10. Exponents

Exercise 10.1

1 A. Question

Express the following numbers in the exponential form:

1728

Answer

We prime factorize the number

$1728 = 3^3 \times 2^6$

$\Rightarrow 1728 = 3^3 \times 4^3$

Since the powers are same so the bases can be multiplied

$\Rightarrow 1728 = 12^3$

1 B. Question

Express the following numbers in the exponential form:

$$\frac{1}{512}$$

Answer

We prime factorize the number 512

$512 = 2^9$

Since reciprocal of a number results in negative power,

So $\frac{1}{512} = 2^{-9}$

1 C. Question

Express the following numbers in the exponential form:

0.000169.

Answer

We prime factorize the number 169

$169 = 13^2$

So we can say that

$0.000169 = 13^2 \times 10^{-6}$

$\Rightarrow 0.000169 = (13 \times 10^{-3})^2$

$\Rightarrow 0.000169 = 0.013^2$

2 A. Question

Write the following numbers using base 10 and exponents:

12345

Answer

The number can be expanded by expressing the number in terms of its digits and multiplying each digit with 10 raised to a specific power according to its place value

$12345 = 1\times10^4 + 2\times10^3 + 3\times10^2 + 4\times10 + 5$

2 B. Question

Write the following numbers using base 10 and exponents:

1010.0101

Answer

$$1010.0101 = 1\times10^3 + 0\times10^2 + 1\times10 + 0 + \frac{0}{10} + \frac{1}{10^2} + \frac{0}{10^3} + \frac{1}{10^4}$$

$$\Rightarrow 1010.0101 = 1\times10^3 + 1\times10 + \frac{1}{10^2} + \frac{1}{10^4}$$

2 C. Question

Write the following numbers using base 10 and exponents:

0.1020304

Answer

$$0.1020304 = 0 + \frac{1}{10} + \frac{0}{10^2} + \frac{2}{10^3} + \frac{0}{10^4} + \frac{3}{10^5} + \frac{0}{10^6} + \frac{4}{10^7}$$

$$\Rightarrow 0.1020304 = \frac{1}{10} + \frac{2}{10^3} + \frac{3}{10^5} + \frac{4}{10^7}$$

3. Question

Find the value of $(-0.2)^{-4}$

Answer

Since the power is negative so

$$(-0.2)^{-4} = \frac{1}{(-0.2)^4}$$

$$\Rightarrow (-0.2)^{-4} = \frac{1}{0.0016}$$

Exercise 10.2

1 A. Question

Simplify:

$3^1 \times 3^2 \times 3^3 \times 3^4 \times 3^5 \times 3^6$.

Answer

Since the bases are same so the powers can be added

$3^1 \times 3^2 \times 3^3 \times 3^4 \times 3^5 \times 3^6 = 3^{(1 + 2 + 3 + 4 + 5 + 6)}$

$\Rightarrow 3^1 \times 3^2 \times 3^3 \times 3^4 \times 3^5 \times 3^6 = 3^{21}$

1 B. Question

Simplify:

$2^2 \times 3^3 \times 2^4 \times 3^5 \times 3^6$.

Answer

Since there are two bases so their powers can be separately added.

$2^2 \times 3^3 \times 2^4 \times 3^5 \times 3^6 = 2^{(2 + 4)} \times 3^{(3 + 5 + 6)}$

$\Rightarrow 2^2 \times 3^3 \times 2^4 \times 3^5 \times 3^6 = 2^6 \times 3^{14}$

2. Question

How many zeros are there in $10^4 \times 10^3 \times 10^2 \times 10$?

Answer

Since the bases are same so the powers can be added

$10^4 \times 10^3 \times 10^2 \times 10 = 10^{(4 + 3 + 2 + 1)}$

$\Rightarrow 10^4 \times 10^3 \times 10^2 \times 10 = 10^{10}$

Since the power of 10 is 10 so there are 10 zeroes.

3. Question

Which is larger: $(5^3 \times 5^4 \times 5^5 \times 5^6)$ or $(5^7 \times 5^8)$?

Answer

$5^3 \times 5^4 \times 5^5 \times 5^6 = 5^{(3 + 4 + 5 + 6)}$

$\Rightarrow 5^3 \times 5^4 \times 5^5 \times 5^6 = 5^{18}$

$5^7 \times 5^8 = 5^{(7 + 8)}$

$\Rightarrow 5^7 \times 5^8 = 5^{15}$

Since the power of 5 in $5^3 \times 5^4 \times 5^5 \times 5^6$ is greater so $5^3 \times 5^4 \times 5^5 \times 5^6$ is greater than $5^7 \times 5^8$

Exercise 10.3

1 A. Question

Simplify:

$10^{-1} \times 10^2 \times 10^{-3} \times 10^4 \times 10^{-5} \times 10^6$;

Answer

Since the bases are same so their powers are solved algebraically

$10^{-1} \times 10^2 \times 10^{-3} \times 10^4 \times 10^{-5} \times 10^6 = 10^{(-1 + 2-3 + 4-5 + 6)}$

On solving we get,

$\Rightarrow 10^{-1} \times 10^2 \times 10^{-3} \times 10^4 \times 10^{-5} \times 10^6 = 10^3$

1 B. Question

Simplify:

$$\frac{2^3 \times 3^2 \times 5^4}{3^3 \times 5^2 \times 2^4}.$$

Answer

$$\frac{2^3 \times 3^2 \times 5^4}{3^3 \times 5^2 \times 2^4}$$

Since the denominator and numerator have common base so their powers can be subtracted

$$\Rightarrow \frac{2^3 \times 3^2 \times 5^4}{3^3 \times 5^2 \times 2^4} = \frac{5^2}{3 \times 2}$$

$$\Rightarrow \frac{2^3 \times 3^2 \times 5^4}{3^3 \times 5^2 \times 2^4} = \frac{25}{6}$$

2. Question

Which is larger: $(3^4 \times 2^3)$ or $(2^5 \times 3^2)$?

