CAT 2022 Question Paper Slot 3

Quant

45. If $c = \frac{16x}{y} + \frac{49y}{x}$ for some non-zero real numbers x and y, then c cannot take the value

A 60

B -50

C -70

D -60

46. Suppose k is any integer such that the equation $2x^2 + kx + 5 = 0$ has no real roots and the equation $x^2 + (k-5)x + 1 = 0$ has two distinct real roots for x. Then, the number of possible values of k is

A 9

B 7

C 8

D 13

47. If $(\sqrt{\frac{7}{5}})^{3x-y} = \frac{875}{2401}$ and $(\frac{4a}{b})^{6x-y} = (\frac{2a}{b})^{y-6x}$, for all non-zero real values of a and b, then the value of $x + y$ is

48. Consider six distinct natural numbers such that the average of the two smallest numbers is 14, and the average of the two largest numbers is 28. Then, the maximum possible value of the average of these six numbers is

A 23

B 24

C 23.5

D 22.5

49. Suppose the medians BD and CE of a triangle ABC intersect at a point O. If area of triangle ABC is 108 sq. cm., then, the area of the triangle EOD, in sq. cm., is

50. If $(3 + 2\sqrt{2})$ is a root of the equation $ax^2 + bx + c = 0$ and $(4 + 2\sqrt{3})$ is a root of the equation $ay^2 + my + n = 0$ where a, b, c, m and n are integers, then the value of $(\frac{b}{m} + \frac{c-2b}{n})$ is

A 0

B 1

C 3

D 4

51. A group of N people worked on a project. They finished 35% of the project by working 7 hours a day for 10 days. Thereafter, 10 people left the group and the remaining people finished the rest of the project in 14 days by working 10 hours a day. Then the value of N is

A 150

B 23

C 36

D 140

52. A glass contains 500 cc of milk and a cup contains 500 cc of water. From the glass, 150 cc of milk is transferred to the cup and mixed thoroughly. Next, 150 cc of this mixture is transferred from the cup to the glass. Now, the amount of water in the glass and the amount of milk in the cup are in the ratio

A 1 : 1

B 10 : 13

C 3 : 10

D 10 : 3

53. Nitu has an initial capital of ₹20,000. Out of this, she invests ₹8,000 at 5.5% in bank A, ₹5,000 at 5.6% in bank B and the remaining amount at x% in bank C, each rate being simple interest per annum. Her combined annual interest income from these investments is equal to 5% of the initial capital. If she had invested her entire initial capital in bank C alone, then her annual interest income, in rupees, would have been

A 700

B 800

C 900

D 1000

54. Two cars travel from different locations at constant speeds. To meet each other after starting at the same time, they take 1.5 hours if they travel towards each other, but 10.5 hours if they travel in the same direction. If the speed of the slower car is 60 km/hr, then the distance traveled, in km, by the slower car when it meets the other car while traveling towards each other, is

A 100

B 90

C 120

D 150

55. The arithmetic mean of all the distinct numbers that can be obtained by rearranging the digits in 1421, including itself, is

A 2222

B 2442

C 2592

D 3333

56. The lengths of all four sides of a quadrilateral are integer valued. If three of its sides are of length 1 cm, 2 cm and 4 cm, then the total number of possible lengths of the fourth side is

A 3

B 4

C 6

D 5

57. The average of all 3-digit terms in the arithmetic progression 38, 55, 72, ..., is

58. In an examination, the average marks of students in sections A and B are 32 and 60, respectively. The number of students in section A is 10 less than that in section B. If the average marks of all the students across both the sections combined is an integer, then the difference between the maximum and minimum possible number of students in section A is

59. Let r be a real number and $f(x) = \begin{cases} 2x - r & if x \geq r \\ r & if x < r \end{cases}$. Then, the equation $f(x) = f(f(x))$ holds for all real values of x where

A $x > r$

B $x \leq r$

C $x \neq r$

D $x \geq r$

60. In a triangle ABC, AB = AC = 8 cm. A circle drawn with BC as diameter passes through A. Another circle drawn with center at A passes through Band C. Then the area, in sq. cm, of the overlapping region between the two circles is

