Sets

Concept of sets

In our daily lives, we talk about different collections such as collection of maths books in a cupboard, collection of toys in a shop, collection of shirts in a shop, students in a school, collection of all natural numbers, etc.

These collections are said to be **sets.**

A set is a well-defined collection of objects.

Sets are usually represented by capital letters A, B, C, D, X, Y, Z, etc. The objects inside a set are called **elements** or members of a set. They are denoted by small letters a, b, c, d, x, y, z, etc.

Now, let us consider the set of natural numbers. We know that 4 is a natural number. However, −1 is not a natural number. We denote it as $4 \in \mathbf{N}$ and $-1 \notin \mathbf{N}$.

If *a* is an element of a set *A* then we say that "*a* belongs to *A*" and mathematically we write it as "$a \in A$"; if *b* is not an element of *A* then we say that "*b* does not belong to *A*" and represent it as "$b \notin A$".

There are three different ways of representing a set:

1. Description method
2. Roster method or listing method or tabular form
3. Set-builder form or rule method

Let us study about them one by one.

Description method: In this method, a description about the set is made and it is enclosed in curly brackets { }.

For example: The set of composite numbers less than 30 is written as

{Composite numbers less than 30}

Roster method or listing method or tabular form : In the roster form, all the elements of a set are listed in such a manner that different elements are separated by commas and enclosed within the curly brackets { }. The roster form enables us to see all the members of a set at a glance.

For example: A set of all integers greater than 5 and less than 9 will be represented in roster form as {6, 7, 8}. However, it must be noted that in roster form, the order in which the elements are listed is immaterial. Hence, the set {6, 7, 8} can also be written as {7, 6, 8}.

Set-builder form or rule method: In set-builder representation of a set, all the elements of the set have a single common property that is exclusive to the elements of the set i.e., no other element outside the set has that property.

We have learnt how to write a set of all integers greater than 5 and less than 9 in roster form. Now, let us understand how we write the same set in set-builder form. Let us denote this set by L.

$L = \{x : x$ is an integer greater than 5 and less than 9$\}$

Hence, in set-builder form, we describe an element of a set by a symbol x (though we may use any other small letter), which is followed by a colon (:). After the colon, we describe the characteristic property possessed by all the elements of that set.

Note:

1. The order of listing the elements in a set can be changed.
2. If one or more elements in a set are repeated, then the set remains the same.
3. Each element of the set is listed once and only once.

Now, consider the following three sets.

$A = \{x: x \in \mathbf{Z}, -18 < x \leq 5\}$

$B = \{x: x \in \mathbf{W}\}$

$C = \{x: x \in \mathbf{N}, -7 < x < -1\}$

Did you observe anything about the number of elements of these sets?

Observe that if we count the elements of set A, then we find that the number of elements is limited in this set. However, the number of elements in set B is not limited and we cannot count the number of elements of this set. Also, observe that set C does not contain any element as there does not exist any natural number lying between −7 and −1.

Therefore, on this basis i.e., on the basis of number of elements, the sets are classified into following categories:

(a) Finite set

(b) Infinite set

(c) Empty set

(d) Singleton set

Let us now study about them one by one.

(a) Finite set − A set that contains limited (countable) number of different elements is called a finite set.

(b) Infinite set − A set that contains unlimited (uncountable) number of different elements is called an infinite set.

(c) Empty set − A set that contains no element is called an empty set. It is also called null (or void) set. An empty set is denoted by Φ or {}. Also, since an empty set has no element, it is regarded as a finite set.

(d) Singleton set − A set having exactly one element is known as singleton set.

Therefore, we can now classify the above discussed sets as follows:

$A = \{-17, -16, -15, \ldots, 0, 1, 2, 3, 4, 5\} \rightarrow$ Finite set

$B = \{0, 1, 2, 3, 4, 5 \ldots\} \rightarrow$ Infinite set

$C = \Phi$ or $\{\} \rightarrow$ Empty set

$D = \{4\} \rightarrow$ Singleton set

Now, again consider set *A*. We have seen that it is a finite set.

Can you find the number of elements in this set?

We have $A = \{-17, -16, -15, -14, -13, -12, -11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

We see that the number of elements in set *A* is 23. This number 23 is known as the **cardinal number** of set *A*.

Cardinal number of a set is defined as:

The number of distinct elements in a finite set *A* is called its cardinal number. It is denoted by *n* (*A*).

Note: The cardinal number of an infinite set is not defined.

Now, can you find what the cardinal number of an empty set is?

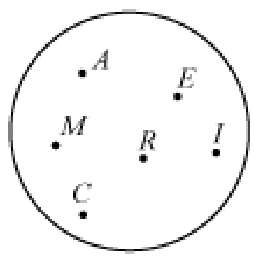
As the empty set has no elements, therefore, its cardinal number is 0 i.e., n (Φ) = 0

We have learnt different ways of representing a set such as description method, roster method, and set-builder method. However, there is one more way of representing a given set and that is through Venn diagrams.

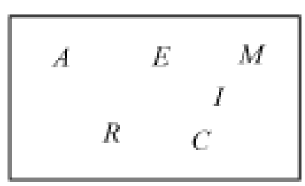
Venn diagrams are closed figures such as square, rectangle, circle, etc. inside which some points are marked. The closed figure represents a set and the points marked inside it represent the elements of the set.

For example, consider the set of all letters in the word AMERICA. This set consists of the letters A, M, E, R, I, and C.

This set can be represented by a Venn diagram as follows:



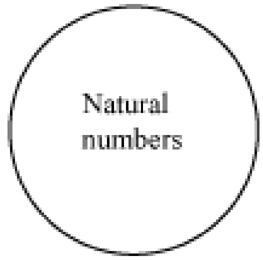
Sometimes, in Venn diagrams, points are not marked, only the elements are written inside the closed figure. For example, the set of letters in the word AMERICA can also be shown as follows:



Now, consider the set of all natural numbers. **How will we represent this set by a Venn diagram?**

In such cases, when the number of elements in a set is large, the description of the set is written in the closed figure.

Therefore, the set of all natural numbers can be shown by a Venn diagram as follows:



Let us now look at some examples to understand the above discussed concepts better.

Example 1:

Which of the following collection are sets?

1. **Collection of rivers in India**
2. **Collection of good dancers in a locality**
3. **Collection of integers which are less than 21**
4. **Collection of best runners**
5. **Collection of all states of America**
6. **Collection of all vowels**

Solution:

1. The collection of rivers is a set because every river of India will be included in it.

2. The collection of good dancers in a locality is not a set because some dancers of the locality may be good from the point of view of one person, but the same may not be good from the point of view of another person.

3. The collection of integers which are less than 21 is a set as the range of integers in the collection is defined.

4. The collection of best runners is not a set because some runners may be good from the point of view of one person, but they may not be good from the point of view of another person.

5. The collection of states of America is a set because all the states of America will be included in it.

6. The collection of all vowels is a set because all the five vowels will be included in it.

Example 2:

Write the roster form for the set *A* = {*x* : *x* is a letter in the word AEROPLANE which has vowels just before and after it}.

Solution:

In the word AEROPLANE, the vowels are A, E, and O.

Now, the third letter (i.e., R) has a vowel (i.e., E) just before it and a vowel (i.e., O) just after it. Hence, it satisfies the given condition.

Now, look at letter N, which has vowel (i.e., A) just before it and a vowel (i.e., E) just after it. Hence, this letter also satisfies the given condition.

