LONG ANSWER QUESTIONS [4 Marks]

Que 1. Prove that if in two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, then two triangles are congruent.

Sol. Given: two triangles ABC and DEF

Such that $\angle B = \angle E, \angle C = \angle F$ and $BC = EF$.

To prove: $\Delta ABC \cong \Delta \mathrm{DEF}$

Proof: For proving the congruence of two triangles, three cases arise.

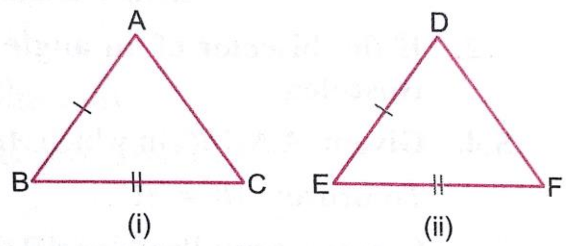

Fig. 7.26

Case I: When $AB = DE$

In this case

$AB = DE$ and $\angle B = \angle E$

$BC = EF$

$\therefore$ $\Delta \mathrm{ABC} \cong \Delta \mathrm{DEF}$ (SAS congruence criterion)

Case II: When $AB < ED$

In this case, take a point P on ED such that $PE = AB$. Join FP.

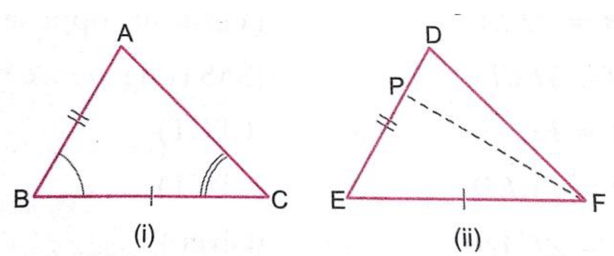

Fig. 7.27

In triangles ABC and PEF, we have

$AB = PE$ (By supposition)

$\angle B = \angle E$ (Given)

And $BC = EF$ (Given)

∴ ΔABC ≅ ΔPEF (SAS criterion of congruence)

⟹ ∠ACB = ∠PFE (CPCT)

But ∠ACB = ∠DFE (Given)

∴ ∠PFE = ∠DFE

This is possible only when P and D coincide.
Therefore, AB must be equal to DE.
Thus, in triangle ABC and DEF, we have

AB = DE (Proved above)
∠B = ∠ (Given)
and BC = EF (Given)
∴ ΔABC ≅ ΔDEF (SAS congruence criterion)

Case III: When AB > ED

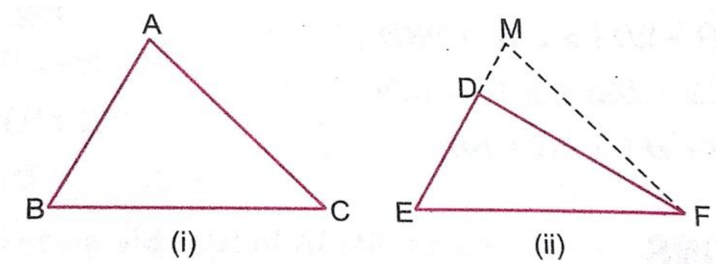

Fig. 7.28

In this case, take a point M on ED produced such that ME = AB. Join FM. Now, repeating the arguments as given in case (II), we can conclude that AB = DE and

So, ΔABC ≅ ΔDEF
Hence, in all the three cases, we have
ΔABC ≅ ΔDEF

Que 2. If the bisector of an angle of a triangle bisects the opposite side, prove that the triangle is isosceles.

Sol. Given: A ΔABC in which AD is the bisector of ∠A which meets BC in D such that BD = DC
To prove: AB = AC
Construction: Produce AD to E such that AD = DE and then join CE.
Proof: In ΔABD and ΔECD, we have

BD = CD (Given)
AD = ED (By construction)

and $\angle ADB = \angle EDC$ (Vertically opposite angles)
Therefore, $\Delta ABD \cong \Delta ECD$ (SAS congruence criterion)
So, $AB = EC$ (CPCT) …..(i)
and $\angle BAD = \angle CED$ (CPCT) ……(ii)
Also, $\angle BAD = \angle CAD$ (Given) ……(iii)

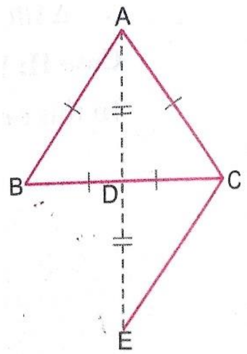

Fig. 7.29

Therefore, from (ii) and (iii)

$\angle CAD = \angle CED$

So, $AC = EC$ (Sides opposite to equal angles) …..(iv)

From (i) and (iv), we get

$AB = AC$

Que 3. In Fig. 7.30, two sides AB and BC and median AM of two triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR. Show that ΔABC ≅ ΔPQR.

