

Exercise 9.1**Question 1:**

Identify the terms, their coefficients for each of the following expressions.

(i) $5xyz^2 - 3zy$

(ii) $1 + x + x^2$

(iii) $4x^2y^2 - 4x^2y^2z^2 + z^2$

(iv) $3 - pq + qr - rp$

(v) $\frac{x}{2} + \frac{y}{2} - xy$

(vi) $0.3a - 0.6ab + 0.5b$

Answer:

The terms and the respective coefficients of the given expressions are as follows.

-	Terms	Coefficients
(i)	$5xyz^2$ $- 3zy$	5 - 3
(ii)	1 x x^2	1 1 1
(iii)	$4x^2y^2$ $- 4x^2y^2z^2$ z^2	4 - 4 1
(iv)	3 $- pq$ qr $- rp$	3 -1 1 -1

(v)	$\frac{x}{2}$ $\frac{y}{2}$ $- xy$	$\frac{1}{2}$ $\frac{1}{2}$ $- 1$
(vi)	$0.3a$ $- 0.6ab$ $0.5b$	0.3 $- 0.6$ 0.5

Question 2:

Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories?

$x + y$, 1000 , $x + x^2 + x^3 + x^4$, $7 + y + 5x$, $2y - 3y^2$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$, $4z - 15z^2$, $ab + bc + cd + da$, pqr , $p^2q + pq^2$, $2p + 2q$

Answer:

The given expressions are classified as

Monomials: 1000 , pqr

Binomials: $x + y$, $2y - 3y^2$, $4z - 15z^2$, $p^2q + pq^2$, $2p + 2q$

Trinomials: $7 + y + 5x$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$

Polynomials that do not fit in any of these categories are

$x + x^2 + x^3 + x^4$, $ab + bc + cd + da$

Question 3:

Add the following.

(i) $ab - bc$, $bc - ca$, $ca - ab$

(ii) $a - b + ab$, $b - c + bc$, $c - a + ac$

(iii) $2p^2q^2 - 3pq + 4$, $5 + 7pq - 3p^2q^2$

(iv) $l^2 + m^2$, $m^2 + n^2$, $n^2 + l^2$, $2lm + 2mn + 2nl$

Answer:

The given expressions written in separate rows, with like terms one below the other and then the addition of these expressions are as follows.

(i)

$$\begin{array}{r}
 ab - bc \\
 + \quad bc - ca \\
 + \quad -ab \quad + ca \\
 \hline
 0
 \end{array}$$

Thus, the sum of the given expressions is 0.

(ii)

$$\begin{array}{r}
 a - b + ab \\
 + \quad b \quad - c + bc \\
 + \quad -a \quad c \quad + ac \\
 \hline
 ab + bc + ac
 \end{array}$$

Thus, the sum of the given expressions is $ab + bc + ac$.

(iii)

$$\begin{array}{r}
 2p^2q^2 - 3pq + 4 \\
 + \quad -3p^2q^2 + 7pq + 5 \\
 \hline
 -p^2q^2 + 4pq + 9
 \end{array}$$

Thus, the sum of the given expressions is $-p^2q^2 + 4pq + 9$.

(iv)

$$\begin{array}{r}
 l^2 + m^2 \\
 + \quad m^2 + n^2 \\
 + \quad l^2 \quad + n^2 \\
 + \quad 2lm + 2mn + 2nl \\
 \hline
 2l^2 + 2m^2 + 2n^2 + 2lm + 2mn + 2nl
 \end{array}$$

Thus, the sum of the given expressions is $2(l^2 + m^2 + n^2 + lm + mn + nl)$.

Question 4:

(a) Subtract $4a - 7ab + 3b + 12$ from $12a - 9ab + 5b - 3$

(b) Subtract $3xy + 5yz - 7zx$ from $5xy - 2yz - 2zx + 10xyz$

(c) Subtract $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$

Answer:

The given expressions in separate rows, with like terms one below the other and then the subtraction of these expressions is as follows.

(a)

$$\begin{array}{r}
 12a - 9ab + 5b - 3 \\
 4a - 7ab + 3b + 12 \\
 \hline
 (-) \quad (+) \quad (-) \quad (-) \\
 8a - 2ab + 2b - 15
 \end{array}$$

(b)

$$\begin{array}{r}
 5xy - 2yz - 2zx + 10xyz \\
 3xy + 5yz - 7zx \\
 \hline
 (-) \quad (-) \quad (+) \\
 2xy - 7yz + 5zx + 10xyz
 \end{array}$$

(c)

$$\begin{array}{r}
 18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q \\
 -10 - 8p + 7q - 3pq + 5pq^2 + 4p^2q \\
 \hline
 (+) \quad (+) \quad (-) \quad (+) \quad (-) \quad (-) \\
 28 + 5p - 18q + 8pq - 7pq^2 + p^2q
 \end{array}$$

Exercise 9.2**Question 1:**

Find the product of the following pairs of monomials.

(i) $4, 7p$ (ii) $-4p, 7p$ (iii) $-4p, 7pq$

(iv) $4p^3, -3p$ (v) $4p, 0$

Answer:

The product will be as follows.

(i) $4 \times 7p = 4 \times 7 \times p = 28p$

(ii) $-4p \times 7p = -4 \times p \times 7 \times p = (-4 \times 7) \times (p \times p) = -28p^2$

(iii) $-4p \times 7pq = -4 \times p \times 7 \times p \times q = (-4 \times 7) \times (p \times p \times q) = -28p^2q$

(iv) $4p^3 \times -3p = 4 \times (-3) \times p \times p \times p \times p = -12p^4$

(v) $4p \times 0 = 4 \times p \times 0 = 0$

Question 2:

Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively.