Answer

We divide the two expressions

$$\frac{3^4 \times 2^3}{2^5 \times 3^2}$$

Since the denominator and numerator have common base so their powers can be subtracted

$$\frac{3^4 \times 2^3}{2^5 \times 3^2} = \left(\frac{3}{2}\right)^2$$

Since the fraction is greater than 1 so $(3^4 \times 2^3)$ is greater than $(2^5 \times 3^2)$

3. Question

Suppose m and n are distinct integers. Can $\frac{3^m \times 2^n}{2^m \times 3^n}$ be an integer? Give reasons.

Answer

We divide the two expressions

$$\frac{3^m \times 2^n}{2^m \times 3^n}$$

Since the denominator and numerator have common base so their powers can be subtracted

$$\frac{3^m \times 2^n}{2^m \times 3^n} = \left(\frac{3}{2}\right)^{m-n}$$

Since m and n are two distinct integers so the power can never become 0 and hence the fraction can never an integer.

4. Question

Suppose b is a positive integer such that $\frac{2^4}{b^2}$ is also an integer. What are the possible values of b?

Answer

Since $\frac{2^4}{b^2}$ is an integer so

the numerator must be greater than denominator

$$\frac{2^4}{b^2} = \left(\frac{4}{b}\right)^2$$

So b has to be either 1,2 or 4 to become an integer.

Exercise 10.4

1 A. Question

Simplify:

$$\frac{(2^5)^6 \times (3^3)^2}{(2^6)^5 \times (3^2)^3};$$

Answer

$$\frac{(2^5)^6 \times (3^3)^2}{(2^6)^5 \times (3^2)^3} = \frac{2^{30} \times 3^6}{2^{30} \times 3^6}$$

$$\Rightarrow \frac{(2^5)^6 \times (3^3)^2}{(2^6)^5 \times (3^2)^3} = 1$$

1 B. Question

Simplify:

$$\frac{(5^{-3})^2 \times 3^4}{(3^{-2})^{-3} \times (5^3)^{-2}}$$

Answer

$$\frac{(5^{-3})^2 \times 3^4}{(3^{-2})^{-3} \times (5^3)^{-2}} = \frac{5^{-6} \times 3^4}{3^{-6} \times 5^{-6}}$$

$$\Rightarrow \frac{(5^{-3})^2 \times 3^4}{(3^{-2})^{-3} \times (5^3)^{-2}} = \frac{3^4}{3^{-6}} = 3^{10}$$

2. Question

Can you find two integers m,n such that $2^{m+n} = 2^{mn}$?

Answer

$2^{m+n} = 2^{mn}$

$\Rightarrow m + n = mn$

Let m = n

So we get

$2m = m^2$

$\Rightarrow m = 2$

Since m = n, so

n = 2

3. Question

If $(2^m)^4 = 4^6$, find the value of m.

Answer

$(2^m)^4 = 4^6$

$\Rightarrow (4^m)^2 = 4^6$

Since the bases are same so their powers can be equated

2m = 6

$\Rightarrow m = 3$

Exercise 10.5

1 A. Question

Simplify:

$$\frac{6^8 \times 5^3}{10^3 \times 3^4};$$

Answer

$$\frac{6^8 \times 5^3}{10^3 \times 3^4} = \frac{(3 \times 2)^8 \times 5^3}{(5 \times 2)^3 \times 3^4}$$

$$\Rightarrow \frac{6^8 \times 5^3}{10^3 \times 3^4} = \frac{3^8 \times 2^8 \times 5^3}{5^3 \times 2^3 \times 3^4}$$

Since the bases are common in numerator and denominator so their powers are subtracted

$$\Rightarrow \frac{6^8 \times 5^3}{10^3 \times 3^4} = 3^4 \times 2^5 = 2592$$

1 B. Question

Simplify:

$$\frac{(15)^{-3} \times (-12)^4}{5^{-6} \times (36)^2};$$

Answer

$$\frac{(15)^{-3} \times (-12)^4}{5^{-6} \times 36^2} = \frac{(5 \times 3)^{-3} \times (-4 \times 3)^4}{5^{-6} \times 6^4}$$

$$\Rightarrow \frac{(15)^{-3} \times (-12)^4}{5^{-6} \times 36^2} = \frac{(5)^{-3} \times 3^{-3} \times (-4)^4 \times 3^4}{5^{-6} \times (3 \times 2)^4}$$

$$\Rightarrow \frac{(15)^{-3} \times (-12)^4}{5^{-6} \times 36^2} = \frac{(5)^{-3} \times (2)^8 \times 3}{5^{-6} \times 3^4 \times 2^4}$$

Since the bases are common in numerator and denominator so their powers are subtracted

$$\Rightarrow \frac{(15)^{-3} \times (-12)^4}{5^{-6} \times 36^2} = \frac{5^3 \times 2^4}{3^3} = \frac{2000}{27}$$

