# Sample Question Paper - 6
# Mathematics-Standard (041)
# Class- X, Session: 2021-22
# TERM II

**Time Allowed: 2 hours** **Maximum Marks: 40**

**General Instructions:**

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

**Section A**

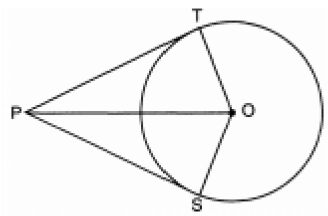
1. If 10 times the $10^{th}$ term of an A.P. is equal to 15 times the $15^{th}$ term, show that $25^{th}$ term of the A.P. is zero. **[2]**

OR

Is 302 a term of the A.P. 3,8,13,...?

2. Determine the positive value of 'k' for which the equation $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will both have real and equal roots. **[2]**

3. In the given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that $\angle$SPT = $120^o$, Prove that OP = 2PS **[2]**




4. A golf ball has diameter equal to 4.2 cm. Its surface has 200 dimples each of radius 2 mm. Calculate the total surface area which is exposed to the surroundings assuming that the dimples are hemispherical. **[2]**

5. The average marks of A, B and C is 33, while the average marks of B, C and D is 37. If A obtains 30 marks, find the marks obtained by D. **[2]**

6. Find the roots of quadratic equation by the factorisation method: $3x^2 + 5\sqrt{5}x - 10 = 0$ **[2]**

OR

Find the value of k for the quadratic equation $2x^2+kx+3=0$, so that they have two real equal roots.

**Section B**

7. Find the median of the following frequency distribution: **[3]**

| **Marks** | 0 - 10 | 10 - 20 | 20 - 30 | 30 - 40 | 40 - 50 |
|---|---|---|---|---|---|
| **Number of students** | 6 | 16 | 30 | 9 | 4 |

8. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of $60^{0}$. **[3]**

9. Find the mean of the following frequency distribution: **[3]**

| **Class interval** | 10-30 | 30-50 | 50-70 | 70-90 | 90-110 | 110-130 |
|---|---|---|---|---|---|---|
| **Frequency** | 5 | 8 | 12 | 20 | 3 | 2 |

10. The angle of elevation of a jet fighter from point A on ground is $60^{0}$. After flying 10 seconds, the angle changes to 30°. If the jet is flying at a speed of 648 km/hour, find the constant height at which the jet is flying. **[3]**

OR

Two ships are there in the sea on either side of a lighthouse in such a way that the ships and the lighthouse are in the same straight line. The angles of depression of two ships are observed from the top of the lighthouse are 60° and 45° respectively. If the height of the lighthouse is 200 m, find the distance between the two ships. (Use $\sqrt{3}$ = 1.73)

**Section C**

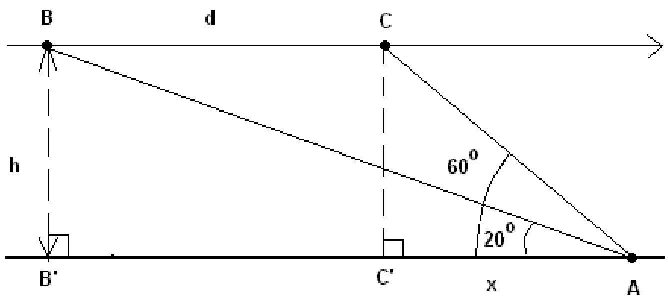
11. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104 cm and the radius of each of the hemispherical ends is 7 cm, find the cost of polishing its surface at the rate of ₹10 per $dm^2$. **[4]**

12. Equal circles with centres O and O' touch each other at X. OO' is produced to meet the circle with centre O' at A and AC is a tangent to the circle with centre O. If O'D is perpendicular to AC, find $\frac{DO'}{CO}$ **[4]**

OR

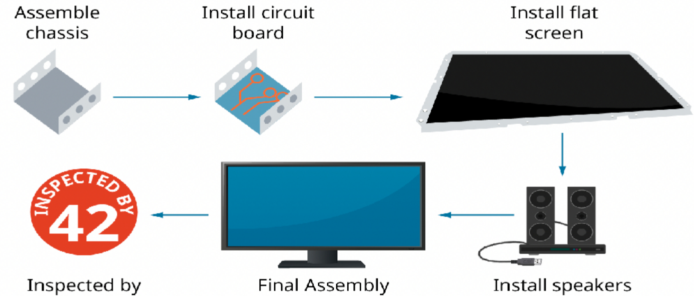
Two circles with centres O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ.

