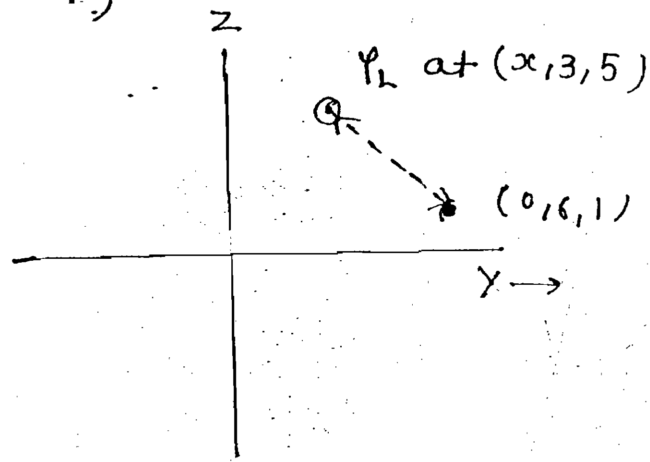
Lecture - 5

2) ρ_L at $y = 3$, $z = 5$ for all x

Given $\vec{E}$ at $(0, 6, 1)$

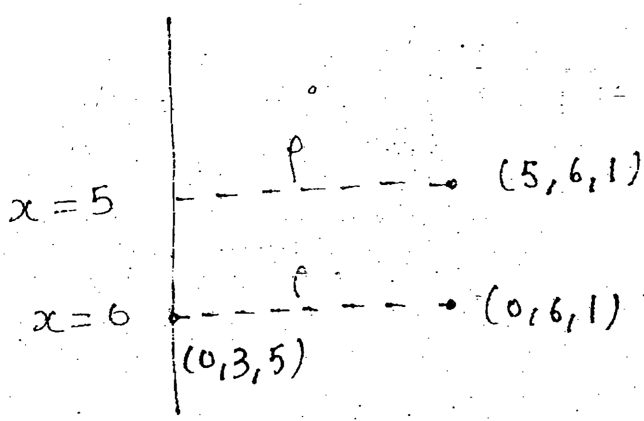
$\vec{E} = ?$ at $(5, 6, 1)$

$$E = \frac{\rho_L}{2\pi\epsilon\rho}\, a_\rho$$



$$E \propto \frac{1}{\rho}$$

ρ_L at $(x,3,5)$



For both point ρ is same

$$\rho = \sqrt{3^2 + 4^2} = 5$$

Note :-

In general z-axis $\longrightarrow (0, 0, z)$

Any point (x, y, z)

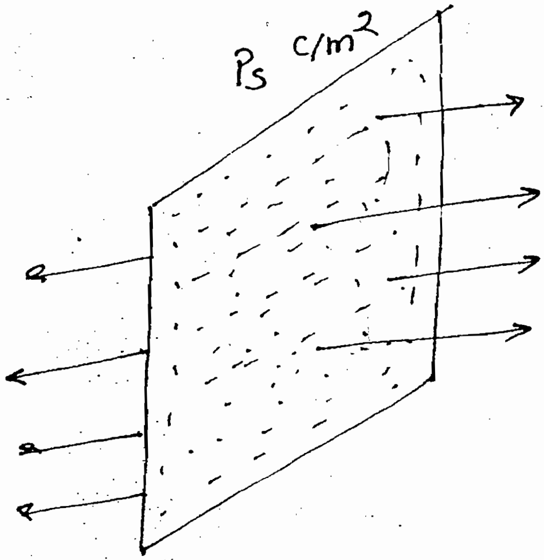
$$\rho = \sqrt{x^2 + y^2}$$

Any line parallel to x-axis $\longrightarrow (x, a, b)$

Any point $\longrightarrow (x, y, z)$

$$\rho = \sqrt{(y-a)^2 + (z-b)^2}$$

Electric field strength of an infinite sheet of ρ_s C/m² charge density :-



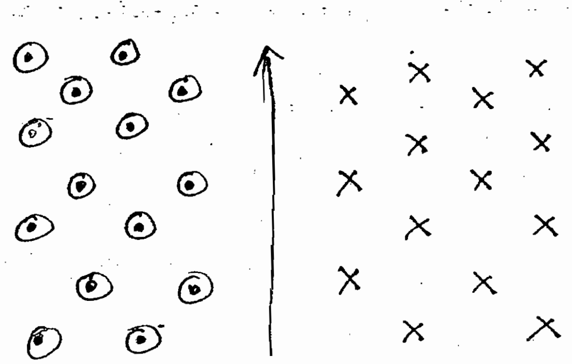
→ The electric field is completely normal to the sheet and the field lines are parallel to themselves

