# Sample Question Paper - 12
# Mathematics (041)
# Class- XII, Session: 2021-22
# TERM II

**Time Allowed: 2 hours** **Maximum Marks: 40**

**General Instructions:**

1. This question paper contains three sections – A, B and C. Each part is compulsory.
2. Section - A has 6 short answer type (SA1) questions of 2 marks each.
3. Section – B has 4 short answer type (SA2) questions of 3 marks each.
4. Section - C has 4 long answer-type questions (LA) of 4 marks each.
5. There is an internal choice in some of the questions.
6. Q 14 is a case-based problem having 2 sub-parts of 2 marks each.

**Section A**

1. Evaluate: $\int_0^{\pi}$ sin 2x cos 3x dx **[2]**

OR

Prove that: $\int\limits_{\pi/6}^{\pi/3} \frac{1}{(1+\sqrt{\tan x})} dx = \frac{\pi}{12}$.

2. Find the general solution for differential equation: $(x-1)\frac{dy}{dx} = 2x^3y$ **[2]**

3. For what value of λ are the vectors $\overrightarrow{\mathrm{a}} = 2\hat{\mathrm{i}} + \lambda\hat{\mathrm{j}} + \hat{\mathrm{k}}$ and $\overrightarrow{\mathrm{b}} = \hat{\mathrm{i}} - 2\hat{\mathrm{j}} + 3\hat{\mathrm{k}}$ perpendicular to each other? **[2]**

4. Find the distance of the point (0, -3, 2) from the plane x + 2y - z = 1, measured parallel to the line $\frac{x+1}{3} = \frac{y+1}{2} = \frac{z}{3}$. **[2]**

5. In a bulb factory, machines A, B and C manufacture 60%, 30% and 10% bulbs respectively. Out of these bulbs 1%, 2% and 3% of the bulbs produced respectively by A , B and C are found to be defective. A bulb is picked up at random from the total production and found to be defective. Find the probability that this bulb was produced by the machine A. **[2]**

6. If $E_1$ and $E_2$ are independent events such that $P(E_1) = 0.3$ and $P(E_2) = 0.4$, find $P(E_1 \cup E_2)$. **[2]**

**Section B**

7. Evaluate: $\int \frac{x+2}{\sqrt{x^2+2x+3}}$ **[3]**

8. Solve the initial value problem: $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y\sin\left(\frac{y}{x}\right) = 0, y(1) = \frac{\pi}{2}$ **[3]**

OR

Solve the differential equation $\frac{dy}{dx} + y\cot x = 2\cos x$, given that y = 0, when $x = \frac{\pi}{2}$.

9. Find $\lambda$ when the projection of $\overrightarrow{\mathrm{a}} = \lambda\hat{\mathrm{i}} + \hat{\mathrm{j}} + 4\hat{\mathrm{k}}$ on $\overrightarrow{\mathrm{b}} = 2\hat{\mathrm{i}} + 6\hat{\mathrm{j}} + 3\hat{\mathrm{k}}$ is 4 units. **[3]**

10. Find the image of the point having position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $r.\left(2\hat{i} - \hat{j} + \hat{k}\right) + 3 = 0$ **[3]**

OR

Find the direction cosines of the unit vector perpendicular to the plane $\vec{r}.\left(6\hat{i}-3\hat{j}-2\hat{k}\right)+1=0$ passing through the origin.

## Section C

11. Evaluate the definite integral $\int_0^{\pi/4}\frac{\sin x+\cos x}{9+16\sin 2x}dx$. **[4]**

12. Find the area of the region {(x, y): $y^2 \leq 6ax$ and $x^2 + y^2 \leq 16a^2$}. Also find the area of the region sketched using method of integration. **[4]**

OR

Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

13. Find the vector and cartesian equations of the plane passing through the line of intersection of the planes **[4]**
$\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k})=7,\ \vec{r}\cdot(2\hat{i}+5\hat{j}+3\hat{k})=9$
such that the intercepts made by the plane on x-axis and z-axis are equal.

