac{\cos^4 x}{\sin^2 x \cos^2 x}\, dx$

$= \int \frac{\sin^2 x}{\cos^2 x}\, dx - \int 1\, dx + \int \frac{\cos^2 x}{\sin^2 x}\, dx$

$= \int \tan^2 x\, dx + \int \cot^2 x\, dx - \int 1\, dx$

➡ અહીં $\sin^6 x + \cos^6 x$

$= (\sin^2 x)^3 + (\cos^2 x)^3$

$= (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x \cos^2 x\, (\sin^2 x + \cos^2 x)$

$(\because a^3 + b^3 = (a + b)^3 - 3ab(a + b))$

$= 1 - 3\sin^2 x \cos^2 x$

$\therefore I = \int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cdot \cos^2 x}\, dx$

$= \int \frac{1 - 3\sin^2 x \cos^2 x}{\sin^2 x \cos^2 x}\, dx$

$$= \int \left(\frac{1}{\sin^2 x \cos^2 x} - 3 \right) dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} \, dx - 3 \int 1 dx$$

$$= \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \right) dx - 3x$$

$$= \int \sec^2 x \, dx + \int \text{cosec}^2 x \, dx - 3x$$

$$\therefore \ \text{I} = \tan x - \cot x - 3x + \text{C}$$

અન્ય રીત :

$$\text{I} = \int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} \, dx$$

$$= \int \frac{(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x)}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^4 x}{\sin^2 x \cos^2 x} \, dx - \int \frac{\sin^2 x \cos^2 x}{\sin^2 x \cos^2 x} \, dx + \int \frac{\cos^4 x}{\sin^2 x \cos^2 x} \, dx$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, dx - \int 1 \, dx + \int \frac{\cos^2 x}{\sin^2 x} \, dx$$

$$= \int \tan^2 x \, dx + \int \cot^2 x \, dx - \int 1 \, dx$$

26. સંકલિત મેળવો : $\displaystyle\int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} \, dx$

➡ $$\text{I} = \int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} \, dx$$

$$= \int \frac{\sqrt{x}}{\sqrt{\left(a^{\frac{3}{2}}\right)^2 - \left(x^{\frac{3}{2}}\right)^2}} \, dx$$

હવે $x^{\frac{3}{2}} = t$ આદેશ લેતાં,

$$\therefore \frac{3}{2} \left(x^{\frac{1}{2}} \right) dx = dt$$

$$\therefore \sqrt{x} \, dx = \frac{2}{3} \, dt$$

$$\text{I} = \int \frac{\frac{2}{3}}{\sqrt{\left(a^{\frac{3}{2}}\right)^2 - t^2}} \, dt$$

$$= \frac{2}{3} \sin^{-1} \left(\frac{t}{a^{\frac{3}{2}}} \right) + \text{C}$$

$$= \frac{2}{3} \sin^{-1} \left(\frac{x^{\frac{3}{2}}}{a^{\frac{3}{2}}} \right) + \text{C}$$

$= \frac{2}{3} \sin^{-1} \left(\frac{x^3}{a^3}\right)^{\frac{1}{2}} + C$

$\therefore I = \frac{2}{3} \sin^{-1} \left(\sqrt{\frac{x^3}{a^3}}\right) + C$

27. સંકલિત મેળવો : $\int \frac{1}{x\sqrt{x^4 - 1}} dx$

➡ $I = \int \frac{1}{x\sqrt{x^4 - 1}} dx$

અહીં $x^2 = \sec\theta$ આદેશ લેતાં,

$\therefore 2x\, dx = \sec\theta \tan\theta\, d\theta$

$dx = \frac{1}{2x}(\sec\theta \tan\theta)\, d\theta$

$\therefore I = \int \frac{\sec\theta \tan\theta}{2x^2\sqrt{\sec^2\theta - 1}} d\theta$

$= \int \frac{\sec\theta \tan\theta}{2 \sec\theta \tan\theta} d\theta$

$= \frac{1}{2} \int 1\, d\theta$

$= \frac{1}{2} (\theta)$

$\therefore I = \frac{1}{2} \sec^{-1}(x^2) + C$

28. $\int_0^2 (x^2 + 3)\, dx$ ને સરવાળાના લક્ષ તરીકે દર્શાવો.