Thus, the set can be written in roster form as

A = {R, N}

Example 3:
State whether each of the following sets is finite or infinite:

Set of multiples of 7
Set of lines passing through the point (1,1) as well as the origin

Solution:

1. The multiples of 7 are 7, 14, 21, 28, 35 …
   Hence, the number of elements in set A = {7, 14, 21, 28, 35…} is not definite. Hence, it is an infinite set.

2. The two given points are (1,1) and (0,0) and we know that there is one and only one line passing through two fixed points. Hence, there will be only one line that passes through the given points.

   Thus, the set contains only one element. Hence, it is a finite set.

Example 4:

Write the following sets in set builder form.

1. **{1, 8, 27, 64, 125}**
2. **{0, 1, 2, 3, 4, 5, 6 ,7}**
3. **{w, x, y, z}**
4. **{3, 6, 9, 12, 15, 18, 21}**
5. **{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}**

Solution:

1. {x : x is the cube of first five natural numbers}

2. $\{x : x \in \mathbf{W}, x < 8\}$
3. {*x* : *x* is a letter amongst the last four letters of the English alphabet}
4. {*x* : *x* is a multiple of 3, $x \leq 21$}
5. {*x* : *x* is a day of a week}

Example 5:

Write the following sets in roster and descriptive forms:

1. **{*x* : *x* is a letter in the word MATHEMATICS}**
2. **{*y* : $y \leq 23$ and it is odd}**

Solution:

1. Roster form: {M, A, T, H, E, I, C, S}

   Descriptive form: {letters of the word MATHEMATICS}

2. Roster form: {1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23}

   Descriptive form: {Odd numbers less than and equal to 23}

Example 6:

Classify the following sets into finite set, infinite set, and empty sets. Also, find the cardinal number in case of finite sets.

(a) *A* = {*x*: *x* is a letter in the word ENGINEER}

(b) *B* = {*x*: *x* is a multiple of 9}

(c) *C* = {*x*: *x* is a factor of 48}

(d) *D* = {*x*: *x* is a vowel in the word RHYTHM}

(e) *E* = {*x*: *x* is a vowel in the word SKY}

(f) *F* = {x: $x \in$ Z}

(g) *G* = {all stars in universe}

(h) *H* = {*x* : *x* > 2, where *x* is an even prime number}

Solution:

(a) A = {E, N, G, I, R}

It is a finite set and n (A) = 5

(b) B = {9, 18, 27, 36, 45, …}

It is an infinite set.

(c) C = {1, 2, 3, 4, 6, 8, 12, 16, 24, 48}

It is a finite set and n (C) = 10

(d) D = Φ

It is an empty set and n (D) = 0

(e) E = Φ

It is an empty set and n (E) = 0

(f) F = {…, − 2, − 1, 0, 1, 2, …}

It is an infinite set.

(g) G = {all stars in universe}

The number of elements in set G is not defined and hence it is an infinite set.

(h) H = {x : x > 2, where x is an even prime number}

We can see that no value of x will satisfy the given property as 2 is the only even prime number and no even number greater than 2 will be a prime number.

Hence, A will be an empty set as it has no elements.

n (H) = 0

Equal Sets and Equivalent Sets

Consider two sets:

A = {−9, −3, 0, 5, 12}

B = {−2, 1, 2, 4, 7}

Did you notice any relation between the sets *A* and *B*?

Let us see.

We have,

$A = \{-9, -3, 0, 5, 12\}$

$B = \{-2, 1, 2, 4, 7\}$

Therefore, we have $n(A) = 5$ and $n(B) = 5$

Observe that both the sets *A* and *B* have same number of elements. Therefore, in this case, we say that the sets *A* and *B* are **equivalent sets** and it can be defined as:

Two finite sets are called equivalent, if they have the same number of elements.

Thus, two finite sets *X* and *Y* are equivalent, if $n(X) = n(Y)$. We write it as $X \leftrightarrow Y$ (read as "*X* is equivalent to *Y*")

Now, consider the two sets:

X = {all letters in the word STONE}

Y = {all letters in the word NOTES}

Did you notice any relation between the sets *X* and *Y*?

Let us see.

We have,

X = {S, T, O, N, E} and *Y* = {N, O, T, E, S}

Observe that both the sets *X* and *Y* have same elements. Therefore, in this case, we say that the sets *X* and *Y* are **equal sets**.

Two sets are called equal, if they have same elements.

When two sets X and Y are equal, we denote it as $X = Y$; and if they are not equal, then we write it as $X \neq Y$

Also, note that $n(X) = 5$ and $n(Y) = 5$

Therefore, we can conclude that:

If A and B are finite sets and $A = B$, then $n(A) = n(B)$ i.e., A and B are equivalent. However, the converse of the above statement may not be true.

For example, if $A = \{2, 4, 6\}$ and $B = \{1, 3, 5\}$, then $n(A) = n(B) = 3$; however, $A \neq B$

In this example we see that no element is common between the two sets A and B. So, we can say that the given sets are **disjoint sets**.

Two sets are called disjoint, if they have no element in common.

For example, if A = Set of students of class III and B = Set of students of class VI
No students can be common to the two classes, hence set A and set B are disjoint.

Let us now look at some more examples to understand the above discussed concepts better.

Example 1:

Which of the following sets are equal?

(a) $X = \{x: x$ is a letter in the word REFRESH$\}$,

$Y = \{$A letter in the word FRESHER$\}$

(b) $X = \{4\}$, $Y = \{x: x \in \mathbf{N}, x - 4 = 0\}$

(c) $X = \{x/x$ is a vowel letter in the word WEIGHT$\}$,

$Y = \{x/x$ is a vowel letter in the word HEIGHT$\}$

(d) $X = \{x: x \in \mathbf{N}, 0 < x < 4\}$, $Y = \{x: x \in \mathbf{W}\}$

Solution:

(a) X = {R, E, F, S, H}, Y = {F, R, E, S, H}

∴ X and Y are equal sets i.e., $X = Y$

(b) X = {4}, Y = {4}

∴ X and Y are equal sets i.e., $X = Y$

(c) X = {E, I}, Y = {E, I}

∴ X and Y are equal sets i.e., $X = Y$

(d) X = {1, 2, 3}, Y = {0, 1, 2, 3, 4, 5 …}

∴ X and Y are not equal sets i.e., $X \neq Y$

Example 2:

Which of the following sets are equivalent?

(a) X = {x: x is a vowel in the word MATRIX}

Y = {A vowel in the word SHIVANI}

(b) X = {5, 0, 1, 0, 2, 1}, Y = {3, 2, 8, 3, 3, 2}

Solution:

(a) X = {A, I}, Y = {A, I}

∴ $n(X) = 2$ and $n(Y) = 2$

This means $n(X) = n(Y)$

Therefore, X and Y are equivalent sets.

(b) X = {5, 0, 1, 0, 2, 1} = {0, 1, 2, 5}, Y = {3, 2, 8, 3, 3, 2} = {2, 3, 8}

∴ $n(X) = 4$ and $n(Y) = 3$

This means $n(X) \neq n(Y)$

Therefore, X and Y are not equivalent sets.

Concept of Subset, Superset and Power Set

Consider the two sets A = {5, 4, 8} and B = {1, 5, 6, 4, 8}.

Do you notice any relation between sets A and B?

We can observe that every element of A is an element of B. In this case, we call A to be a subset of B. Mathematically, we write it as $A \subseteq B$.

If *A* and *B* are any two sets, then set *A* is said to be a subset of set *B* if every element of *A* is also an element of *B*. We write it as $A \subseteq B$ (read as '*A* is a subset of *B*' or '*A* is contained in *B*').