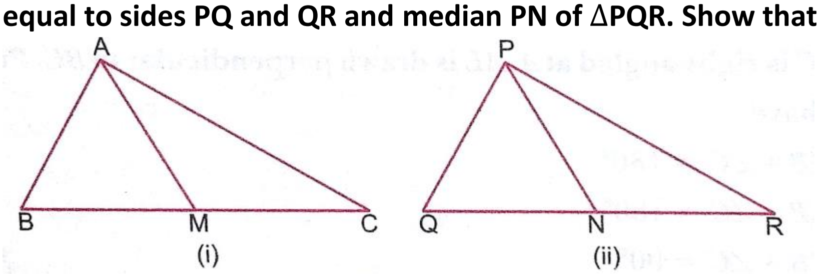

Fig. 7.30

Sol. In ΔABC and ΔPQR,

$BC = QR$ (Given)

$\Rightarrow \quad \frac{1}{2}BC = \frac{1}{2}QR$

$\Rightarrow \quad BM = QN$ In triangle ABM and PQN, we have

$AB = PQ$ (Given)

$BM = QN$ (Proved above)

$AM = PN$ (Given)

∴ $\Delta ABM \cong \Delta PQN$ (SSS congruence criterion)
⇒ $\angle B = \angle Q$ (CPCT)

Now, in triangle ABC and PQR, we have

$\angle B = \angle Q$ (Proved above)
$BC = QR$ (Given)
∴ $\Delta ABC \cong \Delta PQR$ (SAS congruence criterion)

Que 4. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB. Show that ∠BCD is a right angle.

Sol.

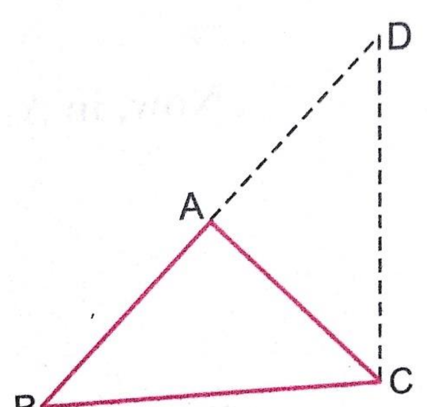

Fig. 7.31

Given: A ΔABC in which AB = AC, side BA is produced to D such that
AD = AB

Construction: Join CD
To prove: ∠BCD, we have
Proof: In ΔABC, we have

$AB = AC$ (Given)
∴ $\angle ACB = \angle ABC$ (Angles opposite to equal sides) …. (i)

Also, $AB = AD \Rightarrow AC = AD$
In ΔADC, we have AD = AC
⇒ $\angle ACD = \angle ADC$ (Angles opposite to equal sides) …..(ii)

Adding (i) and (ii), we get

$\angle ACB + \angle ACD = \angle ABC + \angle ADC$
$\angle BCD = \angle ABC + \angle BDC$

Adding $\angle BCD$ on both sides

$\angle BCD + BCD = \angle ABC + \angle BDC + \angle BCD$

$\Rightarrow \quad 2\angle BCD = 180^\circ \quad \Rightarrow \angle BCD = 90^\circ$

Hence, $\angle BCD$ is a right angle

Que 5. A triangle ABC is right-angled at A. AL is drawn perpendicular to BC. Prove that $\angle BAL = \angle ACB$.

Sol.

