CBSE Board
Class X Mathematics
Sample Paper 2 (Standard) – Solution

Part A

Section I

1. Let $x = 0.\overline{8}$ …… (i)

$\Longrightarrow 10x = 8.\overline{8}$ ….. (ii)

(ii) – (i)

$\Longrightarrow 10x - x = 8$

$\Longrightarrow 9x = 8$

$\Longrightarrow x = 8/9$

OR

$\frac{17}{8}$

Prime factorisation of $8 = 2^3\,5^0$

Since the prime factorisation of 8 is in the form of $2^n\,5^m$

$\Longrightarrow$ The decimal expansion of $\frac{17}{8}$ is terminating.

2. For the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ to be coincident,

we must have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

The given equation of lines are $3x + 6y - 15 = 0$ and $9x + 18y - m = 0$

As $\frac{a_1}{a_2} = \frac{3}{9} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{6}{18} = \frac{1}{3}$

Therefore, $\frac{c_1}{c_2} = \frac{1}{3} \Rightarrow \frac{-15}{-m} = \frac{1}{3} \Rightarrow m = 45$

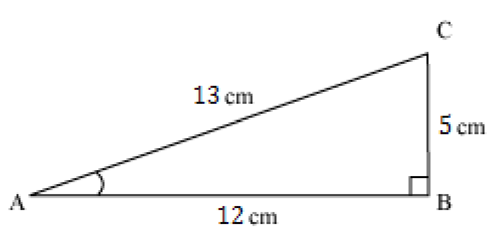
Hence, the value of m is 45.

3. In ΔABC, right angled at B, AB = 12 cm and BC = 5 cm.

Applying Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2 = (12)^2 + (5)^2 = 144 + 25 = 169$

$\Longrightarrow AC = \sqrt{169} = 13$ cm



$$\sin A = \frac{\text{side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5}{13}$$

4. Given sin 2x = 1

$$\Rightarrow 2x = \sin^{-1} 1 = \frac{\pi}{2}$$

$$\Rightarrow x = \frac{\pi}{4}$$

Also, cos y = $\frac{\sqrt{3}}{2}$

$$\Rightarrow y = \cos^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{6}$$

$$\Rightarrow y = \frac{\pi}{6}$$

$$\therefore x - y = \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$$

Hence, the value of x – y is $\frac{\pi}{12}$.

5. Let P(x, y) be the point which divides the line segment AB in the ratio 1: 3.
Coordinates of point P, dividing the line segment joining A(x_1, y_1) & B(x_2, y_2) internally in the ratio m:n is $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

Therefore, $x = \frac{1(5) + 3(-3)}{1+3} = -1$ and $y = \frac{1(2) + 3(6)}{1+3} = 5$

Hence, the coordinates of P are (–1, 5)

OR

Origin is O(0, 0) and given point is P(–24, 7).
Now, distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Therefore, distance between point P and origin is

$$\sqrt{(0+24)^2 + (0-7)^2} = \sqrt{24^2 + 7^2} = \sqrt{576 + 49} = 25$$

Hence, the distance of (–24, 7) from the origin is 25 units.

6. Total number of outcomes = 7
From the given list, numbers which satisfy $|x| < 2$ are −1, 0, 1.

Number of favourable outcomes = 3

$\therefore$ Required probability = $\frac{3}{7}$

OR

All possible outcomes = {Pass, Fail}
$\Longrightarrow$ n(S) = 2
Possible outcomes of passing = A = {Pass}
$\Longrightarrow$ n(A) = 1

$\therefore$ Required probability = $\dfrac{n(A)}{n(S)} = \dfrac{1}{2}$

7. $p(x) = 3x^2 + 7x - 3$ is divided by polynomial $x^2 - 2$

$$\begin{array}{r|l} & \quad 3 \\ x^2 - 2 & \; 3x^2 + 7x - 3 \\ & \; 3x^2 \qquad\; - 6 \\ & \; - \qquad\quad + \\ \hline & \qquad 7x + 3 \end{array}$$

Hence, the remainder is 7x + 3.