So, from the definition, we conclude that:

***A* ⊆ *B* if and only if *x* ∈ *A* implies *x* ∈ *B*.**

Now, if $A \subseteq B$, we can also say that B contains A. In this case, we say that B is a **superset** of A. We write it as $B \supseteq A$ (read as 'B contains A' or 'B is a superset of A').

Now, consider the two sets A = {letters of FOLLOW} and B = {letters of LOWER}.

Then, A = {F, O, L, W} and B = {L, O, W, E, R}

Is set *A* a subset of set *B*?

Observe that there is an element, F in set A which is not a member of set B.

So, A is not a subset of B. We write 'A is not a subset of B' as '$A \not\subseteq B$'. This is also read as 'A is not contained in B'.

If there exists at least one element in *A* which is not an element of *B*, then *A* is not a subset of *B*. Mathematically, we write it as $A \not\subseteq B$.

Another concept you need to know about is that of proper subsets.

To understand what we mean by a proper subset, let us look at the two sets given below.

A = {4, 8, 12}

B = {2, 4, 6, 8, 10, 12, 14}

Observe that A is a subset of B. **Is there any element in B which is not a member of A?**

Yes, 2, 6, 10, 14 $\in B$; however, 2, 6, 10, 14 $\notin A$.

So, in this case, we say that A is a **proper subset** of B.

Let A be any set and B be a non-empty set. Set A is called a proper subset of B if and only if every member of A is also a member of B, and there exists at least one element in B which is not a member of A. We write it as $A \subset B$.

In simple language, we say that A is a proper subset of B, i.e., $A \subset B$, if A is a subset of B and $A \neq B$.

Conversely, if two sets A and B are such that $A \subset B$ and $A \neq B$, then A is called a **proper subset** of B and B is called the **superset** of A.

The following are some important points to be noted.

(a) Every set is a subset of itself.

(b) A subset which is not a proper subset is called an improper subset.
If A and B are two equal sets, then A and B are improper subsets of each other.

(c) Every set has only one improper subset and that is itself.

(d) An empty set is a subset of every set.

(e) An empty set is a proper subset of every set except itself.

(f) Every set is a subset of the universal set.

(g) If $X \subseteq Y$ and $Y \subseteq X$, then $X = Y$

Now, when given any finite set A, we know how to find its cardinal number. **Can we also find the number of subsets and the number of proper subsets of set A?**

Yes, we can.

To know what these numbers are, let us suppose that the cardinal number of the set A is m, i.e., $n(A) = m$, then

The number of subsets of $A = 2^m$

The number of proper subsets of $A = 2^m - 1$

For example:

If $A = \{1, 3, 5\}$, then $n(A) = 3$.

$\therefore$ Number of subsets of $A = 2^3 = 8$

These are: Φ, {1}, {3}, {5}, {1, 3}, {1, 5}, {3, 5}, {1, 3, 5}

Number of proper subsets of $A = 2^3 - 1 = 8 - 1 = 7$

Now, let us discuss about **power set.**

The collection of all subsets of a set *A* is called the **power set** of *A*. It is denoted by P(*A*). In P(*A*), every element is a set.

If the number of elements in set *A* is *m*, then the number of elements in the power set of *A* is 2^m.
i.e., $nP(A) = 2^m$, where $n(A) = m$

Let us go through the given video to understand the above concepts.

We have another type of a set called **universal set** and it can be defined as follows:

A set that contains all the elements under consideration in a given problem is called universal set and it is denoted by *U* or *S* or *ξ*.

Let us consider three sets *X*, *Y*, and *U* as:

X = {the consonants of English alphabets}

Y= {all the letters of the word EDUCATION}

U = {all the letters in English alphabet}

Did you notice any relation between the sets *X*, *Y*, and *U*?

Let us see.

We have, *X* = {B, C, D, F, G, H, J, K, L, M, N, P, Q, R, S, T, V, W, X, Y, Z}, *Y* = {E, D, U, C, A, T, I, O, N} and *U* = {A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z}

Observe that the set *U* contains all the elements of both the sets *X* and *Y*. In this, we say that set *U* is a **universal set** for the sets *X* and *Y***.**

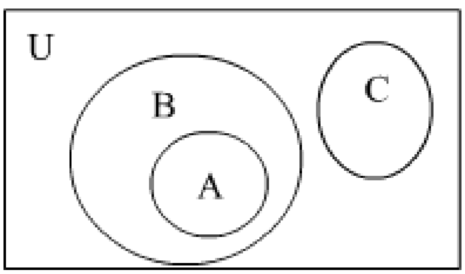
Universal set may vary from problem to problem.

For example, if we consider a set as {0, 1}, then we may consider its universal set as **W** or **Z** or {–1, 0, 1, 2} etc. Similarly, if we consider the set as {Ganga, Yamuna, Saraswati}, then we may consider the universal set as {the rivers in India}.

Therefore, we should always specify the universal set for a given problem.

Now, let us learn to represent the information related to above discussed concepts using Venn diagrams.

Look at the following diagram.



Here, *U*, *A*, *B* and *C* are four sets.

It can be seen that A is the subset of B i.e., $A \subseteq B$.

Also, set *A* is completely contained by set *B* i.e., set $A \subset$ set *B*, we can say that $A \subset B$.

Similarly, set *C* is neither contained by set *B* nor by set *A*. Thus, $C \not\subseteq B$ and $C \not\subseteq A$.

Also, it can be seen that set *U* contains all of the sets *A*, *B* and *C*. Thus, *U* is universal set.

Let's now look at some examples to improve our understanding of the above discussed concepts.

Example 1:

With respect to the three sets: *A* = {5, 10, 15, 20}, *B* = {1, 2, 3, …, 20}, and *C* = {2, 4, 6, 8, 10, 12, 16, 18, 20}, classify the following statements as true or false?

(a) $A \subset B$

(b) B⊆ C

(c) A ⊆ C

(d) Φ⊆A

(e) B⊆ U

Solution:

(a) True

Since 5, 10, 15, 20 ∈ A and 5, 10, 15, 20 ∈ B

Also, there exist many elements which are a member of set B, but not of set A.

So, A⊂ B

(b) False

Since 1, 3, 5, 7, 9, 11, 13, 15, 17, 19 ∈ B and these elements do not belong to C

So, B ⊄ C

(c) False

Since 5, 15 ∈ A, but 5, 15 ∉ C

So, A ⊄ C

(d) True

Since an empty set is a subset of every set

So, Φ ⊆ A

(e) True.

Since every set is a subset of the universal set

So, B ⊆ U

Example 2:

Write all the subsets of the set {3, 6, 9, 12}. Which of these are proper subsets and which are improper subsets?

Solution:

Let *A* = {3, 6, 9, 12}

The subsets of set *A* are:

$$\phi, \{3\}, \{6\}, \{9\}, \{12\}, \{3,6\}, \{3,9\}, \{3,12\}, \{6,9\},$$
$$\{6,12\}, \{9,12\}, \{3,6,9\}, \{3,6,12\}, \{6,9,12\},$$
$$\{9,12,3\}, \{3,6,9,12\}$$

The proper subsets are

$$\phi, \{3\}, \{6\}, \{9\}, \{12\}, \{3,6\}, \{3,9\}, \{3,12\},$$
$$\{6,9\}, \{6,12\}, \{9,12\}, \{3,6,9\}, \{3,6,12\},$$
$$\{6,9,12\}, \{9,12,3\}$$

The improper subset is {3, 6, 9, 12}, i.e., set *A* itself.