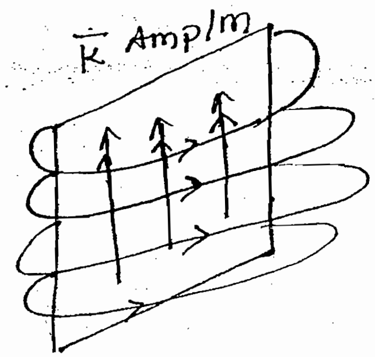
→ The flux density is same everywhere. The field is uniform.

→ $$D = \frac{\rho_s}{2} a_N$$

→ $$E = \frac{D}{\epsilon} = \frac{\rho_s}{2\epsilon} a_N \Rightarrow \boxed{E = \frac{\rho_s}{2\epsilon} a_N}$$ **

Magnetic field strength of an infinite sheet of ρ_s C/m² charge density :-





→ The magnetic field is completely normal to the current and the field lines are parallel to themselves and to the sheet

→ The flux density is same everywhere i.e. field is uniform

→ $$H = \frac{\vec{K}}{2} \times a_N \quad Amp/m$$

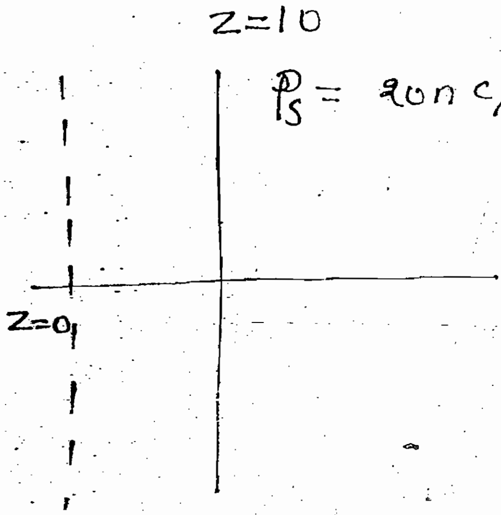
and $$\boxed{B = \frac{\mu \vec{K}}{2} \times a_N.}$$

24. $a_N = \pm a_z$

At $z = 0$, $a_N = -a_z$

$$E = \frac{\rho_s}{2\epsilon} a_N$$

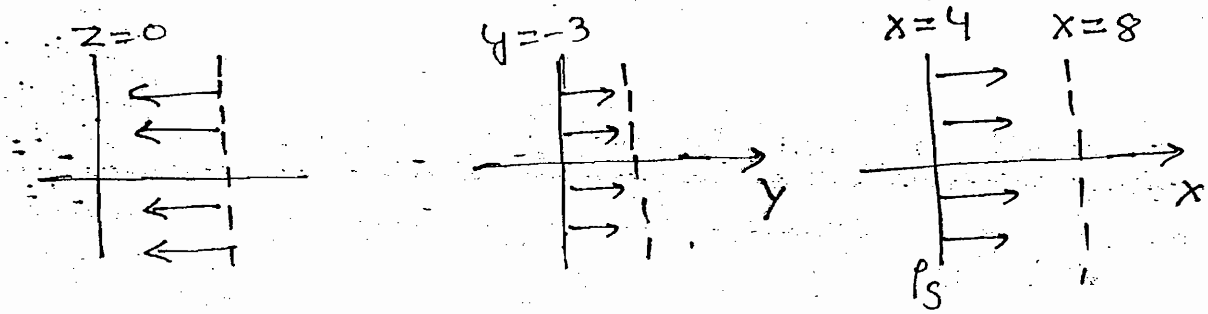
$$= \frac{20 \times 10^{-9}}{2 \times \frac{1}{36\pi \times 10^9}} (-a_z)$$



