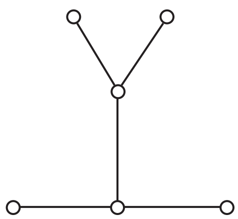
Algorithms Test 5

Number of Questions: 25 **Section Marks: 30**

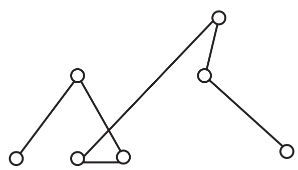
Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Which of the following is NOT a tree?

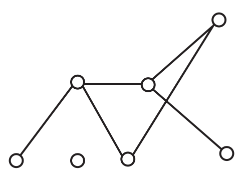
(A)



(B)



(C)



(D) None of the above

2. What is the number of vertices in a tree with 57 edges?

(A) 58 (B) $2^6 - 4$
(C) 56 (D) 57

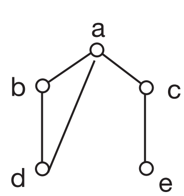
3. Consider 2 graphs G_1 and G_2:

I. G_1 is connected and every edge is a bridge.
II. In G_2 for any pair of vertices in the graph there is one and only one path joining them.

Which of the following is TRUE?

(A) G_1 is a tree but G_2 is not a tree
(B) G_1 is not a tree but G_2 is a tree
(C) Both G_1 and G_2 are trees
(D) Both G_1 and G_2 are not trees

4. How many spanning trees does the following graph contain, (Assume that all edges have same weight)?



(A) 2 (B) 3
(C) 4 (D) 5

5. Suppose we have a graph where each edge value appears atmost twice, then what will be the number (atmost) of minimum spanning Trees of this graph?

(A) 2 (B) 3
(C) 4 (D) 6

6. If we run Dijkstra's algorithm starting from 'S' to find shortest path 'T', consider the following statements:

I. Dijkstra's algorithm returns shortest path with minimum total weight.
II. Dijkstra's algorithm returns shortest path, with minimum number of edges.

Which of the following is TRUE?

(A) I only (B) II only
(C) Both I and II (D) None of the above

7. The reason for using Bellman-Ford algorithm for finding the shortest path between 2 vertices in a graph, when Dijkstras algorithm does the same thing:

I. Bellman-Ford Algorithm is faster than Dijkstra's Algorithm.
II. Bellman-Ford Algorithm works on any directed graph, with negative weights also.

Which of the following is TRUE?

(A) I only (B) II only
(C) Both I and II (D) None of these

8. Consider the following statements:

I. By applying Breadth First search, on a tree with all edges having equal weight; finds minimum distance from root to any node.
II. Use of greedy algorithm to solve the knapsack with fractions is optimal.

Which of the following is TRUE?

(A) I only (B) II only
(C) Both I and II (D) None of the above

9. Consider the set of keys {1, 4, 5, 10, 16, 17, 21}, Draw a binary search tree with height '2,' Then what values will appear at internal nodes? [height of root node is '0']

(A) 10, 5, 16 (B) 10, 4, 17
(C) 4, 17, 21 (D) 5, 10, 17

10. Consider the set of keys {2, 5, 6, 11, 17, 18, 22}, Draw a binary search tree with height '6', then, what will be the child node values (height of root node is 0).

(A) 2 (B) 22
(C) Either 2 or 22 (D) None of the above

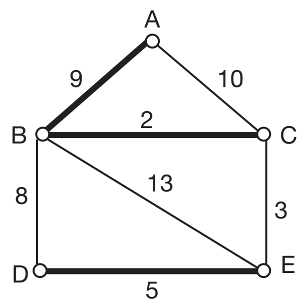
11. The following is a directed graph

$G = (V, E)$:
$V = \{a, b, c, d, e, f, g\}$
$E = \{(a, b), (c, a), (b, c), (c, d), (d, e), (e, f), (e, g), (f, g), (d, g)\}$

Identify the correct strongly connected components, which of the following is TRUE?

(A) There are 3 strongly connected components.
(B) There are 4 strongly connected components.
(C) There are 5 strongly connected components.
(D) None of the above

12. Consider the given graph, with some edges in bold:



The bold edges given in the above graph cannot form a spanning Tree because

(A) The bold edges are not connected
(B) The bold edges are not least weight edges
(C) The kruskals algorithm is not implemented on the above graph.
(D) Both (A) and (B)

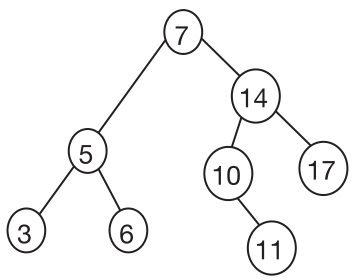
13. Consider the given statements:

I. A digraph is a graph with exactly 2 vertices.
II. A spanning tree of a graph must contain atleast $\frac{n}{2}$ edges always.
III. The sorted edges algorithm for solving the travelling salesman problem always gives optimal result.