Example 3:

Three sets are defined as *A* = {1, 3, 4}, *B* = {3, 4} and *C* = {3, 4, 2, 1}. Prove that *A* and *C* are the super sets of *B*.

Solution:

We have,

A = {1, 3, 4},

B = {3, 4} and

C = {3, 4, 2, 1}

It can be seen that all the elements of *B* are also in sets *A* and *C*.
Also, $B \subset A$ and $B \subset C$. This means that *B* is a proper subset of *A* as well as *C*.
i.e., $B \subset A$ and $B \subset C$.
Thus, *A* and *C* are the supersets of *B.*

Example 4:

If *A* = {–1, –2, –3, –4, –5}, then find the number of subsets of set *A*. Also, find the number of proper subsets of set *A*.

Solution:

A = {–1, –2, –3, –4, –5},

$\therefore$ *n* (*A*) = 5 = *m* (say)

Number of subsets of *A* = $2^m = 2^5 = 32$

Number of proper subsets of *A* = $2^m - 1 = 32 - 1 = 31$

Example 5:
Write the power set for the set *A* = {1, 3, 5}.

Solution:
First, let us write all the subsets of set *A*.
{Φ}, {1}, {3}, {5}, {1,3}, {1,5}, {3,5}, {1,3,5}
P(*A*) = [{Φ}, {1}, {3}, {5}, {1,3}, {1,5}, {3,5}, {1,3,5}]

Example 6:

Given set *A* = {*x*: *x* is a natural number less than 10} and set *B* = {*y*: *y* is an even number less than 9}. Is set *B* a subset of set *A*? If yes then draw the Venn-diagram depicting the given sets.

Solution:

Given, *A* = {*x*: *x* is a natural number less than 10}

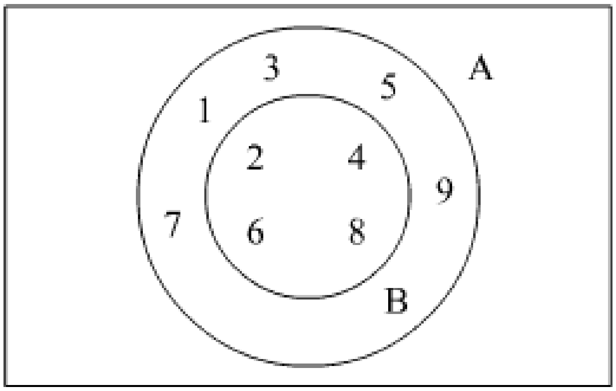
i.e., *A* = {1, 2, 3, 4, 5, 6, 7, 8, 9}

B = {*y*: *y* is an even number less than 9}

i.e., *B* = {2, 4, 6, 8}

It is clear that $B \subset A$.

Venn-diagram for $B \subset A$ is shown below:



Example 7:
Let *A* and *B* be two finite sets such that *n*(*A*) = *m* and *n*(*B*) = *n*. If the ratio of the number of elements of power sets of *A* and *B* is 64 and *n*(*A*) + *n*(*B*) = 32, find the value of *m* and *n*.

Solution:
Given that
$n(A) = m$
$n(B) = n$

The ratio of the number of elements of power sets of *A* and *B* is 64.

$\therefore 2^m : 2^n = 64$
$\Rightarrow 2^{m-n} = 2^6$

$\therefore m - n = 6 \quad ...(1)$

Also, $n(A) + (B) = 32$
$\therefore m + n = 32 \quad ...(2)$

From equations (1) and (2), we get

***m* =** 19

***n* =** 13

Union and Intersection of Sets

Consider the sets *A* = {7, 9, 5} and *B* = {4, 6, 8}

What will we obtain, if we write all the elements of the sets *A* and *B* together in another set?

We will obtain the set {4, 5, 6, 7, 8, 9}.

This set, which we have obtained by writing all the elements of the sets A and B together, is the **union** of the two given sets A and B.

The union of two sets is defined as follows:

The union of two sets A and B is the set that consists of all the elements of A, all the elements of B, and the common elements taken only once.

The symbol '∪' is used for denoting the union. For example, if $X = \{2, 4, 6, 8, 10\}$ and $Y = \{4, 8, 12\}$, then the union of X and Y is given by $X \cup Y = \{2, 4, 6, 8, 10, 12\}$

Now, consider the sets $A = \{4, 5, 9, 14\}$ and $B = \{2, 4, 8, 10, 12, 14\}$

Are there any elements, which are common to both the sets *A* and *B*?

We can observe that the elements 4 and 14 are common to both the sets A and B. The set, which consists of the common elements i.e., the set {4, 14}, is the **intersection** of the sets A and B.

The intersection of two sets is defined as follows:

The intersection of sets A and B is the set of all elements that are common to both A and B.

The symbol '∩' is used for denoting the intersection. For example, if $X = \{A, E, I, O, U\}$ and $Y = \{A, B, C, D, E\}$, then the intersection of the sets X and Y is given by $X \cap Y = \{A, E\}$

There are some properties of union of two sets:

1. $A \cup B = B \cup A$ (Commutative Law)
2. $A \cup \Phi = A$ (Law of identity element Φ)
3. $A \cup A = A$ (Idempotent Law)
4. $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative Law)
5. $U \cup A = U$ (Law of universal set, U)

The properties of the intersection of two sets are given as follows:

1. $A \cap B = B \cap A$ (Commutative Law)
2. $\Phi \cap A = \Phi$ (Law of identity element Φ)
3. $A \cap A = A$ (Idempotent Law)
4. $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law)
5. $U \cap A = A$ (Law of U)
6. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law)

Have you ever observe some properties involving both union and intersection.

Some properties are given below.

- Distributive law of union over intersection of sets:

  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

- Distributive law of intersection over union of sets

  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

These can be verified by taking any three sets.

Now, let us again consider the above discussed sets X = {A, E, I, O, U} and Y = {A, B, C, D, E}. We found that the elements A and E are common to both these sets. Such type of sets, which have one or more elements in common, are called **overlapping sets**.

Two sets are called overlapping (or joint) sets, if they have at least one element in common.

Now consider the sets {1, 2, 3} and {4, 5}. **Is there any element common to these sets?**

We can observe that there is no element common to these sets. These types of sets are called by a special name, which is **disjoint sets**.

Disjoint sets are defined as follows:

If two sets A and B are such that $A \cap B = \Phi$ i.e., they have no element in common, then A and B are called disjoint sets.

Now let us study some formulae used in set theory.

For any two finite sets A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

If A and B are disjoint sets, i.e., $A \cap B = \Phi$ then $n(A \cup B) = n(A) + n(B)$.

To understand the proof of the above formulae, let's go through the following video.

Suppose a set A has 55 elements and a set B has 42 elements. It is also given that the set $A \cup B$ has 85 elements. Can you find the number of elements in $A \cap B$?
Just look at the formula described above.

We know that $n(A \cup B)$, $n(A)$, $n(B)$ and we have to find $n(A \cap B)$.

Hence, we just substitute and solve.

$85 = 55 + 42 - n(A \cap B)$
$n(A \cap B) = 12$

For any three finite sets A, B and C, we have
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

To know the proof of this formula, let us go through the following video.