$(p, q); (10m, 5n); (20x^2, 5y^2); (4x, 3x^2); (3mn, 4np)$

Answer:

We know that,

Area of rectangle = Length $\times$ Breadth

Area of 1st rectangle = $p \times q = pq$

Area of 2nd rectangle = $10m \times 5n = 10 \times 5 \times m \times n = 50mn$

Area of 3rd rectangle = $20x^2 \times 5y^2 = 20 \times 5 \times x^2 \times y^2 = 100x^2y^2$

Area of 4th rectangle = $4x \times 3x^2 = 4 \times 3 \times x \times x^2 = 12x^3$

Area of 5th rectangle = $3mn \times 4np = 3 \times 4 \times m \times n \times n \times p = 12mn^2p$

Question 3:

Complete the table of products.

First monomial $\rightarrow$	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
Second monomial $\downarrow$						

$2x$	$4x^2$	...	...	...	...	...
$- 5y$	...	...	$- 15x^2y$	...	...	...
$3x^2$	...	...	...	...	...	...
$- 4xy$	...	...	...	...	...	...
$7x^2y$	...	...	...	...	...	...
$- 9x^2y^2$	...	...	...	...	...	...

Answer:

The table can be completed as follows.

<u>First monomial →</u> <u>Second monomial ↓</u>	$2x$	$- 5y$	$3x^2$	$- 4xy$	$7x^2y$	$- 9x^2y^2$
$2x$	$4x^2$	$- 10xy$	$6x^3$	$- 8x^2y$	$14x^3y$	$- 18x^3y^2$
$- 5y$	$- 10xy$	$25 y^2$	$- 15x^2y$	$20xy^2$	$- 35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$- 15x^2y$	$9x^4$	$- 12x^3y$	$21x^4y$	$- 27x^4y^2$
$- 4xy$	$- 8x^2y$	$20xy^2$	$- 12x^3y$	$16x^2y^2$	$- 28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$- 35x^2y^2$	$21x^4y$	$- 28x^3y^2$	$49x^4y^2$	$- 63x^4y^3$
$- 9x^2y^2$	$- 18x^3y^2$	$45 x^2y^3$	$- 27x^4y^2$	$36x^3y^3$	$- 63x^4y^3$	$81x^4y^4$

Question 4:

Obtain the volume of rectangular boxes with the following length, breadth and height respectively.

- (i) $5a, 3a^2, 7a^4$ (ii) $2p, 4q, 8r$ (iii) $xy, 2x^2y, 2xy^2$

(iv) $a, 2b, 3c$

Answer:

We know that,

Volume = Length $\times$ Breadth $\times$ Height

(i) Volume = $5a \times 3a^2 \times 7a^4 = 5 \times 3 \times 7 \times a \times a^2 \times a^4 = 105 a^7$

(ii) Volume = $2p \times 4q \times 8r = 2 \times 4 \times 8 \times p \times q \times r = 64pqr$

(iii) Volume = $xy \times 2x^2y \times 2xy^2 = 2 \times 2 \times xy \times x^2y \times xy^2 = 4x^4y^4$

(iv) Volume = $a \times 2b \times 3c = 2 \times 3 \times a \times b \times c = 6abc$

Question 5:

Obtain the product of

(i) xy, yz, zx (ii) $a, -a^2, a^3$ (iii) $2, 4y, 8y^2, 16y^3$

(iv) $a, 2b, 3c, 6abc$ (v) $m, -mn, mnp$

Answer:

(i) $xy \times yz \times zx = x^2y^2z^2$

(ii) $a \times (-a^2) \times a^3 = -a^6$

(iii) $2 \times 4y \times 8y^2 \times 16y^3 = 2 \times 4 \times 8 \times 16 \times y \times y^2 \times y^3 = 1024 y^6$

(iv) $a \times 2b \times 3c \times 6abc = 2 \times 3 \times 6 \times a \times b \times c \times abc = 36a^2b^2c^2$

(v) $m \times (-mn) \times mnp = -m^3n^2p$

Exercise 9.3**Question 1:**

Carry out the multiplication of the expressions in each of the following pairs.

(i) $4p, q + r$ (ii) $ab, a - b$ (iii) $a + b, 7a^2b^2$

(iv) $a^2 - 9, 4a$ (v) $pq + qr + rp, 0$

Answer:

(i) $(4p) \times (q + r) = (4p \times q) + (4p \times r) = 4pq + 4pr$

(ii) $(ab) \times (a - b) = (ab \times a) + [ab \times (-b)] = a^2b - ab^2$

(iii) $(a + b) \times (7a^2b^2) = (a \times 7a^2b^2) + (b \times 7a^2b^2) = 7a^3b^2 + 7a^2b^3$

(iv) $(a^2 - 9) \times (4a) = (a^2 \times 4a) + (-9) \times (4a) = 4a^3 - 36a$

(v) $(pq + qr + rp) \times 0 = (pq \times 0) + (qr \times 0) + (rp \times 0) = 0$

Question 2:

Complete the table

---	First expression	Second Expression	Product
(i)	a	$b + c + d$	-
(ii)	$x + y - 5$	$5xy$	-
(iii)	p	$6p^2 - 7p + 5$	-
(iv)	$4p^2q^2$	$p^2 - q^2$	-
(v)	$a + b + c$	abc	-

Answer:

The table can be completed as follows.