A 16π

B $16(\pi - 1)$

C $32(\pi - 1)$

D 32π

61. A school has less than 5000 students and if the students are divided equally into teams of either 9 or 10 or 12 or 25 each, exactly 4 are always left out. However, if they are divided into teams of 11 each, no one is left out. The maximum number of teams of 12 each that can be formed out of the students in the school is

62. The minimum possible value of $\frac{x^2 - 6x + 10}{3 - x}$, for $x < 3$, is

A $-\frac{1}{2}$

B 2

C $\frac{1}{2}$

D -2

63. A donation box can receive only cheques of ₹100, ₹250, and ₹500. On one good day, the donation box was found to contain exactly 100 cheques amounting to a total sum of ₹15250. Then, the maximum possible number of cheques of ₹500 that the donation box may have contained, is

64. Moody takes 30 seconds to finish riding an escalator if he walks on it at his normal speed in the same direction. He takes 20 seconds to finish riding the escalator if he walks at twice his normal speed in the same direction. If Moody decides to stand still on the escalator, then the time, in seconds, needed for him to finish riding the escalator is

65. Two ships are approaching a port along straight routes at constant speeds. Initially, the two ships and the port formed an equilateral triangle with sides of length 24 km. When the slower ship travelled 8 km, the triangle formed by the new positions of the two ships and the port became right-angled. When the faster ship reaches the port, the distance, in km, between the other ship and the port will be

A 4

B 12

C 8

D 6

66. Bob can finish a job in 40 days, if he works alone. Alex is twice as fast as Bob and thrice as fast as Cole in the same job. Suppose Alex and Bob work together on the first day, Bob and Cole work together on the second day, Cole and Alex work together on the third day, and then, they continue the work by repeating this three - day roster, with Alex and Bob working together on the fourth day, and so on. Then, the total number of days Alex would have worked when the job gets finished, is

Answers

Quant

45.**B**	46.**A**	47.**14**	48.**D**	49.**9**	50.**D**	51.**D**	52.**A**
53.**B**	54.**B**	55.**A**	56.**D**	57.**548**	58.**63**	59.**B**	60.**C**
61.**150**	62.**B**	63.**12**	64.**60**	65.**B**	66.**11**		

Explanations

Quant

45. **B**

Let $\frac{x}{y}$ $be\ t$

Therefore, $c = 16t\ +\ \frac{49}{t}$

Applying AM>= GM

$\frac{\left(16t + \frac{49}{t}\right)}{2} \geq \left(16t \times \frac{49}{t}\right)^{\frac{1}{2}}$

$16t\ +\ \frac{49}{t} \geq 56$

When t is positive then c is greater than equal to 56.

When t is negative then c is less than equal to -56.

Therefore $c\ \in\ (-\infty,\ -56]\ \cup\ [56, \infty\ ]$

As -50 is not in the range of c so it is the answer

46. **A**

$2x^2 + kx + 5 = 0$ has no real roots so D<0

$k^2 - 40\ < 0$

$\left(k - \sqrt{40}\right)\left(k + \sqrt{40}\right) < 0$

$k \in \left(-\sqrt{40}, \sqrt{40}\right)$

$x^2 + (k-5)x + 1 = 0$ has two distinct real roots so D>0

$(k-5)^2 - 4 > 0$

$k^2 - 10k + 21 > 0$

$(k-3)\,(k-7) > 0$

$k \in (-\infty\ , 3) \cup (7, \infty\ )$

Therefore possibe value of k are -6, -5, -4, -3, -2, -1, 0, 1, 2

In 9 total 9 integer values of k are possible.

47. **14**

$(\sqrt{\frac{7}{5}})^{3x-y} = \frac{875}{2401}$

$\left(\frac{7}{5}\right)^{\frac{(3x-y)}{2}} = \frac{125}{343}$

$\left(\frac{7}{5}\right)^{\frac{(3x-y)}{2}} = \left(\frac{7}{5}\right)^{-3}$

3x-y = -6

$(\frac{4a}{b})^{6x-y} = (\frac{2a}{b})^{y-6x}$

Therefor, y=6x as the bases are different so the power should be zero for the results to be equal.