13. Mr. Vinod is a pilot in Air India. During the Covid-19 pandemic, many Indian passengers were stuck at Dubai Airport. The government of India sent special aircraft to take them. Mr. Vinod was leading this operation. He is flying from Dubai to New Delhi with these passengers. His airplane is approaching point A along a straight line and at a constant altitude h. At 10:00 am, the angle of elevation of the airplane is $20^{0}$ and at 10:01 am, it is $60^{0}$. **[4]**




i. What is the distance 'd' is covered by the airplane from 10:00 am to 10:01 am if the speed of the airplane is constant and equal to 600 miles/hour?
ii. What is the altitude 'h' of the airplane? (round answer to 2 decimal places).

14. Elpis Technology is a laptop manufacturer. The company works for many branded laptop companies and also provides them with spare parts. Elpis Technology produced 6000 units in 3rd year and 7000 units in the 7th year. **[4]**




Assuming that production increases uniformly by a fixed number every year, find

i. the production in the 1st year, **(2)**

ii. the production in the 5th year, **(1)**

iii. the total production in 7 years. **(1)**

## Solution

## MATHEMATICS STANDARD 041

## Class 10 - Mathematics

### Section A

1. Let first term = a and common difference = d
   Then as per given
   $10 \times a_{10} = 15 \times a_{15}$
   10[a + (10 - 1)d] = 15[a + (15 - 1)d]
   10(a + 9d) = 15(a + 14d)
   10a+90d=15a+210d
   5a+120d=0
   or a + 24d=0 .............(1)
   To prove: $a_{25}$ = 0
   $a_{25}$= a + (25 - 1)d
   = a + 24d
   = 0 [From (1)]
   $\text{Hence } \mathrm{a}_{25} = 0$

   OR

   Here the given AP is 3,8,13...........
   a=3 and d=8-3=5
   $\text{Let the } \mathrm{n}^{\mathrm{th}} \text{ term of the give AP} = 302$
   Then, $a_n$ = 302
   a + (n - 1)d = 302
   $3 + (n - 1) \times 5 = 302$
   3 + 5n - 5 = 302
   5n = 302 + 5 - 3
   5n = 304
   $\mathrm{n} = \frac{304}{5} = 60.8$
   Since, n is not a whole number hence 302 is not a term of given A.P.
2. For equation $x^2 + kx + 64 = 0$
   $b^2 - 4ac = 0$
   or, $k^2 - 4 \times 1 \times 64 = 0$
   or, $k^2 - 256 = 0$
   or, $k = \pm 16$ ..(i)
   and for equation $x^2 - 8x + k = 0$
   $b^2 - 4ac = 0$
   or, $(-8)^2 - 4 \times 1 \times k = 0$
   or, 64 = 4k
   or, $k = \frac{64}{4} = 16 \ldots (ii)$
   From (i) and (ii), we get k=16
   For k= 16, given equations have equal roots.
3. Given that $\angle SPT = 120^\circ$
   or, $\angle OPS = \frac{120^\circ}{2} = 60^\circ$ (as OP bisect $\angle SPT$)
   Also, $\angle PTO = 90^\circ$ (as radius $\perp$ tangent)
   $\therefore$ In right triangle POS.
   $\cos \angle OPS = \frac{PS}{OP}$
   or, $\frac{1}{2} = \frac{PS}{OP}$
   or, OP = 2 PS

4. We have,

Surface area of the ball = $4\pi\left(\frac{4.2}{2}\right)^2 cm^2$ = $4\pi \times 4.41$ = $17.64\pi\ cm^2$

In case of each dimple, surface area equal to $\pi r^2$ is removed from the surface of the ball where as the surface area of hemisphere i.e, $2\pi r^2$ is exposed to the surroundings

$\therefore$ Total surface area exposed to the surroundings

= Surface area of the ball - 200 $\times$ $\pi r^2$ +200 $\times$ $2\pi r^2$

= $17.64\pi$ + $200\pi r^2$ = $17.64\pi$ + $200\pi \times (\frac{2}{10})^2$ =$17.64\pi$ + $8\pi$ = $25.64\pi$

= 25.64 $\times$ $\frac{22}{7}$ = 80.58 $cm^2$

5. According to the question,

$\frac{A+B+C}{3} = 33$

$\Rightarrow B + C = 99 - A$

$B + C = 99 - 30 = 69$......(i) ($\because$A = 30)