## CASE-BASED/DATA-BASED

14. In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03. **[4]**

**Based on the above information, answer the following questions.**

i. The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Govind is

ii. Let A be the event of committing an error in processing the form and let $E_1$, $E_2$ and $E_3$ be the events that Govind, Priyanka and Tahseen processed the form. The value of $\sum_{i=1}^{3}P\left(E_i \mid A\right)$?

**Solution**

**MATHEMATICS 041**

**Class 12 - Mathematics**

**Section A**

1. Let I = $\int_0^\pi$ sin2x cos3x xdx, then

I = $\frac{1}{2}\int_0^\pi$ (sin5x - sinx) dx

$= \frac{1}{2}\left[\frac{-\cos 5x}{5} + \cos x\right]$

$= \frac{1}{2}\left[-\frac{\cos(5\pi)}{5} + \cos(\pi)\right] - \frac{1}{2}\left[-\frac{\cos(0)}{5} + \cos(0)\right]$

$= \frac{1}{2}\left[\frac{-(-1)}{5} - 1\right] - \frac{1}{2}\left[-\frac{1}{5} + 1\right]$

$= \frac{1}{2}\left[\frac{-4}{5} - \frac{4}{5}\right]$

$= \frac{1}{2}2\left(-\frac{4}{5}\right)$

$= -\frac{4}{5}$

OR

Let $y = \int\limits_{\pi/6}^{\pi/3} \frac{1}{(1+\sqrt{\tan x})}dx$

$y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ....... (i)

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{(\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)})}dx$ ....... (ii)

Adding eq.(i) and eq.(ii)

$2y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{(\sqrt{\sin x}+\sqrt{\cos x})}dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{(\sqrt{\cos x}+\sqrt{\sin x})}dx$

$2y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{(\sqrt{\sin x}+\sqrt{\cos x})}dx$

$2y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1dx$

$2y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1dx$

$y = \frac{\pi}{12}$

2. We have,$(x-1)\frac{dy}{dx} = 2x^3y$

Separating the variables we get:

$\Rightarrow \frac{dy}{y} = 2x^3\frac{dx}{(x-1)}$

$\Rightarrow \frac{dy}{y} = \frac{2\left((x-1)\left(x^2+x+1\right)+1\right)}{(x-1)}dx$

$\Rightarrow \frac{dy}{y} = 2\left(x^2+x+1+\frac{1}{x-1}\right)dx$

Integrating both the sides we get,

$\Rightarrow \int\frac{dy}{y} = \int 2\left(x^2+x+1+\frac{1}{x-1}\right)dx + c$

$\Rightarrow \log|y| = \frac{2x^3}{3} + \frac{2x^2}{2} + 2x + 2\log|x-1| + c$

$\Rightarrow \log|y| = \frac{2x^3}{3} + x^2 + 2x + 2\log|x-1| + c$

$\log|y| = \frac{2x^3}{3} + x^2 + 2x + 2\log|x-1| + c$

3. Let $\overrightarrow{\mathrm{a}}=2\hat{\mathrm{i}}+\lambda\hat{\mathrm{j}}+\hat{\mathrm{k}}$ and $\overrightarrow{\mathrm{b}}=\hat{\mathrm{i}}-2\hat{\mathrm{j}}+3\hat{\mathrm{k}}$

For $\vec{a}$ to be perpendicular to $\vec{b}$

$$then\ \cos\theta=0$$

i.e. $\vec{a}\cdot\vec{b}=0$ [vector dot product]

$\left(2\hat{\imath}+\lambda\hat{\jmath}+\hat{k}\right)(\hat{\imath}-2\hat{\jmath}+3\hat{k})=0$

2 - 2$\lambda$+ 3 = 0

5 - 2$\lambda$ = 0

Hence, $\lambda$=$\frac{5}{2}$

4. Equation of line passing through (0, -3, 2) and parallel to the line

$\frac{x+1}{3}=\frac{y+1}{2}=\frac{z}{3}$ is $\frac{x}{3}=\frac{y+3}{2}=\frac{z-2}{3}=k$

