Chapter 2. Relations and Functions

Question-1

Find x and y, if (2x, x+y) = (6, 2).

Solution:

$2x = 6$

$\therefore x = 3$

$x + y = 2$

$3 + y = 2$

$\therefore y = -1$

Question-2

Find the domain of the following function : $f(x) = x|x|$

Solution:

The domain of the function $f(x) = x|x|$ is R.

Question-3

Let A = {a, b, c} and B = {p, q}. Find

(i) A × B
(ii) B × A
(iii) A × A
(iv) B × B

Solution:

(i) A × B = {(a,p), (a,q), (b,p), (b,q), (c,p), (c,q)}
(ii) B × A = {(p,a), (q,a), (p,b), (q,b), (p,c), (q,c)}
(iii) A × A = {(a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c)}
(iv) B × B = {(p,p), (p,q), (q,p), (q,q)}

Question-4

Find the domain of the following function : $f(x) = \frac{1}{\sqrt{x + |x|}}$

Solution:

$x+|x| = 0$ for $x < 0$ or $x = 0$.

$\therefore$ The domain of the function $f(x) = \frac{1}{\sqrt{x + |x|}}$ is $(0, \infty)$

Question-5

Let A = {1, 2, 3}, B = {2, 3, 4} and C = {4, 5}. Verify that

(i) A × (B ∩ C) = (A × B) ∩ (A × C)

(ii) A × (B ∪ C) = (A × B) ∪ (A × C)

Solution:

(i) L.H.S = A × (B ∩ C) = {1, 2, 3} × {4} = {(1,4), (2,4), (3,4)}

R.H.S = (A × B) ∩ (A × C)

= {(1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4)} ∩ {(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)}

= {(1,4), (2,4), (3,4)}∴ A × (B ∩ C) = (A × B) ∩ (A × C)

(ii) L.H.S = A × (B ∪ C)

= {1, 2, 3} × {2, 3, 4, 5}

= {(1,2), (1, 3), (1, 4), (1,5), (2, 2), (2,3), (2,4), (2,5), (3,2), (3,3), (3,4), (3,5)}

R.H.S = (A × B) ∪ (A × C)

= {(1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4)} ∪ {(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)}

= {(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5) (3,2), (3,3),(3,4), (3,5)}

∴ A × (B ∪ C) = (A × B) ∪ (A × C)

Question-6

If R is the relation "less than" from A = {1, 2, 3, 4, 5} to B = {1, 4, 5}, write down the set of ordered pairs corresponding to R. Find the inverse relation to R.

Solution:

R = {(x, y) / x ∈A, y ∈B and x < y}

= {(1,4),(1, 5),(2,4), (2,5), (3,4), (3,5), (4,5)}

∴ Inverse relation corresponds to the Cartesian product {(4,1),(5, 1),(4,2), (5,2), (4,3), (5,3), (5,4)} and corresponds to the relation 'greater than' from B to A.

Question-7

Prove that A ∩ (B - C) = (A ∩ B) - (A ∩ C) .

Solution:

Let $x \in A \cap (B - C)$

$\Rightarrow x \in A$ and $x \in (B - C)$

$\Rightarrow x \in A$ and $\{x \in B$ and $x \notin C\}$

$\Rightarrow x \in A$ and $\{x \in B$ and $x \notin C\}$

$\Rightarrow x \in A$ and $x \in B$ or $x \in A$ and $x \notin C$

$\Rightarrow (A \cap B) - (A \cap C)$

Question-8

If A= {1, 2, 3} , B = {4}, C = {5}, then verify that

(i) A×(B ∪ C) = (A×B) ∪ (A×C)

(ii) A×(B - C) = (A×B) - (A×C)

Solution:

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

$A \times (B \cup C) = \{1, 2, 3\} \times \{4, 5\}$
$= \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$

$(A \times B) \cup (A \times C) = \{(1, 4), (2, 4), (3, 4)\} \cup \{(1, 5), (2,5), (3, 5)\}$
$= \{(1, 4), (2, 4), (3, 4), (1, 5), (2,5), (3, 5)\}$

$\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

$A \times (B - C) = \{1, 2, 3\} \times \{4\}$
$= \{(1,4), (2,4), (3,4)\}$

$(A \times B) - (A \times C) = \{(1,4), (2,4), (3,4)\} - \{(1, 5), (2, 5), (3, 5)\}$
$= \{(1,4), (2,4), (3,4)\}$

Question-9

If R is the relation in N x N defined by (a,b) R (c,d) if and only if a + d = b + c, show that R is an equivalence relation.