8. Dimensions of the resulting cuboid will be 4 cm, 4 cm, 8 cm.
Surface area of the cuboid

$= 2(lb + bh + lh)$

$= 2(4 \times 4 + 4 \times 8 + 4 \times 8)$

$= 2(16 + 32 + 32)$

$= 2(16 + 64)$

$= 2 \times 80 = 160 \text{ cm}^2$

Hence, the surface area of the resulting cuboid is 160 cm

9. Let PT = x cm
Given: In ΔPQR, ST || QR
Using Basic proportionality theorem, we have

$\dfrac{PS}{SQ} = \dfrac{PT}{TR}$

$\Rightarrow \dfrac{2.5}{5} = \dfrac{x}{6}$

$\therefore x = 3$

Hence, the length of PT is 3 cm.

10. $x^2 - 3\sqrt{3}x + 6 = 0$

$\Rightarrow x^2 - \sqrt{3}x - 2\sqrt{3}x + 6 = 0$

$\Rightarrow x\left(x - \sqrt{3}\right) - 2\sqrt{3}\left(x - \sqrt{3}\right) = 0$

$\Rightarrow \left(x - \sqrt{3}\right)\left(x - 2\sqrt{3}\right) = 0$

$\Rightarrow x - \sqrt{3} = 0 \text{ or } x - 2\sqrt{3} = 0$

$\Rightarrow x = \sqrt{3} \text{ or } x = 2\sqrt{3}$

OR

Given equation is $x^2 - 2x + 1 = 0$
Comparing with $ax^2 + bx + c = 0$
$a = 1$, $b = -2$ and $c = 1$
Discriminant $= b^2 - 4ac = (-2)^2 - 4 = 0$

11. According to the question,

$$a - \frac{4}{5} = 2 - a$$

$$\Rightarrow 2a = 2 + \frac{4}{5}$$

$$\Rightarrow 2a = \frac{14}{5}$$

$$\Rightarrow a = \frac{7}{5}$$

OR

Given that the first and last terms of an A.P. are 1 and 11, i.e., $a = 1$ and $l = 11$.
Sum of its n terms $= S_n = 36$

$$S_n = \frac{n}{2}(a + l)$$

$$\Longrightarrow 36 = \frac{n}{2}(1 + 11)$$

$$\Longrightarrow n = \frac{36}{6} = 6$$

Thus, the number of terms in the A.P. is 6.

12. Given, AR = 5 cm, BR = 4 cm and AC = 11 cm
We know that the tangents drawn from a point outside a circle are equal in length.
$\Longrightarrow$ AR = AQ = 5 cm and BR = BP = 4 cm
And PC = QC = AC – AQ = 11 – 5 = 6 cm
Therefore, BC = BP + PC = 4 cm + 6 cm = 10 cm

13. Given system of equations has infinitely many solutions.

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{2}{4} = \frac{3}{k} = \frac{-5}{-10}$$

$$\Rightarrow k = \frac{4 \times 3}{2} = 6$$

14. We may find cumulative frequencies with their respective class intervals as below

Weight (in kg)	Frequency (fi)	Cumulative frequency
40 – 45	2	2
45 – 50	3	2 + 3 = 5
50 – 55	8	5 + 8 = 13
55 – 60	6	13 + 6 = 19
60 – 65	6	19 + 6 = 25
65 – 70	3	25 + 3 = 28
70 – 75	2	28 + 2 = 30
Total (n)	30	

Cumulative frequency just greater than $\frac{n}{2}\left(\text{i.e.} \frac{30}{2} = 15\right)$ is 19,

belonging to class interval 55 – 60.
Median class = 55 – 60
Lower limit (l) of median class = 55
Hence, the lower limit is 55.

15. Vertices of a triangle are (7, –2), (5, 1) and (3, 2)
Area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\Rightarrow \text{Area} = \frac{1}{2}\{7(1-2) + 5(2-(-2)) + 3(-2-1)\}$$

$$= \frac{1}{2}\{-7 + 20 - 9\}$$

$$= \frac{1}{2}\{4\} = 2$$

Hence, the area of the triangle is 2 sq. units.