$\frac{\mathbf{B+C+D}}{3} = 37$

$\Rightarrow B + C = 111 - D$......(ii)

by Using (i) and (ii), we get

$69 = 111 - D$

$\Rightarrow D = 111 - 69 = 42$

6. Given, $3x^2 + 5\sqrt{5}x - 10 = 0$

By splitting the middle term, we have

$3x^2 + 6\sqrt{5}x - \sqrt{5}x - 10 = 0$

$\Rightarrow 3x(x + 2\sqrt{5}) - \sqrt{5}(x + 2\sqrt{5}) = 0$

$\Rightarrow (3x - \sqrt{5})(x + 2\sqrt{5}) = 0$

$\therefore 3x - \sqrt{5} = 0$ or $x + 2\sqrt{5} = 0$

$\therefore \mathrm{x} = \frac{\sqrt{5}}{3}$ or $\mathrm{x} = -2\sqrt{5}$

OR

The given quadratic equation is

$2x^2 + kx + 3 = 0$

Here, a = 2, b = k, c = 3

Therefore, discriminant = $b^2 - 4ac$

= $(k)^2 - 4(2)(3) = k^2 - 24$

If the given quadratic equation has two equal real roots, then

$b^2 - 4ac = 0$

$\Rightarrow \mathrm{k}^2 - 24 = 0 \Rightarrow \mathrm{k}^2 = 24$

$\Rightarrow \mathrm{k} = \pm\sqrt{24} \Rightarrow \mathrm{k} \pm 2\sqrt{6}$

Hence, the required values of k are $\pm 2\sqrt{6}$ .

i.e., $2\sqrt{6}$ and $-2\sqrt{6}$

**Section B**

7.

| Class Interval | Frequency | Cumulative Frequency |
|---|---|---|
| 0 - 10 | 6 | 6 |
| 10 - 20 | 16 | 22 |
| 20 - 30 | 30 | 52 |
| 30 - 40 | 9 | 61 |
| 40 - 50 | 4 | 65 |

Here, N = 65 $\Rightarrow \frac{N}{2} = 32.5$

The cumulative frequency just greater than 32.5 is 52.

Hence, median class is 20 - 30.

$\therefore$ l = 20, h = 10, f = 30, cf = cf of preceding class = 22

Now, Median = $l+\left\{h\times\frac{\left(\frac{N}{2}-cf\right)}{f}\right\}$

$=20+\left\{10\times\frac{(32.5-22)}{30}\right\}$

$=20+\left\{10\times\frac{10.5}{30}\right\}$

= 20 + 3.5

= 23.5

Thus, the median of the data is 23.5.

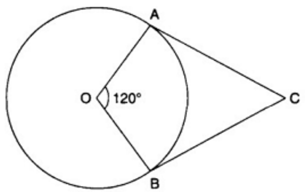
8. Required: To draw a pair of tangents to a circle of radius 5cm which are inclined to each other at a angle of $60^{0}$.

   Steps of construction:

   i. Draw a circle of radius 5 cm with centre O.

   ii. Draw an angle AOB of $120^{0}$.

   iii. At A and B, draw $90^{0}$ angles which meet at C.




Then AC and BC are the required tangents which are inclined to each other at an angle of $60^{0}$.

Justification:

$\therefore \angle OAC=90^{\circ}$ [By construction]

And OA is a radius

∴ AC is a tangent to the circle.

$\therefore \angle OBC=90^{\circ}$ [By construction]

and OB is a radius

∴ BC is a tangent to the circle.

Now, in quadrilateral OACB

$\angle AOB+\angle OAC+\angle OBC+\angle ACB=360^{\circ}$

[Angle sum property of a quadrilateral]

$\Rightarrow 120^{\circ}+90^{\circ}+90^{\circ}+\angle ACB=360^{\circ}$

$\Rightarrow 300^{\circ}+\angle ACB=360^{\circ}$

$\Rightarrow \angle ACB=360^{\circ}-300^{\circ}=60^{\circ}$

9. Let the assumed mean (A) = 60

| **Class Interval** | **Mid value $x_i$** | **$d_i = x_i - 60$** | **$u_i = \frac{(x_i-60)}{20}$** | **Frequency $f_i$** | **$f_iu_i$** |
|---|---|---|---|---|---|
| 10 – 30 | 20 | -40 | -2 | 5 | -10 |
| 30 – 50 | 40 | -20 | -1 | 8 | -8 |
| 50 – 70 | 60 | 0 | 0 | 12 | 0 |
| 70 – 90 | 80 | 20 | 1 | 20 | 20 |
| 90 – 110 | 100 | 40 | 2 | 3 | 6 |
| 110 – 130 | 120 | 60 | 3 | 2 | 6 |
| | | | | **N = 50** | **Sum = 14** |

We have

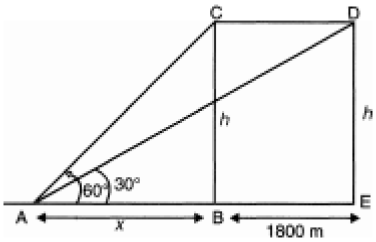
A = 60, h = 20

Mean = A + h$\frac{sum}{N}$

= 60 + 20$\left(\frac{14}{5}\right)$

= 60 + 5.6

= 65.6

10. 