-	First expression	Second Expression	Product
(i)	a	$b + c + d$	$ab + ac + ad$
(ii)	$x + y - 5$	$5xy$	$5x^2y + 5xy^2 - 25xy$

(iii)	p	$6p^2 - 7p + 5$	$6p^3 - 7p^2 + 5p$
(iv)	$4p^2q^2$	$p^2 - q^2$	$4p^4q^2 - 4p^2q^4$
(v)	$a + b + c$	abc	$a^2bc + ab^2c + abc^2$

Question 3:

Find the product.

(i) $(a^2) \times (2a^{22}) \times (4a^{26})$

(ii) $\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right)$

(iii) $\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right)$

(iv) $x \times x^2 \times x^3 \times x^4$

Answer:

(i) $(a^2) \times (2a^{22}) \times (4a^{26}) = 2 \times 4 \times a^2 \times a^{22} \times a^{26} = 8a^{50}$

(ii) $\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right) = \left(\frac{2}{3}\right) \times \left(\frac{-9}{10}\right) \times x \times y \times x^2 \times y^2 = \frac{-3}{5}x^3y^3$

(iii) $\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right) = \left(-\frac{10}{3}\right) \times \left(\frac{6}{5}\right) \times pq^3 \times p^3q = -4p^4q^4$

(iv) $x \times x^2 \times x^3 \times x^4 = x^{10}$

Question 4:

(a) Simplify $3x(4x - 5) + 3$ and find its values for (i) $x = 3$, (ii) $x = \frac{1}{2}$.

(b) $a(a^2 + a + 1) + 5$ and find its values for (i) $a = 0$, (ii) $a = 1$, (iii) $a = -1$.

Answer:

(a) $3x(4x - 5) + 3 = 12x^2 - 15x + 3$

$$\begin{aligned}
 \text{(i) For } x = 3, 12x^2 - 15x + 3 &= 12(3)^2 - 15(3) + 3 \\
 &= 108 - 45 + 3 \\
 &= 66
 \end{aligned}$$

$$x = \frac{1}{2}, 12x^2 - 15x + 3 = 12\left(\frac{1}{2}\right)^2 - 15\left(\frac{1}{2}\right) + 3$$

(ii) For

$$= 12 \times \frac{1}{4} - \frac{15}{2} + 3$$

$$= 3 - \frac{15}{2} + 3 = 6 - \frac{15}{2}$$

$$= \frac{12-15}{2} = \frac{-3}{2}$$

$$(b) a(a^2 + a + 1) + 5 = a^3 + a^2 + a + 5$$

$$(i) \text{ For } a = 0, a^3 + a^2 + a + 5 = 0 + 0 + 0 + 5 = 5$$

$$(ii) \text{ For } a = 1, a^3 + a^2 + a + 5 = (1)^3 + (1)^2 + 1 + 5$$

$$= 1 + 1 + 1 + 5 = 8$$

$$(iii) \text{ For } a = -1, a^3 + a^2 + a + 5 = (-1)^3 + (-1)^2 + (-1) + 5$$

$$= -1 + 1 - 1 + 5 = 4$$

Question 5:

$$(a) \text{ Add: } p(p - q), q(q - r) \text{ and } r(r - p)$$

$$(b) \text{ Add: } 2x(z - x - y) \text{ and } 2y(z - y - x)$$

$$(c) \text{ Subtract: } 3l(l - 4m + 5n) \text{ from } 4l(10n - 3m + 2l)$$

$$(d) \text{ Subtract: } 3a(a + b + c) - 2b(a - b + c) \text{ from } 4c(-a + b + c)$$

Answer:

$$(a) \text{ First expression} = p(p - q) = p^2 - pq$$

$$\text{Second expression} = q(q - r) = q^2 - qr$$

$$\text{Third expression} = r(r - p) = r^2 - rp$$

Adding the three expressions, we obtain

$$\begin{array}{r} p^2 - pq \\ + \quad q^2 - qr \\ + \quad r^2 - rp \\ \hline p^2 - pq + q^2 - qr + r^2 - rp \end{array}$$

Therefore, the sum of the given expressions is $p^2 + q^2 + r^2 - pq - qr - rp$.

(b) First expression = $2x(z - x - y) = 2xz - 2x^2 - 2xy$

Second expression = $2y(z - y - x) = 2yz - 2y^2 - 2yx$

Adding the two expressions, we obtain

$$\begin{array}{r} 2xz - 2x^2 - 2xy \\ + \quad -2yx + 2yz - 2y^2 \\ \hline 2xz - 2x^2 - 4xy + 2yz - 2y^2 \end{array}$$

Therefore, the sum of the given expressions is $-2x^2 - 2y^2 - 4xy + 2yz + 2zx$.

(c) $3l(l - 4m + 5n) = 3l^2 - 12lm + 15ln$

$4l(10n - 3m + 2l) = 40ln - 12lm + 8l^2$

Subtracting these expressions, we obtain

$$\begin{array}{r} 40ln - 12lm + 8l^2 \\ 15ln - 12lm + 3l^2 \\ (-) \quad (+) \quad (-) \\ \hline +25ln \quad +5l^2 \end{array}$$

Therefore, the result is $5l^2 + 25ln$.