Which of the following is TRUE?

(A) I and II (B) II and III
(C) I and III (D) Only II

14. Consider the given AVL-tree,



If the element with value '12' is inserted into above AVL tree, How many rotations are required to balance the tree?

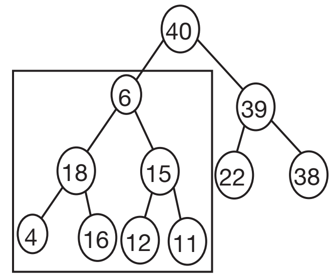
(A) 0 (B) 1
(C) 2 (D) 3

15. In which order, the element must be inserted into an AVL tree, so that, no single Rotation is required, for the given

Elements = {1, 2, 3, 4, 5, 6, 7}

(A) {4, 2, 6, 1, 3, 5, 7}
(B) {4, 2, 1, 3, 5, 6, 7}
(C) {7, 6, 5, 4, 2, 1, 3}
(D) {4, 1, 2, 3, 5, 6, 7}

16. Consider the given tree:



The MAX HEAP property is applied on the above tree, (box area), at node with value '6', which of the following is BEST suitable?

(A) First compare 6 and 18 then swap them if required
(B) First compare 6 and 15 then swap them if required.
(C) First compare 18 and 15, then compare the greater element with '6' and swap them if required.
(D) Any one of the above

17. Consider the following statements about Depth First traversal:

I. Suppose we run DFS (Depth First search) on an undirected graph and find exactly 15 back edges. Then the graph is guaranteed to have at least one cycle.
II. DFS on a directed graph with 'n' vertices and at least 'n' edges is guaranteed to find at least one back edge.

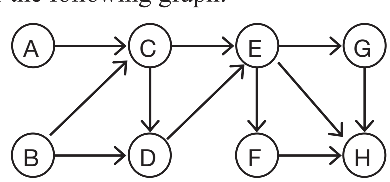
Which of the following is TRUE?

(A) I only (B) II only
(C) Both I and II (D) None of the above

18. Suppose 'G' is a connected, undirected graph, whose edges have positive weights. Let M be a minimum spanning Tree of this graph. We modify the graph by adding '6' to the weight of each edge, which of the following is TRUE?

(A) The order of edges added to minimum spanning tree using kruskal's algorithm, will change.
(B) The modification adds $6(|V| - 1)$ to the total weight of all spanning trees.
(C) The order of edges added to minimum spanning tree using prim's algorithm, will change.
(D) None of the above

19. Consider the following graph:



Which of the following orderings not a valid topological sort of the graph?

(A) BACEDFGH (B) ABCDEFGH
(C) ABCDEGFH (D) BACDEFGH

20. What is the time complexity to print all the keys of a binary search tree in sorted order?

(A) $O(\log n)$ (B) $\theta(n)$
(C) $\theta(n^2)$ (D) $O(n + \log n)$

21. Let T_1, T_2, T_3, T_4, T_4, T_5, T_6 are tasks given along with their deadlines and profits:

Tasks	T_1	T_2	T_3	T_4	T_5	T_6
Deadlines	2	1	3	2	3	1
Profit	6	4	3	7	2	8

Which of the following tasks are not completed?

(A) T_6, T_4, T_3 (B) T_6, T_1, T_2
(C) T_1, T_2, T_5 (D) T_4, T_5, T_6

22. Consider the below program fragment:

```
for i ← 1 to length (A) - 1
{
element ← A [i]
pos ← i
while pos > 0 and A [pos - 1]
{
    A[pos] ← A[pos - 1]
    pos ← pos -1
{
A[pos] ← element.
}
```