16. If r, h and *l* denote respectively the radius of base, height and slant height of a right circular cone, then total surface area is $\pi r^2 + \pi rl$.

Part B

Section II

17.

(a) Mid-point of I and H = $\left(\frac{9+12}{2}, \frac{16+15}{2}\right) = \left(\frac{21}{2}, \frac{31}{2}\right)$

(b) The distance of the point Q from the x – axis is 6m.

(c) The distance between A and D is 16m.

(d) Coordinates of A are (1, 8) and that of B are (5, 10).

Coordinates of a point dividing AB in the ratio 3: 1 is

$$\left(\frac{3\times 5+1\times 1}{1+3}, \frac{3\times 10+1\times 8}{1+3}\right) = \left(4, \frac{19}{2}\right) = (4.0, 9.5)$$

(e) (x, y) is equidistant from Q(9, 8) and S(17, 8).

$\Rightarrow (9 - x)^2 + (8 - y)^2 = (17 - x)^2 + (8 - y)^2$

$\Rightarrow 81 - 18x = 289 - 34x$

$\Rightarrow 16x = 208$

$\Rightarrow x = 13$

$\Rightarrow x - 13 = 0$

18.

(a) Width of the scale model = $\frac{1}{5}\times \text{width of the boat} = \frac{1}{5}\times 60 = 12\text{cm}$

(b) If any two polygons are not the mirror image of one another then there similarity will effect.

(c) We know that,

Scale factor = $\frac{\text{length in image}}{\text{corresponding length in object}}$

If two similar triangles have a scale factor of a: b then their altitudes have a ratio a: b

(d) This is an example of similarity.

$$\Rightarrow \frac{10}{4} = \frac{22.5}{\text{Shadow of a tree}}$$

$$\Rightarrow \text{Shadow of a tree} = \frac{90}{10} = 9\text{m}$$

(e) Here, ΔTEF and ΔTAB are similar triangles as they form the equal angles.

Therefore, the ratio of their corresponding sides is same.

As E and F are the midpoints TA and TB, so TE = 6m and TF = 6m

$$\frac{EF}{AB} = \frac{TE}{TA} = \frac{1}{2} \Rightarrow EF = 6\text{m}$$

19.

(a) $x^2 - 10x + 16 = x^2 - 8x - 2x + 16 = x(x-8) - 2(x-8) = 0$

$\Longrightarrow (x-8)(x-2) = 0$

$\Longrightarrow x = 8$ or $x = 2$

(b) Zeroes of a polynomial can be expressed graphically. Number of zeroes of polynomial is equal to number of points where the graph of polynomial **Intersects** **x – axis.**

(c) Graph of a quadratic polynomial is a Parabola.

(d) A highway underpass is parabolic in shape and a parabola is the graph that results from $p(x)=ax^2+bx+c$ which has two zeroes. (As it is a quadratic polynomial)

Product of zeroes = 36 and one of the zero = 6 $\Longrightarrow$ other zero = 6

x^2 – (sum of zeroes)x + product of zeroes

$= x^2 - 12x + 36$

(e) $f(x) = (x-3)^2 + 4 = x^2 - 6x + 13$ is a Quadratic Polynomial.

The number of zeroes that f(x) can have is 2.

20.

(a)

Time (in sec)	No. od students(f)	X	fx
0 – 20	8	10	80
20 – 40	10	30	300
40 – 60	13	50	650
60 – 80	6	70	420
80 – 100	3	90	270
	$\Sigma f = 40$		$\Sigma fx = 1720$

Mean time taken by a student to finish the race = 1720/40 = 43 seconds

(b) The modal class is 40 – 60 as it has the highest frequency i.e 13.

Lower limit of the modal class = 40

(c) The construction of cumulative frequency table is useful in determining the Median.