Solution:

Reflexive

$(a,b)\ R\ (a,b) \Leftrightarrow a + b = b + a$ for $a,b \in N$

$\Leftrightarrow b + a = a + b$ (Transposing)

$\Leftrightarrow (a,b)\ R\ (a,b)$ for $a,b \in N$

$\therefore (a,b)\ R\ (a,b) \Leftrightarrow (a,b)\ R\ (a,b)$ for $a,b \in N$

Symmetric

If $(a,b)\ R\ (c,d) \Leftrightarrow a + d = b + c$ for $a,b,c,d \in N$

$\Leftrightarrow b + c = a + d$ (transposing)

$\Leftrightarrow (c,d)\ R\ (a,b)$ for $a,b,c,d \in N$

$\therefore (a,b)\ R\ (c,d) \Leftrightarrow (c,d)\ R\ (a,b)$ for $a,b,c,d \in N$

Transitive

If $(a,b)\ R\ (c,d) \Leftrightarrow a + d = b + c$ for $a,b,c,d \in N$

and $(c,d)\ R\ (e,f) \Leftrightarrow c + f = d + e$ for $c,d,e,f \in N$

then $(a,b)\ R\ (c,d) \Leftrightarrow a + d = b + c$ for $a,b,c,d \in N$

$\Leftrightarrow a + d + e + f = b + c + e + f$

$\Leftrightarrow a + (d + e) + f = b + c + e + f$ (since $c + f = d + e$)

$\Leftrightarrow a + \cancel{c} + f + \cancel{d} = b + \cancel{c} + e + \cancel{d}$

$\Leftrightarrow a + f = b + e$

$\Leftrightarrow (a,b)\ R\ (e,f)$ for $a,b,e,f \in N$

$\therefore (a,b)\ R\ (c,d) \Leftrightarrow (a,b)\ R\ (e,f)$ for $a,b,c,d,e,f \in N$

$\therefore$ the relation defined by $(a,b)\ R\ (c,d)$ if and only if $a + d = b + c$ is an equivalence relation.

Question-10

Find the domain of the following function : $f(x) = \frac{x}{x^2 - 3x + 2}$

Solution:

$x^2-3x+2 = 0$ for $x = 2,1$ $\therefore$ The domain of the function : $f(x) = \frac{x}{x^2 - 3x + 2}$ is R- {1,2}.

Question-11

Let A = {1, 2, 3, 4} and S = {(a, b): a ∈ A, b ∈ A, a divides b}. Write S explicitly.

Solution:

S = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)}

Question-12

Find the domain of the following function : $f(x) = e^{x+\sin x}$

Solution:

The domain of the function : $f(x) = e^{x+\sin x}$ is R .

Question-13

Find the domain of the following function : $f(x) = \frac{x+7}{x^2-8x+4}$

Solution:

$x^2 - 8x+4 = 0$ for $x = \frac{8 \pm \sqrt{64-16}}{2} = \frac{8 \pm \sqrt{48}}{2} = \frac{8 \pm 4\sqrt{3}}{2} = 4 \pm 2\sqrt{3}$

The domain of the function: $f(x) = \frac{x+7}{x^2-8x+4}$ is $R - \{4 \pm 2\sqrt{3}\}$

Question-14

Let A = {1, 2} and B = {3, 4}. Write all subsets of A × B.

Solution:

A × B = {(1,3), (1,4), (2,3), (2,4)}.

∴ The subsets of A × B are φ , {(1,3)}, {(1,4)}, {(2,3)}, {(2,4)},

{(1,3), (1,4)}, {(1,3), (2,3)}, {(1,3), (2,4)}, {(1,4), (2,3)}, {(1,4), (2,4)}, {(2,3), (2,4)},

{(1,3), (1,4), (2,3)}, {(1,3), (1,4), (2,4)}, {(1,3), (2,3), (2,4)}, {(1,4), (2,3), (2,4)},

{(1,3), (1,4), (2,3), (2,4)}.

Question-15

Find the domain of the following function : f(x) = [x] + x

Solution:

The domain of the function f(x) = [x] + x is R.