(d) $3a(a + b + c) - 2b(a - b + c) = 3a^2 + 3ab + 3ac - 2ba + 2b^2 - 2bc$

$= 3a^2 + 2b^2 + ab + 3ac - 2bc$

$4c(-a + b + c) = -4ac + 4bc + 4c^2$

Subtracting these expressions, we obtain

$$\begin{array}{r} -4ac + 4bc + 4c^2 \\ 3ac - 2bc \quad + 3a^2 + 2b^2 + ab \\ (-) \quad (+) \quad (-) \quad (-) \quad (-) \\ \hline -7ac + 6bc + 4c^2 - 3a^2 - 2b^2 - ab \end{array}$$

Therefore, the result is $-3a^2 - 2b^2 + 4c^2 - ab + 6bc - 7ac$.

Exercise 9.4**Question 1:**

Multiply the binomials.

(i) $(2x + 5)$ and $(4x - 3)$ (ii) $(y - 8)$ and $(3y - 4)$

(iii) $(2.5l - 0.5m)$ and $(2.5l + 0.5m)$ (iv) $(a + 3b)$ and $(x + 5)$

(v) $(2pq + 3q^2)$ and $(3pq - 2q^2)$

(vi) $\left(\frac{3}{4}a^2 + 3b^2\right)$ and $4\left(a^2 - \frac{2}{3}b^2\right)$

Answer:

(i) $(2x + 5) \times (4x - 3) = 2x \times (4x - 3) + 5 \times (4x - 3)$

$$= 8x^2 - 6x + 20x - 15$$

$$= 8x^2 + 14x - 15 \text{ (By adding like terms)}$$

(ii) $(y - 8) \times (3y - 4) = y \times (3y - 4) - 8 \times (3y - 4)$

$$= 3y^2 - 4y - 24y + 32$$

$$= 3y^2 - 28y + 32 \text{ (By adding like terms)}$$

(iii) $(2.5l - 0.5m) \times (2.5l + 0.5m) = 2.5l \times (2.5l + 0.5m) - 0.5m (2.5l + 0.5m)$

$$= 6.25l^2 + 1.25lm - 1.25lm - 0.25m^2$$

$$= 6.25l^2 - 0.25m^2$$

(iv) $(a + 3b) \times (x + 5) = a \times (x + 5) + 3b \times (x + 5)$

$$= ax + 5a + 3bx + 15b$$

(v) $(2pq + 3q^2) \times (3pq - 2q^2) = 2pq \times (3pq - 2q^2) + 3q^2 \times (3pq - 2q^2)$

$$= 6p^2q^2 - 4pq^3 + 9pq^3 - 6q^4$$

$$= 6p^2q^2 + 5pq^3 - 6q^4$$

(vi) $\left(\frac{3}{4}a^2 + 3b^2\right) \times \left[4\left(a^2 - \frac{2}{3}b^2\right)\right] = \left(\frac{3}{4}a^2 + 3b^2\right) \times \left(4a^2 - \frac{8}{3}b^2\right)$

$$= \frac{3}{4}a^2 \times \left(4a^2 - \frac{8}{3}b^2\right) + 3b^2 \times \left(4a^2 - \frac{8}{3}b^2\right)$$

$$= 3a^4 - 2a^2b^2 + 12b^2a^2 - 8b^4$$

$$= 3a^4 + 10a^2b^2 - 8b^4$$

Question 2:

Find the product.

(i) $(5 - 2x)(3 + x)$ (ii) $(x + 7y)(7x - y)$

(iii) $(a^2 + b)(a + b^2)$ (iv) $(p^2 - q^2)(2p + q)$

Answer:

$$\begin{aligned} \text{(i)} \quad (5 - 2x)(3 + x) &= 5(3 + x) - 2x(3 + x) \\ &= 15 + 5x - 6x - 2x^2 \\ &= 15 - x - 2x^2 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (x + 7y)(7x - y) &= x(7x - y) + 7y(7x - y) \\ &= 7x^2 - xy + 49xy - 7y^2 \\ &= 7x^2 + 48xy - 7y^2 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad (a^2 + b)(a + b^2) &= a^2(a + b^2) + b(a + b^2) \\ &= a^3 + a^2b^2 + ab + b^3 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad (p^2 - q^2)(2p + q) &= p^2(2p + q) - q^2(2p + q) \\ &= 2p^3 + p^2q - 2pq^2 - q^3 \end{aligned}$$

Question 3:

Simplify.

(i) $(x^2 - 5)(x + 5) + 25$

(ii) $(a^2 + 5)(b^3 + 3) + 5$

(iii) $(t + s^2)(t^2 - s)$

(iv) $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$

(v) $(x + y)(2x + y) + (x + 2y)(x - y)$

(vi) $(x + y)(x^2 - xy + y^2)$

(vii) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$

(viii) $(a + b + c)(a + b - c)$

Answer:

$$\begin{aligned} \text{(i)} \quad (x^2 - 5)(x + 5) + 25 &= x^2(x + 5) - 5(x + 5) + 25 \\ &= x^3 + 5x^2 - 5x - 25 + 25 \\ &= x^3 + 5x^2 - 5x \end{aligned}$$