(d)

Time (in sec)	No. od students(f)	Cf
0 – 20	8	8
20 – 40	10	18
40 – 60	13	31
60 – 80	6	37
80 – 100	3	40
	N= $\Sigma f = 40$	

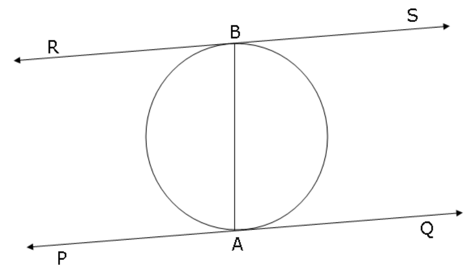
Here $N/2 = 40/2 = 20$, Median Class = 40 – 60, Modal Class = 40 – 60
Sum of upper limits of median class and modal class = 60 + 60 = 120

(e) Number of students who finished the race within 80 seconds
= 8 + 10 + 13 + 6= 37

Part B

Section III

21. Let AB be the diameter of the given circle, and let PQ and RS be the tangent lines drawn to the circle at points A and B respectively.

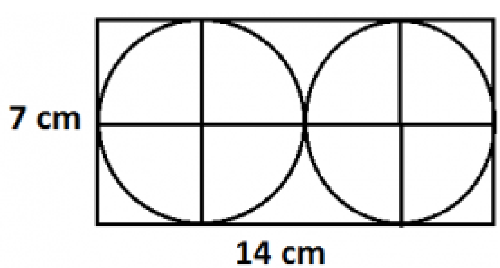


Since tangent at a point to a circle is perpendicular to the radius through the point of contact.
Therefore, AB is perpendicular to both PQ and RS.
$\Longrightarrow \angle PAB = 90^\circ$ and $\angle ABS = 90^\circ$
$\Longrightarrow \angle PAB = \angle ABS$
But, these are a pair of alternate interior angles.
Therefore, PQ is parallel to RS.

22. Circumference of first circle = $C_1 = 2\pi \times 19 = 38\pi$
Circumference of second circle = $C_2 = 2\pi \times 9 = 18\pi$
$C_1 + C_2 = 56\pi$
Let R be the radius of the new circle.
$\therefore 2\pi R = 56\pi$
$\therefore 2R = 56$
$\therefore R = 28$ cm

OR

Dimension of the rectangular cardboard = 14 cm × 7 cm
Since, two circular pieces of equal radii and maximum area touching each other are cut from the rectangular card board, therefore,
the diameter of each circular piece is 14/ 2 = 7 cm.



$\therefore$ Area of each circle $= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2}$ cm^2

$\Rightarrow$ Area of two circles $= 2 \times \frac{77}{2} = 77$ cm^2

Area of rectangular cardboard $= 14 \times 7 = 98$ cm^2

Thus, area of remaining cardboard

= Area of rectangular cardboard − Area of two circles

$= (98 - 77)$ cm^2

$= 21$ cm^2

23. Given: Height of the cylindrical part (h) = 10cm
Radius (r) of cylindrical part = radius (r) of hemispherical part = 3.5cm
Surface area of article = CSA of cylindrical part + 2×CSA of hemispherical part

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi \times 3.5 \times 10 + 2 \times 2\pi \times 3.5 \times 3.5$

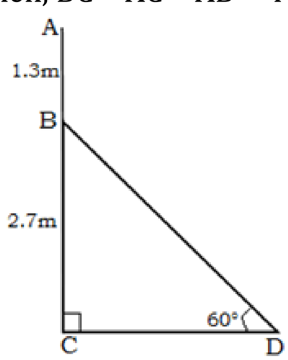
$= 70\pi + 49\pi$

$= 119\pi$

$= 17 \times 22 = 374$ cm^2

Hence, the surface area of the article is 374 cm^2.

24. Let AC be the electric pole of height 4 m. Let B be a point 1.3 m below the top A of the pole AC.
Then, BC = AC − AB = 4 − 1.3 = 2.7 m



Let BD be the length of ladder inclined at an angle of 60°.