Question-16

Let A and B be two sets such that n(A) = 3 and n(B) = 2. If (x, 1), (y, 2), (z, 1) are in A × B, find A and B, where x, y, z are distinct elements.

Solution:

A = {x, y, z} and B = {1, 2}

Question-17

Find the domain of the following function : f(x) = $\frac{\sin^{-1} x}{x}$

Solution:

The domain of the function : f(x) = $\frac{\sin^{-1} x}{x}$ is $[-1,1]-\{0\}$

Question-18

Let A = {1,2}, B = {1, 2, 3, 4}, C = {5, 6} and D= {5, 6, 7, 8}. Verify that A × C ⊂ B × D.

Solution:

A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}
B × D = {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}
∴ A × C ⊂ B × D.

Question-19

Find the range of each of the following function: f(x) = $|x-3|$

Solution:

f(x) =|x-3| is positive for all values of x in R.
The range of the function f(x) = $|x-3|$ is$(0, \infty)$.

Question-20

Let A be a non empty set such that A × B = A × C. show that B = C.

Solution:

Let a ∈ A. Since B ≠ φ , there exists b ∈ B. Now, (a, b) ∈ A × B = A × C implies b ∈ C.
∴ every element in B is in C giving B ⊂ C. Similarly, C ⊂ B. Hence B = C.

Question-21

Find the range of the following function: $f(x) = 1-|x-2|$

Solution:

$|x-2| \geq 0 \Rightarrow 1-|x-2| \leq 1$

The range of the function $f(x) = 1-|x-2|$ is $(-\infty,1)$

Question-22

Find the range of the following function: $f(x) = \frac{|x-4|}{x-4}$

Solution:

$f(x) = \frac{|x-4|}{x-4} = 1$, if $x - 4>0$

$= -1$, if $x - 4< 0$

∴ The range of the function $f(x) = \frac{|x-4|}{x-4}$ *is* $(-1,1)$

Question-23

Let A = {1, 2, 3, 4} and B = {x, y, z}. Let R be a relation from A to B defined by R = {(1, x), (1, z), (3, x), (4, y)}. Find the domain and range of R.

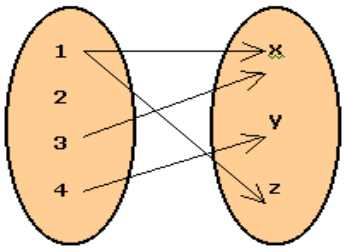
Solution:

Domain of R = {1, 3, 4} and Range R = {x, y, z} = B

Question-24

Let A = {1, 2, 3, 4} and B = {x, y, z}.
Let R be a relation from A to B defined by R = {(1, x), (1, z), (3, x), (4, y)}.
Draw the arrow diagram of relation R.

Solution:



Question-25

Find the range of the following function: $f(x) = \sqrt{16-x^2}$

Solution:

The range of the function : $f(x) = \sqrt{16-x^2}$ is $[0,4]$.

Question-26

Find the range of the following function: f(x)= $\frac{1}{\sqrt{x-5}}$

Solution:

The range of the function is (0, ∞)

Question-27

In N×N, show that the relation defined by (a,b)R(c,d) if and only if ad = bc is an equivalence relation.

Solution:

Reflexive

(a,b)R(a,b) Û ab = ba for a,bÎN

Û ba = ab (Transposing)

Û (a,b)R(a,b) for a,bÎN

∴ (a,b)R(a,b) ⇔ (a,b)R(a,b) for a,bÎN.

Symmetric

(a,b)R(c,d) Û ad = bc a,b,c,dÎN

Û bc = ad (Transposing)

Û (c,d)R(a,b) for a,b,c,dÎN

∴ (a,b)R(c,d) ⇔ (c,d)R(a,b) for a,b,c,dÎN

Transitive

If (a,b)R(c,d) ⇔ ad = bc a,b,c,dÎN

and (c,d)R(e,f) ⇔ c = de c,d,e,fÎN

Then (a,b)R(c,d) ⇔ ad = bc a,b,c,dÎN

Û adef = bcef (Multiplying both sides by ef)

Û adef = be(cf)

Û adef = bede (Since cf = de)

Û af = be

Û (a,b) R (e,f) a,b,e,fÎN

∴ (a,b)R(c,d) Û (a,b) R (e,f) a,b,e,fÎN

∴ the relation defined by (a,b)R(c,d) if and only if ad = bc is an equivalence relation.