$$(ii) (a^2 + 5) (b^3 + 3) + 5$$

$$= a^2 (b^3 + 3) + 5 (b^3 + 3) + 5$$

$$= a^2 b^3 + 3a^2 + 5b^3 + 15 + 5$$

$$= a^2 b^3 + 3a^2 + 5b^3 + 20$$

$$(iii) (t + s^2) (t^2 - s)$$

$$= t (t^2 - s) + s^2 (t^2 - s)$$

$$= t^3 - st + s^2 t^2 - s^3$$

$$(iv) (a + b) (c - d) + (a - b) (c + d) + 2 (ac + bd)$$

$$= a (c - d) + b (c - d) + a (c + d) - b (c + d) + 2 (ac + bd)$$

$$= ac - ad + bc - bd + ac + ad - bc - bd + 2ac + 2bd$$

$$= (ac + ac + 2ac) + (ad - ad) + (bc - bc) + (2bd - bd - bd)$$

$$= 4ac$$

$$(v) (x + y) (2x + y) + (x + 2y) (x - y)$$

$$= x (2x + y) + y (2x + y) + x (x - y) + 2y (x - y)$$

$$= 2x^2 + xy + 2xy + y^2 + x^2 - xy + 2xy - 2y^2$$

$$= (2x^2 + x^2) + (y^2 - 2y^2) + (xy + 2xy - xy + 2xy)$$

$$= 3x^2 - y^2 + 4xy$$

$$(vi) (x + y) (x^2 - xy + y^2)$$

$$= x (x^2 - xy + y^2) + y (x^2 - xy + y^2)$$

$$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$$

$$= x^3 + y^3 + (xy^2 - xy^2) + (x^2y - x^2y)$$

$$= x^3 + y^3$$

$$(vii) (1.5x - 4y) (1.5x + 4y + 3) - 4.5x + 12y$$

$$= 1.5x (1.5x + 4y + 3) - 4y (1.5x + 4y + 3) - 4.5x + 12y$$

$$= 2.25x^2 + 6xy + 4.5x - 6xy - 16y^2 - 12y - 4.5x + 12y$$

$$= 2.25x^2 + (6xy - 6xy) + (4.5x - 4.5x) - 16y^2 + (12y - 12y)$$

$$= 2.25x^2 - 16y^2$$

$$(viii) (a + b + c) (a + b - c)$$

$$= a (a + b - c) + b (a + b - c) + c (a + b - c)$$

$$= a^2 + ab - ac + ab + b^2 - bc + ca + bc - c^2$$

$$= a^2 + b^2 - c^2 + (ab + ab) + (bc - bc) + (ca - ca)$$

$$= a^2 + b^2 - c^2 + 2ab$$

Exercise 9.5

Question 1:

Use a suitable identity to get each of the following products.

(i) $(x + 3)(x + 3)$ (ii) $(2y + 5)(2y + 5)$

(iii) $(2a - 7)(2a - 7)$ (iv) $\left(3a - \frac{1}{2}\right)\left(3a - \frac{1}{2}\right)$

(v) $(1.1m - 0.4)(1.1m + 0.4)$ (vi) $(a^2 + b^2)(-a^2 + b^2)$

(vii) $(6x - 7)(6x + 7)$ (viii) $(-a + c)(-a + c)$

(ix) $\left(\frac{x}{2} + \frac{3y}{4}\right)\left(\frac{x}{2} + \frac{3y}{4}\right)$ (x) $(7a - 9b)(7a - 9b)$

Answer:

The products will be as follows.

$$\begin{aligned} \text{(i)} \quad (x + 3)(x + 3) &= (x + 3)^2 \\ &= (x)^2 + 2(x)(3) + (3)^2 \quad [(a + b)^2 = a^2 + 2ab + b^2] \\ &= x^2 + 6x + 9 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (2y + 5)(2y + 5) &= (2y + 5)^2 \\ &= (2y)^2 + 2(2y)(5) + (5)^2 \quad [(a + b)^2 = a^2 + 2ab + b^2] \\ &= 4y^2 + 20y + 25 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad (2a - 7)(2a - 7) &= (2a - 7)^2 \\ &= (2a)^2 - 2(2a)(7) + (7)^2 \quad [(a - b)^2 = a^2 - 2ab + b^2] \\ &= 4a^2 - 28a + 49 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad \left(3a - \frac{1}{2}\right)\left(3a - \frac{1}{2}\right) &= \left(3a - \frac{1}{2}\right)^2 \\ &= (3a)^2 - 2(3a)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 \quad [(a - b)^2 = a^2 - 2ab + b^2] \\ &= 9a^2 - 3a + \frac{1}{4} \end{aligned}$$

(v) $(1.1m - 0.4)(1.1m + 0.4)$

$$= (1.1m)^2 - (0.4)^2 [(a + b)(a - b) = a^2 - b^2]$$

$$= 1.21m^2 - 0.16$$

$$(vi) (a^2 + b^2)(-a^2 + b^2) = (b^2 + a^2)(b^2 - a^2)$$

$$= (b^2)^2 - (a^2)^2 [(a + b)(a - b) = a^2 - b^2]$$

$$= b^4 - a^4$$

$$(vii) (6x - 7)(6x + 7) = (6x)^2 - (7)^2 [(a + b)(a - b) = a^2 - b^2]$$

$$= 36x^2 - 49$$

$$(viii) (-a + c)(-a + c) = (-a + c)^2$$

$$= (-a)^2 + 2(-a)(c) + (c)^2 [(a + b)^2 = a^2 + 2ab + b^2]$$

$$= a^2 - 2ac + c^2$$

$$(ix) \left(\frac{x}{2} + \frac{3y}{4}\right)\left(\frac{x}{2} + \frac{3y}{4}\right) = \left(\frac{x}{2} + \frac{3y}{4}\right)^2$$

$$= \left(\frac{x}{2}\right)^2 + 2\left(\frac{x}{2}\right)\left(\frac{3y}{4}\right) + \left(\frac{3y}{4}\right)^2 [(a + b)^2 = a^2 + 2ab + b^2]$$

$$= \frac{x^2}{4} + \frac{3xy}{4} + \frac{9y^2}{16}$$

$$(x) (7a - 9b)(7a - 9b) = (7a - 9b)^2$$

$$= (7a)^2 - 2(7a)(9b) + (9b)^2 [(a - b)^2 = a^2 - 2ab + b^2]$$

$$= 49a^2 - 126ab + 81b^2$$

Question 2:

Use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following products.

$$(i) (x + 3)(x + 7) \quad (ii) (4x + 5)(4x + 1)$$

$$(iii) (4x - 5)(4x - 1) \quad (iv) (4x + 5)(4x - 1)$$

$$(v) (2x + 5y)(2x + 3y) \quad (vi) (2a^2 + 9)(2a^2 + 5)$$

$$(vii) (xyz - 4)(xyz - 2)$$

Answer:

The products will be as follows.

$$(i) (x + 3)(x + 7) = x^2 + (3 + 7)x + (3)(7)$$

$$= x^2 + 10x + 21$$

$$(ii) (4x + 5)(4x + 1) = (4x)^2 + (5 + 1)(4x) + (5)(1)$$

$$= 16x^2 + 24x + 5$$

$$(iii) (4x - 5)(4x - 1) = (4x)^2 + [(-5) + (-1)](4x) + (-5)(-1)$$

$$= 16x^2 - 24x + 5$$

$$(iv) (4x + 5)(4x - 1) = (4x)^2 + [(5) + (-1)](4x) + (5)(-1)$$

$$= 16x^2 + 16x - 5$$

$$(v) (2x + 5y)(2x + 3y) = (2x)^2 + (5y + 3y)(2x) + (5y)(3y)$$

$$= 4x^2 + 16xy + 15y^2$$

$$(vi) (2a^2 + 9)(2a^2 + 5) = (2a^2)^2 + (9 + 5)(2a^2) + (9)(5)$$

$$= 4a^4 + 28a^2 + 45$$

$$(vii) (xyz - 4)(xyz - 2)$$

$$= (xyz)^2 + [(-4) + (-2)](xyz) + (-4)(-2)$$

$$= x^2y^2z^2 - 6xyz + 8$$

Question 3:

Find the following squares by using the identities.

$$(i) (b - 7)^2 \quad (ii) (xy + 3z)^2 \quad (iii) (6x^2 - 5y)^2$$

$$(iv) \left(\frac{2}{3}m + \frac{3}{2}n\right)^2 \quad (v) (0.4p - 0.5q)^2 \quad (vi) (2xy + 5y)^2$$

Answer:

$$(i) (b - 7)^2 = (b)^2 - 2(b)(7) + (7)^2 \quad [(a - b)^2 = a^2 - 2ab + b^2]$$

$$= b^2 - 14b + 49$$

$$(ii) (xy + 3z)^2 = (xy)^2 + 2(xy)(3z) + (3z)^2 \quad [(a + b)^2 = a^2 + 2ab + b^2]$$

$$= x^2y^2 + 6xyz + 9z^2$$

$$(iii) (6x^2 - 5y)^2 = (6x^2)^2 - 2(6x^2)(5y) + (5y)^2 \quad [(a - b)^2 = a^2 - 2ab + b^2]$$

$$= 36x^4 - 60x^2y + 25y^2$$

$$(iv) \left(\frac{2}{3}m + \frac{3}{2}n\right)^2 = \left(\frac{2}{3}m\right)^2 + 2\left(\frac{2}{3}m\right)\left(\frac{3}{2}n\right) + \left(\frac{3}{2}n\right)^2 \quad [(a + b)^2 = a^2 + 2ab + b^2]$$

$$= \frac{4}{9}m^2 + 2mn + \frac{9}{4}n^2$$

$$(v) (0.4p - 0.5q)^2 = (0.4p)^2 - 2(0.4p)(0.5q) + (0.5q)^2$$

$$[(a - b)^2 = a^2 - 2ab + b^2]$$

$$= 0.16p^2 - 0.4pq + 0.25q^2$$

$$(vi) (2xy + 5y)^2 = (2xy)^2 + 2(2xy)(5y) + (5y)^2$$

$$[(a + b)^2 = a^2 + 2ab + b^2]$$

$$= 4x^2y^2 + 20xy^2 + 25y^2$$

Question 4:

Simplify.

