INTRODUCTION TO THREE DIMENSIONAL GEOMETRY

- In three dimensions, the coordinate axes of a rectangular Cartesian coordinate system are three mutually perpendicular lines. The axes are called the x, y and z axes.

- The three planes determined by the pair of axes are the coordinate planes, called XY, YZ and ZX-planes.
- The three coordinate planes divide the space into eight parts known as octants.

- The coordinates of a point P in three dimensional Geometry is always written in the form of triplet like (x, y, z). Here x, y and z are the distances from the XY, YZ and ZX-planes.
  (i) Any point on x-axis is of the form $(x, 0, 0)$
  (ii) Any point on y-axis is of the form $(0, y, 0)$
  (iii) Any point on z-axis is of the form $(0, 0, z)$
  The coordinates of the origin O are $(0, 0, 0)$

- Signs of the coordinates in eight octant :

Octants → / Coordinates ↓	I	II	III	IV	V	VI	VII	VIII
x	+	−	−	+	+	−	−	+
y	+	+	−	−	+	+	−	−
z	+	+	+	+	−	−	−	−

- Distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- The coordinates of the point R which divides the line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ internally and externally in the ratio $m : n$ is given by

$$\left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right] \text{ and } \left[\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right]$$

Case I : The coordinates of the mid-point of the line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $\left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right]$

Case II : The coordinates of the point R which divides PQ in the ratio $k : 1$ are obtained by taking $k = \frac{m}{n}$ which are as given below $\left[\frac{kx_2 + x_1}{1+k}, \frac{ky_2 + y_1}{1+k}, \frac{kz_2 + z_1}{1+k}\right]$

- The coordinates of the centroid of the triangle, whose vertices are $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3) are

$$\left[\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right]$$