3x-y=-6

or, 3x - 6x = -6

or x= 2

y= 6x = 12

x+y = 14

48. **D**

Let the six numbers be a, b, c, d, e, f in ascending order

a+b = 28

e+f = 56

If we want to maximise the average then we have to maximise the value of c and d and maximise e and minimise f

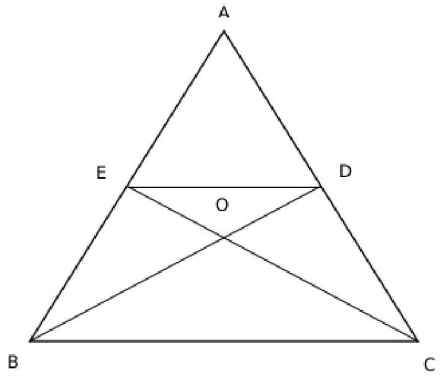
e+f = 56

As e and f are distinct natural numbers so possible values are 27 and 29

Therefore c and d will be 25 and 26 respecitively

So average = $\frac{(a+b+c+d+e+f)}{6} = \frac{(28+25+26+56)}{6} = \frac{135}{6} = 22.5$

49. **9**



Area of ABD : Area of BDC = 1:1

Therefore, area of ABD = 54

Area of ADE : Area of EDB = 1:1

Therefore, area of ADE = 27

O is the centroid and it divides the medians in the ratio of 2:1

Area of BEO : Area of EOD = 2:1

Area of EOD = 9

50. **D**

a, b, c, m and n are integers so if one root is $3 + 2\sqrt{2}$ then the other root is $3 - 2\sqrt{2}$

Sum of roots = 6 = -b/a or b= -6a

Product of roots = 1 = c/a or c=a

a, b, c, m and n are integers so if one root is $4 + 2\sqrt{3}$ then the other root is $4 - 2\sqrt{3}$

Sum of roots = 8 = -m/a or m = -8a

product of roots = 4 = n/a or n = 4a

$(\frac{b}{m} + \frac{c-2b}{n})$

= $\frac{6a}{8a} + \frac{(a+12a)}{4a} = \frac{3}{4} + \frac{13}{4} = \frac{16}{4} = 4$

51.**D**

Let the unit of work done by 1 man in 1 hour and 1 day be 1 MDH unit (Man Day Hour).

Thus, in 7 hours per day for 10 days, the work done by N people $N \times 7 \times 10$ MDH units.

Since this is equal to 35% of the total work,

35% of the total work =$N \times 7 \times 10$ MDH units.

Total work = $\frac{(N \times 100 \times 7 \times 10)}{35} = 200 \times N$ MDH units.

The work left =$200N - 70N = 130N$ MDH units.

Now, 10 people left the job. So, the number of people left = (N-10)

Since (N-10) people completed the rest of work in 14 days by working 10 hours a day,

$(N - 10) \times 14 \times 10 = 130N$

$10N = 1400$

N = 140

Thus, the correct option is D.

52.**A**

Initially: a glass 500cc milk and a cup 500cc water

Step 1: 150 cc of milk is transferred to the cup from glass

After step 1: Glass - 350 cc milk, Cup - 150 cc milk and 500 cc water

Step 2: 150 cc of this mixture is transferred from the cup to the glass

After step 2:

Glass - 350 cc milk + 150 cc mixture with milk:water ratio 3:10

Cup - 500 cc mixture with milk:water ratio 3:10

water in glass : milk in cup =$\frac{10}{13} \times 150 \ : \ \frac{3}{13} \times 500 = 1 : 1$

The answer is option A.

53.**B**

It is given,

$\frac{5.5 \times 1 \times 8000}{100} + \frac{5.6 \times 1 \times 5000}{100} + \frac{x \times 1 \times 7000}{100} = \frac{5}{100} \times 20000$

$440 + 280 + 70x = 1000$

x = 4%

Interest = $\frac{20000 \times 4 \times 1}{100}$ = Rs 800

The answer is option B.

54. **B**

Both the cars take 1.5 hrs to meet when they travel towards each other.

It is given, speed of slower car is 60 km/hr

Therefore, distance covered by slower car before they meet = 60*1.5 = 90 km

The answer is option B.

55. **A**

The number of 4-digit numbers possible using 1,1,2, and 4 is $\frac{4!}{2!} = 12$

Number of 1's, 2's and 4's in units digits will be in the ratio 2:1:1, i.e. 6 1's, 3 2's and 3 4's.