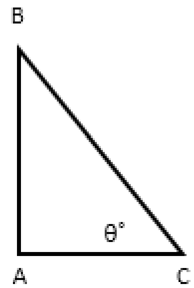
$\sin 60° = \frac{BC}{BD}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{2.7}{BD}$

$\Rightarrow BD = \frac{5.4}{\sqrt{3}} = 1.8\sqrt{3}$ or $\frac{9\sqrt{3}}{5}$

Thus, the length of the ladder is $\frac{9\sqrt{3}}{5}$ m.

OR



Let AB be the pole and let AC be its shadow.

Let the angle of elevation of the sun be θ°.

$\angle ACB = \theta$, $\angle CAB = 90°$

AB = 10 m and AC = $10\sqrt{3}$ m

In ΔCAB,

$$\tan\theta = \frac{AB}{AC} = \frac{10}{10\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \theta = 30$

Hence, the angular elevation of the sun is 30°.

25. The given AP is $\sqrt{2}, \sqrt{8}, \sqrt{18}, \ldots$ or $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2} \ldots$

Common difference d = $2\sqrt{2} - \sqrt{2} = \sqrt{2}$

Term next to $3\sqrt{2} = 3\sqrt{2} + d = 3\sqrt{2} + \sqrt{2} = 4\sqrt{2} = \sqrt{16 \times 2} = \sqrt{32}$

26. A die is thrown at once then S = {1, 2, 3, 4, 5, 6}

$\therefore$ n(S) = 6

Let E be the event of getting a prime number.

$\therefore$ E = {2, 3, 5}

$\therefore$ n(E) = 3

$\therefore$ P(E) = $\frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

Section IV

27. Suppose the line 3x + y − 9 = 0 divides the line segment joining the points A(1, 3) and B(2, 7) in the ratio k : 1 at point C.

Then, the co-ordinates of C are $\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$

But, C lies on 3x + y − 9 = 0.

Therefore,

$$\left[3\left(\frac{2k+1}{k+1}\right)\right] + \left[\frac{7k+3}{k+1}\right] - 9 = 0$$

$\Rightarrow$ 6k + 3 + 7k + 3 − 9k − 9 = 0

$\Rightarrow 4k - 3 = 0$

$\Rightarrow k = \frac{3}{4}$

So, the required ratio is 3: 4.

OR

Given, the point P divides the join of (2, 1) and (−3, 6) in the ratio 2: 3.

Co-ordinates of the point P $\equiv \left(\frac{2\times(-3)+3\times 2}{2+3}, \frac{2\times 6+3\times 1}{2+3}\right) \equiv \left(\frac{-6+6}{5}, \frac{12+3}{5}\right) \equiv (0, 3)$

Now, the given equation is x − 5y + 15 = 0.

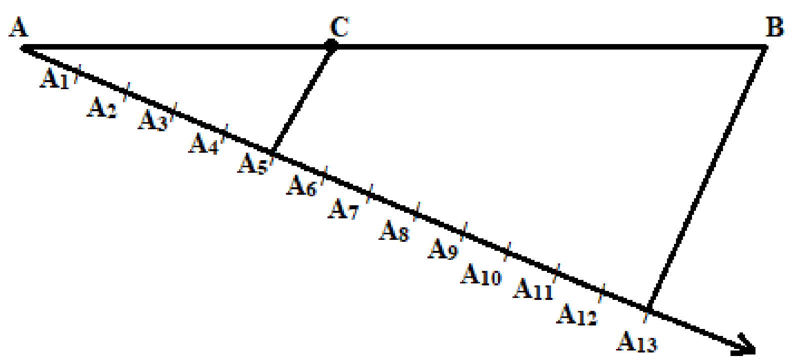
Substituting x = 0 and y = 3 in this equation, we have

L.H.S. = 0 − 5(3) + 15 = −15 + 15 = 0 = R.H.S.

Hence, the point P lies on the line x − 5y + 15 = 0.

28.

1. Draw a line segment of AB length 7.6 cm.
2. Draw a ray AX making acute angle with AB and take 13 points $A_1, A_2, \ldots A_{13}$ on it at equal distances
3. Join A_{13} and B. Draw line parallel to $A_{13}B$ from A_5 meeting AB at C.
4. AC: CB = 5:8. Measuring the lengths, we get AC = 2.9 cm and CB = 4.7 cm.



