Factorization – 1

Exercise

Solution 1:

1. $15x^3y = 3 \times \underline{5} \times \underline{x} \times x \times x \times y$

2. $5x^4 - x^3 = x^3\underline{(5x-1)}$

3. $-5a^2 + 10a = \underline{-5a}(a-2)$

4. $ab + a - 2b - 2 = (a-2)\underline{(b+1)}$
   $ab + a - 2b - 2 = a(b+1) - 2(b+1) = (a-2)(b+1)$

5. $16a^2 + \underline{8a} + 1 = (4a + 1)^2$
   Middle term $= \pm 2 \times \sqrt{\text{First term}} \times \sqrt{\text{Last term}}$
   $= \pm 2 \times \sqrt{16a^2} \times \sqrt{1}$
   $= \pm 2 \times 4a \times 1$
   $= \pm 8a$

6. $\underline{x^2} + 10x + 25 = (x+5)^2$
   $\text{First term} = \frac{(\text{Middle term})^2}{4 \times (\text{Last term})} = \frac{10x \times 10x}{4 \times 25} = x^2$

7. $4y^2 - \underline{12y} + 9 = (2y-3)^2$
   Middle term $= \pm 2 \times \sqrt{\text{First term}} \times \sqrt{\text{Last term}}$
   $= \pm 2 \times \sqrt{4y^2} \times \sqrt{9}$
   $= \pm 2 \times 2y \times 3$
   $= \pm 12y$

8. $16x^2 - 72x + 81$ is perfect square of $\underline{4x-9}$.
   $16x^2 - 72x + 81 = (4x)^2 - 2(4x)(9) + (9)^2 = (4x-9)^2$

9. $a^2 - \underline{0.4a} + 0.04 = (a - 0.2)^2$
   Middle term $= \pm 2 \times \sqrt{\text{First term}} \times \sqrt{\text{Last term}}$
   $= \pm 2 \times \sqrt{a^2} \times \sqrt{0.04}$
   $= \pm 2 \times a \times 0.2$
   $= \pm 0.4a$

10. Adding $\underline{\pm 6x}$ to $9x^2 + 1$ it will become perfect square trinomial.
    First term $= 9x^2$, Last term $= 1$
    Middle term $= \pm 2 \times \sqrt{\text{First term}} \times \sqrt{\text{Last term}}$
    $= \pm 2 \times \sqrt{9x^2} \times \sqrt{1}$
    $= \pm 2 \times 3x \times 1$
    $= \pm 6x$

Solution 2(1):

4ab + 8a – b – 2

= 4a(b + 2) – 1(b + 2)

= (4a – 1)(b + 2)

Solution 2(2):

$x^2y – 3x^2 + y – 3$

= $x^2(y – 3) + 1(y – 3)$

= $(x^2 + 1)(y – 3)$

Solution 2(3):

$2x^2 – 5a – 5x + 2ax$

= $2x^2 + 2ax – 5x – 5a$

= 2x(x + a) – 5(x + a)

= (2x – 5)(x + a)

Solution 2(4):

3ab + 12 – 4a – 9b

= 3ab – 4a – 9b + 12

= a(3b – 4) – 3(3b – 4)

= (a – 3)(3b – 4)

Solution 2(5):

$x^2 + 49 + 14x$

= $x^2 + 14x + 49$

= $(x)^2 + 2(x)(7) + (7)^2$

= $(x + 7)^2$

Solution 2(6):

$16a^2 + 40ab + 25b^2$

= $(4a)^2 + 2(4a)(5b) + (5b)^2$

= $(4a + 5b)^2$

Solution 2(7):

$m^4 – 16m^2 + 64$

$= (m^2)^2 – 2(m^2)(8) + (8)^2$

$= (m^2 – 8)^2$

Solution 2(8):

$4y^3 – 28y^2 + 49y = y(4y^2 – 28y + 49)$

$= y[(2y)^2 – 2(2y)(7) + (7)^2]$

$= y(2y – 7)^2$

Solution 2(9):

$25x^2 + 4y^2 + 9z^2 + 20xy + 12yz + 30zx$

$= (5x)^2 + (2y)^2 + (3z)^2 + 2(5x)(2y) + 2(2y)(3z) + 2(3z)(5x)$

$= (5x + 2y + 3z)^2$

Solution 2(10):