$\Rightarrow$ x = 3k, y = 2k - 3 and z = 3k + 2

Substituting x = 3k, y = 2k - 3

And z = 3k + 2 in x + 2 y - z = 1 we have 3k + 2(2k - 3) - (3k + 2) = 1

$\Rightarrow$ 3k + 4k - 3k - 6 - 2 = 1

$\Rightarrow$ 4k - 8 = 1

$\Rightarrow$ 4k = 9

$\Rightarrow$ k = $\frac{9}{4}$then,we get

$\therefore$ x = $3\times\frac{9}{4}=\frac{27}{4}$

y = $2\times\frac{9}{4}-3=\frac{3}{2}$

And z = $3\times\frac{9}{4}+2=\frac{35}{4}$

Therefore $\left(\frac{27}{4},\frac{3}{2},\frac{35}{4}\right)$ is the point of intersation at line through (0, - 3,2) which is parallel to the line

$\frac{x+1}{3}=\frac{y+1}{2}=\frac{z}{3}$ and the plane x + 2 y - z = 1

Now, Required distance = $\sqrt{\left(\frac{27}{4}-0\right)^2+\left(\frac{3}{2}+3\right)^2+\left(\frac{35}{4}-2\right)^2}$ units

$=\sqrt{\frac{729}{16}+\frac{81}{4}+\frac{729}{16}}$ units

$=\sqrt{\frac{729+324+729}{16}}=\sqrt{\frac{1782}{16}}=\frac{42.21}{4}$ = 10.55 units

5. Let A: bulb manufactured from machine A

B :bulb Manufactured from machine B

C :bulb Manufactured from machine C

D : Defective bulb

We want to find $P(\frac{B}{AD})$ i.e. probability of selected defective bulb is from machine A.

Therefore,by Baye's theorm,we have,

$P(\frac{B}{AD})$ = $\frac{\mathrm{P(A)\cdot P(D|A)}}{\mathrm{P(A)\cdot P(D|A)+P(B)\cdot P(D|B)+P(C)\cdot P(D|C)}}$

= $\frac{\left(\frac{60}{100}\right)\left(\frac{1}{100}\right)}{\left(\frac{60}{100}\right)\left(\frac{1}{100}\right)+\left(\frac{30}{100}\right)\left(\frac{2}{100}\right)+\left(\frac{10}{100}\right)\left(\frac{3}{100}\right)}$

= $\frac{6}{15}=\frac{2}{5}$

Conclusion: Therefore, the probability of selected defective bulb is from machine A is $\frac{2}{5}$

6. Given: $E_1$ and $E_2$ are two independent events such that P($E_1$) = 0.3 and P($E_2$) = 0.4

To find: P($E_1 \cup E_2$) when $E_1$ and $E_2$ are independent

By addition theorm of probability, we have,

P($E_1 \cup E_2$) = P($E_1$) + P($E_2$) - P($E_1 \cap E_2$)

= 0.3 + 0.4 - (0.3 $\times$ 0.4)

= 0.58

Therefore , P($E_1 \cup E_2$) = 0.58 when $E_1$ and $E_2$ are Independent.

**Section B**

7. Let I = $\int \frac{x+2}{\sqrt{x^2+2x+3}}$

x + 2 = $A\frac{d}{dx}$ [$x^2$ + 2x + 3] + B

$\Rightarrow$ x + 2 = 2Ax + 2A + B

Comparing the coefficients, we have, 2A = 1 and 2A + B = 2

$\Rightarrow$ A = $\frac{1}{2}$

Substituting the value of A in 2 A + B = 2, we have, $2 \times \frac{1}{2}$ + B = 2

$\Rightarrow$ 1 + B = 2

$\Rightarrow$ B = 2 - 1

$\Rightarrow$ B = 1

Thus we have, x + 2 = $\frac{1}{2}[2x+2]+1$

Hence, using values of A , and B, we have

$I = \int \frac{x+2}{\sqrt{x^2+2x+3}}dx$

$= \int \frac{\left[\frac{1}{2}[2x+2]+1\right]}{\sqrt{x^2+2x+3}}\Bigg] dx$

$= \int \frac{\left[\frac{1}{2}[2x+2]\right]}{\sqrt{x^2+2x+3}}dx + \int \frac{dx}{\sqrt{x^2+2x+3}}$