25. $x = 4$ $\quad \rho_{s_1} = 18\,nc/m^2 \longrightarrow E_1 = \pm a_x$

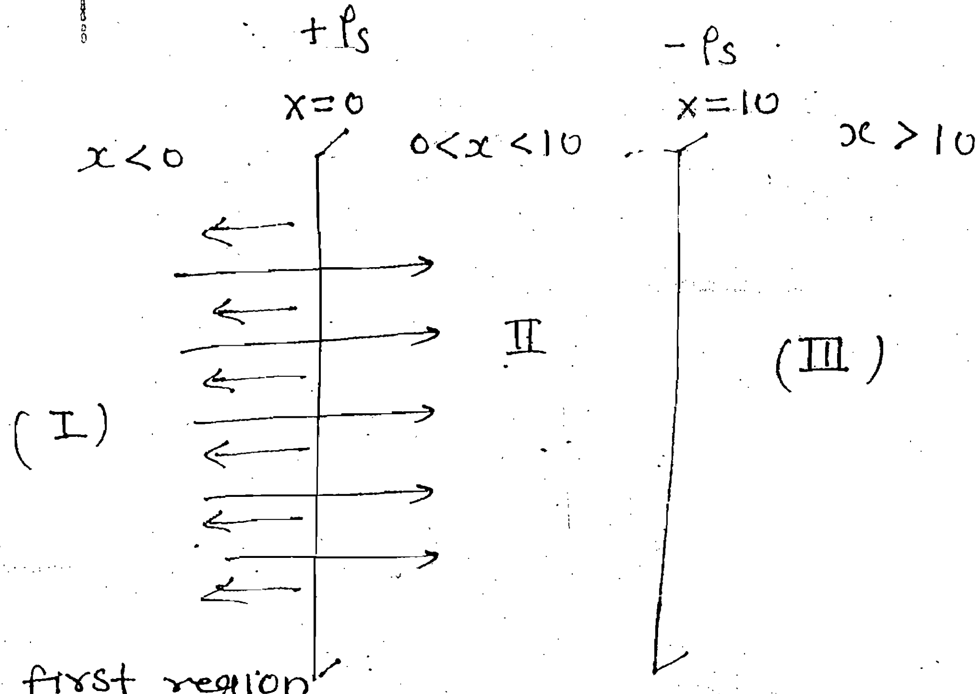
$y = -3$ $\quad \rho_{s_2} = 9\,nc/m^2 \longrightarrow E_2 = \pm a_y$

$z = 0$ $\quad \rho_{s_3} = -24\,nc/m^2 \longrightarrow E_3 = \pm a_z$



$$E_3 = \frac{24}{2\epsilon}(-a_z) \qquad E_2 = \frac{9}{2\epsilon} a_y \frac{nN}{C} \qquad E_1 = \frac{18}{2\epsilon} a_x \frac{nN}{C}$$

$$E_T = \frac{3}{2\epsilon}\left(6a_x + 3a_y - 8a_z\right) \frac{nN}{C}$$



<u>In first region</u>

$$E_1 = \frac{\rho_s}{2\epsilon}(-a_x)$$

$$E_2 = \frac{\rho_s}{2\epsilon}(a_x)$$

$$\vec{E}_{Net} = 0$$

<u>In third region</u>

$$\vec{E}_{Net} = 0$$

<u>In second region</u> :-

$$E_1 = \frac{\rho_s}{2\epsilon} a_x \qquad E_2 = \frac{\rho_s}{2\epsilon} a_x$$

$$E = \frac{\rho_s}{\epsilon} a_x \quad 0 < x < 10$$

$$= 0 \quad \text{elsewhere}$$

7. $\vec{K} = 30\, a_z$ mA/m $\longrightarrow$ $Y=0$ plane $\Rightarrow$ zx plane