$4m^2 + 9n^2 + p^2 – 12mn + 6np – 4pm$

$= (2m)^2 + (-3n)^2 + (-p)^2 + 2(2m)(-3n) + 2(-3n)(-p) + 2(-p)(2m)$

$= (2m – 3n – p)^2$

Practice 1:

Solution 1:

1. $2x^2y^2 =$ _2_ × x × x ×y × y
2. $10a^2b = 2 \times$ _5_ × a × _a_ × b
3. $6xy =$ _2_× 3 × x ×y
4. $15mn^2 = 3 \times$ _5_ × m × _n_ × n

Solution 2:

1. $25 = 5 \times 5$
2. $6x^2y = 2 \times 3 \times x \times x \times y$
3. $20x^2y^4 = 2 \times 2 \times 5 \times x \times x \times y \times y \times y \times y$
4. $24x^3y^2 = 2 \times 2 \times 2 \times 3 \times x \times x \times x \times y \times y$
5. $26xy = 2 \times 13 \times x \times y$
6. $18a^3b = 2 \times 3 \times 3 \times a \times a \times a \times b$

Practice 2:

Solution 1:

1. $x^2 – x = x(x – 1)$_
   $x^2 – x =$ _$x \times x$_- $x = x(x – 1)$
2. $8x^3 + 4x^2 = 4x^2(2x + 1)$
   $8x^3 + 4x^2 = 4 \times 2 \times x \times x \times x \times 4 \times x \times x = 4x^2(2x + 1)$
3. $3a^2 – 6 = 3(a^2 – 2)$
   $3a^2 – 6 = 3 \times a \times a – 3 \times 2 = 3(a^2 – 2)$

4. $xy - xz = x(y - z)$
   $xy - yz = x \times y - x \times z = x(y - z)$

Solution 2:

1. $10x + 5 = 2 \times 5 \times x + 5$
   $= 5(2x + 1)$ ….(Taking ‘5’ common)
2. $5x^2 + 15$
   $= 5 \times x \times x + 3 \times 5 = 5(x^2 + 3)$ ….(Taking ‘5’ common)
3. $7a - 7b$
   $= 7 \times a - 7 \times b = 7(a - b)$ ….(Taking ‘7’ common)
4. $-3x + 6$
   $= -3 \times x + 2 \times 3 = -3(x - 2)$ ….(Taking ‘-3’ common)
5. $6x^3y^2 - 3x$
   $= 2 \times 3 \times x \times x \times x \times y \times y - 3 \times x = 3x(2x^2y^2 - 1)$ ….(Taking ‘3x’ common)
6. $9xy^2 - 18x^2$
   $= 9 \times x \times y \times y - 2 \times 9 \times x \times x = 9x(y^2 - 2x)$ ….(Taking ‘9x’ common)
7. $8 - 4xy$
   $= 2 \times 4 - 4 \times x \times y = 4(2 - xy)$ ….(Taking ‘4’ common)
8. $9x - 27xyz$
   $= 9 \times x - 3 \times 9 \times x \times y \times z = 9x(1 - 3yz)$ ….(Taking ‘9x’ common)
9. $12a^2b - 18ab^2$
   $= 2 \times 6 \times a \times a \times b - 3 \times 6 \times a \times b \times b = 6ab(2a - 3b)$ ….(Taking ‘6ab’ common)

Practice 3:

Solution 1:

$\underline{xy + 2x} + \underline{4y + 8}$

$= x(y + 2) + 4(y + 2)$

$= (x + 4)(y + 2)$

Solution 2:

$\underline{xy - 4x} + \underline{3y - 12}$

$= x(y - 4) + 3(y - 4)$

$= (x + 3)(y - 4)$

Solution 3:

$= \underline{x^2y + 5x^2} + \underline{y + 5}$

$= x^2(y + 5) + 1(y + 5)$

$= (x^2 + 1)(y + 5)$

Solution 4:

$\underline{6x^2 + 4xy} - \underline{3x - 2y}$

$= 2x(3x + 2y) - 1(3x + 2y)$

$= (2x - 1)(3x + 2y)$

Solution 5:

$15x - 4a + 6 - 10ax$

$= \underline{15x + 6} - \underline{10ax - 4a}$

$= 3(5x + 2) - 2a(5x + 2)$

$= (5x + 2)(3 - 2a)$

Solution 6:

$10m^2n + 9 + 6m + 15mn$

$= \underline{10m^2n + 15mn} + \underline{6m + 9}$

$= 5mn(2m + 3) + 3(2m + 3)$

$= (5mn + 3)(2m + 3)$

Practice 4:

Solution 1(1):

First term $= x^2 = (x)^2$

Last term $= 4 = (2)^2$

Here, the first and the last terms are perfect positive squares and have a positive sign.