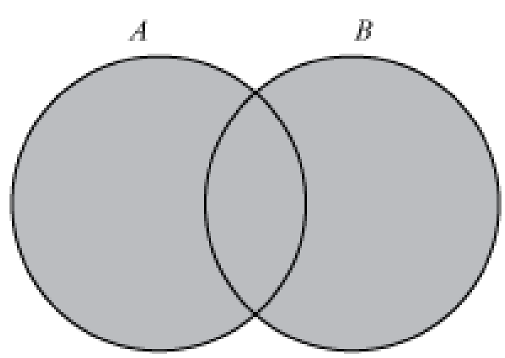
We know that if A and B are two sets, then there can be the following relationships between A and B:

1. A and B can be overlapping.
2. A and B can be disjoint.
3. Either of the sets A or B will be contained in the other set.

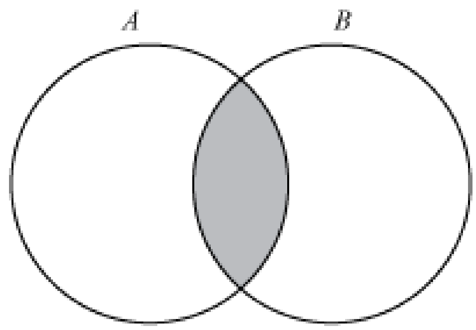
The Venn diagrams representing the intersection and union of the sets A and B in the above cases can be shown as follows:

1. When the sets are overlapping, they can be shown by two intersecting closed figures.

When the sets A and B are overlapping, the Venn diagram representing $A \cup B$ can be shown as:

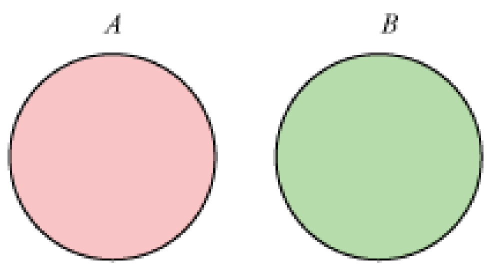


When the sets A and B are overlapping, the set $A \cap B$ is the shaded portion of the following the Venn diagram.

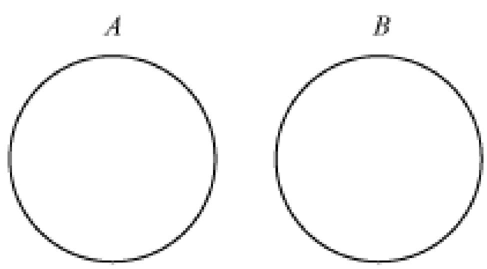


2. When the sets are disjoint, they can be shown by two separate figures drawn side by side.

   When the sets *A* and *B* are disjoint, the Venn diagrams representing $A \cup B$ can be shown as:

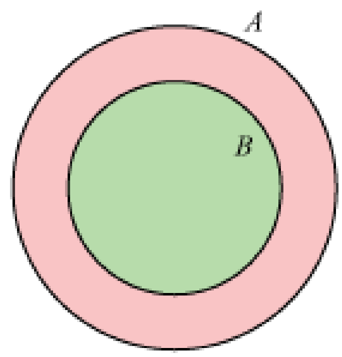


   When the sets *A* and *B* are disjoint, the Venn diagrams representing $A \cap B$ can be shown as:

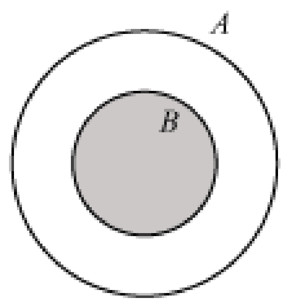


3. When all the elements of one set are present in the second set, they can be represented by drawing one circle inside the other.

   When set *B* is fully contained in set *A*, the Venn diagrams representing $A \cup B$ can be shown as:



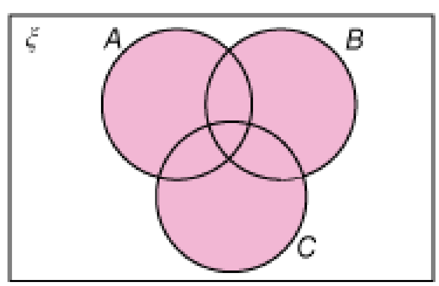
When set B is fully contained in set A, the Venn diagrams representing $A \cap B$ can be shown as:



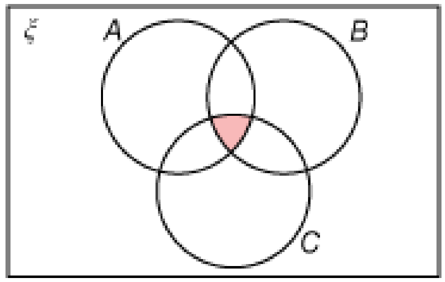
Now, if three sets A, B and C are given, then **how will we represent the union and the intersection of these three sets?**

Let's see.

The union of the three sets A, B and C, i.e., $A \cup B \cup C$, is represented by the shaded portion of the following Venn diagram.



The intersection of the three sets A, B and C, i.e., $A \cap B \cap C$ is represented by the shaded portion of the following Venn diagram.



The above proved formulae are extremely relevant in real-world situations. One of the applications of the formula can be understood by going through the given video.

Let us now look at some examples to understand these concepts better.

Example 1:

Three sets *A*, *B*, and *C* are defined as *A* = {3, 6, 8, 2, 11, 13, 12}, *B* = {7, 9, 3, 2, 10, 14, 15} and *C* = {1, 2, 3, 6, 8, 10, 11}. Find $A \cap (B \cup C)$. Also, prove the associative law of intersection and union using these sets.

Solution:

We have to find $A \cap (B \cup C)$. Hence, let us first find the union of *B* and *C*, and then its intersection with *A*.

$B = \{7, 9, 3, 2, 10, 14, 15\}$

$C = \{1, 2, 3, 6, 8, 10, 11\}$

$B \cup C = \{1, 2, 3, 6, 7, 8, 9, 10, 11, 14, 15\}$

Let us represent the set $B \cup C$ as *D.*

$D = \{1, 2, 3, 6, 7, 8, 9, 10, 11, 14, 15\}$

Now, we have to find $A \cap D$.

$A = \{3, 6, 8, 2, 11, 13, 12\}$

$\therefore A \cap D = \{3, 6, 8, 2, 11\}$

Hence, we have

$A \cap (B \cup C) = \{3, 6, 8, 2, 11\}$

Now, let us prove the associative laws:

We have,

$A = \{3, 6, 8, 2, 11, 13, 12\}$

$B = \{7, 9, 3, 2, 10, 14, 15\}$

$C = \{1, 2, 3, 6, 8, 10, 11\}$

$A \cup B = \{2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$A \cap B = \{2, 3\}$

$B \cup C = \{1, 2, 3, 6, 7, 8, 9, 10, 11, 14, 15\}$

$B \cap C = \{2, 3, 10\}$

Now, $(A \cap B) \cap C = \{2, 3\} \cap \{1, 2, 3, 6, 8, 10, 11\} = \{2, 3\}$

and $A \cap (B \cap C) = \{3, 6, 8, 2, 11, 13, 12\} \cap \{2, 3, 10\} = \{2, 3\}$

Thus, $\boldsymbol{(A \cap B) \cap C = A \cap (B \cap C)}$

Also, $(A \cup B) \cup C = \{2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\} \cup \{1, 2, 3, 6, 8, 10, 11\}$

$= \{1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

and $A \cup (B \cup C) = \{3, 6, 8, 2, 11, 13, 12\} \cup \{1, 2, 3, 6, 7, 8, 9, 10, 11, 14, 15\}$

$= \{1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

Thus, $\boldsymbol{(A \cup B) \cup C = A \cup (B \cup C)}$

Example 2:

Three sets A, B and C are defined as $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $B = \{4, 8, 12, 16, 20, 24\}$ and $C = \{1, 4, 12, 15\}$. Their universal set is given as $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 20, 22, 24\}$.