29. Since, the highest frequency is between 35 – 45 we have l = 35, h = 10, f_0 = 21, f_1 = 23, f_2 = 14

Mode = $l + (f_1 - f_0)/(2f_1 - f_0 - f_2) \times h = 35 + (23 - 21)/(2 \times 23 - 21 - 14) \times 10 = 36.8$

Mean $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2830}{80} = 35.375$

OR

From the data given as above we may observe that maximum class frequency is 12 belonging to class interval 65 – 75.

So, modal class = 65 – 75

Lower class limit (l) of modal class = 65

Frequency (f_1) of modal class = 12

Frequency (f_0) of class preceding the modal class = 11

Frequency (f_2) of class succeeding the modal class = 9

Class size (h) = 10

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 65 + \left(\frac{12 - 11}{2(12) - 11 - 9}\right) \times 10$$

$$= 65 + \frac{1}{4} \times 10 = 65 + 2.5 = 67.5$$

30.

The numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called rational numbers.

Let $x = 3.\overline{1416}$

$\Rightarrow x = 3.141614161416.....\quad ....(i)$

Since there are four repeating digits,

we multiply by 1000.

$\Rightarrow 10000x = 31416.14161416.....\quad ....(ii)$

Subtracting (i) from (ii), we get

$9999x = 31413$

$\Rightarrow x = \frac{31413}{9999}$ which is of the form $\frac{p}{q}$.

So, $3.\overline{1416}$ is a rational number.

31. Dividend, $p(x) = x^3 - 3x^2 + x + 2$

Quotient = (x – 2)

Remainder = (– 2x + 4)

Let g(x) be the divisor.

According to the division algorithm,

Dividend = Divisor × Quotient + Remainder

$x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$

$\Rightarrow x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x - 2)$

$\Rightarrow x^3 - 3x^2 + 3x - 2 = g(x)(x - 2)$

Now, g(x) is the quotient when $x^3 - 3x^2 + 3x - 2$ is divided by x – 2

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x - 2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & - \quad + \\
\hline
 & \quad -x^2 + 3x - 2 \\
 & \quad -x^2 + 2x \\
 & \quad + \quad - \\
\hline
 & \qquad\quad x - 2 \\
 & \qquad\quad x - 2 \\
 & \qquad\quad - \quad + \\
\hline
 & \qquad\quad 0 \\
\hline
\end{array}
$$

$\therefore g(x) = x^2 - x + 1$

32. Let the fraction be $\frac{x}{y}$.

According to the question,

$x + y = 8$ ....(1)

$$\frac{x+3}{y+3} = \frac{3}{4}$$

$\Rightarrow 4x + 12 = 3y + 9$

$\Rightarrow 4x - 3y = -3$ ....(2)

Multiplying (1) by 3, we get

$3x + 3y = 24$ ....(3)

Adding (2) and (3), we get

$7x = 21$

$\Rightarrow x = 3$

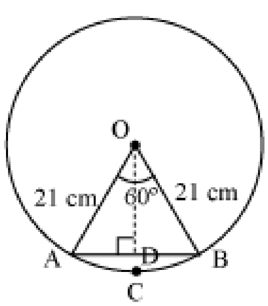
$\Rightarrow y = 8 - x = 8 - 3 = 5$

Thus, the fraction is $\frac{3}{5}$.

33. Radius (r) of circle = 21 cm

Angle subtended by given arc = 60°

Length of an arc of a sector of angle $\theta = \frac{\theta}{360°} \times 2\pi r$



a. Length of arc ACB = $\frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21$

$= \frac{1}{6} \times 2 \times 22 \times 3$ = 22 cm

b. Area of sector OACB = $\frac{60°}{360°} \times \pi r^2$

$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21$

$= 231 \text{ cm}^2$

c. Now in ΔOAB

∠OAB = ∠OBA (as OA = OB)…(i)

∠OAB + ∠AOB + ∠OBA = 180°

⇒ 2∠OAB + 60° = 180° ….from(i)

⇒ ∠OAB = 60°

So, ΔOAB is an equilateral triangle.