Sum = 6(1) + 3(2) + 3(4) = 24

Similarly, in tens digit, hundreds digit and thousands digit as well.

Therefore, sum = 24 + 24(10) + 24(100) + 24(1000) = 24(1111)

Mean = $\frac{24(1111)}{12} = 2222$

The answer is option A.

56. **D**

Sum of the three sides of a quadrilateral is greater than the fourth side.

Therefore, let the fourth side be

1+2+4>d or d<7

1+2+d>4 or d>1

Possible values of d are 2, 3, 4, 5 and 6.

57. **548**

General term = 38 + (n-1)17 = 17n + 21 = 17(n+1) + 4 = 17k + 4

Each term is in the form of 17k + 4

Least 3-digit number in the form of 17k + 4 is at k = 6, i.e. 106

Highest 3-digit number in the form of 17k + 4 is at k = 58, i.e. 990

106, 123, 140,....., 990

990 = 106 + 17(n-1)

n = 53

Sum = $\frac{53}{2}(106 + 990) = 53 \times 548$

Average = $53 \times \frac{548}{53} = 548$

58. **63**

Let the number of students in section A and B be a and b, respectively.

It is given, a = b - 10

$\frac{32a+60b}{a+b}$ is an integer

$\frac{32a+60(a+10)}{a+a+10} = k$

$\frac{46a+300}{a+5} = k$

$k = \frac{46(a+5)}{a+5} + \frac{70}{a+5}$

$k = 46 + \frac{70}{a+5}$

a can take values 2, 5, 9, 30, 65

Difference = 65 - 2 = 63

59. **B**

When x< r

f(x) = r

f(x) = f(f(x))

r = f(r)

r= 2r-r

r=r

When x>=r

f(x) = 2x-r

f(x) = f(f(x))

2x-r = f(2x-r)

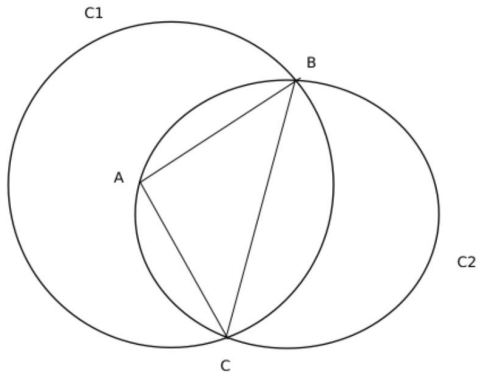
2x-r = 2(2x-r) - r

2x-r = 4x-3r

or, x=r

Therefore x<= r

60. **C**



BC is the diameter of circle C2 so we can say that $\angle BAC = 90^\circ$ as angle in the semi circle is 90°

Therefore overlapping area = $\frac{1}{2}$(Area of circle C2) + Area of the minor sector made be BC in C1

AB= AC = 8 cm and as $\angle BAC = 90^\circ$, so we can conclude that BC= $8\sqrt{2}$ cm

Radius of C2 = Half of length of BC = $4\sqrt{2}$ cm

Area of C2 = $\pi \left(4\sqrt{2}\right)^2 = 32\pi \ cm^2$

A is the centre of C1 and C1 passes through B, so AB is the radius of C1 and is equal to 8 cm

Area of the minor sector made be BC in C1 = $\frac{1}{4}$(Area of circle C1) - Area of triangle ABC = $\frac{1}{4}\pi \left(8\right)^2 - \left(\frac{1}{2} \times 8 \times 8\right) = 16\pi - 32 \ cm^2$

Therefore,

Overlapping area between the two circles= $\frac{1}{2}$(Area of circle C2) + Area of the minor sector made be BC in C1

= $\frac{1}{2}\left(32\pi\right) + (16\pi - 32) = 32\left(\pi - 1\right) cm^2$

61. **150**

Since the total number of students, when divided by either 9 or 10 or 12 or 25 each, gives a remainder of 4, the number will be in the form of LCM(9,10,12,25)k + 4 = 900k + 4.

It is given that the value of 900k + 4 is less than 5000.

Also, it is given that 900k + 4 is divided by 11.

It is only possible when k = 2 and total students = 1804.

So, the number of 12 students group = 1800/12 = 150.