Verify the following results:

1. $\boldsymbol{U \cap A = A}$ **and** $\boldsymbol{U \cup B = U}$
2. $\boldsymbol{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$
3. $\boldsymbol{n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)}$

Solution:

1.

$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $B = \{4, 8, 12, 16, 20, 24\}$ and

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 20, 22, 24\}$

$\therefore U \cap A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 20, 22, 24\} \cap \{1, 2, 3, 4, 5, 6, 7, 8\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

$= A$

Similarly,

$U \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 20, 22, 24\} \cup \{4, 8, 12, 16, 20, 24\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 20, 22, 24\}$

$= U$

2.

$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $B = \{4, 8, 12, 16, 20, 24\}$

Therefore, $n(A) = 8$ and $n(B) = 6$

We have:

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 12, 16, 20, 24\}$

$A \cap B = \{4, 8\}$

$\therefore n(A \cup B) = 12$ and $n(A \cap B) = 2$

Now, $n(A) + n(B) - n(A \cap B) = 8 + 6 - 2 = 12 = n(A \cup B)$

Thus, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

3.

We have

$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $B = \{4, 8, 12, 16, 20, 24\}$ and $C = \{1, 4, 12, 15\}$

Therefore, $n(A) = 8$, $n(B) = 6$ and $n(C) = 4$

Also, $A \cap B = \{4, 8\}$, $B \cap C = \{4, 12\}$, $C \cap A = \{1, 4\}$, $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 12, 15\ 16, 20, 24\}$ and $A \cap B \cap C = \{4\}$

Therefore, $n(A \cup B \cup C) = 13$, $n(A \cap B) = 2$, $n(B \cap C) = 2$, $n\ (C \cap A) = 2$ and $n\ (A \cap B \cap C) = 1$

Now,

$n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n\ (C \cap A) + n\ (A \cap B \cap C)$

$= 8 + 6 + 4 - 2 - 2 - 2 + 1$

$= 13$

= $n(A \cup B \cup C)$

Thus, $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n\ (C \cap A) + n\ (A \cap B \cap C)$

Example 3:

Classify the following sets as disjoint or overlapping sets:

1. **$A = \{x : 17 < x \leq 30\}$ and B = {Prime numbers lying less than 20}**
2. **X = {x : x is a letter in the word COMPUTER} and Y = {y : y is the letter in the word BAG}**

Solution:

1. We have $A = \{18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$ and

$B = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$A \cap B = \{19\}$

Therefore, the sets A and B have one element in common. Hence, they are overlapping sets.

2. We have X = {C, O, M, P, U, T, E, R} and Y = {B, A, G}

The sets X and Y have no element in common. Hence, they are disjoint sets.

Example 4:
Let A = {1, 2, 4, 6, 8}, B = {1, 3, 6, 9, 12} and C = {1, 3, 5, 7, 9, 11}.

Verify the following.

(a) Distributive law of union over intersection of sets:

(b) Distributive law of intersection over union of sets

Solution:

(a)
B ∩ C = {1, 3, 6, 9, 12} ∩ {1, 3, 5, 7, 9, 11} = {1, 3, 9}
A ∪ (B ∩ C) = {1, 2, 4, 6, 8} ∪ {1, 3, 9} = {1, 2, 3, 4, 6, 8, 9}

A ∪ B = {1, 2, 4, 6, 8} ∪ {1, 3, 6, 9, 12} = {1, 2, 3, 4, 6, 8, 9, 12}
A ∪ C = {1, 2, 4, 6, 8}∪{1, 3, 5, 7, 9, 11} = {1, 2, 3, 4, 5, 6, 7, 8, 9, 11}
(A ∪ B) ∩ (A ∪ C) = {1, 2, 3, 4, 6, 8, 9, 12} ∩ {1, 2, 3, 4, 5, 6, 7, 8, 9, 11} = {1, 2, 3, 4, 6, 8, 9}
Thus, A ∪ (B ∩ C) = (A ∪B) ∩ (A ∪C)

Hence, distributive law of union over intersection of sets is verified.

(b)

B ∪ C = {1, 3, 6, 9, 12} ∪ {1, 3, 5, 7, 9, 11} = {1, 3, 5, 6, 7, 9, 11, 12}

A ∩ (B ∪ C) = {1, 2, 4, 6, 8} ∩ {1, 3, 5, 6, 7, 9, 11, 12} = {1, 6}

A ∩ B = {1, 2, 4, 6, 8} ∩ {1, 3, 6, 9, 12} = {1, 6}

A ∩ C = {1, 2, 4, 6, 8} ∩ {1, 3, 5, 7, 9, 11} = {1}

∴ (A ∩ B) ∪(A ∩ C) = {1, 6} ∪{1} = {1, 6}

A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

Hence, distributive law of intersection over union of sets is verified.

Example 5:

If *A* and *B* are two disjoint sets such that *n* (*A*) = 17 and *n* (*A*∪*B*) = 25, then what is the cardinal number of the set *B*?

Solution:

Since A and B are disjoint sets,

$n(A \cup B) = n(A) + n(B)$

$\Rightarrow 25 = 17 + n(B)$

$\Rightarrow n(B) = 25 - 17 = 8$

Therefore, the cardinal number of set B is 8.

Example 6:

In a group of 60 people, 38 people prefer to take coffee and 29 people prefer to take tea. If a person can choose at least one between tea and coffee, then how many people prefer to take both tea and coffee?

Solution:

Let the sets A and B denote

A= {people who prefer to take coffee}

B = {people who prefer to take tea}

Clearly, $A \cup B$ = {all people}

$A \cap B$ = {people who prefer both tea and coffee}

Now, we have

$n(A \cup B) = 60$, $n(A) = 38$, $n(B) = 29$

We know that,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow 60 = 38 + 29 - n(A \cap B)$

$\Rightarrow 60 = 67 - n(A \cap B)$

$\Rightarrow 60 - 67 = -n(A \cap B)$

$\Rightarrow -7 = -n(A \cap B)$

$\Rightarrow n\ (A \cap B) = 7$

Therefore, 7 people prefer to take both tea and coffee.

Example 7:

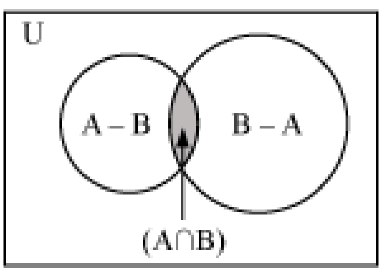
In a survey, 150 people liked winter, 200 liked summer and 50 liked both summer and winter. Find the number of people who liked

1. **Winter but not summer**
2. **Winter or summer**

Solution:

1) Let *A* denote the set of people who like winter and *B* denote the set of people who like summer.

$n\ (A) = 150$ $n\ (B) = 200$ $n\ (A \cap B) = 50$



1) From the Venn diagram, we have

$A = (A - B) \cup (A \cap B)$

$n\ (A) = n\ (A - B) + n\ (A \cap B)$

$n\ (A - B) = n\ (A) - n\ (A \cap B)$

$= 150 - 50$

$= 100$

Thus, the number of people who like winter but not summer is 100.

2) The number of people who like winter or summer is simply the union of the two sets *A* and *B*.

$n\ (A \cup B) = n\ (A) + n\ (B) - n\ (A \cap B)$

= $150 + 200 - 50$

$= 300$

Thus, the number of people who like winter or summer is 300.