Question-28

Find the domain and the range of the following function : f(x) = $\frac{1}{\sqrt{x-[x]}}$

Solution:

We know that $0 \le x - [x] \le 1$ for all $x \in R$. Also, $x - [x] = 0$ for $x \in Z$.

$f(x) = \frac{1}{\sqrt{x-[x]}}$ is defined if $x - [x] > 0$

i.e., $x \in R - Z$.

Hence the domain of the function is $R - Z$.

Question-29

Find the domain and the range of the following function : f(x)= $\frac{1}{\sqrt{4+3\sin x}}$

Solution:

$-1 \le \sin x \le 1 \Rightarrow -3 £ 3 \sin x \le 3$

i.e $-1 \le 4 + 3 \sin x \le 7$

$\therefore \frac{1}{\sqrt{7}} \le y \le 1$

The domain of the function is R; Range : $\frac{1}{\sqrt{7}} \le y \le 1$

Question-30

Let R be the relation on Z defined by a R b if and only if a – b is an even integer. Find (i) R, (ii) domain R, (iii) range of R.

Solution:

(i) $R = \{(a, b): a \text{ and } b \text{ are even integers}\} \cup \{(c, d) : c \text{ and } d \text{ are odd integers}\}$

(ii) Domain = Z

(iii) Range = Z

Question-31

Find the domain and the range of the following function : f(x) = 1 - |x – 3|

Solution:

The domain of the function is R ; Range : $(-\infty, 1)$

Question-32

Let R be the relation on Z defined by $R = \{(a, b): a \in Z, b \in Z, a^2 = b^2\}$. Find (i) R, (ii) domain R, (iii) range of R.

Solution:

(i) $R = \{(a, a): a \in Z\} \cup \{(a, -a): a \in Z\}$

(ii) Domain = Z

(iii) Range = Z

Question-33

Find the domain and the range of the following function : f(x) = x!

Solution:

The domain of the function is N ∪ {0}; Range : {n! : n = 0,1,2…..}

Question-34

Determine the domain and the range of the relation R defined by R = {(x+1, x+5): x ∈ {0,1 ,2, 3, 4,5}}

Solution:

Domain = {1, 2, 3, 4, 5, 6}, Range = {5, 6, 7, 8, 9 ,10}

Question-35

Determine the domain and the range of the relation R, where R = {(x, x^3) : x is a prime number less than 10}.

Solution:

Domain = {2, 3, 5, 7}, Range = {8, 27, 125, 343}

Question-36

Find the domain and the range of the following function : f(x) = $\sin^2(x^3)$ + $\cos^2(x^3)$

Solution:

$\sin^2(x^3) + \cos^2(x^3) = 1$

∴ The domain of the function is R ; Range : {1}

Question-37

Is inclusion of a subset in another, i.e., ARB if and only if A ⊂ B, in the context of a universal set, an equivalence relation in the class of subsets of the universal set? Justify your answer.

Solution:

Let U be the universal set .Let R be the relation ' is a subset of' or 'is included in ' between the subsets of U.

Since every set is a subset of itself i.e., for every subset A in U, A ⊆ A or A R A.
∴ R is reflexive.

Now let A and B be two subsets of U such that A ⊆ B, then it is not necessary that B must also be a subset of A.

∴ A R B need not imply B R A.
∴ R is not symmetric.
Hence R is not an equivalence relation.

Question-38

Find the domain and the range of the following function : f(x) = $\frac{x^2-9}{x-3}$

Solution:

f(x) = $\frac{x^2-9}{x-3}$ = x+3
The domain of the function is R ; Range : R .

Question-39

Determine the domain and range of the following relations

(i) {(1, 2), (1, 4), (1, 6), (1, 8)}
(ii) {(x, y) : x ∈ N, y ∈ N and x + y = 10}
(iii) {(x, y) : x ∈ N, x<5, y = 3}
(iv) {(x, y) : y = |x − 1|, x ∈ Z and |x| ≤ 3}

Solution:

(i) Domain = {1}, Range = {2, 4, 6, 8}
(ii) Domain = {1, 2, 3, 4, 5, 6, 7, 8, 9}, Range = {9, 8, 7, 6, 5 ,4, 3, 2, 1}
(iii) Domain = {1, 2, 3, 4}, Range = {3}
(iv) Domain = {-3, -2, -1, 0, 1, 2, 3}, Range = {4, 3, 2, 1, 0}

Question-40

How many relations are possible from a set A of m elements to another set B of n elements? Why?