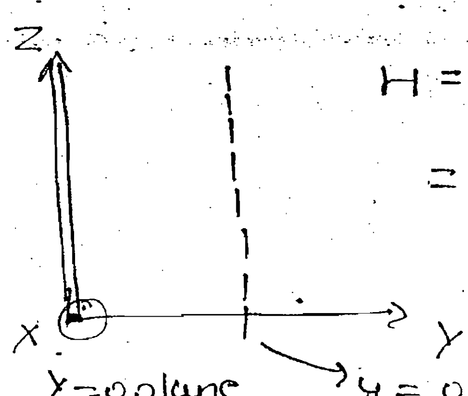
$H = \frac{\vec{K}}{2} \times a_N$

at $y = 2$

$a_N = \pm a_y$

for $y = 20$

$a_N = a_y$



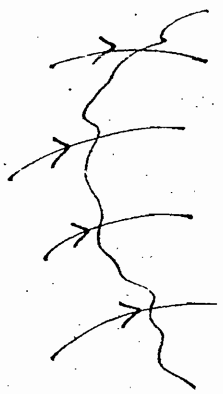
$H = \frac{30}{2} a_z \times a_y$

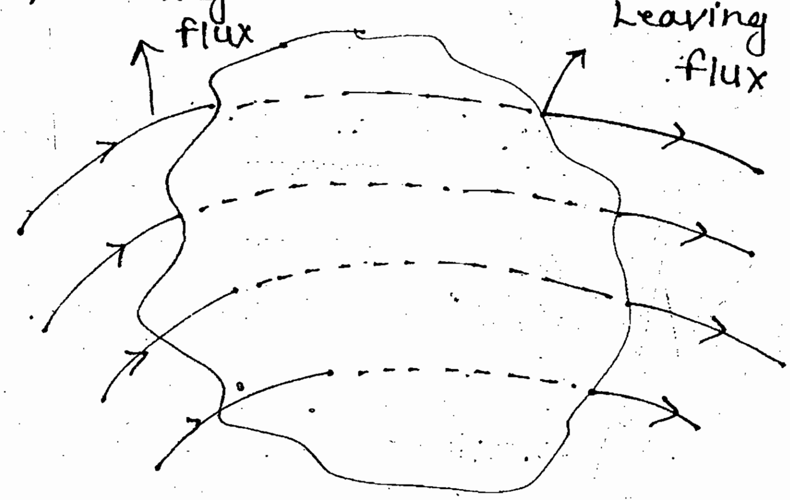
$= -15 a_x$ mA/m

Ans-(A)

Closed surface Integral of B — Maxwell's III Equation :-

→ Magnetic field always forms closed lines around the current and has no starting / ending point for flux lines i.e. No sources / sinks exists for B flux lines





$$\int B \cdot ds = \psi_m$$

This is not Maxwell's equation

$$\oint B \cdot ds = 0$$

This is Maxwell III equation

→ For any closed surface

Entering flux = Leaving flux

$$\oint B \cdot ds = 0$$

B field is solenoidal (No divergence)

$$\nabla \cdot B = 0$$

Note :-

$$\oint D \cdot ds = 0 \quad \begin{cases} \to Q = 0 \\ \to +Q, -Q \end{cases} \Rightarrow \boxed{\oint B \cdot ds = 0}$$

Always

→ By comparison, equals of charge in E fields donot exist in B fields i.e. every cause of B field is a dipole, i.e. Magnetic monopoles don't exist.

eg:- Bar Magnet , N/S – Dipole

best example:- I current carrying wire

→ I always flows only when both the polarities exist and are connected hence it is always treated as magnetic dipoles.

→ I always flows in closed circuits and every closed I wire is a magnetic dipole

Clockwise → South pole

Anticlockwise → North pole

→ Cause – current – closed

Entering current = Leaving current (Junction)

Effect – B field – closed around the cause

Entering flux = Leaving flux (Closed surface)

$$\boxed{\oint B \cdot ds = 0} \longrightarrow \text{KCL in Magnetic Field}$$

Potential, Gradient, closed line Integral in E field :-

Potential is a ~~square~~ scalar measure of E field strength in terms of energy at any point in the field

$$\text{V (Volts) Potential at any point} = \frac{\text{Workdone by the charge to reach the point}}{\text{charge}}$$

$$V = \frac{W}{Q} = \frac{\text{Joules}}{\text{Coulomb}}$$

$\Rightarrow$ Work = Force . displacement

$$\Rightarrow dW = \vec{F}\cdot\vec{dl} = Q\vec{E}\cdot\vec{dl}$$

= Workdone in field / force direction

= Workdone on the charge

→ Work is done on the charge when it moves in the field direction and hence the charge acquires energy

$$\boxed{V = \frac{W}{Q} = -\int \vec{E}\cdot\vec{dl}}$$ → Scalar potential function of space

$$V_{AB} = -\int_{\text{Ref }B}^{\text{Ref }A} \vec{E}\cdot\vec{dl}$$ = Potential diff b/w A & B

If ref. B has $V_B = 0$, then V_A is called as absolute potential at A.