Middle term $= \pm 2 \times \sqrt{\text{First term}} \times \sqrt{\text{Last term}}$

$= \pm 2 \times \sqrt{x^2} \times \sqrt{4}$

$= \pm 2 \times x \times 2$

$= \pm 4x$

But the middle term of the given polynomial is $+ 2x$.

So, the given polynomial is not a perfect square.

Solution 1(2):

First term $= x^2 = (x)^2$

Last term $= 49 = (7)^2$

Here, the first and the last terms are perfect positive squares and have a positive sign.

Middle term $= \pm 2 \times \sqrt{\text{First term}} \times \sqrt{\text{Last term}}$

$= \pm 2 \times \sqrt{x^2} \times \sqrt{49}$

$= \pm 2 \times x \times 7$

$= \pm 14x$

The middle term of the given polynomial is $- 14x$.

So, the given polynomial is a perfect square.

Solution 1(3):

First term $= a^2 = (a)^2$

Last term $= 25 = (5)^2$

Here, the first and the last terms are perfect positive squares and have a positive sign.

Middle term $= \pm 2 \times \sqrt{\text{First term}} \times \sqrt{\text{Last term}}$

$= \pm 2 \times \sqrt{a^2} \times \sqrt{25}$

$= \pm 2 \times a \times 5$

$= \pm 10a$

The middle term of the given polynomial is $+ 10a$.

So, the given polynomial is a perfect square.

Solution 1(4):

First term $= 9x^2y^2 = (3xy)^2$

Thus, the first term is a perfect positive perfect square.

Last term = 8, which is not a perfect square.

$\therefore$ The given polynomial is not a perfect square.

Solution 1(5):

First term = $25x^2 = (5x)^2$

Last term = $49 = (7)^2$

Here, the first and the last terms are perfect positive squares and have a positive sign.

Middle term = $\pm 2 \times \sqrt{\text{First term}} \times \sqrt{\text{Last term}}$

$= \pm 2 \times \sqrt{25x^2} \times \sqrt{49}$

$= \pm 2 \times 5x \times 7$

$= \pm 70x$

But, the middle term of the given polynomial is $-35x$.

So, the given polynomial is not a perfect square.

Solution 1(6):

First term = $4x^2 = (2x)^2$

Last term = $1 = (1)^2$

Here, the first and the last terms are perfect positive squares and have a positive sign.

Middle term = $\pm 2 \times \sqrt{\text{First term}} \times \sqrt{\text{Last term}}$

$= \pm 2 \times \sqrt{4x^2} \times \sqrt{1}$

$= \pm 2 \times 2x \times 1$

$= \pm 4x$

The middle term of the given polynomial is $+4x$.

So, the given polynomial is a perfect square.

Solution 1(7):

First term = $x^2 = (x)^2$

Last term = $\frac{1}{x^2} = \left(\frac{1}{x}\right)^2$

Here, the first and the last terms are perfect positive squares and have a positive sign.

Middle term = $\pm 2 \times \sqrt{\text{First term}} \times \sqrt{\text{Last term}}$

$= \pm 2 \times \sqrt{x^2} \times \sqrt{\frac{1}{x^2}}$

$= \pm 2 \times x \times \frac{1}{x}$

$= \pm 2$

The middle term of the given polynomial is $+2$.

So, the given polynomial is a perfect square.

Solution 1(8):

First term = $x^2 = (x)^2$

Last term = $16 = (4)^2$

Here, the first and the last terms are perfect positive squares and have a positive sign.

Middle term = $\pm 2 \times \sqrt{\text{First term}} \times \sqrt{\text{Last term}}$

$= \pm 2 \times \sqrt{x^2} \times \sqrt{16}$

$= \pm 2 \times x \times 4$

$= \pm 8x$

The middle term of the given polynomial is $-8x$.

So, the given polynomial is a perfect square.