Solution:

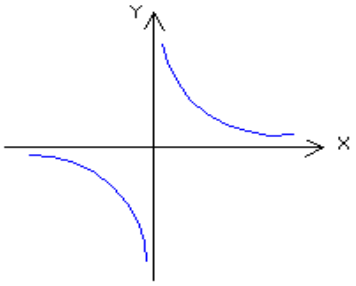
Number of elements in A = m.
Number of elements in B = n
∴ Number of elements in A×B = mn
Number of subsets of A×B = 2^{mn}

Since every subset of A×B is a relation from A to B therefore 2^{mn} relations are possible from A to B.

Question-41

Draw the graph of the following function: f(x) = $\frac{1}{x}$,x ≠ 0

Solution:



Question-42

Let A = {1, 2}. List all the relations on A.

Solution:

$A \times A = \{(1,1),(1,2),(2,1),(2,2)\}$

The relations on A are $\phi, \{(1,1)\}, \{(1,2)\}, \{(2,1)\}, \{(2,2)\},$

$\{(1,1),(1,2)\}, \{(1,1),(2,1)\}, \{(1,1),(2,2)\},$
$\{(1,2),(2,1)\}, \{(1,2),(2,2)\}\{(2,1),(2,2)\},$
$\{(1,1),(1,2),(2,1)\}, \{(1,1),(1,2),(2,2)\},$
$\{(1,1),(2,1),(2,2)\}, \{(1,2),(2,1),(2,2)\},$
$\{(1,1),(1,2),(2,1),(2,2)\}$

Question-43

Let A = {x, y, z} and B = {1, 2}. Find the number of relations from A into B.

Solution:

n (A) = 3 and n (B) = 2

∴n (A × B) = 2 × 3 = 6

∴the number of relations from A into B are $2^6 = 64$.

Question-44

Which of the following relations are functions? If it is a function, determine its domain and range:

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}
(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}
(iii) {(0, 0), (1, 1), (1, -1), (4, 2), (4, -2), (9, 3), (9, -3), (16, 4), (16, -4)}
(iv) {(1, 2), (1, 3), (2, 5)}
(v) {(2, 1), (3, 1), (5, 2)}
(vi) {(1, 2), (2, 2), (3, 2)}

Solution:

(i) Domain = {2, 5, 8, 11, 14, 17}, Range = {1}
(ii) Domain = {2, 4, 6, 8, 10, 12, 14}, Range = {1, 2, 3, 4, 5, 6, 7}
(iii) No, As there are four pairs of ordered pairs which have the same first element.
(iv) No, As two ordered pairs which have the same first element.
(v) Domain = {2, 3, 5}, Range = {1, 2}
(vi) Domain = {1, 2, 3}, Range = {2}

Question-45

If A = {1,2,3}, B = {a, b}, find A × A .

Solution:

A × A = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

Question-46

Find the domain and range of the following functions:

(i) $\left\{\left(x, \frac{x^2-1}{x-1}\right): x \in R, x \neq 1\right\}$

(ii) {(x, -|x|): x ∈ R}

Solution:

(i) Domain = R − {1}, Range = R − {2}
(ii) Domain = R, Range = {y: y ∈ R and y ≤ 0}

Question-47

If A = {1,2,3}, B = {a, b}, find A × B

Solution:

A × B = {(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)}

Question-48

Find the domain and range of the following functions:

(i) $\{(x, \sqrt{9-x^2}): x \in R\}$

(ii) $\left\{\left(x, \frac{1}{1-x^2}\right): x \in R, x \neq 1\right\}$

Solution:

(i) Domain = {x: x ∈ R and −3 ≤ x ≤ 3}, Range = {y: y ∈ R and −3 ≤ y ≤ 3}

(ii) Domain = R − {1, -1}, Range = {y: y ∈ R, y ≠ 0, y<0 and y ≥ 1}

Question-49

If A = {1,2,3}, B = {a, b}, find B × B.

Solution:

B × B = {(a, a), (a, b), (b, a), (b, b)}