➡ અહીં $a = 0,\ b = 2$

$\therefore h = \frac{b - a}{h} = \frac{2 - 0}{h}$

$\therefore h = \frac{2}{h}$ અર્થાત્ $nh = 2$

હવે $\int_a^b f(x)\, dx = \lim_{\substack{h \to 0 \\ h \to \infty}} h \sum_{i=1}^{h} f(a + ih)$

(સરવાળાના લક્ષનું સૂત્ર)

$f(x) = x^2 + 3$

$\therefore f(a + ih) = (a + ih)^2 + 3$

$= (0 + ih)^2 + 3 \quad (\because a = 0$ છે.)

$= i^2 h^2 + 3$

$\therefore \int_0^2 f(x)\, dx = \lim_{n \to \infty} h \sum_{i=1}^{h} (i^2 h^2 + 3)$

$= \lim_{n \to \infty} \left(h \sum_{i=1}^{n} i^2 h^2 + 3h\, \Sigma 1 \right)$

$= \lim_{n \to \infty} h^3 \sum_{i=1}^{n} i^2 + 3h\, \Sigma 1$

$$= \lim_{n \to \infty} \frac{8}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + 3\left(\frac{2}{n}\right) \cdot n$$

$$= \frac{4}{3} \lim_{n \to \infty} \frac{n\left(1 + \frac{1}{n}\right) \cdot n\left(2 + \frac{1}{n}\right)}{n^2} + 6$$

$$= \frac{4}{3} \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) + 6$$

$$= \frac{4}{3} (1 + 0)(2 + 0) + 6$$

$$= \frac{8}{3} + 6$$

➥ અહીં $a = 0$, $b = 2$

$\therefore h = \frac{b - a}{h} = \frac{2 - 0}{h}$

$\therefore h = \frac{2}{h}$ અર્થાત્ $nh = 2$

હવે $\int_a^b f(x)\,dx = \lim_{\substack{h \to 0 \\ h \to \infty}} h \sum_{i=1}^{h} f(a + ih)$

(સરવાળાના લક્ષનું સૂત્ર)

$f(x) = x^2 + 3$

$\therefore f(a + ih) = (a + ih)^2 + 3$

$= (0 + ih)^2 + 3$ $\quad (\because a = 0$ છે.)

$= i^2 h^2 + 3$

$$\therefore \int_0^2 f(x)\,dx = \lim_{n \to \infty} h \sum_{i=1}^{h} (i^2 h^2 + 3)$$

$$= \lim_{n \to \infty} \left(h \sum_{i=1}^{n} i^2 h^2 + 3h\,\Sigma 1\right)$$

$$= \lim_{n \to \infty} h^3 \sum_{i=1}^{n} i^2 + 3h\,\Sigma 1$$

$$= \lim_{n \to \infty} \frac{8}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + 3\left(\frac{2}{n}\right) \cdot n$$

$$= \frac{4}{3} \lim_{n \to \infty} \frac{n\left(1 + \frac{1}{n}\right) \cdot n\left(2 + \frac{1}{n}\right)}{n^2} + 6$$

$$= \frac{4}{3} \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) + 6$$

$$= \frac{4}{3} (1 + 0)(2 + 0) + 6$$

$$= \frac{8}{3} + 6$$

29. $\int_0^2 e^x\,dx$ **ને સરવાળાના લક્ષ તરીકે દર્શાવો.**

➥ અહીં $a = 0$, $b = 2$

$\therefore h = \dfrac{b-a}{h} = \dfrac{2-0}{h} = \dfrac{2}{h} \quad \therefore h = \dfrac{2}{h}$

