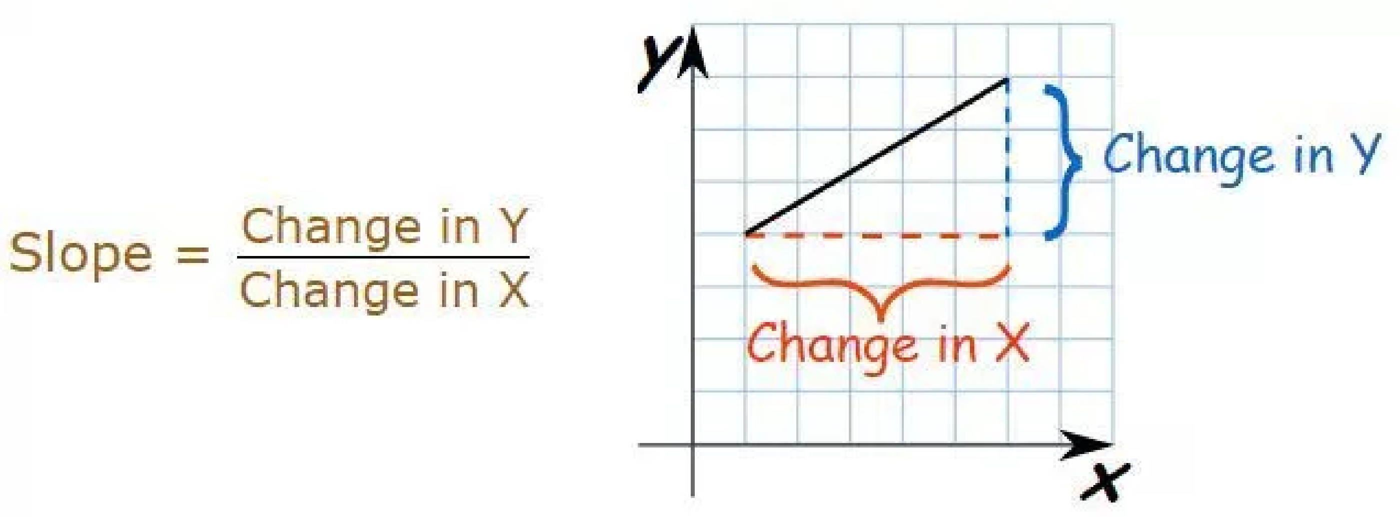
# 2. Derivative Rules

## Derivative Rules

The rate of change of one quantity with respect to some another quantity has a great importance.
The rate of change of a quantity 'y' with respect to another quantity 'x' is called the **derivative** or differential coefficient of y with respect to x.




The **Derivative** means the slope of a function at any point.

## Some Standard Differentiation Formulae

**(1) Differentiation of some common functions:**

| Common Functions | Function | Derivative |
|---|---|---|
| Constant | c | 0 |
| Line | x | 1 |
| | ax | a |
| Square | $x^2$ | 2x |

**(2) Differentiation of algebraic functions:**

In particular

(i) $\frac{d}{dx}x^n = nx^{n-1}$

(ii) $\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}f'(x)$

(iii) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

(iv) $\frac{d}{dx}\left(\frac{1}{x^n}\right) = -\frac{n}{x^{n+1}}$

**(3) Differentiation of trigonometric functions:**

(i) $\frac{d}{dx}\sin x = \cos x$

(ii) $\frac{d}{dx}\cos x = -\sin x$

(iii) $\frac{d}{dx}\tan x = \sec^2 x$

(iv) $\frac{d}{dx}\sec x = \sec x \tan x$

(v) $\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$

(vi) $\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

**(4) Differentiation of logarithmic and exponential functions:**

(i) $\frac{d}{dx}\log x = \frac{1}{x}$, for $x > 0$

(ii) $\frac{d}{dx}e^x = e^x$

(iii) $\frac{d}{dx}a^x = a^x \log a$, for $a > 0$

(iv) $\frac{d}{dx}\log_a x = \frac{1}{x \log a}$, for $x > 0, a > 0, a \neq 1$

**(5) Differentiation of inverse trigonometrical functions:**

(i) $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$, for $-1 < x < 1$

(ii) $\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$, for $-1 < x < 1$

(iii) $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$, for $|x| > 1$

(iv) $\frac{d}{dx}\operatorname{cosec}^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}}$, for $|x| > 1$