<u>Potential function of a point charge</u> :—

$$V = -\int \vec{E}\cdot\vec{dl}$$

Spherical coord.

$$= -\int \frac{Q}{4\pi\epsilon r^2} a_r \, dr \, a_r = \frac{Q}{4\pi\epsilon_0 r}$$

$$\Rightarrow \boxed{V = \frac{Q}{4\pi\epsilon r}}$$

$\vec{E} \rightarrow$ Intensity $- \frac{1}{r^2}$ decrease

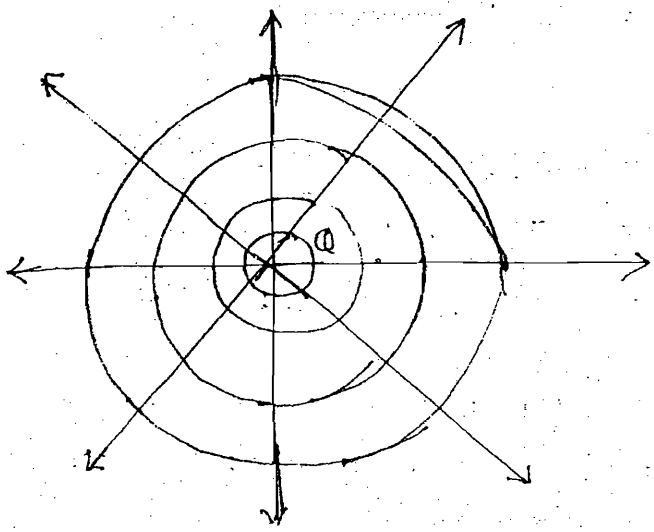
a_r directed vector

$V \longrightarrow$ Potential $- \frac{1}{r}$ decrease

— scalar —

If $r =$ constant then $V =$ constant

$\longrightarrow$ cocentric sphere $\rightarrow$ surface



→ The family of ~~cocentri~~ equipotential surfaces represent the potential distribution and variation of potential in the region and similar to vector intensity line

Potential function of a line Charge:—

$$V = -\int \vec{E} \cdot \vec{dl}$$

$$= -\int \frac{\rho_L}{2\pi\epsilon\rho} a_\rho \, d\rho \, a_\rho$$

$$\Rightarrow V = \frac{\rho_L}{2\pi\epsilon} \ln\left(1/\rho\right)$$

If $\rho =$ constant $\Rightarrow$ $V =$ constant

Equipotential surfaces are cocentric cylinders

$\vec{E}$ – Intensity – $\frac{1}{\rho}$ decrease

a_ρ directed vector

$V \rightarrow$ Potential – logarithmic decrease

– scalar

<u>Potential function of a sheet charge (Uniform):–</u>

$$V = -\int \vec{E} \cdot \vec{dl}$$

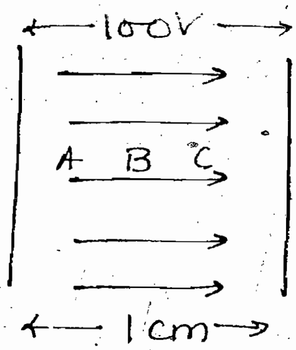
$$\Rightarrow V = -Ed$$

$$\Rightarrow E = -\frac{V}{d}$$

$\vec{E} \rightarrow$ Intensity – Uniform

V – Potential – linear decrease

eg:– Capacitor Plates

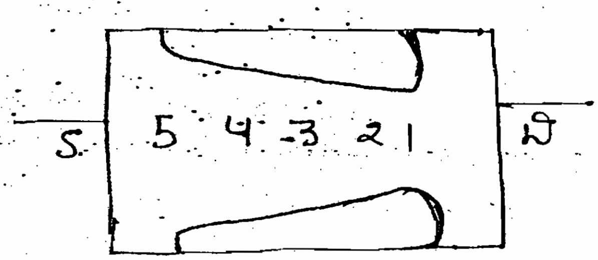


E_A or E_B or E_C

$= \frac{100}{10} = 10 \text{ kV/m}$

$V_A > V_B > V_C$

<u>FET channel</u>



28.