62. **B**

Let $\frac{x^2-6x+10}{3-x} = p$

$x^2 - 6x + 10 = 3p - px$

$x^2 - (6 - p)\, x + 10 - 3p = 0$

Since the equation will have real roots,

$(6 - p)^2 - 4 \times (10 - 3p) \geq 0$

$p^2 - 12p + 12p + 36 - 40 \geq 0$

$p^2 \geq 4$

$p \geq 2\ ,\ p \leq -2$

Now, when $p = -2, x = 4$. Since it is given that $x < 3$, thus this value will be discarded.

Now, $\frac{1}{2}$ and $-\frac{1}{2}$ do not come in the mentioned range.

when $p = 2, x = 2$

Thus, the minimum possible value of p will be 2.

Thus, the correct option is B.

Alternate explanation:

Since $x < 3$,

$3 - x > 0$

Let $3 - x = y$. So, $y > 0$.

Now, $\frac{x^2-6x+10}{3-x} = \frac{x^2-6x+9+1}{3-x}$

=> $\frac{(3-x)^2+1}{3-x}$

Since $3 - x = y$, the equation will transform to $\frac{y^2+1}{y}$ or $y + \frac{1}{y}$

The minimum value of the expression $y + \frac{1}{y}$ for $y > 0$ will at $y = 1$

i.e., **Minimum value** = $1 + 1 = 2$

Thus, the correct option is B.

63. **12**

Let the number of 100 cheques, 250 cheques and 500 cheques be x, y and z respectively.

We need to find the maximum value of z.

x + y + z = 100 ..... (1)

100x + 250y + 500z = 15250

2x + 5y + 10z = 305 ..... (2)

2x + 2y + 2z = 200 ..... (1)

(2) - (1), we get

3y + 8z = 105

At z = 12, x = 3

Therefore, maximum value z can take is 12.

64. **60**

Let the speed of Moody be 'x' steps/sec and that of the escalator be 'y' steps/sec.

In 30 seconds, Moody willfinish riding the escalator when going in the same direction.

Thus, total steps = 30(x+y)

If Moody's speed becomes twice, the time becomes 20 seconds.

Thus, total steps = 20(2x+y)

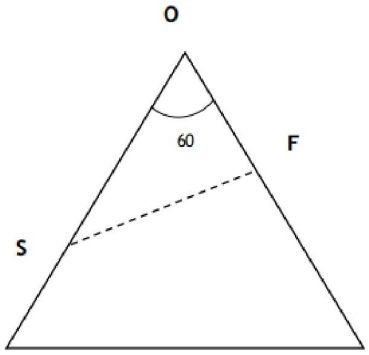
Or 30x + 30y = 40x + 20y

Or x = y

So, total steps = 60y.

Time taken by only escalator= 60y/y = 60s.

65. **B**



Let S be the slower ship and F be the faster ship.

It is given that when S travelled 8 km, the positions of ships with the port is forming a right triangle.

Since one of the angles is 60(since one vertex is still part of the equilateral triangle),

the other two vertexes will have angles of 30 and 90.

The distance between O and S = 24 - 8 = 16

In triangle OFS, $\cos 60^0 = \frac{OF}{OS}$

Thus, OF = 8.

Thus in the time, S covered 8 km, F will cover 24 - 8 = 16 km.

Thus, the ratio of their speeds is 2:1,

Thus, when F covers 24 km, S will cover 12 km.

The correct option is B.

66. **11**

Let the efficiency of Bob be 3 units/day. So, Alex's efficiency will be 6 units/day, and Cole's will be 2 units/day.

Since Bob canfinish the job in 40 days, the total work will be 40*3 = 120 units.

Since Alex and Bob work on thefirst day, the total work done = 3 + 6 = 9 units.

Similarly, for days 2 and 3, it will be 5 and 8 units, respectively.

Thus, in the first 3 days, the total work done = 9 + 5 + 8 = 22 units.

The work done in thefirst 15 days = 22*5 = 110 units.

Thus, the work will befinished on the 17th day(since 9 + 5 = 14 units are greater than the remaining work).

Since Alex works on two days of every 3 days, he will work for 10 days out of the first 15 days.

Then he will also work on the 16th day.

The total number of days = 11.