1 C. Question

Simplify:

$$\frac{(0.22)^4 \times (0.222)^3}{(0.2)^5 \times (0.2222)^2}.$$

Answer

$$\frac{(0.22)^4 \times (0.222)^3}{(0.2)^5 \times (0.2222)^2} = \frac{2^4 \times (0.11)^4 \times 2^3 \times (0.111)^3}{2^5 \times (0.1)^5 \times 2^2 \times (0.1111)^2}$$

$$\Rightarrow \frac{(0.22)^4 \times (0.222)^3}{(0.2)^5 \times (0.2222)^2} = \frac{2^4 \times (11 \times 10^{-2})^4 \times 2^3 \times (111 \times 10^{-3})^3}{2^5 \times (10^{-1})^5 \times 2^2 \times (1111 \times 10^{-4})^2}$$

$$\Rightarrow \frac{(0.22)^4 \times (0.222)^3}{(0.2)^5 \times (0.2222)^2} = \frac{2^4 \times 11^4 \times 10^{-8} \times 2^3 \times 111^3 \times 10^{-9}}{2^5 \times 10^{-5} \times 2^2 \times 1111^2 \times 10^{-8}}$$

$$\Rightarrow \frac{(0.22)^4 \times (0.222)^3}{(0.2)^5 \times (0.2222)^2} = \frac{11^4 \times 10^{-4} \times 111^3}{1111^2}$$

$$\Rightarrow \frac{(0.22)^4 \times (0.222)^3}{(0.2)^5 \times (0.2222)^2} = \frac{11^2 \times 10^{-4} \times 111^3}{101^2} = \frac{165483351 \times 10^{-4}}{10201}$$

2. Question

Is $\frac{(10^4)^3}{5^{13}}$ an integer? Justify your answer.

Answer

$$\frac{(10^4)^3}{5^{13}} = \frac{(5^4 \times 2^4)^3}{5^{13}}$$

$$\Rightarrow \frac{(10^4)^3}{5^{13}} = \frac{5^{12} \times 2^{12}}{5^{13}}$$

$$\Rightarrow \frac{(10^4)^3}{5^{13}} = \frac{2^{12}}{5}$$

Since after reducing the expression there is a 5 in the denominator but no multiple of 5 in the numerator so it is not an integer

3. Question

Which is larger : $(100)^4$ or $(125)^3$?

Answer

We divide the two numbers

$$\frac{100^4}{125^3} = \frac{(25 \times 4)^4}{(5^3)^3}$$

$$\Rightarrow \frac{100^4}{125^3} = \frac{5^8 \times 2^8}{5^9}$$

$$\Rightarrow \frac{100^4}{125^3} = \frac{2^8}{5}$$

Since the numerator is greater than the denominator so 100^4 is greater than 125^3

Exercise 10.6

1 A. Question

Simplify:

$$\left(\frac{2}{3}\right)^8 \times \left(\frac{6}{4}\right)^3;$$

Answer

$$\left(\frac{2}{3}\right)^8 \times \left(\frac{6}{4}\right)^3 = \frac{2^8 \times (2 \times 3)^3}{3^8 \times (2^2)^3}$$

$$\left(\frac{2}{3}\right)^8 \times \left(\frac{6}{4}\right)^3 = \frac{2^8 \times 2^3 \times 3^3}{3^8 \times 2^6}$$

$$\left(\frac{2}{3}\right)^8 \times \left(\frac{6}{4}\right)^3 = \frac{2^{11} \times 3^3}{3^8 \times 2^6}$$

Since bases are same so the powers can be subtracted

$$\left(\frac{2}{3}\right)^8 \times \left(\frac{6}{4}\right)^3 = \frac{2^5}{3^5} = \frac{32}{243}$$

1 B. Question

Simplify:

$(1.8)^6 \times (4.2)^{-3}$

Answer

$(1.8)^6 \times (4.2)^{-3} = (18\times10^{-1})^6\times(42\times10^{-1})^{-3}$

$\Rightarrow (1.8)^6 \times (4.2)^{-3} = (3^2\times2\times10^{-1})^6\times(7\times3\times2\times10^{-1})^{-3}$

$\Rightarrow (1.8)^6 \times (4.2)^{-3} = 3^{12}\times2^6\times10^{-6}\times7^{-3}\times3^{-3}\times2^{-3}\times10^3$

$\Rightarrow (1.8)^6 \times (4.2)^{-3} = 3^9\times2^3\times7^{-3}\times10^{-3}$

$$\Rightarrow (1.8)^6 \times (4.2)^{-3} = \frac{3^9 \times 2^3}{7^3 \times 10^3} = \frac{157464}{343000}$$

$$\Rightarrow (1.8)^6 \times (4.2)^{-3} = \frac{19683}{42875}$$

1 C. Question

Simplify:

$$\frac{(0.0006)^9}{(0.015)^{-4}}.$$

Answer

$$\frac{(0.0006)^9}{(0.015)^{-4}} = \frac{6^9 \times 0.0001^9}{15^{-4} \times 0.001^{-4}}$$

$$\Rightarrow \frac{(0.0006)^9}{(0.015)^{-4}} = \frac{(3 \times 2)^9 \times (10^{-4})^9}{(3 \times 5)^{-4} \times (10^{-3})^{-4}}$$

$$\Rightarrow \frac{(0.0006)^9}{(0.015)^{-4}} = \frac{(3 \times 2)^9 \times (5 \times 2)^{-36}}{(3 \times 5)^{-4} \times (5 \times 2)^{12}}$$

$$\Rightarrow \frac{(0.0006)^9}{(0.015)^{-4}} = \frac{3^9 \times 2^9 \times 5^{-36} \times 2^{-36}}{3^{-4} \times 5^{-4} \times 5^{12} \times 2^{12}}$$

$$\Rightarrow \frac{(0.0006)^9}{(0.015)^{-4}} = \frac{3^{13}}{5^{44} \times 2^{39}}$$

2. Question

Can it happen that for some integer m ≠ 0, $\left(\frac{4}{25}\right)^m = \left(\frac{2}{5}\right)^{m^2}$?