$= \frac{1}{2}\int \frac{[2x+2]}{\sqrt{x^2+2x+3}}dx + \int \frac{dx}{\sqrt{x^2+2x+3}}$

Substituting t = $x^2$ + 2x + 3 and dt = 2 x + 2

in the first integrand, we have, I = $\frac{1}{2}\int \frac{dt}{\sqrt{t}} + \int \frac{dx}{\sqrt{x^2+2x+3}}$

= $\frac{1}{2} \times 2\sqrt{t} + \int \frac{dx}{\sqrt{x^2+2x+1+2}} + C$

= $\sqrt{t} + \int \frac{dx}{\sqrt{(x+1)^2+(\sqrt{2})^2}} + c$

I = $\sqrt{x^2+2x+3}$ + log [|x + 1| + $\sqrt{(x+1)^2+(\sqrt{2})^2}\,] + C$

$\Rightarrow \mathrm{I} = \sqrt{x^2+2x+3}$ + log [|x + 1| $\sqrt{x^2+2x+3}$] + c

8. Given that, $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y\sin\left(\frac{y}{x}\right) = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{x - y\sin\left(\frac{y}{x}\right)}{x\sin\left(\frac{y}{x}\right)}$

This is a homogeneous differential equation.

Putting y = vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$, given equation reduces to

$v + x\frac{dv}{dx} = -\frac{1 - v\sin v}{\sin v}$

$\Rightarrow x\frac{dv}{dx} = -\frac{1 - v\sin v}{\sin v} - v$

$\Rightarrow x\frac{dv}{dx} = -\frac{1}{\sin v}$

$\Rightarrow$ sin v dv = $-\frac{1}{x}$ dx, if x $\neq$ 0

Integrating both sides,

$\Rightarrow \int$ sin v dv = $-\int \frac{1}{x}$ dx

$\Rightarrow$ - cos v = - log |x| + C

$\Rightarrow$ - cos $\left(\frac{y}{x}\right)$ + log |x| = C ...(i)

It is given that y (1) = $\frac{\pi}{2}$ i.e., when x = 1, y = $\frac{\pi}{2}$.

Putting x = 1 and y = $\frac{\pi}{2}$ in (i), we get

$\Rightarrow$ -cos $\frac{\pi}{2}$ + log 1 = C $\Rightarrow$ C = 0

Putting C = 0 in (i), we get

-cos $\left(\frac{y}{x}\right)$ + log |x| = 0

$\Rightarrow$ log |x| = cos $\left(\frac{y}{x}\right)$

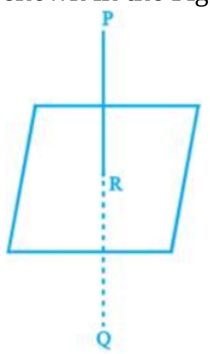
Hence, log |x| = cos $\left(\frac{y}{x}\right)$, is the required solution.

OR

Given:
$\frac{dy}{dx}+y\cot x=2\cos x$
which is a linear differential equation of the form
$\frac{dy}{dx}+Py=Q$
Here, p = cot x and Q = 2 cos x
$\therefore \text{IF}=e^{\int Pdx}=e^{\int \cot x dx}=e^{\log|\sin x|}\Rightarrow \text{IF}=\sin x$
The general solution is given by
$y\times \text{IF}=\int(\text{IF}\times Q)dx+C$
$\Rightarrow\quad y\sin x=\int 2\sin x\cos x dx+C$
$\Rightarrow\quad y\sin x=\int \sin 2x dx+C$
$\Rightarrow\quad y\sin x=-\frac{\cos 2x}{2}+C$ ...(i)
Also, given that y = 0, when $x=\frac{\pi}{2}$.
On putting $x=\frac{\pi}{2}$ in Eq. (i), we get,
$0\sin\frac{\pi}{2}=-\frac{\cos\left(2\frac{\pi}{2}\right)}{2}+C$
$\Rightarrow C-\frac{\cos\pi}{2}=0\Rightarrow C+\frac{1}{2}=0\quad[\because \cos\pi=-1]$
$\therefore\quad C=-\frac{1}{2}$
On putting the value of C in Eq. (i). we get
$y\sin x=-\frac{\cos 2x}{2}-\frac{1}{2}$
$\therefore 2y\sin x+\cos 2x+1=0$
which is the required solution.