$$(i) (a^2 - b^2)^2 \quad (ii) (2x + 5)^2 - (2x - 5)^2$$

$$(iii) (7m - 8n)^2 + (7m + 8n)^2 \quad (iv) (4m + 5n)^2 + (5m + 4n)^2$$

$$(v) (2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$$

$$(vi) (ab + bc)^2 - 2ab^2c \quad (vii) (m^2 - n^2m)^2 + 2m^3n^2$$

Answer:

$$(i) (a^2 - b^2)^2 = (a^2)^2 - 2(a^2)(b^2) + (b^2)^2 \quad [(a - b)^2 = a^2 - 2ab + b^2]$$

$$= a^4 - 2a^2b^2 + b^4$$

$$(ii) (2x + 5)^2 - (2x - 5)^2 = (2x)^2 + 2(2x)(5) + (5)^2 - [(2x)^2 - 2(2x)(5) + (5)^2]$$

$$[(a - b)^2 = a^2 - 2ab + b^2]$$

$$[(a + b)^2 = a^2 + 2ab + b^2]$$

$$= 4x^2 + 20x + 25 - [4x^2 - 20x + 25]$$

$$= 4x^2 + 20x + 25 - 4x^2 + 20x - 25 = 40x$$

$$(iii) (7m - 8n)^2 + (7m + 8n)^2$$

$$= (7m)^2 - 2(7m)(8n) + (8n)^2 + (7m)^2 + 2(7m)(8n) + (8n)^2$$

$$[(a - b)^2 = a^2 - 2ab + b^2 \text{ and } (a + b)^2 = a^2 + 2ab + b^2]$$

$$= 49m^2 - 112mn + 64n^2 + 49m^2 + 112mn + 64n^2$$

$$= 98m^2 + 128n^2$$

$$(iv) (4m + 5n)^2 + (5m + 4n)^2$$

$$= (4m)^2 + 2(4m)(5n) + (5n)^2 + (5m)^2 + 2(5m)(4n) + (4n)^2$$

$$[(a + b)^2 = a^2 + 2ab + b^2]$$

$$= 16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2$$

$$= 41m^2 + 80mn + 41n^2$$

$$(v) (2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$$

$$= (2.5p)^2 - 2(2.5p)(1.5q) + (1.5q)^2 - [(1.5p)^2 - 2(1.5p)(2.5q) + (2.5q)^2]$$

$$[(a - b)^2 = a^2 - 2ab + b^2]$$

$$= 6.25p^2 - 7.5pq + 2.25q^2 - [2.25p^2 - 7.5pq + 6.25q^2]$$

$$= 6.25p^2 - 7.5pq + 2.25q^2 - 2.25p^2 + 7.5pq - 6.25q^2$$

$$= 4p^2 - 4q^2$$

$$(vi) (ab + bc)^2 - 2ab^2c$$

$$= (ab)^2 + 2(ab)(bc) + (bc)^2 - 2ab^2c [(a + b)^2 = a^2 + 2ab + b^2]$$

$$= a^2b^2 + 2ab^2c + b^2c^2 - 2ab^2c$$

$$= a^2b^2 + b^2c^2$$

$$(vii) (m^2 - n^2m)^2 + 2m^3n^2$$

$$= (m^2)^2 - 2(m^2)(n^2m) + (n^2m)^2 + 2m^3n^2 [(a - b)^2 = a^2 - 2ab + b^2]$$

$$= m^4 - 2m^3n^2 + n^4m^2 + 2m^3n^2$$

$$= m^4 + n^4m^2$$

Question 5:

Show that

$$(i) (3x + 7)^2 - 84x = (3x - 7)^2 \quad (ii) (9p - 5q)^2 + 180pq = (9p + 5q)^2$$

$$(iii) \left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$$

$$(iv) (4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$$

$$(v) (a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$$

Answer:

$$(i) \text{ L.H.S} = (3x + 7)^2 - 84x$$

$$= (3x)^2 + 2(3x)(7) + (7)^2 - 84x$$

$$= 9x^2 + 42x + 49 - 84x$$

$$= 9x^2 - 42x + 49$$

$$\text{R.H.S} = (3x - 7)^2 = (3x)^2 - 2(3x)(7) + (7)^2$$

$$= 9x^2 - 42x + 49$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\begin{aligned} \text{(ii) L.H.S} &= (9p - 5q)^2 + 180pq \\ &= (9p)^2 - 2(9p)(5q) + (5q)^2 + 180pq \\ &= 81p^2 - 90pq + 25q^2 + 180pq \\ &= 81p^2 + 90pq + 25q^2 \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= (9p + 5q)^2 \\ &= (9p)^2 + 2(9p)(5q) + (5q)^2 \\ &= 81p^2 + 90pq + 25q^2 \end{aligned}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\begin{aligned} \text{(iii) L.H.S} &= \left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn \\ &= \left(\frac{4}{3}m\right)^2 - 2\left(\frac{4}{3}m\right)\left(\frac{3}{4}n\right) + \left(\frac{3}{4}n\right)^2 + 2mn \\ &= \frac{16}{9}m^2 - 2mn + \frac{9}{16}n^2 + 2mn \\ &= \frac{16}{9}m^2 + \frac{9}{16}n^2 = \text{R.H.S.} \end{aligned}$$

$$\begin{aligned} \text{(iv) L.H.S} &= (4pq + 3q)^2 - (4pq - 3q)^2 \\ &= (4pq)^2 + 2(4pq)(3q) + (3q)^2 - [(4pq)^2 - 2(4pq)(3q) + (3q)^2] \\ &= 16p^2q^2 + 24pq^2 + 9q^2 - [16p^2q^2 - 24pq^2 + 9q^2] \\ &= 16p^2q^2 + 24pq^2 + 9q^2 - 16p^2q^2 + 24pq^2 - 9q^2 \\ &= 48pq^2 = \text{R.H.S} \end{aligned}$$

$$\begin{aligned} \text{(v) L.H.S} &= (a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) \\ &= (a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = 0 = \text{R.H.S.} \end{aligned}$$

Question 6:

Using identities, evaluate.