Answer

$$\left(\frac{4}{25}\right)^m = \left(\frac{2}{5}\right)^{m^2}$$

$$\left(\frac{2}{5}\right)^{2m} = \left(\frac{2}{5}\right)^{m^2}$$

Since the bases are same so the powers can be equated

$2m = m^2$

$\Rightarrow m = 2$

So it can happen when m = 2

3. Question

Find all positive integers m,n such that $(3^m)^n = 3^m \times 3^n$.

Answer

$(3^m)^n = 3^m \times 3^n$

$\Rightarrow 3^{mn} = 3^{m+n}$

Since the bases are same so the powers can be equated

mn = m + n

Let m = n

$\Rightarrow m^2 = 2m$

$\Rightarrow m = 2$

Answer: m = 2, n = 2

Exercise 10.7

1 A. Question

Use the laws of exponents and simplify.

$$\frac{(12)^6}{162}$$

Answer

$12^6 = (4 \times 3)^6$

$\Rightarrow 12^6 = 2^{12} \times 3^6$

On prime factorizing 162 we get

$162 = 2 \times 3^4$

$$\frac{12^6}{162} = \frac{2^{12} \times 3^6}{2 \times 3^4}$$

$$\Rightarrow \frac{12^6}{162} = 2^{11} \times 3^2 = 18432$$

1 B. Question

Use the laws of exponents and simplify.

$$\frac{3^{-4} \times 10^{-5} \times (625)}{5^{-3} \times 6^{-4}}$$

Answer

$$\frac{3^{-4} \times 10^{-5} \times 625}{5^{-3} \times 6^{-4}} = \frac{3^{-4} \times (5 \times 2)^{-5} \times 5^4}{5^{-3} \times (3 \times 2)^{-4}}$$

$$\Rightarrow \frac{3^{-4} \times 10^{-5} \times 625}{5^{-3} \times 6^{-4}} = \frac{3^{-4} \times 5^{-5} \times 2^{-5} \times 5^{4}}{5^{-3} \times 3^{-4} \times 2^{-4}}$$

$$\Rightarrow \frac{3^{-4} \times 10^{-5} \times 625}{5^{-3} \times 6^{-4}} = \frac{5^{2}}{2} = \frac{25}{2}$$

1 C. Question

Use the laws of exponents and simplify.

$$\frac{2^{3^2}}{(2^3)^2}$$

Answer

$$\frac{2^{3^2}}{(2^3)^2} = \frac{2^9}{2^6}$$

$$\Rightarrow \frac{2^{3^2}}{(2^3)^2} = 2^3 = 8$$

2. Question

What is the value of $\frac{(10^3)^2 \times 10^{-4}}{10^2}$?

Answer

$$\frac{(10^3)^2 \times 10^{-4}}{10^2} = \frac{10^6 \times 10^{-4}}{10^2}$$

$$\Rightarrow \frac{(10^3)^2 \times 10^{-4}}{10^2} = \frac{10^2}{10^2} = 1$$

3. Question

Simplify:

$$\left(\frac{b^{-3} \cdot b^{7} \cdot (b^{-1})^{2}}{(-b)^{2} \cdot (b^{2})^{3}}\right)^{-2}.$$

Answer

$$\left(\frac{b^{-3} \times b^{7} \times (b^{-1})^{2}}{(-b)^{2} \times (b^{2})^{3}}\right)^{-2} = \left(\frac{b^{-3} \times b^{7} \times b^{-2}}{b^{2} \times b^{6}}\right)^{-2}$$

$$\Rightarrow \left(\frac{b^{-3} \times b^7 \times (b^{-1})^2}{(-b)^2 \times (b^2)^3}\right)^{-2} = \left(\frac{b^2}{b^8}\right)^{-2}$$

$$\Rightarrow \left(\frac{b^{-3} \times b^7 \times (b^{-1})^2}{(-b)^2 \times (b^2)^3}\right)^{-2} = \left(\frac{1}{b^6}\right)^{-2} = b^{12}$$

4 A. Question

Find the value of each of the following expressions:

$(3^2)^2 - ((-2)^3)^2 - (-(5^2))^2$;

Answer

$(3^2)^2 = 9^2 = 81$

$((-2)^3)^2 = (-8)^2 = 64$

$(-(5^2))^2 = (-25)^2 = 625$

So the value of the expression is

$(3^2)^2 - ((-2)^3)^2 - (-(5^2))^2 = 81-64-625 = -608$

4 B. Question

Find the value of each of the following expressions:

$((0.6)^2)^0 - ((4.5)^0)^{-2}$;