$\therefore f(x) = e^x$

$\therefore f(a+ih) = e^{a+ih}$

$= e^{0+ih}$

$= e^{ih}$

$$\therefore \int_0^2 e^x \, dx = \lim_{h\to 0} h \sum_{i=1}^{h} f(a+ih)$$

$$= \lim_{h\to 0} h \sum_{i=1}^{h} e^{ih}$$

$$= \lim_{h\to 0} h\left(e^h + e^{2h} + \ldots\ldots\ldots. + e^{nh}\right)$$

અહીં પ્રથમ પદ = e^h

સામાન્ય ગુણોત્તર $r = e^h > 1$

$$= \lim_{h\to 0} h\left[\frac{a(r^h-1)}{r-1}\right]$$

$$= \lim_{h\to 0} h\left[\frac{e^h(e^{nh}-1)}{e^h-1}\right]$$

$$= \lim_{h\to 0} \frac{e^h(e^2-1)}{\frac{e^h-1}{h}} \left(\because h = \frac{2}{n} \Rightarrow nh = 2\right)$$

$$= \frac{e^0(e^2-1)}{\log_e e}$$

$= e^2 - 1 \qquad (\because e^0 = 1, \log_e e = 1$ છે.)

30. નિયત સંકલિત મેળવો : $\displaystyle\int_0^1 \frac{dx}{e^x + e^{-x}}$

➥ $$I = \int_0^1 \frac{dx}{e^x + e^{-x}} = \int \frac{1}{e^x + \frac{1}{e^x}}\, dx$$

$$= \int_0^1 \frac{e^x\, dx}{e^{2x}+1}$$

હવે $e^x = t$ આદેશ મૂકો.

$\therefore e^x\, dx = dt$ તથા $x = 0$ તો $t = 1$

અને $x = 1$ તો $t = e$

$$\therefore I = \int_1^e \frac{dt}{t^2+1}$$

$$= \left[\tan^{-1}(t)\right]_1^e$$

$$= \tan^{-1}(e) - \tan^{-1}(1)$$

$$= \tan^{-1}\left(\frac{e-1}{1+(e)(1)}\right)$$

$$\left[\because\ \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$

$\therefore$ I $= \tan^{-1}\left(\frac{e-1}{e+1}\right)$ અથવા $\tan^{-1}(e) - \tan^{-1}(1)$

$\therefore$ I $= \tan^{-1}(e) - \frac{\pi}{4}$

31. **નિયત સંકલિત મેળવો :** $\int_0^{\frac{\pi}{2}} \frac{\tan x}{1 + m^2 \tan^2 x}\, dx$

➡ $$I = \int_0^{\frac{\pi}{2}} \frac{\tan x}{1 + m^2 \tan^2 x}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\frac{\sin x}{\cos x}}{1 + m^2\left(\frac{\sin^2 x}{\cos^2 x}\right)}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\cos^2 x + m^2 \sin^2 x}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{1 - \sin^2 x + m^2 \sin^2 x}\, dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{1 - (1 - m^2)\sin^2 x}\, dx$$

હવે $\sin^2 x = t$ આદેશ લેતાં,

$\therefore\ 2\sin x \cos x\, dx = dt$

$\therefore\ \sin x \cos x\, dx = \frac{1}{2} dt$

તથા $x = 0$ તો $t = \sin 0$

$\therefore\ t = 0$

અને $x = \frac{\pi}{2}$ તો $t = \sin\frac{\pi}{2}$

$\therefore\ t = 1$

$$\therefore I = \frac{1}{2}\int_0^1 \frac{dt}{1 - (1 - m^2)t}$$

$$= \frac{1}{2}\left\{\frac{\log|1 - (1 - m^2)t|}{-(1 - m^2)}\right\}_0^1 + C$$

$$= \frac{-1}{2(1 - m^2)} = \left\{\log|1 - (1 - m^2)(1) - \log|1 - (1 - m^2)O\right\}$$

$$= \frac{-1}{2(m^2 - 1)} = \left\{\log m^2 - \log(1)\right\}$$

➡ $$I = \int_0^{\frac{\pi}{2}} \frac{\tan x}{1 + m^2 \tan^2 x}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\frac{\sin x}{\cos x}}{1 + m^2\left(\frac{\sin^2 x}{\cos^2 x}\right)} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\cos^2 x + m^2 \sin^2 x} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{1 - \sin^2 x + m^2 \sin^2 x} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{1 - (1 - m^2) \sin^2 x} dx$$