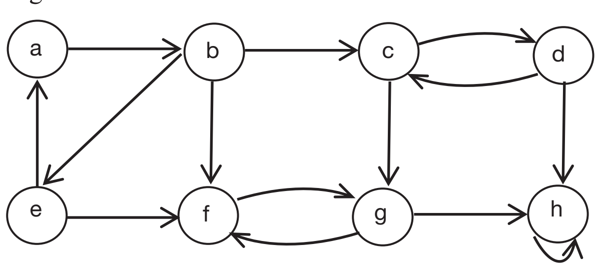
This code implements

(A) Bubble sort (B) Insertion sort
(C) Heap sort (D) Bucket sort

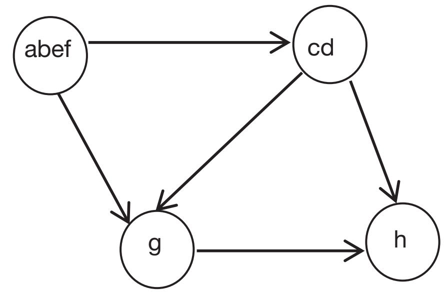
23. What is the best case time complexity of given algorithm in Q. 22?

(A) $\theta(n^2)$ (B) $\theta(\log n)$
(C) $\theta(n)$ (D) $\theta(n \log n)$

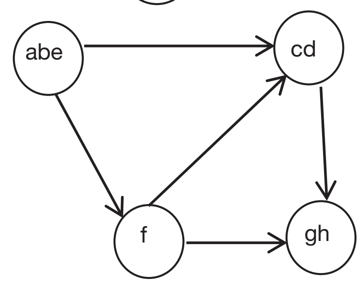
24. Which of the following is an acyclic component graph of given graph using strongly connected components algorithm?



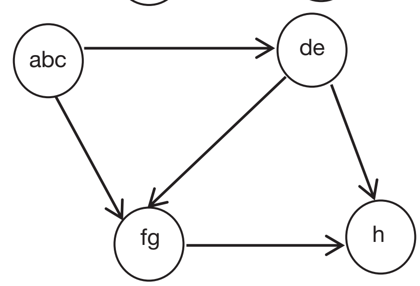
(A)



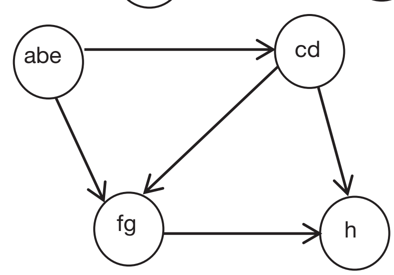
(B)



(A)



(D)



25. Consider an undirected graph

$$G = (V, E)$$
$$V = \{r, s, t, u, v, w, x, y\}$$
$$E = \{(r,s), (r,v), (s,w), (w,t), (w,x), (t,u), (t,x), (u,y), (u,x)\ (x,y)\}$$

Which node has the highest degree?

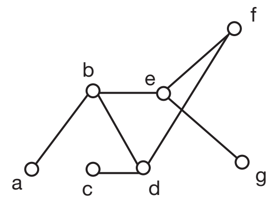
(A) w (B) x
(C) u (D) t

Answer Keys

1. C	**2.** A	**3.** C	**4.** B	**5.** C	**6.** A	**7.** B	**8.** C	**9.** B	**10.** C
11. C	**12.** A	**13.** D	**14.** B	**15.** A	**16.** C	**17.** A	**18.** B	**19.** A	**20.** B
21. C	**22.** B	**23.** C	**24.** D	**25.** B					

Hints and Explanations

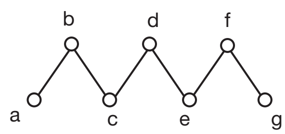
1. A graph without any cycles is called a Tree graph.
Option (C)



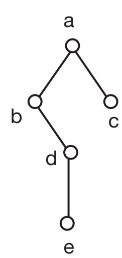
There is a cycle ($b - e - f - d - b$) in the graph.
Choice (C)

2. A tree with 'n' vertices contains (n – 1) edges.
$n - 1 = 57$
Number of vertices, $n = 58$ Choice (A)

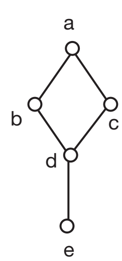
3. **G_1: Example:**



∴ No cycles, It is a tree
G_2: Example:

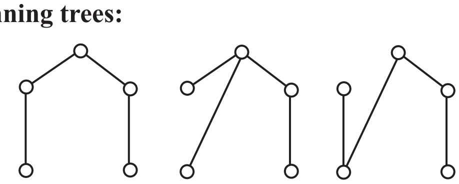


∴ No cycles,
If we have cycle



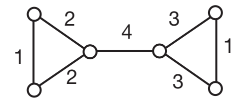
There will be different paths from ($a - e$)
(i) $a - b - d - e$
(ii) $a - c - d - e$
∴ G_2 is also a tree. Choice (C)