Answer

The value of any number raised to the power zero is always 1

$((0.6)^2)^0 = 1$

$(4.5)^0 = 1$

$\Rightarrow 1^{-2} = 1$

So the solution of the expression is

$((0.6)^2)^0 - ((4.5)^0)^{-2} = 1-1 = 0$

4 C. Question

Find the value of each of the following expressions:

$$(4^{-1})^4 \times 2^5 \times \left(\frac{1}{16}\right)^3 \times (8^{-2})^5 \times (64^2)^3;$$

Answer

$(4^{-1})^4 = 4^{-4}$

$\Rightarrow (4^{-1})^4 = 2^{-8}$

$\left(\frac{1}{16}\right)^3 = (2^{-4})^3$

$\left(\frac{1}{16}\right)^3 = 2^{-12}$

$(8^{-2})^5 = (2^{-6})^5$

$\Rightarrow (8^{-2})^5 = 2^{-30}$

$(64^2)^3 = (2^{12})^3$

$(64^2)^3 = 2^{36}$

$(4^{-1})^4 \times 2^5 \times \left(\frac{1}{16}\right)^3 \times (8^{-2})^5 \times (64^2)^3;$ $= 2^{-8} \times 2^5 \times 2^{-12} \times 2^{-30} \times 2^{36}$

$\Rightarrow (4^{-1})^4 \times 2^5 \times \left(\frac{1}{16}\right)^3 \times (8^{-2})^5 \times (64^2)^3;$ $= 2^{-9}$

Additional Problems 10

1 A. Question

The value of $(3^m)^n$, for every pair of integers (m,n), is

A. 3^{m+n}

B. 3^{mn}

C. 3^{mn}

D. $3^m + 3^n$

Answer

Using the third law of exponents, If a ≠0 is a number and m, n are integers, then$(a^m)^n = a^{mn}$

So, $(3^m)^n = 3^{mn}$

1 B. Question

If x,y, $2x+\frac{y}{2}$ are nonzero real numbers, then

$$\left(2x+\frac{y}{2}\right)^{-1}\left\{(2x)^{-1}+\left(\frac{y}{2}\right)^{-1}\right\}$$

equals

A. 1

B. x · y

C. x · y

D. 1/xy

Answer

$$\left(2x+\frac{y}{2}\right)^{-1}\{(2x)^{-1}+(\frac{y}{2})^{-1}\}$$

For a number a ≠ 0, we define

$$a^{-1}=\frac{1}{a}$$

It is given that x,y, $2x+\frac{y}{2}$ are nonzero real numbers,

$$\Rightarrow \frac{1}{(2x+\frac{y}{2})}\times(\frac{1}{2x}+\frac{2}{y})$$

$$\Rightarrow \frac{2}{4x+y}\times\left(\frac{y+4x}{2xy}\right)$$

$$\Rightarrow \frac{1}{xy}$$

1 C. Question

If $2^x - 2^{x-2} = 192$, the value of x is

A. 5

B. 6

C. 7

D. 8

Answer

Given $2^x - 2^{x-2} = 192$

$\Rightarrow 2^x - 2^x.2^{-2} = 192$

Taking 2^x common from both the terms,

$\Rightarrow 2^x (1 - 2^{-2}) = 192$

For a number a ≠ 0and natural number n, we define

$$a^{-n} = (\frac{1}{a})^n$$

$$\because 2^{-2} = (\frac{1}{2})^2 = \frac{1}{4}$$

$$\Rightarrow 2^x \left(1 - \frac{1}{4}\right) = 192$$

$$\Rightarrow 2^x \times \frac{3}{4} = 192$$

$$\Rightarrow 2^x = 192 \times \frac{4}{3}$$

$\Rightarrow 2^x = 64 \times 4$

$\Rightarrow 2^x = 2^6 \times 2^2$

For any a ≠ 0, and integers m,n, $a^m \times a^n = a^{m+n}$

$\Rightarrow 2^x = 2^8$

Hence, x = 8

1 D. Question

The number $\left(6^{\left(6^6\right)}\right)^{1/6}$ is equal to

A. 6^6

B. 6^{6^6-1}

C. $6^{\left(6^5\right)}$

D. $6^{\left(5^6\right)}$

Answer

$(6^{(6^6)})^{\frac{1}{6}}$

Using the third law of exponents, If a ≠0 is a number and m, n are integers, then $(a^m)^n = a^{mn}$

$\Rightarrow (6^{(6^6)})^{\frac{1}{6}} = (6)^{6^6 \times \frac{1}{6}}$

For a number a ≠ 0, we define

$a^{-1} = \frac{1}{a}$

$\Rightarrow (6)^{6^6 \times \frac{1}{6}} = (6)^{6^6 \times 6^{-1}}$

For any a ≠ 0, and integers m,n, $a^m \times a^n = a^{m+n}$

$\Rightarrow (6)^{6^6 \times 6^{-1}} = (6)^{6^{6-1}} = (6)^{6^5}$

1 E. Question

The number of pairs positive integers (m,n)such that $m^n = 25$ is

A. 0

B. 1

C. 2

D. more than 2

Answer

Given $m^n = 25$

$\Rightarrow m^n = 5^2$

⇒ (m, n) = (5, 2)

Hence, only 1 such pair of positive integers is possible.

2. Question

The diameter of the Sun is 1.4×10^9 meters and that of the Earth is about 1.2768×10^7 meters. Find the approximate ratio of the diameter of the Sun to that of the Earth.