(v) $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$, for $x \in R$

(vi) $\frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}$, for $x \in R$

**(6) Differentiation of hyperbolic functions:**

(i) $\frac{d}{dx}\sinh x = \cosh x$

(ii) $\frac{d}{dx}\cosh x = \sinh x$

(iii) $\frac{d}{dx}\tanh x = \text{sech}^2 x$

(iv) $\frac{d}{dx}\coth x = -\text{cosech}^2 x$

(v) $\frac{d}{dx}\text{sech}\, x = -\text{sech}\, x \tanh x$

(vi) $\frac{d}{dx}\text{cosech}\, x = -\text{cosech}\, x \coth x$

(vii) $\frac{d}{dx}\sinh^{-1} x = 1/\sqrt{(1+x^2)}$

(viii) $\frac{d}{dx}\cosh^{-1} x = 1/\sqrt{(x^2-1)}$

(ix) $\frac{d}{dx}\tanh^{-1} x = 1/(x^2-1)$ (x) $\frac{d}{dx}\coth^{-1} x = 1/(1-x^2)$

(xi) $\frac{d}{dx}\text{sech}^{-1} x = -1/x\sqrt{(1-x^2)}$

(xii) $\frac{d}{dx}\text{cosech}^{-1} x = -1/x\sqrt{(1+x^2)}$

**(7) Suitable substitutions**

| Function | Substitution | Function | Substitution |
|---|---|---|---|
| $\sqrt{a^2 - x^2}$ | $x = a\sin\theta$ or $a\cos\theta$ | $\sqrt{x^2 + a^2}$ | $x = a\tan\theta$ or $a\cot\theta$ |
| $\sqrt{x^2 - a^2}$ | $x = a\sec\theta$ or $a\operatorname{cosec}\theta$ | $\sqrt{\frac{a-x}{a+x}}$ | $x = a\cos 2\theta$ |
| $\sqrt{\frac{a^2 - x^2}{a^2 + x^2}}$ | $x^2 = a^2\cos 2\theta$ | $\sqrt{ax - x^2}$ | $x = a\sin^2\theta$ |
| $\sqrt{\frac{x}{a+x}}$ | $x = a\tan^2\theta$ | $\sqrt{\frac{x}{a-x}}$ | $x = a\sin^2\theta$ |
| $\sqrt{(x-a)(x-b)}$ | $x = a\sec^2\theta - b\tan^2\theta$ | $\sqrt{(x-a)(b-x)}$ | $x = a\cos^2\theta + b\sin^2\theta$ |

## Rules for Differentiation

Let f(x), g(x) and u(x) be differentiable functions

1. If at all points of a certain interval, f'(x) = o, then the function f(x) has a constant value within this interval.
2. **Chain rule**
   **(i) Case I:** if y is a function of u and u is a function of x, then derivative of y with respect to x is
   $$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \text{ or } y = f(u) \Rightarrow \frac{dy}{dx} = f'(u)\frac{du}{dx}.$$
   **(ii) Case II:** If y and x both are expressed in terms of t, y and x both are differentiable with respect to t, then
   $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}.$$
3. **Sum and difference rule: Using linear property**
   $$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$$

4. **Product rule**

(i) $$\frac{d}{dx}(f(x).g(x)) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

(ii) $$\frac{d}{dx}(u.v.w.) = u.v.\frac{dw}{dx} + v.w.\frac{du}{dx} + u.w.\frac{dv}{dx}$$

5. **Scalar multiple rule:**

$$\frac{d}{dx}(k\,f(x)) = k\,\frac{d}{dx}f(x)$$

6. **Quotient rule:**

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$