1 hr =3600 sec

Hence in 3600 sec distance travelled by plane = 648 km = 648000 m

In 10 sec distance travelled by plane = $\frac{648000}{3600} \times 10$ = 1800 m

So BE = CD= 1800 m

In $\triangle$ABC,

$\frac{h}{x} = \tan 60^{\circ}$

$\frac{h}{x} = \sqrt{3}$

$\Rightarrow h = x\sqrt{3}$......(i)

In $\triangle$ADE we have

$\frac{h}{x+1800} = \tan 30^{\circ}$

$\frac{h}{x+1800} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad h = \frac{x+1800}{\sqrt{3}}$.........(ii)

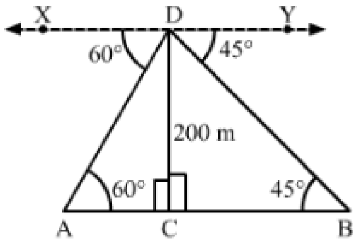
From equation (i) and (ii) we get

$x\sqrt{3} = \frac{x+1800}{\sqrt{3}}$

3x = x + 1800

x = 900 m So h = 900√3 meter

OR




Let CD be the lighthouse and A and B be the positions of the two ships.

Height of the lighthouse, CD = 200 m

Now,

$\angle$CAD = $\angle$ADX = 60° (Alternate angles)

$\angle$CBD = $\angle$BDY = 45° (Alternate angles)

In right $\triangle$ACD,

tan 60$^{\circ}$ = $\frac{\text{CD}}{\text{AC}}$

$\Rightarrow \sqrt{3} = \frac{200}{\text{AC}}$

$\Rightarrow$ AC = $\frac{200}{\sqrt{3}} = \frac{200\sqrt{3}}{3}$ m

In right $\triangle$BCD,

tan 45$^{\text{o}}$ = $\frac{\text{CD}}{\text{BC}}$

$\Rightarrow$ 1 = $\frac{200}{BC}$

$\Rightarrow$ BC = 200 m

$\therefore$ Distance between the two ships, AB = BC + AC

= 200 + $\frac{200\sqrt{3}}{3}$

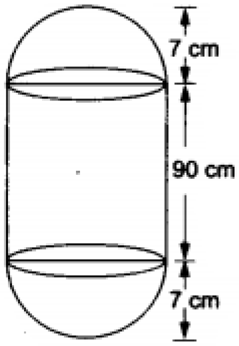
= 200 + $\frac{200\times1.73}{3}$

= 200 + 115.33

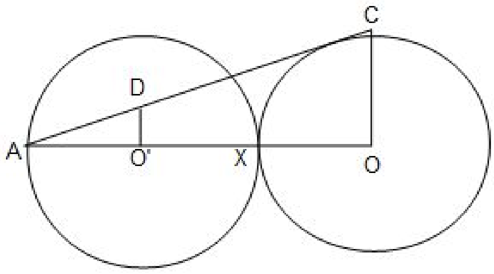
= 315.33 m (approx)

Hence, the distance between the two ships is approximately 315.33 m.

**Section C**

11. 

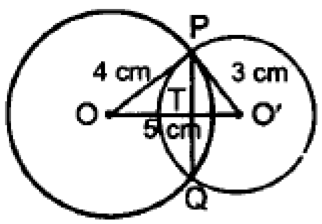

Radius of each hemispherical end = 7 cm.
Height of each hemispherical part = its radius = 7 cm.
Height of the cylindrical part = (104 - 2 $\times$ 7) cm = 90 cm.
Area of surface to be polished = 2(curved surface area of the hemisphere) + (curved surface area of the cylinder)
= [2 ($2\pi r^2$) + $2\pi rh$] sq units
$= \left[\left(4 \times \frac{22}{7} \times 7 \times 7\right) + \left(2 \times \frac{22}{7} \times 7 \times 90\right)\right] \text{cm}^2$
= (616 + 3960) $cm^2$ = 4576 $cm^2$
= $\left(\frac{4576}{10 \times 10}\right) \text{dm}^2$ = 45.76 $dm^2$ [∵ 10 cm = 1 dm].
∴ cost of polishing the surface of the solid
= ₹(45.76 $\times$10) = ₹ 457.60.