$$V = -\int \vec{E}\cdot\vec{dl} = -\int_{(0,0,0)} (y a_x + x a_y)\cdot(dx\, a_x + dy\, a_y).$$

$$V = -\int_{x=0}^{4} y\, dx - \int_{y=0}^{1} x\, dy$$

Using the straight line path or equation

$$\frac{y-1}{1-0} = \frac{x-4}{4-0}$$

$$x = 4y$$

$$= -\int_{0}^{4} \frac{x}{4}\, dx - \int_{0}^{1} 4y\, dy$$

$$= -\frac{1}{8}\left(16^2\right) - \frac{4}{2} \times 1$$

$$= -4V \text{ , Ans.}$$

Note :—

In line integral,

$\int f(x,y,z)\, dx$ We have to transform y & z in terms of x using the known relationship

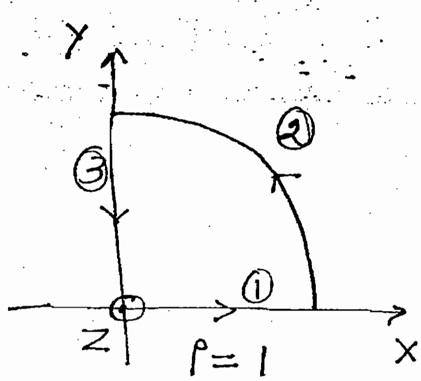
i.e. path of integration

$$\int f(x,y,z)\, dx = \int g(x)\, dx$$

29.

$$\vec{A} = 2\rho\cos\phi\, a_\rho$$

$$\oint A\cdot dl = 3 \longrightarrow \int A\cdot dl$$



(1) $dl = d\rho\, a_\rho$

$\phi = 0$

$A = 2\rho\, a_\rho$

$$\int A\cdot dl = \int_{\rho=0}^{1} 2\rho\, a_\rho\, d\rho \cdot a_\rho$$

$$= \rho^2\Big|_0^1 = 1$$

2 → $dl = \rho\, d\phi\, \underline{a_\phi}$ $(\because a_\phi \cdot a_\rho = 0)$

$$\int A\cdot dl = 0$$

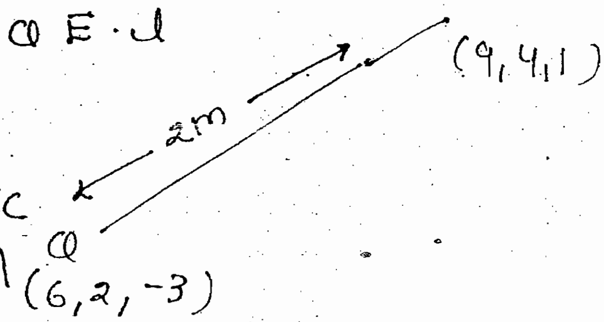
3 → $dl = d\rho\, a_\rho$

ρ from 1 to 0

$\phi = 90 \Rightarrow A = 0$

30 → $\vec{E} = 4a_x - 3a_y + 5a_z$ ⟶ Uniform

$$W = Q\int \vec{E}\cdot \vec{dl} = Q\, E\cdot l$$



Unit length vector $= \dfrac{3a_x + 2a_y + 4a_z}{\sqrt{3^2+2^2+4^2}} \times 2$ 5C

$(\because$ 2m length vector$)$

$$W = 5\,(4a_x - 3a_y + 5a_z)\cdot \frac{(3a_x + 2a_y + 4a_z)\times 2}{\sqrt{29}}$$