9. Given vectors are, $\vec{a}=\lambda\hat{i}+\hat{j}+4\hat{k}, \vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$
The projection of $\vec{a}$ along $\vec{b}$
$=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$
$\vec{a}.\vec{b}=(\lambda\hat{i}+\hat{j}+4\hat{k})\cdot(2\hat{i}+6\hat{j}+3\hat{k})$
$=2\lambda+6+12$
$=2\lambda+18$
$|\vec{b}|=\sqrt{2^2+6^2+3^2}$
$=\sqrt{49}=7$
Given, the projection of vector a along vector b is 4.
$\therefore\quad \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=4$
$\Rightarrow\quad \frac{2\lambda+18}{7}=4$
$\Rightarrow\quad 2\lambda+18=28$
$\Rightarrow\quad 2\lambda=10$
$\Rightarrow\quad \lambda=5$

10. Let the given point be $P\left(\hat{i}+3\hat{j}+4\hat{k}\right)$ and Q be the image of P in the plane $\hat{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)+3=0$as shown in the Fig.




Then PQ is the normal to the plane. Since PQ passes through P and is normal to the given plane, so the equation of PQ is given by

$\vec{r} = \left(\hat{i} + 3\hat{j} + 4\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$

Since Q lies on the line PQ, the position vector of Q can be expressed as $\left(\hat{i} + 3\hat{j} + 4\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$

i.e., $(1 + 2\lambda)\, i + (3 - \lambda)\, \hat{j} + 4\,(4 + \lambda)\, \hat{k}$

Since R is the mid point of PQ, the position vector of R is $\frac{\left[(1+2\lambda)\hat{i}+(3-\lambda)\hat{j}+(4+\lambda)\hat{k}\right]+\left[\hat{i}+3\hat{j}+4\hat{k}\right]}{2}$

i.e., $(\lambda + 1)\,\hat{i} + \left(3 - \frac{\lambda}{2}\right)\hat{j} + \left(4 + \frac{\lambda}{2}\right)\hat{k}$

Again, since R lies on the plane $\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) + 3 = 0$, we have

$\left\{(\lambda - 1)\,\hat{i} + \left(3 - \frac{\lambda}{2}\right)\hat{j} + \left(4 + \frac{\lambda}{2}\right)\hat{k}\right\}.\left(2\hat{i} - \hat{j} - \hat{k}\right) + 3 = 0$

$\Rightarrow \lambda = -2$

Hence, the position vector of Q is $\left(\hat{i} + 3\hat{j} + 4\hat{k}\right) - 2\left(2\hat{i} - \hat{j} + \hat{k}\right)$, i.e., $-3\hat{i} + 5\hat{j} + 2\hat{k}$.

OR

The equation of given plane can be rewritten as,

$\vec{r}.\left(6\hat{i} - 3\hat{j} - 2\hat{k}\right) = -1$

$\Rightarrow \hat{r}.\left(-6\hat{i} + 3\hat{j} + 2\hat{k}\right) = 1$ ...(1)

Now, $\left|-6\hat{i} + 3\hat{j} + 2\hat{k}\right| = \sqrt{36 + 9 + 4} = 7$

Dividing equation (1) by 7, we get,

$\vec{r}.\left(\frac{-6}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k}\right) = \frac{1}{7}$

$\therefore \hat{n} = \frac{-6}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k}\ [\because \vec{r}.\vec{n} = d]$