12. 


We know that ∠ADO' = 90° ( since O'D is perpendicular to AC)
∠ACO= 90° ( OC(radius)perpendicular to AC(tangent))
In triangles ADO'and ACO ,
∠ADO' = ∠ACO ( each 90°)
∠DAO = ∠CAO (common)
by AA criterion, triangles ADO' and ACO are similar to each other.
$\frac{AO'}{AO} = \frac{DO'}{CO}$ ( corresponding sides of similar triangles)
AO = AO' + O'X + OX
= 3AO' (since AO'=O'X=OX because radii of the two circles are equal )
$\frac{AO'}{AO} = \frac{AO'}{3AO} = \frac{1}{3}$
$\frac{DO'}{CO} = \frac{AO'}{AO} = \frac{1}{3}$
$\frac{DO'}{CO} = \frac{1}{3}$.

OR

Given, OP is tangent of the circle having center O'




So, ∠OPO' = $90^{o}$

In right angled $\triangle$OPO'

OP = 4 cm

O'P = 3 cm

By pythagoras theorem, we get

$OO'^2 = OP^2 + O'P^2$

$= 4^2 + 3^2$

= 16 + 9 = 25

$OO' = 5cm.$

Let O'T = x , then OT = 5 - x

In right angled $\triangle$PTO

By pythagoras theorem, we get

$OP^2 = OT^2 + PT^2$

$\Rightarrow PT^2 = OP^2 - OT^2$

$PT^2 = 4^2 - (5-x)^2$...(i)

In right angled $\triangle$PTO'

By pythagoras theorem, we get

$O'P^2 = O'T^2 + PT^2$

$\Rightarrow PT^2 = O'P^2 - O'T^2$

$PT^2 = 3^2 - x^2$...(ii)

From (i) and (ii) , we get

$3^2 - x^2 = 4^2 - (5-x)^2$

$9 - x^2 = 16 - 25 - x^2 + 10x$

18 = 10x

$\Rightarrow$ x = $\frac{18}{10}$ = 1.8

Substitute x in (ii), we get

$PT^2 = 3^2 - 1.8^2 = 9 - 3.24 = 5.76$

$PT = \sqrt{5.76} = 2.4$

$\Rightarrow$ PQ = 2 PT

= 2 $\times$ 2.4

$\therefore$ PQ= 4.8cm

13. i. Time covered 10.00 am to 10.01 am = 1 minute = $\frac{1}{60}$ hour

    Given: Speed = 600 miles/hour

    Thus, distance d = 600 $\times$ $\frac{1}{60}$ = 10 miles

    ii. Now, tan $20^o$ = $\frac{BB'}{B'A}$ = $\frac{h}{10+x}$ ....eq(1)

    And tan $60^o$ = $\frac{CC'}{C'A} = \frac{BB'}{C'A}$ = $\frac{h}{x}$

    x = $\frac{h}{tan 60^o}$ = $\frac{h}{\sqrt{3}}$

    Putting the value of x in eq(1), we get,

    tan $20^o$ = $\frac{h}{10+\frac{h}{\sqrt{3}}} = \frac{\sqrt{3}h}{10\sqrt{3}+h}$

    $0.364(10\sqrt{3} + h) = \sqrt{3}h$

    6.3 + 0.364 h = 1.732 h

    1.368 h = 6.3

    h = 4.6

    Thus, the altitude 'h' of the airplane is 4.6 miles.

14. i. Let production in a 1st year be a unit and increase in production (every year) be d units.

    $\because$ Increase in production is constant, therefore unit produced every year forms an AP.

    Now, $a_3$ = 6000

    a + 2d = 6000 $\Rightarrow$ a = 6000 - 2d ..(1)

    and $a_7$ = 7000 $\Rightarrow$ a + 6d = 7000

$\Rightarrow$ (6000 - 2d) + 6d = 7000 $\Rightarrow$ 4d = 1000 [using eq. (1)]

$\Rightarrow$ d = 250

When d = 250, eq. (1) becomes

a = 6000 - 2(250) = 5500

$\therefore$ Production in 1st year = 5500

ii. Production in fifth year

$a_5$ = a + 4d = 5500 + 4(250) = 6500

iii. Total production in 7 years = $\frac{7}{2}$(5500 + 7000) = 43750.