Answer

Given the diameter of the Sun = 1.4×10^9 meter

And the diameter of the Earth is = 1.2768×10^7 meters

Ratio of the diameter of the sun and that of the earth = $\frac{1.4 \times 10^9}{1.2768 \times 10^7}$

For any number a ≠ 0 and positive integers m and n, not necessarily distinct,

$$\frac{a^m}{a^n} = a^{m-n}$$

Ratio of the diameter of the sun and that of the earth = $\frac{1.4 \times 10^{9-7}}{1.2768}$

Ratio of the diameter of the sun and that of the earth =
$\frac{1.4 \times 10^2}{1.2768} = \frac{140}{1.2768} = 109.65$

3 A. Question

Find the value of each of the following expressions:

$$(-0.75)^3 + (0.3)^{-3} - \left(-\frac{3}{2}\right)^{-3};$$

Answer

$$(-0.75)^3 + (0.3)^{-3} - (-\frac{3}{2})^{-3}$$

$$= (-0.75 \times -0.75 \times -0.75) + (0.3 \times 0.3 \times 0.3)^{-1} - (-\frac{2}{3})^3$$

{Using For a number a ≠ 0, we define

$a^{-1} = \frac{1}{a}$}

$$\Rightarrow (-0.75)^3 + (0.3)^{-3} - (-\frac{3}{2})^{-3} = -0.421875 + (0.027)^{-1} - (\frac{-8}{27})$$

$$\Rightarrow (-0.75)^3 + (0.3)^{-3} - (-\frac{3}{2})^{-3} = -0.421875 + \frac{1}{0.027} + \frac{8}{27}$$

$$\Rightarrow (-0.75)^3 + (0.3)^{-3} - (-\frac{3}{2})^{-3} = -0.421875 + \frac{1000}{27} + \frac{8}{27}$$

$$\Rightarrow (-0.75)^3 + (0.3)^{-3} - (-\frac{3}{2})^{-3} = -0.421875 + \frac{1008}{27}$$

$$\Rightarrow (-0.75)^3 + (0.3)^{-3} - (-\frac{3}{2})^{-3} = -0.421875 + 37.333333$$

$$\Rightarrow (-0.75)^3 + (0.3)^{-3} - (-\frac{3}{2})^{-3} = 36.91$$

3 B. Question

Find the value of each of the following expressions:

$$\frac{\left(8\times\left(4^{2}\right)^{4}\times3^{3}\times27^{2}\right)+\left(9\times6^{3}\times4^{7}\times\left(3^{2}\right)^{3}\right)}{\left(24\times\left(6^{2}\right)^{4}\times\left(2^{4}\right)^{2}\right)+\left(144+\left(2^{3}\right)^{4}\times\left(9^{2}\right)^{2}\times4^{2}\right)}$$

Answer

Using the third law of exponents, If a ≠0 is a number and m, n are integers, then $(a^m)^n = a^{mn}$

$$\frac{(8\times(4^2)^4\times3^3\times27^2)+(9\times6^3\times4^7\times(3^2)^3)}{(24\times(6^2)^4\times(2^4)^2)+(144\times(2^3)^4\times(9^2)^2\times4^2)}$$

$$=\frac{\left(2^3\times(4)^8\times3^3\times3^{3^2}\right)+(3^2\times(2.3)^3\times2^{2^7}\times(3)^6)}{(8\times3\times(6)^8\times(2)^8)+(12^2\times(2)^{12}\times(3^2)^4\times2^{2^2})}$$

Using For a ≠0 and b ≠ 0, and integer m,

$(a \times b)^m = a^m \times b^m$

$$\Rightarrow\frac{(8\times(4^2)^4\times3^3\times27^2)+(9\times6^3\times4^7\times(3^2)^3)}{(24\times(6^2)^4\times(2^4)^2)+(144\times(2^3)^4\times(9^2)^2\times4^2)}$$

$$=\frac{(2^3\times(2^2)^8\times3^3\times3^6)+(3^2\times(2)^3\times3^3\times2^{14}\times(3)^6)}{(2^3\times3\times(2.3)^8\times(2)^8)+((4.3)^2\times(2)^{12}\times(3)^8\times2^4)}$$

$$\Rightarrow\frac{(8\times(4^2)^4\times3^3\times27^2)+(9\times6^3\times4^7\times(3^2)^3)}{(24\times(6^2)^4\times(2^4)^2)+(144\times(2^3)^4\times(9^2)^2\times4^2)}$$

$$=\frac{(2^3\times(2)^{16}\times3^3\times3^6)+(3^2\times(2)^3\times3^3\times2^{14}\times(3)^6)}{(2^3\times3\times(2)^8\times3^8\times(2)^8)+(2^4\times(3)^2\times(2)^{12}\times(3)^8\times2^4)}$$

Using For any a ≠ 0, and integers m,n,

$a^m \times a^n = a^{m+n}$

$$\Rightarrow\frac{(8\times(4^2)^4\times3^3\times27^2)+(9\times6^3\times4^7\times(3^2)^3)}{(24\times(6^2)^4\times(2^4)^2)+(144\times(2^3)^4\times(9^2)^2\times4^2)}$$

$$=\frac{(2^{3+16}\times3^{3+6})+(3^{2+3+6}\times(2)^{3+14})}{(2^{3+8+8}\times3^{8+1})+((2)^{12+4+4}\times(3)^{8+2})}$$

$$\Rightarrow\frac{(8\times(4^2)^4\times3^3\times27^2)+(9\times6^3\times4^7\times(3^2)^3)}{(24\times(6^2)^4\times(2^4)^2)+(144\times(2^3)^4\times(9^2)^2\times4^2)}$$