Hence direction cosines of $\hat{n}$ are $\frac{-6}{7}, \frac{3}{7}, \frac{2}{7}$

**Section C**

11. According to the question, $I = \int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16\sin 2x} dx$

$\Rightarrow \quad I = \int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16(1 + \sin 2x - 1)} dx$

$\Rightarrow \quad I = \int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16[1 - (1 - \sin 2x)]} dx$

$\Rightarrow I = \int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16\left[1 - \left(\cos^2 x + \sin^2 x - 2\sin x\cos x\right)\right]} dx [\because 1 = \cos^2 x + \sin^2 x]$ and $[\because \sin 2x = 2\sin x\cos x]$

$\Rightarrow \quad I = \int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16\left[1 - (\cos x - \sin x)^2\right]} dx$

put , $cosx - sinx = t$

$\Rightarrow (-sinx - cosx)dx = dt$

$\Rightarrow (sinx + cosx)dx = -dt$

Lower limit , when x = 0, then t = cos 0 - sin 0 = 1

Upper limit , when x = $\frac{\pi}{4}$, then $t = \cos\frac{\pi}{4} - \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0.$

$\therefore \quad I = \int_1^0 \frac{-dt}{9 + 16\left(1 - t^2\right)}$

$\Rightarrow \quad I = \int_0^1 \frac{dt}{9 + 16\left(1 - t^2\right)}$

$= \int_0^1 \frac{dt}{25 - 16t^2}$

$= \frac{1}{16}\int_0^1 \frac{dt}{\left(\frac{5}{4}\right)^2 - t^2}$

$= \frac{1}{2 \times \frac{5}{4} \times 16}\left[\log\left|\frac{5+4t}{5-4t}\right|\right]_0^1 \left[\because \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C\right]$

$= \frac{1}{40}\left[\log\left|\frac{5+4}{5-4}\right| - \log\left|\frac{5}{5}\right|\right]$

$= \frac{1}{40}\left[\log\left(\frac{9}{1}\right) - \log\left(\frac{5}{5}\right)\right]$

$= \frac{1}{40}(\log 9 - \log 1)$

$= \frac{1}{40}(\log 9)$ [∵ log 1 = 0]

$\Rightarrow I = \frac{1}{40}\log(3)^2$

$= \frac{2}{40}\log 3$ [∵log $a^n$ = nlog a]

$\therefore \quad I = \frac{1}{20}\log 3$

12. We have, $y^2 \leq 6ax$, which represents the region interior to parabola $y^2 = 6ax$ towards focus.

    And $x^2 + y^2 \leq 16a^2$, which represents the region interior to circle $x^2 + y^2 = 16a^2$. Solving circle and parabola, we get

    $x^2 + 6ax = 16a^2$

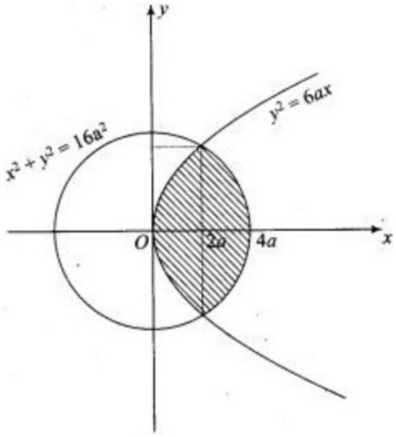
    $\Rightarrow x^2 + 6ax - 16a^2 = 0$

    $\Rightarrow (x - 2a)(x + 8a) = 0$

    $\Rightarrow x = 2a$

    (as x = -8a is not possible)

    Putting x = 2a in a parabola, we get the graph of functions are as shown in the adjacent figure.