4. **Spanning trees:**

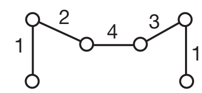


Choice (B)

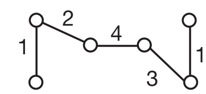
5. Assume the following graph:



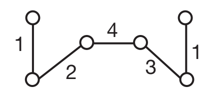
Spanning Tree 1:



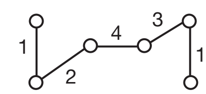
Spanning Tree 2:



Spanning Tree 3:



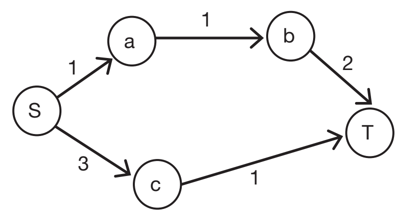
Spanning Tree 4:



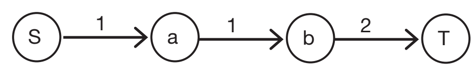
Atmost 4 minimum spanning trees are possible.
Choice (C)

6. I. Dijkstra's algorithm always returns shortest path with minimum total weight.

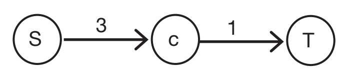
II.



Dijkstra's Algorithm gives, shortest path (length = 3)



But the shortest path

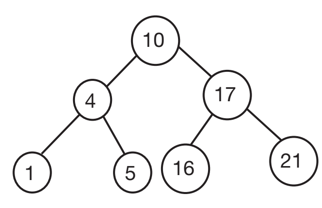


has length = 2
I–TRUE, II–False Choice (A)

7. Dijkstra's is faster than Bellman-Ford, but It cannot work on graphs with negative weight edges.
Choice (B)

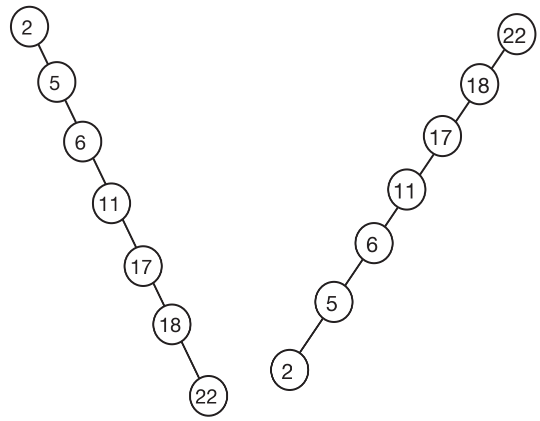
8. Applying BFS on a tree with equal weights results the same tree and it gives minimum distance from root to other nodes. Knapsack fractional problem is optimally solved using greedy design strategy. Choice (C)

9. 1, 4, 5, 10, 16, 17, 21



With height = 2, Internal nodes are 10, 4, 17.
Choice (B)

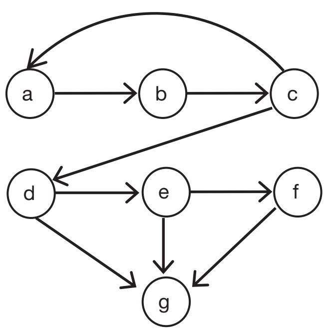
10.



Tree 1: Child 22 Tree 2: Child 2

Choice (C)

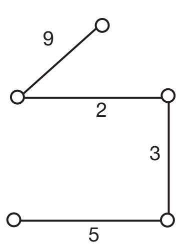
11.



Strongly connected components are:
$\{abc\}$ $\{d\}$ $\{e\}$ $\{f\}$ $\{g\}$
There are five strongly connected components.

Choice (C)

12. Kruskals algorithm is been implemented on the given graph. The bold edges are least weight edges, because the complete spanning tree will contain following edges:



Choice (A)

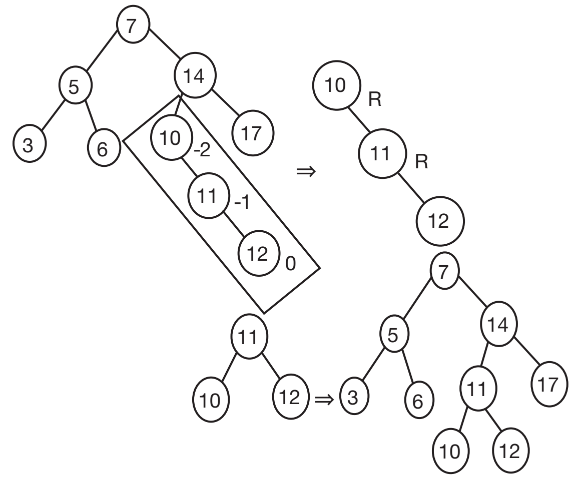
13. I. Digraph:
There is no limit on number of vertices, In digraph edges will have directions.
I–False

II. Example: **Spanning tree**

II–TRUE

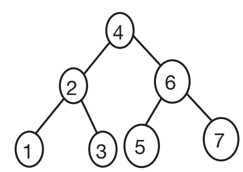
III. There is no optimal solution for Travelling sales person problem.
$\therefore$ Only II is TRUE Choice (D)

14.