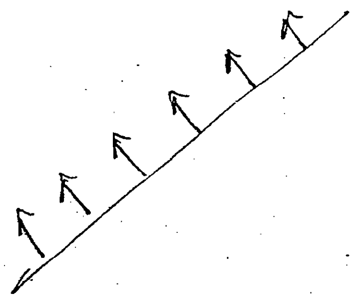
$$= \frac{260}{\sqrt{29}}\ \text{J}$$

Potential Gradient :-

Scalar surface equation (f) $\xrightarrow{\nabla f}$ Vector (Normal) to the surface (direction)

eg:- Linear surface

$$f = 4x + 7y - 15z = 18$$



$$\nabla f = 4a_x + 7a_y - 15a_z$$

eg:- Non-linear surface

$$f = 4xy - 15x^2yz$$

$$\nabla f = (4y - 30xyz)a_x + (4x - 15xz^2)a_y + 15x^2y\, a_z$$

$$V = -\int \vec{E} \cdot \vec{dl}$$

$$dV = -\vec{E}\, dl \cos\theta$$

$$\Rightarrow \boxed{\frac{dV}{dl} = -E\cos\theta}$$

Case-(1) :-

If $\theta = 90^\circ \Rightarrow V = \text{constant}$

→ The direction of E field is always normal to equi-potential surfaces.

Case-(II):-

If $\theta = 0^\circ/180^\circ \Rightarrow \left.\frac{dV}{dl}\right|_{max} = |E|$

→ The magnitude of E intensity is always the maximum rate of change of potential per unit length ~~If~~

Case-(III):-

If $\theta = 0^\circ \Rightarrow |E| = -\left.\frac{dV}{dl}\right|_{max}$

→ The direction of E intensity is always the direction in which potential decreases by maximum

Note:-

The operation is physically called as gradient

Hence $E = -\nabla V$ = potential gradient is field intensity.

→ Gradient signifies the maximum rate of change of scalar along with the direction of change.

eg:- Given $V = 4(x^2-y^2)$ for all z

Find the equation of equipotential surface passing through (3,1,9)

Soln:- V at (3,1,9) = 4(9−1) = 32V

All x & y when V = 32V is the equipotential surface

$\Rightarrow x^2 - y^2 = 8$

Summary:-

Equation of equipotential surface OR Voltage function (V) $\longrightarrow \nabla V \rightarrow$ Vector Intensity function (E) which is normal to equi-potential surfaces

Mathematically, ∇V.

$$\boxed{\nabla V = \frac{1}{h_1}\frac{\partial V}{\partial u}a_u + \frac{1}{h_2}\frac{\partial V}{\partial v}a_v + \frac{1}{h_3}\frac{\partial V}{\partial w}a_w}$$

eg:- If $V = \dfrac{4\cos\theta}{r^2}$

Find $\vec{E}$ at $(2, \pi/4, \pi)$

Soln:- $E = -\nabla V$

$$\nabla V = \frac{1}{1}\frac{\partial}{\partial r}\left(\frac{4\cos\theta}{r^2}\right)a_r + \frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{4\cos\theta}{r^2}\right)a_\theta$$

$$= 4\cos\theta\left(\frac{-2}{r^3}\right)a_r + \frac{4}{r^3}(-\sin\theta)a_\theta$$

$$E = \frac{4}{r^3}\left(2\cos\theta\, a_r + \sin\theta\, a_\theta\right)\Big|_{(2,\pi/4,\pi)}$$

$$= \frac{1}{2}\left(\sqrt{2}\,a_r + \frac{1}{\sqrt{2}}a_\theta\right)$$

32.

(a) $V = 0$ $\quad \nabla V = 6xy\,a_x + (3x^2 - z)\,a_y - y\,a_z$

$\neq 0$ at $(1, 0, -1)$

(b) $x^2y = 1$ in XY plane

$V = 3(1) - y(0)$

$V = 3 \longrightarrow$ equipotential surface

(c) At $(2, -1, 4) \Rightarrow V = -8$

$V = 3(4)(-1) - (-1)4 = -8$

(d) Normal vector at P $= \dfrac{\nabla V}{|\nabla V|}\Big|_{(2,-1,4)} = \dfrac{-12a_x + 8a_y - a_z}{\sqrt{12^2 + 8^2 + 1}}$

$= (-0.83\hat{x} + 0.55\hat{y} + 0.07\hat{z})$