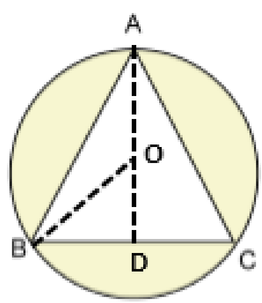
Area of ΔOAB = $\frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times (21)^2 = \frac{441\sqrt{3}}{4} \text{ cm}^2$

Area of segment ACB = Area of sector OACB – Area of ΔOAB

$= \left(231 - \frac{441\sqrt{3}}{4}\right) \text{cm}^2$

OR



Let O be the centre of the circumcircle.

Join OB and draw AD ⊥ BC.

Then, OB = 42 cm and ∠OBD = 30°

In ΔOBD,

$\sin 30° = \frac{OD}{OB}$

$\Rightarrow \frac{1}{2} = \frac{OD}{42}$

$\Rightarrow OD = 21 \text{ cm}$

Now, $BD^2 = OB^2 - OD^2 = 42^2 - 21^2 = (42 + 21)(42 - 21) = 63 \times 21$

$\Rightarrow BD = \sqrt{63 \times 21} = \sqrt{3 \times 21 \times 21} = 21\sqrt{3}$ cm

$\Rightarrow BC = 2 \times 21\sqrt{3} = 42\sqrt{3}$ cm

Now, area of the shaded region

= Area of the circle − Area of an equilateral ΔABC

$= \frac{22}{7} \times 42 \times 42 - \frac{\sqrt{3}}{4} \times 42\sqrt{3} \times 42\sqrt{3}$

$= (5544 - 2291.5)\ cm^2$

$= 3252.5\ cm^2$

Section V

34. Let B be the window of a house AB and let CD be the other house.

Then, AB = EC = h metres.

Let CD = H metres.

Then, ED = (H − h) m

In ΔBED,

$\cot\alpha = \frac{BE}{ED}$

BE = (H − h) cot α ... (a)

In ΔACB,

$\frac{AC}{AB} = \cot\beta$

AC = h. cot β .... (b)

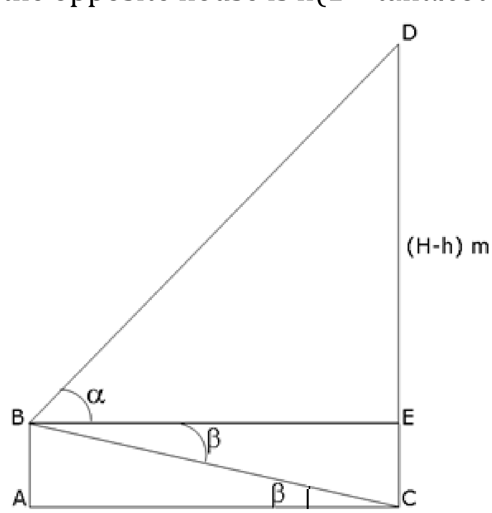
But BE = AC

∴ (H − h) cot α = h cot β ....[From (a) and (b)]

$H = h\,\frac{(\cot\alpha + \cot\beta)}{\cot\alpha}$

H = h(1 + tan α cot β)

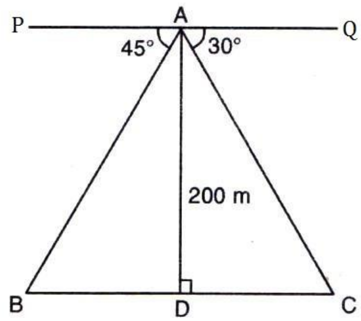
Thus, the height of the opposite house is h(1 + tanα.cot β) metres



OR

Let AD represent the light house.

Let the points B and C denote the ships based on the opposite sides of the light house.