હવે $\sin^2 x = t$ આદેશ લેતાં,

$\therefore\ 2 \sin x \cos x\, dx = dt$

$\therefore\ \sin x \cos x\, dx = \frac{1}{2} dt$

તથા $x = 0$ તો $t = \sin 0$

$\therefore\ t = 0$

અને $x = \frac{\pi}{2}$ તો $t = \sin\frac{\pi}{2}$

$\therefore\ t = 1$

$$\therefore I = \frac{1}{2} \int_0^1 \frac{dt}{1 - (1 - m^2)t}$$

$$= \frac{1}{2} \left\{\frac{\log|1 - (1 - m^2)t|}{-(1 - m^2)}\right\}_0^1 + C$$

$$= \frac{-1}{2(1-m^2)} = \left\{\log|1-(1-m^2)(1) - \log|1-(1-m^2)0\right\}$$

$$= \frac{-1}{2(m^2 - 1)} = \left\{\log m^2 - \log(1)\right\}$$

32. નિયત સંકલિત મેળવો : $\int_0^1 \frac{x}{\sqrt{1 + x^2}} dx$

➥ $\int_0^1 \frac{x}{\sqrt{1 + x^2}} dx$

અહીં $1 + x^2 = t$ આદેશ લેતાં,

$\therefore\ 2x\, dx = dt$

તથા જો $x = 0$ તો $t = 1$

અને $x = 1$ તો $t = 2$

$$\therefore I = \frac{1}{2} \int_1^2 \frac{dt}{\sqrt{t}}$$

$$= \int_1^2 \frac{dt}{2\sqrt{t}}$$

$= \left(\sqrt{t}\right)_1^2$ $\left(\because \frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}\ \text{થાય.}\right)$

$\therefore \text{I} = \sqrt{2} - 1$

33. **નિયત સંકલિત મેળવો :** $\int_0^{\pi} x \sin x \cos^2 x \, dx$

➡ $\text{I} = \int_0^{\pi} x \sin x \cos^2 x \, dx$

$= \int_0^{\pi} (\pi - x) \sin(\pi - x) \cos^2(\pi - x) \, dx$

$\left(\because \int_0^{a} f(x) \, dx = \int_0^{a} f(a - x) \, dx \right)$

$\therefore \text{I} = \int_0^{\pi} (\pi - x) \sin x \cos^2 x \, dx$

$\therefore \text{I} = \int_0^{\pi} \pi \sin x \cos^2 x \, dx - \int_0^{\pi} x \sin x \cos^2 x \, dx$

$= \pi \int_0^{\pi} \sin x \cos^2 x \, dx - \text{I}$

$\therefore 2\text{I} = \pi \int_0^{\pi} \sin x \cos^2 x \, dx$

હવે $\cos x = t$ આદેશ મૂકો.

$\therefore -\sin x \, dx = dt$

તથા જ્યારે $x = 0$ તો $t = \cos 0$

$\therefore t = 1$

અને જ્યારે $x = \pi$ તો $t = \cos \pi$

$\therefore t = -1$

$\therefore 2\text{I} = -\pi \int_1^{-1} t^2 \, dt$

$= \pi \int_{-1}^{1} t^2 \, dt = \pi \left(\frac{t^3}{3} \right)_{-1}^{1}$

$= \pi \left(\frac{1}{3} - \left(-\frac{1}{3} \right) \right)$

➡ $\text{I} = \int_0^{\pi} x \sin x \cos^2 x \, dx$

$= \int_0^{\pi} (\pi - x) \sin(\pi - x) \cos^2(\pi - x) \, dx$

$\left(\because \int_0^{a} f(x) \, dx = \int_0^{a} f(a - x) \, dx \right)$

$\therefore \text{I} = \int_0^{\pi} (\pi - x) \sin x \cos^2 x \, dx$

$\therefore \text{I} = \int_0^{\pi} \pi \sin x \cos^2 x \, dx - \int_0^{\pi} x \sin x \cos^2 x \, dx$

$$= \pi \int_0^{\pi} \sin x \cos^2 x \, dx - I$$

$$\therefore 2I = \pi \int_0^{\pi} \sin x \cos^2 x \, dx$$

હવે $\cos x = t$ આદેશ મૂકો.