From the figure, the area of the shaded region

A = $2\left[\int_0^{2a}\sqrt{6ax}dx + \int_{2a}^{4a}\sqrt{(4a)^2 - x^2}dx\right]$

= $2\left[\sqrt{6a}\left(\frac{2}{3}x^{3/2}\right)_0^{2a} + \left(\frac{x}{2}\sqrt{(4a)^2 - x^2} + \frac{(4a)^2}{2}\sin^{-1}\frac{x}{4a}\right)_{2a}^{4a}\right]$

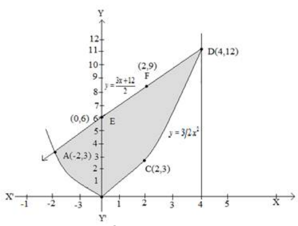
= $2\left[\sqrt{6a}\frac{2}{3}(2a)^{3/2} + 8a^2 \cdot \frac{\pi}{2} - \frac{2a}{2}\sqrt{16a^2 - 4a^2} - 8a^2 \cdot \frac{\pi}{6}\right]$

= $2\left[\sqrt{6a}\frac{2}{3} \cdot 2\sqrt{2}a^{3/2} + 4\pi a^2 - a \cdot 2\sqrt{3}a - \frac{4a^2}{3}\pi\right]$

= $2\left[\frac{8}{3}\sqrt{3}a^2 + 4\pi a^2 - 2\sqrt{3}a^2 - \frac{4a^2\pi}{3}\right]$

= $2\left[\frac{2}{3}\sqrt{3}a^2 + \frac{8a^2\pi}{3}\right] = \frac{4}{3}a^2[\sqrt{3} + 4\pi]$

OR




$4y = 3x^2$......(1)

2y = 3x + 12......(2)

From (2), $y = \frac{3x+12}{2}$

Using this value of y in (1), we get,

$x^2 - 6x - 8 = 0$

$\Rightarrow (x + 2)(x - 4) = 0$

$\Rightarrow x = -2, 4$

From (2),

When, x = -2, y = 3
When, x = 4, y = 12
Thus, points of intersection are, (-2, 3) and (4, 12).
Area $= \int_{-2}^{4} \frac{3x+12}{2} dx - \int_{-2}^{4} \frac{3}{4} x^2 dx$
$= \frac{1}{2}[\frac{3x^2}{2} + 12x]_{-2}^{4} - \frac{3}{4}[\frac{x^3}{3}]_{-2}^{4}$
$\frac{1}{2}[(24+48) - (6-24)] - \frac{1}{4}[64 - (-8)]$
= 45 - 18 = 27 sq units.

13. Equation of plane through the given line is
$\{\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) - 7\} + \lambda\{\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) - 9\} = 0$....(i)
$\vec{r} \cdot \{(2+2\lambda)\hat{i} + (2+5\lambda)\hat{j} + (-3+3\lambda)\hat{k}\} = (7+9\lambda)$...(ii)
Here x intercept = z intercept
$\therefore \frac{7+9\lambda}{2+2\lambda} = \frac{7+9\lambda}{-3+3\lambda}$
or $\lambda = 5$
$\therefore$ Equation of plane in vector form is obtained by putting the value of $\lambda$ in equation (ii), we get
$\hat{r} \cdot (12\hat{i} + 27\hat{j} + 12\hat{k}) = 52$
and equation of plane in cartesian form is given as:
$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (12\hat{i} + 27\hat{j} + 12\hat{k}) = 52$
i.e., 12x + 27y + 12z - 52 = 0

**CASE-BASED/DATA-BASED**

14. Let A be the event of commiting an error and $E_1$, $E_2$ and $E_3$ be the events that Govind, Priyanka and Tahseen processed the form.
   i. Using Bayes' theorem, we have
   $P(E_1 \mid A) = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)}$
   $= \frac{0.5 \times 0.06}{0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03} = \frac{30}{47}$
   $\therefore$ Required probability $= P\left(\bar{E}_1 \mid A\right)$
   $= 1 - P(E_1 \mid A) = 1 - \frac{30}{47} = \frac{17}{47}$
   ii. $\sum_{i=1}^{3} P(E_i \mid A) = P(E_1 \mid A) + P(E_2 \mid A) + P(E_3 \mid A)$
   = 1 [$\because$ Sum of posterior probabilities is 1]