$\angle ABD = \angle PAB = 45°$ (interior alternate angle)

$\angle ACD = \angle QAC = 30°$ (interior alternate angle)

$\therefore \tan 45° = \dfrac{AD}{BD} \Rightarrow 1 = \dfrac{200}{BD} \Rightarrow BD = 200 \text{ m}$

Also, $\tan 30° = \dfrac{AD}{DC}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{200}{DC} \Rightarrow DC = 200\sqrt{3} \text{ m}$

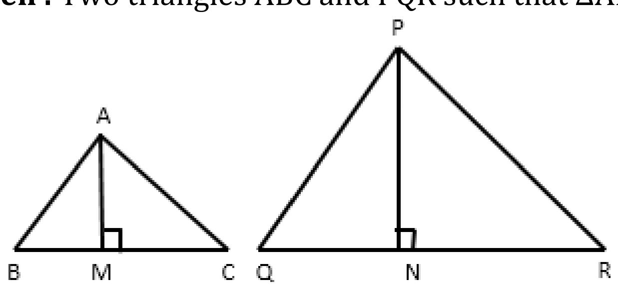
$\Rightarrow DC = 200 \times 1.732 = 346.4$ m

$\therefore BC = BD + DC = (200 + 346.4) \Rightarrow BC = 546.4$ m

Distance between two ships = 546.4 m

35. **Statement :** Ratio of the areas of two similar triangles is equal to the ration of the squares of their corresponding sides.

Given : Two triangles ABC and PQR such that $\Delta ABC \sim \Delta PQR$



To prove : $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{CA^2}{RP^2}$

Proof :

For finding the areas of the two triangles, we draw altitudes AM and PN of the triangles.

Now, $ar(\Delta ABC) = \dfrac{1}{2} BC \times AM$

And $ar(\Delta PQR) = \dfrac{1}{2} QR \times PN$

So, $\frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \frac{BC \times AM}{QR \times PN}$ ....(1)

Now, in ΔABM and ΔPQN.

$\angle B = \angle Q$ (As $\Delta ABC \sim \Delta PQR$)

$\angle M = \angle N$ (Each = $90°$)

So, $\Delta ABM \sim \Delta PQN$ (AA similarity criterion)

Therefore, $\frac{AM}{PN} = \frac{AB}{PQ}$ ....(2)

Also, $\Delta ABC \sim \Delta PQR$

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ ....(3)

$$\frac{ar(ABC)}{ar(PQR)} = \frac{AB}{PQ} \times \frac{AM}{PN} \qquad [\text{from } (1) \text{ and } (3)]$$

$$= \frac{AB}{PQ} \times \frac{AB}{PQ} \qquad [\text{from } (2)]$$

$$= \left(\frac{AB}{PQ}\right)^2$$

Now using (3), we get

$$\frac{ar(ABC)}{ar(PQR)} = \frac{AB^2}{PQ^2}$$

Similarly

$$\frac{ar(ABC)}{ar(PQR)} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$$

36. Length of the cylinder = 24 cm

Diameter of copper wire = 4 mm = 0.4 cm

Therefore, the number of rounds required for a wire to cover the length of cylinder

$$= \frac{\text{Length of cylinder}}{\text{Thickness of wire}}$$

$$= \frac{24 \text{ cm}}{0.4 \text{ cm}}$$

$$= 60$$

Now, diameter of cylinder = 20 cm

Therefore, length of the wire required to complete one round = circumference of base of the cylinder = πd $= \frac{22}{7} \times 20 = \frac{440}{7}$ cm

Length of wire for covering the whole surface of cylinder

= length of wire in completing 60 rounds

$= 60 \times \frac{440}{7} = 3771.428\ cm$

Radius of copper wire = $\frac{0.4}{2} cm = 0.2\ cm$

Therefore, volume of wire = $\pi r^2 h$ = $\frac{22}{7} \times (0.2)^2 \times 3771.428$ = 474.122 cm^3

Weight of wire = volume × density

= 474.122 × 8.68 = 4115.38 g = 4.11538 kg $\approx$ 4.12 kg