$\therefore -\sin x \, dx = dt$

તથા જ્યારે $x = 0$ તો $t = \cos 0$

$\therefore t = 1$

અને જ્યારે $x = \pi$ તો $t = \cos \pi$

$\therefore t = -1$

$$\therefore 2I = -\pi \int_1^{-1} t^2 \, dt$$

$$= \pi \int_{-1}^{1} t^2 \, dt = \pi \left(\frac{t^3}{3}\right)_{-1}^{1}$$

$$= \pi \left(\frac{1}{3} - \left(-\frac{1}{3}\right)\right)$$

34. નિયત સંકલિત મેળવો : $\int_0^{\frac{1}{2}} \frac{dx}{(1 + x^2)\sqrt{1 - x^2}}$

➡ $I = \int_0^{\frac{1}{2}} \frac{dx}{(1 + x^2)\sqrt{1 - x^2}} dx$

$x = \sin\theta$ આદેશ લેતાં,

$\therefore dx = \cos\theta \, d\theta$

તથા જ્યારે $x = 0$ તો $\sin\theta = 0$

$\therefore \theta = 0$

$x = \frac{1}{2}$ તો $\sin\theta = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

$\therefore \theta = \frac{\pi}{6}$

$$\therefore I = \int_0^{\frac{\pi}{6}} \frac{\cos\theta \, d\theta}{(1 + \sin^2\theta)\sqrt{1 - \sin^2\theta}}$$

$$= \int_0^{\frac{\pi}{6}} \frac{\cos\theta}{(1 + \sin^2\theta)\cos\theta} d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{1}{1 + \sin^2\theta} d\theta$$

હવે અંશ તથા છેદના દરેક પદને $\cos^2\theta$ વડે ભાગતાં,

$$\therefore I = \int_0^{\frac{\pi}{6}} \frac{\frac{1}{\cos^2\theta}}{\frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}} d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sec^2\theta}{\sec^2\theta + \tan^2\theta}\, d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sec^2\theta}{(1 + \tan^2\theta) + \tan^2\theta}\, d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sec^2\theta}{1 + 2\tan^2\theta}\, d\theta$$

➥ $I = \int_0^{\frac{1}{2}} \frac{dx}{(1 + x^2)\sqrt{1 - x^2}}\, dx$

$x = \sin\theta$ આદેશ લેતાં,

$\therefore\ dx = \cos\theta\ d\theta$

તથા જ્યારે $x = 0$ તો $\sin\theta = 0$

$\therefore\ \theta = 0$

$x = \frac{1}{2}$ તો $\sin\theta = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

$\therefore \theta = \frac{\pi}{6}$

$$\therefore I = \int_0^{\frac{\pi}{6}} \frac{\cos\theta\ d\theta}{(1 + \sin^2\theta)\sqrt{1 - \sin^2\theta}}$$

$$= \int_0^{\frac{\pi}{6}} \frac{\cos\theta}{(1 + \sin^2\theta)\cos\theta}\, d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{1}{1 + \sin^2\theta}\, d\theta$$

હવે અંશ તથા છેદના દરેક પદને $\cos^2\theta$ વડે ભાગતાં,

$$\therefore I = \int_0^{\frac{\pi}{6}} \frac{\frac{1}{\cos^2\theta}}{\frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}}\, d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sec^2\theta}{\sec^2\theta + \tan^2\theta}\, d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sec^2\theta}{(1 + \tan^2\theta) + \tan^2\theta}\, d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sec^2\theta}{1 + 2\tan^2\theta}\, d\theta$$