$$= \frac{(2^{19} \times 3^9) + (3^{11} \times (2)^{17})}{(2^{19} \times 3^9) + ((2)^{20} \times (3)^{10})}$$

$$\Rightarrow \frac{(8 \times (4^2)^4 \times 3^3 \times 27^2) + (9 \times 6^3 \times 4^7 \times (3^2)^3)}{(24 \times (6^2)^4 \times (2^4)^2) + (144 \times (2^3)^4 \times (9^2)^2 \times 4^2)}$$

$$= \frac{(2)^{17}(2^2 \times 3^9) + 3^9(3^2 \times (2)^{17})}{(2)^{17}(2^2 \times 3^9) + (2)^{17} \times 3^9((2)^3 \times 3)}$$

$$\Rightarrow \frac{(8 \times (4^2)^4 \times 3^3 \times 27^2) + (9 \times 6^3 \times 4^7 \times (3^2)^3)}{(24 \times (6^2)^4 \times (2^4)^2) + (144 \times (2^3)^4 \times (9^2)^2 \times 4^2)} = \frac{4+9}{4+8 \times 3}$$

$$\Rightarrow \frac{(8 \times (4^2)^4 \times 3^3 \times 27^2) + (9 \times 6^3 \times 4^7 \times (3^2)^3)}{(24 \times (6^2)^4 \times (2^4)^2) + (144 \times (2^3)^4 \times (9^2)^2 \times 4^2)} = \frac{13}{4+8 \times 3}$$

$$\Rightarrow \frac{(8 \times (4^2)^4 \times 3^3 \times 27^2) + (9 \times 6^3 \times 4^7 \times (3^2)^3)}{(24 \times (6^2)^4 \times (2^4)^2) + (144 \times (2^3)^4 \times (9^2)^2 \times 4^2)} = \frac{13}{28}$$

3 C. Question

Find the value of each of the following expressions:

$$\frac{\left(2^{19} \times 27^3\right) + \left(15 \times 4^9 \times 9^4\right)}{\left(6^9 \times 2^{10}\right) + 12^{10}}$$

Answer

$$\frac{(2^{19} \times 27^3) + (15 \times 4^9 \times 9^4)}{(6^9 \times 2^{10}) + 12^{10}} = \frac{\left(2^{19} \times {3^3}^3\right) + (3 \times 5 \times {2^2}^9 \times {3^2}^4)}{((2 \times 3)^9 \times 2^{10}) + (3 \times 2^2)^{10}}$$

Using For a ≠0 and b ≠ 0, and integer m,

$(a \times b)^m = a^m \times b^m$

$$\Rightarrow \frac{(2^{19} \times 27^3) + (15 \times 4^9 \times 9^4)}{(6^9 \times 2^{10}) + 12^{10}} = \frac{(2^{19} \times 3^9) + (3 \times 5 \times 2^{18} \times 3^8)}{(2^9 \times 3^9 \times 2^{10}) + (3^{10} \times 2^{20})}$$

Using For any a ≠ 0, and integers m, n,

$a^m \times a^n = a^{m+n}$

$$\Rightarrow \frac{(2^{19} \times 27^3) + (15 \times 4^9 \times 9^4)}{(6^9 \times 2^{10}) + 12^{10}} = \frac{(2^{19} \times 3^9) + (5 \times 2^{18} \times 3^9)}{(2^{19} \times 3^9) + (3^{10} \times 2^{20})}$$

$$\Rightarrow \frac{(2^{19} \times 27^3) + (15 \times 4^9 \times 9^4)}{(6^9 \times 2^{10}) + 12^{10}} = \frac{2^{18} \times 3^9(2) + 2^{18} \times 3^9(5)}{2^{18} \times 3^9(2) + 2^{18} \times 3^9(3 \times 2^2)}$$

$$\Rightarrow \frac{(2^{19} \times 27^3) + (15 \times 4^9 \times 9^4)}{(6^9 \times 2^{10}) + 12^{10}} = \frac{2+5}{2+12}$$

$$\Rightarrow \frac{(2^{19} \times 27^3) + (15 \times 4^9 \times 9^4)}{(6^9 \times 2^{10}) + 12^{10}} = \frac{7}{14} \Rightarrow \frac{(2^{19} \times 27^3) + (15 \times 4^9 \times 9^4)}{(6^9 \times 2^{10}) + 12^{10}} = \frac{1}{2}$$

4. Question

How many digits are there in the number $2^3 \times 5^4 \times 20^5$?

Answer

$2^3 \times 5^4 \times 20^5 = 2^3 \times 5^4 \times (4\times 5)^5$

Using For a ≠0 and b ≠ 0, and integer m,

$(a \times b)^m = a^m \times b^m$

$\Rightarrow 2^3 \times 5^4 \times 20^5 = 2^3 \times 5^4 \times 4^5 \times 5^5$

$\Rightarrow 2^3 \times 5^4 \times 20^5 = 2^3 \times 5^4 \times (2^2)^5 \times 5^5$

Using third law of exponents, If a ≠0 is a number and m, n are integers, then $(a^m)^n = a^{mn}$

$\Rightarrow 2^3 \times 5^4 \times 20^5 = 2^3 \times 5^4 \times 2^{10} \times 5^5$

Using For any a ≠ 0, and integers m, n,

$a^m \times a^n = a^{m+n}$

$\Rightarrow 2^3 \times 5^4 \times 20^5 = 2^{3+10} \times 5^{4+5}$

$\Rightarrow 2^3 \times 5^4 \times 20^5 = 2^{13} \times 5^9$

We know that $2\times 5 = 10$

So, $2^3 \times 5^4 \times 20^5 = (2^9 \times 5^9)\times 2^4$

$\Rightarrow 2^3 \times 5^4 \times 20^5 = 10^9 \times 16$

$\Rightarrow 2^3 \times 5^4 \times 20^5 = 16000000000$

Hence, there are 11 digits in the given number.