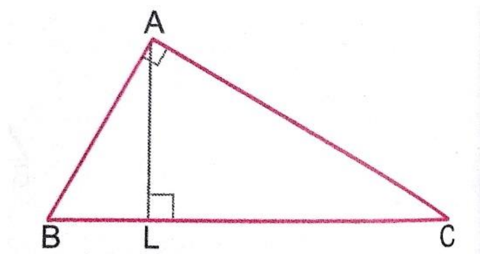

Fig. 7.32

In ΔABC, we have

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \quad 90^\circ + \angle B + \angle C = 180^\circ$

$\Rightarrow \quad \angle B + \angle C = 90^\circ$

$\Rightarrow \quad \angle C = 90^\circ - \angle B \quad$ ....(i)

In ΔABL, we have

$\angle ALB + \angle BAL + \angle B = 180^\circ$

$\Rightarrow \quad 90^\circ + \angle BAL + \angle B = 180^\circ \quad \Rightarrow \angle BAL + \angle B = 90^\circ$

$\angle BAL = 90^\circ - \angle B \quad$ ....(ii)

From (i) and (ii), we get

$\angle BAL = \angle ACB$

Que 6. ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC, AB = AC and DB = DC. Show that AD is the perpendicular bisector of BC.

Sol.

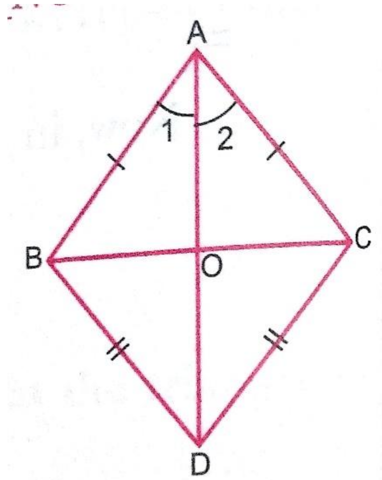

Fig. 7.33

Let AD intersect BC at O

Then we have to prove $\angle AOB = \angle AOC = 90^\circ$

and BO = OC

In ΔABD and ΔACD

	AB = AC	(Given)
	AD = DA	(Common)
	BD = DC	(Given)
∴	$\Delta ABD \cong \Delta ACD$	(By SSS congruence)
⇒	$\angle 1 = \angle 2$	

Now, in ΔAOB and ΔAOC

	AB = AC	(Given)
	AO = OA	(Common)
	$\angle 1 = \angle 2$	(Proved above)
∴	$\Delta AOB \cong \Delta AOC$	(By SAS congruence)
⇒	BO = OC and $\angle AOC$	

But $\angle AOB + \angle AOC = 180^\circ$ (Linear Pair)

⇒ $\angle AOB + \angle AOB = 180^\circ$

⇒ $\angle AOB = 90^\circ$

Hence, $AD \perp BC$ and AD bisects BC, i.e., AD is the perpendicular bisector of BC.

Que 7. ΔABC is a right triangle such that AB = AC and bisector of angle C intersects the side AB at D. Prove that AC + AD = BC.

Sol.

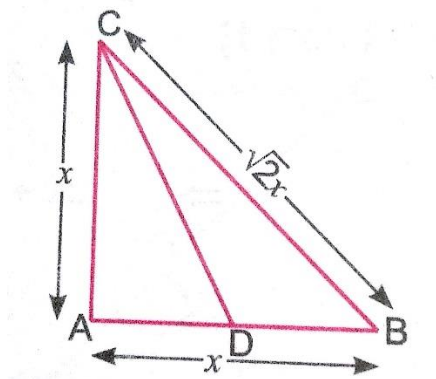

Fig. 7.34

Let AB = AC = X
By Pythagoras theorem

$$BC = \sqrt{AB^2 + AC^2} = \sqrt{x^2 + x^2} \Rightarrow BC = \sqrt{2}x$$

Again by Bisector theorem

$$\frac{AC}{BC} = \frac{AD}{BD} \quad \Rightarrow \quad \frac{BC}{AC} = \frac{BD}{AD}$$

$$\Rightarrow \frac{BC}{AC} + 1 = \frac{BD}{AD} + 1 \quad \Rightarrow \quad \frac{BC + AC}{AC} = \frac{BD + AD}{AD}$$

$$\Rightarrow \frac{BC + AC}{AC} = \frac{AB}{AD} \quad \Rightarrow \quad \frac{\sqrt{2}x + x}{x} = \frac{x}{AD}$$

$$\Rightarrow \frac{\sqrt{2} + 1}{1} = \frac{x}{AD} \quad \Rightarrow \quad AD = \frac{x}{\sqrt{2} + 1}$$

$$\therefore AC + AD = x + \frac{x}{\sqrt{2} + 1} = \frac{\sqrt{2}x + x + x}{\sqrt{2} + 1} = \frac{\sqrt{2}x + 2x}{\sqrt{2} + 1} = \frac{\sqrt{2}x(1 + \sqrt{2})}{(\sqrt{2} + 1} = \sqrt{2}x = BC$$