$$\text{(i) } 71^2 \quad \text{(ii) } 99^2 \quad \text{(iii) } 102^2 \quad \text{(iv) } 998^2$$

$$\text{(v) } (5.2)^2 \quad \text{(vi) } 297 \times 303 \quad \text{(vii) } 78 \times 82$$

$$\text{(viii) } 8.9^2 \quad \text{(ix) } 1.05 \times 9.5$$

Answer:

$$\begin{aligned} \text{(i)} \quad 71^2 &= (70 + 1)^2 \\ &= (70)^2 + 2(70)(1) + (1)^2 [(a + b)^2 = a^2 + 2ab + b^2] \\ &= 4900 + 140 + 1 = 5041 \\ \text{(ii)} \quad 99^2 &= (100 - 1)^2 \\ &= (100)^2 - 2(100)(1) + (1)^2 [(a - b)^2 = a^2 - 2ab + b^2] \\ &= 10000 - 200 + 1 = 9801 \\ \text{(iii)} \quad 102^2 &= (100 + 2)^2 \\ &= (100)^2 + 2(100)(2) + (2)^2 [(a + b)^2 = a^2 + 2ab + b^2] \\ &= 10000 + 400 + 4 = 10404 \\ \text{(iv)} \quad 998^2 &= (1000 - 2)^2 \\ &= (1000)^2 - 2(1000)(2) + (2)^2 [(a - b)^2 = a^2 - 2ab + b^2] \\ &= 1000000 - 4000 + 4 = 996004 \\ \text{(v)} \quad (5.2)^2 &= (5.0 + 0.2)^2 \\ &= (5.0)^2 + 2(5.0)(0.2) + (0.2)^2 [(a + b)^2 = a^2 + 2ab + b^2] \\ &= 25 + 2 + 0.04 = 27.04 \\ \text{(vi)} \quad 297 \times 303 &= (300 - 3) \times (300 + 3) \\ &= (300)^2 - (3)^2 [(a + b)(a - b) = a^2 - b^2] \\ &= 90000 - 9 = 89991 \\ \text{(vii)} \quad 78 \times 82 &= (80 - 2)(80 + 2) \\ &= (80)^2 - (2)^2 [(a + b)(a - b) = a^2 - b^2] \\ &= 6400 - 4 = 6396 \\ \text{(viii)} \quad 8.9^2 &= (9.0 - 0.1)^2 \\ &= (9.0)^2 - 2(9.0)(0.1) + (0.1)^2 [(a - b)^2 = a^2 - 2ab + b^2] \\ &= 81 - 1.8 + 0.01 = 79.21 \\ \text{(ix)} \quad 1.05 \times 9.5 &= 1.05 \times 0.95 \times 10 \\ &= (1 + 0.05)(1 - 0.05) \times 10 \\ &= [(1)^2 - (0.05)^2] \times 10 \\ &= [1 - 0.0025] \times 10 [(a + b)(a - b) = a^2 - b^2] \\ &= 0.9975 \times 10 = 9.975 \end{aligned}$$

Question 7:

Using $a^2 - b^2 = (a + b)(a - b)$, find

(i) $51^2 - 49^2$ (ii) $(1.02)^2 - (0.98)^2$ (iii) $153^2 - 147^2$

(iv) $12.1^2 - 7.9^2$

Answer:

$$\begin{aligned} \text{(i) } 51^2 - 49^2 &= (51 + 49)(51 - 49) \\ &= (100)(2) = 200 \end{aligned}$$

$$\begin{aligned} \text{(ii) } (1.02)^2 - (0.98)^2 &= (1.02 + 0.98)(1.02 - 0.98) \\ &= (2)(0.04) = 0.08 \end{aligned}$$

$$\begin{aligned} \text{(iii) } 153^2 - 147^2 &= (153 + 147)(153 - 147) \\ &= (300)(6) = 1800 \end{aligned}$$

$$\begin{aligned} \text{(iv) } 12.1^2 - 7.9^2 &= (12.1 + 7.9)(12.1 - 7.9) \\ &= (20.0)(4.2) = 84 \end{aligned}$$

Question 8:

Using $(x + a)(x + b) = x^2 + (a + b)x + ab$, find

(i) 103×104 (ii) 5.1×5.2 (iii) 103×98 (iv) 9.7×9.8

Answer:

$$\begin{aligned} \text{(i) } 103 \times 104 &= (100 + 3)(100 + 4) \\ &= (100)^2 + (3 + 4)(100) + (3)(4) \\ &= 10000 + 700 + 12 = 10712 \end{aligned}$$

$$\begin{aligned} \text{(ii) } 5.1 \times 5.2 &= (5 + 0.1)(5 + 0.2) \\ &= (5)^2 + (0.1 + 0.2)(5) + (0.1)(0.2) \\ &= 25 + 1.5 + 0.02 = 26.52 \end{aligned}$$

$$\begin{aligned} \text{(iii) } 103 \times 98 &= (100 + 3)(100 - 2) \\ &= (100)^2 + [3 + (-2)](100) + (3)(-2) \\ &= 10000 + 100 - 6 \\ &= 10094 \end{aligned}$$

$$\begin{aligned} \text{(iv) } 9.7 \times 9.8 &= (10 - 0.3)(10 - 0.2) \\ &= (10)^2 + [(-0.3) + (-0.2)](10) + (-0.3)(-0.2) \\ &= 100 + (-0.5)10 + 0.06 = 100.06 - 5 = 95.06 \end{aligned}$$

