LIMITS AND DERIVATIVES

Limit : If the right and left hand limits equal, then that common value is called the limit of $f(x)$ at $x = a$ and denote it by $\lim_{x \to a} f(x)$.

$\lim_{x \to a^-} f(x)$ left hand limit of f at a.

$\lim_{x \to a^-} f(x)$ right hand limit of $f(x)$ at a.

Theorem 1 : Let f and g be two functions such that both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then

(i) $\boxed{\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)}$

(ii) $\boxed{\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)}$

(iii) $\boxed{\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a}}{\lim_{x \to a}}}$

Theorem 2 : For any positive integer n, $\boxed{\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}}$

Theorem 3 : Let f and g be any two real valued functions with the same domain such that $f(x) \le g(x)$ for all x in the domain of definition, for some a, if both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then $\lim_{x \to a} f(x) \le \lim_{x \to a} g(x)$.

Theorem 4 : (Sandwich theorem) : Let f, g and h be real functions such that $f(x) \le g(x) \le h(x)$ for all x in the common domain of definition. For some real number a, if $\lim_{x \to a} f(x) = l = \lim_{x \to a} h(x)$, then $\lim_{x \to a} g(x) = l$.

Theorem 5 : The following are two important limits (i) $\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1}$ (ii) $\boxed{\lim_{x \to 0} \frac{1 - \cos x}{x} = 0}$

Derivative : The derivative of a function f at a is defined by $\boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$

Derivative of a function f at a point x is defined by $\boxed{f'(x) = \frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$

first principle of derivative

Note : $\boxed{\lim_{x \to a} [(\lambda \cdot f)(x)] = \lambda \lim_{x \to a} f(x)}$

Limits of polynomials and rational functions : A function f is said to be a polynomial function if $f(x)$ is zero function or if $f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$. where a_is are real numbers such that $a_n \neq 0$ for some natural number n.

Theorem 6 : For functions u and v the following holds : (Leibnitz rule)

(i) $(u \pm v)' = u' \pm v'$ (ii) $(uv)' = u'v + uv'$ (iii) $\left[\frac{u}{v}\right]' = \frac{u'v - uv'}{v^2}$ provided all are defined

Theorem 7 : $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$ for any positive integer n.

Theorem 8 : $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ be a polynomial function where a_is are all real numbers and $a_n \neq 0$. Then the derivative function is given by

$$\boxed{\frac{df(x)}{dx} = na_n x^{n-1} + (n-1) a_{n-1} x^{x-2} + \ldots + 2a_2 x + a_1}$$

$\boxed{\frac{d}{dx}(x^n) = nx^{n-1}}$ $\boxed{\frac{d}{dx}(\sin x) = \cos x}$ $\boxed{\frac{d}{dx} = -\sin x}$