5. Question

If $a^7 = 3$, find the value of $\frac{(a^{-2})^{-3} \times (a^3)^4 \times (a^{-17})^{-1}}{a^7}$.

Answer

Given: $a^7 = 3$

$$\frac{(a^{-2})^{-3} \times (a^3)^4 \times (a^{-17})^{-1}}{a^7} = \frac{(a^6) \times (a^{12}) \times (a^{17})}{a^7}$$

{Using the third law of exponents, If a ≠0 is a number and m, n are integers, then $(a^m)^n = a^{mn}$}

$$\Rightarrow \frac{(a^{-2})^{-3} \times (a^3)^4 \times (a^{-17})^{-1}}{a^7} = \frac{(a^{6+12+17})}{a^7}$$

{UsingFor any a ≠ 0, and integers m, n,

$a^m \times a^n = a^{m+n}$}

$$\Rightarrow \frac{(a^{-2})^{-3} \times (a^3)^4 \times (a^{-17})^{-1}}{a^7} = \frac{(a^{35})}{a^7}$$

For any number a ≠ 0 and positive integers m and n, not necessarily distinct,

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\Rightarrow \frac{(a^{-2})^{-3} \times (a^3)^4 \times (a^{-17})^{-1}}{a^7} = a^{35-7}$$

$$\Rightarrow \frac{(a^{-2})^{-3} \times (a^3)^4 \times (a^{-17})^{-1}}{a^7} = a^{28}$$

$$\Rightarrow \frac{(a^{-2})^{-3} \times (a^3)^4 \times (a^{-17})^{-1}}{a^7} = (a^7)^4$$

$$\Rightarrow \frac{(a^{-2})^{-3} \times (a^3)^4 \times (a^{-17})^{-1}}{a^7} = (3)^4 = 81$$

6. Question

If $2^m \times a^2 = 2^8$, where a,m are positive integers, find all possible values of a + m.

Answer

Given: $2^m \times a^2 = 2^8$

$\Rightarrow 2^m \times a^2 = 2^8 \times 1^2$

Then a + m = 8 + 1 = 9

Also, $2^m \times a^2 = 2^4 \times 4^2$

Then a + m = 4 +4 = 8

Similarly, $2^m \times a^2 = 2^2 \times 8^2$

Then a + m = 2 +8 = 10

7. Question

Suppose $3^k \times b^2 = 6^4$ for some positive integers k, b. Find all possible values of k + b.

Answer

Given: $3^k \times b^2 = 6^4$

$\Rightarrow 3^k \times b^2 = (2 \times 3)^4$

Using For a ≠0 and b ≠ 0, and integer m,

$(a \times b)^m = a^m \times b^m$

$\Rightarrow 3^k \times b^2 = 2^4 \times 3^4$

$\Rightarrow 3^k \times b^2 = 4^2 \times 3^4$

On comparing,

k = 4, b = 4

⇒ k+b = 8

8. Question

Find the value of $\frac{(625)^{6.25} \times (25)^{2.60}}{(625)^{7.25} \times (5)^{1.20}}$.

Answer

$$\frac{625^{6.25} \times 25^{2.60}}{625^{7.25} \times 5^{1.20}} = \frac{(5^4)^{6.25} \times (5^2)^{2.60}}{(5^4)^{7.25} \times 5^{1.20}}$$

Using the third law of exponents, If a ≠0 is a number and m, n are integers, then $(a^m)^n = a^{mn}$

$$\Rightarrow \frac{625^{6.25} \times 25^{2.60}}{625^{7.25} \times 5^{1.20}} = \frac{5^{25} \times 5^{5.2}}{5^{29} \times 5^{1.20}}$$

Using For any a ≠ 0, and integers m, n,

$a^m \times a^n = a^{m+n}$

$$\Rightarrow \frac{625^{6.25} \times 25^{2.60}}{625^{7.25} \times 5^{1.20}} = \frac{5^{25+5.2}}{5^{29+1.2}}$$

$$\Rightarrow \frac{625^{6.25} \times 25^{2.60}}{625^{7.25} \times 5^{1.20}} = \frac{5^{30.2}}{5^{30.2}}$$

$$\Rightarrow \frac{625^{6.25} \times 25^{2.60}}{625^{7.25} \times 5^{1.20}} = 1$$

9. Question

A person had some rupees which is a power of 5. He gave a part of it to his friend which is also a power of 5. He was left with ₹ 500. How much did money he have?

Answer

Let the money he have be Rs 5^x and that he gave to his friend be Rs 5^y, such that x>y.

According to the question,

$5^x - 5^y = 500$

We can see from the equation that $5^x > 500$ because 500 is the money that he is left with.

So $5^x = 625$

$\Rightarrow x = 4$

Then $625 - 5^y = 500$

$\Rightarrow 5^y = 125$

$\Rightarrow y = 3$

So, the money he had = Rs 625 and that he gave to his friend = Rs 125