Solution 2(1):

$9a^2 + \underline{24a} + 16$

$\text{Middle term} = \pm 2 \times \sqrt{\text{First term}} \times \sqrt{\text{Last term}}$

$= \pm 2 \times \sqrt{9a^2} \times \sqrt{16}$

$= \pm 2 \times 3a \times 4$

$= \pm 24a$

Solution 2(2):

$\underline{4x^2} - 12x + 9$

$\text{First term} = \frac{(\text{Middle term})^2}{4 \times \text{Last term}}$

$= \frac{(-12x)^2}{4 \times 9}$

$= \frac{12x \times 12x}{4 \times 9}$

$= 4x^2$

Solution 2(3):

$9x^2 + 30xy + \underline{25y^2}$

$\text{Last term} = \frac{(\text{Middle term})^2}{4 \times \text{First term}}$

$= \frac{(30xy)^2}{4 \times 9x^2}$

$= \frac{30xy \times 30xy}{4 \times 9x^2}$

$= 25y^2$

Solution 2(4):

$\underline{x^2y^2} + 4xy + 4$

$\text{First term} = \frac{(\text{Middle term})^2}{4 \times \text{Last term}}$

$= \frac{(4xy)^2}{4 \times 4}$

$= \frac{4xy \times 4xy}{4 \times 4}$

$= x^2y^2$

Solution 2(5):

$81x^2 + \underline{36x} + 4$

$\text{Middle term} = \pm 2 \times \sqrt{\text{First term}} \times \sqrt{\text{Last term}}$

$= \pm 2 \times \sqrt{81x^2} \times \sqrt{4}$

$= \pm 2 \times 9x \times 2$

$= \pm 36x$

Solution 2(6):

$4a^2 + \underline{2} + \frac{1}{4a^2}$

Middle term $= \pm 2 \times \sqrt{\text{First term}} \times \sqrt{\text{Last term}}$

$= \pm 2 \times \sqrt{4a^2} \times \sqrt{\frac{1}{4a^2}}$

$= \pm 2 \times 2a \times \frac{1}{2a}$

$= \pm 2$

Solution 3:

1. $x^2 + 12x + 36$
   $= (x)^2 + 2(x)(6) + (6)^2$
   $= (x + 6)^2$
2. $4x^2 + 12xy + 9y^2$
   $= (2x)^2 + 2(2x)(3y) + (3y)^2$
   $= (2x + 3y)^2$
3. $9x^2 + 48x + 64$
   $= (3x)^2 + 2(3x)(8) + (8)^2$
   $= (3x + 8)^2$
4. $x^2 – 8x + 16$
   $= (x)^2 – 2(x)(4) + (4)^2$
   $= (x – 4)^2$
5. $25x^2y^2 – 20xy + 4$
   $= (5xy)^2 – 2(5xy)(2) + (2)^2$
   $= (5xy – 2)^2$
6. $16x^2 + 40x + 25$
   $= (4x)^2 + 2(4x)(5) + (5)^2$
   $= (4x + 5)^2$
7. $81 – 90xy + 25x^2y^2$
   $= (9)^2 – 2(9)(5xy) + (5xy)^2$
   $= (9 – 5xy)^2$
8. $3x^3 – 30x^2 + 75x$
   $= 3x(x^2 – 10x + 25)$
   $= 3x[(x)^2 – 2(x)(5) + (5)^2]$
   $= 3x(x – 5)^2$

Practice 5:

Solution 1:

$9x^2 + 4y^2 + 1 + 12xy + 4y + 6x$

$= (3x)^2 + (2y)^2 + (1)^2 + 2(3x)(2y) + 2(2y)(1) + 2(1)(3x)$

$= (3x + 2y + 1)^2$

Solution 2:

$16a^2 + 9b^2 + c^2 – 24ab + 6bc – 8ca$

$= (4a)^2 + (-3b)^2 + (-c)^2 + 2(4a)(-3b) + 2(-3b)(-c) + 2(-c)(4a)$

$= (4a – 3b – c)^2$

Solution 3:

$a^4 + 4b^2 + 9 + 4a^2b – 12b – 6a^2$

$= (a^2)^2 + (2b)^2 + (-3)^2 + 2(a^2)(2b) + 2(2b)(-3) + 2(-3)(a^2)$

$= (a^2 + 2b – 3)^2$

Solution 4:

$9x^2 + 16y^2 + 25 + 24xy – 40y – 30x$

$= (3x)^2 + (4y)^2 + (-5)^2 + 2(3x)(4y) + 2(4y)(-5) + 2(-5)(3x)$

$= (3x + 4y – 5)^2$

Solution 5:

$a^2 + 4b^2 + c^2 – 4ab – 4bc + 2ca$

$= (a)^2 + (-2b)^2 + (c)^2 + 2(a)(-2b) + 2(-2b)(c) + 2(c)(a)$

$= (a – 2b + c)^2$