Example 8:

The Department of Non-Conventional Energy Resources conducted a survey of 1110 families to know the number of families who use different types of fuels as the means for cooking food.

In the survey, it was reported that 600 families use LPG, 400 families use coal and 300 families use wood. 20 families use all the three types of fuels, 100 families use both LPG and coal, and 80 families use both LPG and wood.

Also, each family uses at least one of the three fuels. How many families use both coal and wood as a fuel for cooking food?

Solution:

Let A, B, C denote the sets of families who use LPG, coal, and wood respectively as a fuel for cooking food. Accordingly, we have

$n(A \cup B \cup C) = 1110$, $n(A) = 600$, $n(B) = 400$, $n(C) = 300$, $n(A \cap B \cap C) = 20$,

$n(A \cap B) = 100$, $n(A \cap C) = 80$

We know that

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C)$
$+ n(A \cap B \cap C)$

$\Rightarrow 1110 = 600 + 400 + 300 - 100 - 80 - n(B \cap C) + 20$

$\Rightarrow 1110 = 1140 - n(B \cap C)$

$\Rightarrow n(B \cap C) = 30$

Thus, 30 families use both coal and wood as a fuel for cooking food.

Difference between Sets

Let us consider two sets $A = \{2, 3, 4, 5, 6, 7, 8, 9\}$ and $B = \{2, 4, 6, 8, 10\}$.

Do these two sets have any element in common?

Observe that the elements 2, 4, 6 and 8 are common to both the sets. **Which set will we get if we exclude these elements from set *A*?**

We will get the set {3, 5, 7, 9}. We denote this set as $A - B$, and it is said to be the difference between the sets A and B (in that order).

The difference between sets A and B (in that order), i.e., $A - B$ is the set of elements belonging to A, but not to B. Thus, $A - B = \{x : x \in A \text{ and } x \notin B\}$.

Similarly, we define the set $B - A$ as the set consisting of all elements which belong to B, but not to A,

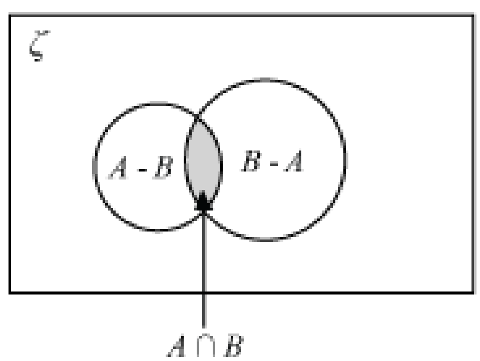
i.e., $B - A = \{x : x \in B \text{ and } x \notin A\}$

So, with respect to the given sets, we find that $B - A = \{10\}$

Here, we observe that $A - B \neq B - A$.

Note that sets $A - B$, $A \cap B$ and $B - A$ are mutually disjoint sets. This means that if we find the intersection of any of these sets, then we will get a null set as our answer.

If U is the universal set for the sets A, B and C, then the sets $A - B$, $A \cap B$ and $B - A$ can be shown diagrammatically as



Properties of difference of two sets:

(i) $A - B \neq B - A$

(ii) $A - B \subseteq A$

(iii) If $A \subseteq B$, then $A - B = \phi$

(iv) If $A \cap B = \phi$, then $A - B = A$

There are some results related to the cardinal number of the difference between two sets. These can be listed as follows:

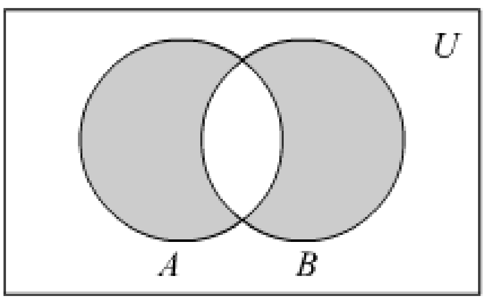
If A and B are two sets, then

1. $n(A - B) = n(A \cup B) - n(B) = n(A) - n(A \cap B)$
2. $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

One can verify these results by taking any two sets.

Symmetric Difference : If A and B are two sets, the their symmetric difference is $(A - B) \cup (B - A)$ and denoted by $A \Delta B$.

Thus, $A \Delta B = (A - B) \cup (B - A) = \{x : x \notin A \cap B\}$.



Let us now look at some examples to understand this concept better.

Example 1:

Two sets are given as $A = \{1, 2, 5, 8, 10, 13\}$ and $B = \{2, 5, 7, 10, 14, 15\}$. Find $A - B$, $B - A$ and $A \Delta B$.

Solution:

$A = \{1, 2, 5, 8, 10, 13\}$

$B = \{2, 5, 7, 10, 14, 15\}$

The common elements are 2, 5 and 10.

For $A - B$, we write all the elements of A and skip the elements common to both A and B. $A - B = \{1, 8, 13\}$

Similarly, for $B - A$, we write all the elements of B and skip the elements common to both the sets.

$B - A = \{7, 14, 15\}$

Now $A \Delta B = (A - B) \cup (B - A)$
$= \{1, 8, 13\} \cup \{7, 14, 15\}.$
$= \{1, 7, 8, 13, 14, 15\}$

Example 2:

If for two sets A and B, $n(A - B) = 10$, $n(B - A) = 7$ and $n(A \cap B) = 3$, then find

1. $n(A \cup B)$
2. $n(A)$
3. $n(B)$

Solution:

We know that $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

$\therefore n(A \cup B) = 10 + 7 + 3 = 20$

Also, we know that $n(A - B) = n(A) - n(A \cap B)$

$\therefore n(A) = n(A - B) + n(A \cap B)$

$\Rightarrow n(A) = 10 + 3 = 13$

We also know that $n(A - B) = n(A \cup B) - n(B)$

$\therefore n(B) = n(A \cup B) - n(A - B)$

$\Rightarrow n(B) = 20 - 10 = 10$

Example 3:

Two sets are given as A = {letters of the word AUTOMOBILE} and B = {vowels in the word MATHEMATICS}

Verify that:

1. $n(A - B) = n(A \cup B) - n(B)$
2. $n(A - B) = n(A) - n(A \cap B)$
3. $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

Solution:

We have

A = {letters of the word AUTOMOBILE} = {A, U, T, O, M, B, I, L, E}

B = {vowels in the word MATHEMATICS} = {A, E, I}

$A - B$ = {U, T, O, M, B, L}

$B - A = \Phi$

$A \cup B$ = {A, U, T, O, M, B, I, L, E}

$A \cap B$ = {A, E, I}

$\therefore n(A) = 9, n(B) = 3, n(A - B) = 6, n(B - A) = 0, n(A \cup B) = 9, n(A \cap B) = 3$

1. $n(A \cup B) - n(B) = 9 - 3 = 6 = n(A - B)$

   $\therefore n(A - B) = n(A \cup B) - n(B)$

2. $n(A) - n(A \cap B) = 9 - 3 = 6 = n(A - B)$

   $\therefore n(A - B) = n(A) - n(A \cap B)$

3. $n(A - B) + n(B - A) + n(A \cap B) = 6 + 0 + 3 = 9 = n(A \cup B)$

   $\therefore n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

Example 4:

From the adjoining Venn-diagram, find the following sets:

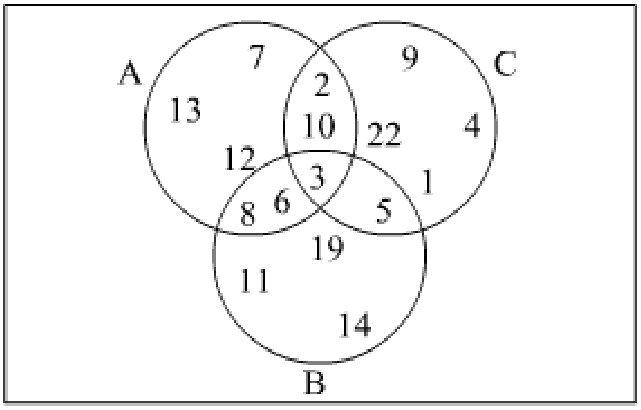
(i) *A*

(ii) *C*

(iii) *A − B*

(iv) *B − C*

(v) *A − C*



Solution:

From the given Venn-diagram, we find that

(i) *A* = {2, 3, 6, 7, 8, 10, 12, 13}

(ii) *C* = {1, 2, 3, 4, 5, 9, 10, 22}

(iii) *A* − *B* = {2, 7, 10, 12, 13}

(iv) *B* − *C* = {6, 8, 11, 14, 19}

(v) *A* − *C* = {6, 7, 8, 12, 13}

Complement of a set

Let us consider a set *X* as

X = {2, 3, 6, 8}

Let us consider its universal set *ξ* as

ξ = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

Can we find the set of elements in set *ξ*, which are not in *X*?

We can find this by taking the difference of *X* from *ξ* as

ξ − *X* = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} − {2, 3, 6, 8} = {0, 1, 4, 5, 7, 9}

This set (consisting of all the elements *ξ*, which do not belong to *X*) is known as the complement of set *X* and we denote it by X' or X^c.

Let X be any set and ξ be its universal set. The complement of set X is the set consisting of all the elements of ξ, which do not belong to X. It is denoted by X' or X^c (read as complement of set X).

Thus, $X' = \{x | x \in \xi \text{ and } x \notin X\}$ or $X' = \xi - X$

For the above sets X and ξ, we may observe that $n(X) = 4$, $n(\xi) = 10$, and $n(X') = 6$.

Can we find any relation among them?

We observe that:

$n(X') = n(\xi) - n(X)$

This relation holds true for a set, its complement, and a universal set.

The other properties of a set and its complement are as follows.

(a) $X \cap X' = \phi$

(b) $X \cup X' = \xi$

(c) $\xi' = \phi$

(d) $\phi' = \xi$

(e) $(X')' = X$

(f) If $X \subseteq Y$ then $X' \subseteq Y'$

Apart from these properties, there are two more properties for two sets A and B. They are:

(a) $(A \cap B)' = A' \cup B'$

(b) $(A \cup B)' = A' \cap B'$

These are also known as **De Morgan's laws**.

Let us prove the first one.

Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, and $\xi = \{1, 2, 3, 4, 5, 6\}$

Now, $A \cap B = \{2, 3\}$

Therefore, $(A \cap B)' = \{1, 4, 5, 6\}$

Also, $A' = \{4, 5, 6\}$ and $B' = \{1, 5, 6\}$

$\therefore A' \cup B' = \{1, 4, 5, 6\}$

Clearly, we have

$$(A \cap B)' = A' \cup B'$$

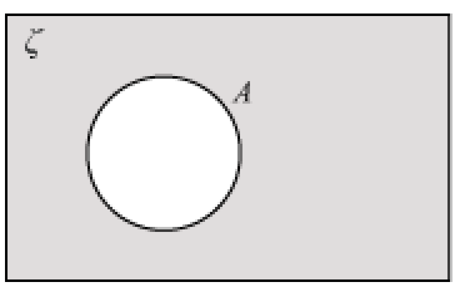
Similarly, we can prove the second one.

Now, how will we represent the complement of a set A with the help of a Venn diagram?

We know that if A is a set and ξ is a universal set for the set A, then the complement of the set A is $A^c = \xi - A$.

If we represent the sets ξ and A by a Venn diagram, then we can easily represent A^c on it.

For this, we represent the set A by using a circle and ξ by using a rectangle (or a square which is bigger and encloses the circle). Now, the portion outside the set A, but inside the set ξ, represents the set A^c. This can be shown as follows:



A^C (Shaded portion)

Let us look at some examples in order to understand these concepts better.

Example 1:

If *A* and *B* are two sets and *ξ* is their universal set such that $n(A')=3,\ n(A)=5$, and *n* (*B*) = 6, then how many elements are there in the complement of set *B*?

Solution:

We know that,

$$\begin{aligned} n(A') &= n(\xi)-n(A) \\ \Rightarrow 3 &= n(\xi)-5 \\ \Rightarrow 3+5 &= n(\xi) \\ \Rightarrow n(\xi) &= 8 \end{aligned}$$

We also know that,

$$\begin{aligned} n(B') &= nn(\xi)-n(B) \\ &= 8-6 \\ &= 2 \end{aligned}$$

Therefore, the complement of set *B* contains 2 elements.

Example 2:

If *A* = {*x*, 1, 2, 3, *y*}, *B* = {2, 4, 5, *y*}, and *ξ*= {*x*, *y*, *z*, 1, 2, 3, 4, 5, 6}, then show that $(A\cup B)^c = A^c \cap B^c$

Solution:

Now, $A^c = \xi - A$ = {*x, y, z,* 1, 2, 3, 4, 5, 6} − {*x*, 1, 2, 3, *y*}= {*z*, 4, 5, 6}

$B^c = \xi - B$ = {*x*, *y*, *z*, 1, 2, 3, 4, 5, 6} − {2, 4, 5, *y*} = {*x, y*, 1, 3, 6}

$A \cup B$ = {*x, y,* 1, 2, 3, 4, 5}

$\therefore (A \cup B)^c$ = {*z*, 6}

Now, $A^c \cap B^c$ = {*z*, 6}

Clearly, $(A \cup B)^c = A^c \cap B^c$

Example 3:

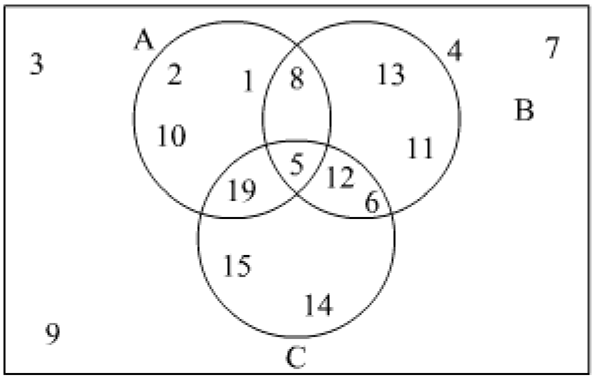
Find the following sets from the adjoining Venn-diagram.

(i) $(A \cap B)^c$

(ii) A^c

(iii) $(A \cup C)^c$

(iv) ξ



Solution:

From the given Venn-diagram, we find that

(i) $(A \cap B)^c = \{1, 2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 14, 15, 19\}$

(ii) $A^c = \{3, 4, 6, 7, 9, 11, 12, 13, 14, 15\}$

(iii) $(A \cup C)^c = \{3, 4, 7, 9, 11, 13\}$

(iv) $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 19\}$

Example 4:

Taking the set of first ten natural numbers as the universal set, find the set

$(B - A)' \cap B'$, where $A = \{1, 2, 4, 9\}$ and $B = \{2, 5, 7, 9, 8, 10, 1, 3\}$

Solution:

$B - A = \{5, 7, 8, 10, 3\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$(B - A)' = \{1, 2, 4, 6, 9\}$

$B' = \{4, 6\}$

$\therefore (B - A)' \cap B' = \{4, 6\}$