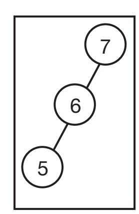


RR–Imbalance: perform Left Rotation.
$\therefore$ 1 Rotation. Choice (B)

15. Option (A):

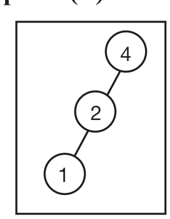


Option (C):



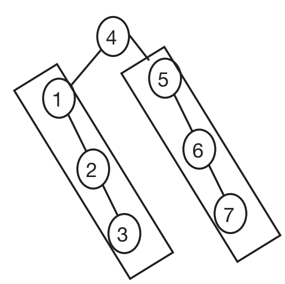
requires LL rotation
for 1st 3 elements

Option (B): **Option (D):**



requires LL rotation

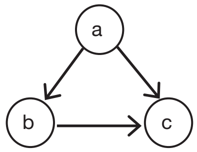
for 1st 3 elements



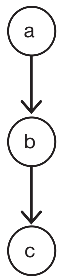
If we have Imbalance, we need to perform Rotation.
Option (A) has no Imbalance. Choice (A)

16. First compare the children of a node. If it is a max heap, then compare the child with greater value to parent node, if required swap them. Choice (C)

17. I. If the graph has a back edge, then it has a cycle.

II. 

DFS on the above graph starting at 'a' does not find any back edge.



∴ I–TRUE, II–False Choice (A)

18. The order of edges added to minimum spanning tree does not change, because we are adding same weight to all the edges.
To the total weight we have to add $6(V-1)$, because in a spanning tree, there will be $(V-1)$ edges, For every edge additional weight '6' is added. Choice (B)

19. **Option (A):**
BACEDFGH
There is an edge from $D \rightarrow E$
Where as in the order E appears first and D appears next. Which is invalid.
Check the other options in the same manner.
Choice (A)

20. We can print all the keys of a binary search tree in sorted order using inorder traversal which will take $\theta(n)$ time. Choice (B)

21. Arrange the given tasks according to their profits

Tasks	$T6$	$T4$	$T1$	$T2$	$T3$	$T5$
Profit	8	7	6	4	3	2
Deadline	1	2	2	1	3	3

Order of execution of tasks to maximize the profit

8	7	3		
T6	T4	T3		

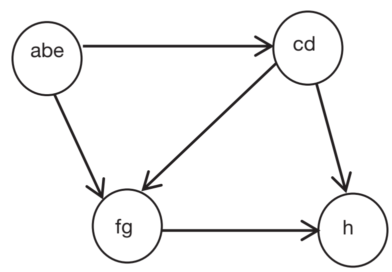
0 1 2 3 4 5

The uncompleted tasks are $T1$, $T2$, $T5$
Choice (C)

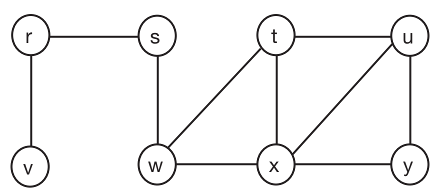
22. The given code implements insertion sort. In insertion sort, every iteration removes an element from the input data and insert it into the correct position in the already sorted list until no input elements remain. Choice (B)

23. The best case input is when the array is already in sorted order. Then time complexity is $\theta(n)$ because in each iteration, the first remaining element of input is only compared with the right-most element of sorted list of the array. Choice (C)

24. Strongly connected components of a directed graph $G = (V, E)$ is a maximal set of vertices $C \subseteq V$ such that for every pair of vertices u and v in C, vertices u and v are reachable from each other. In the given graph the connected components are $\{a, b, e\}$, $\{f, g\}$, $\{c, d\}$ and $\{h\}$.
Hence, the required acyclic component graph will be



Choice (D)

25. 

Highest degree node = x(degree = 4) Choice (B)