SUPPLEMENTARY MATERIAL

CHAPTER 5

Theorem 5 (To be on page 173 under the heading Theorem 5)

(i) Derivative of Exponential Function $f(x) = e^x$.

If $f(x) = e^x$, then

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{e^{x+\Delta x} - e^x}{\Delta x}$$

$$= e^x \cdot \lim_{\Delta x \to 0} \frac{e^{\Delta x} - 1}{\Delta x}$$

$$= e^x \cdot 1 \quad \left[\text{since } \lim_{h \to 0} \frac{e^h - 1}{h} = 1\right]$$

Thus, $\frac{d}{dx}(e^x) = e^x$.

(ii) Derivative of logarithmic function $f(x) = \log_e x$.

If $f(x) = \log_e x$, then

$$f'(x) = \lim_{\Delta x \to 0} \frac{\log_e(x + \Delta x) - \log_e x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\log_e\left(1 + \frac{\Delta x}{x}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{x} \frac{\log_e\left(1 + \frac{\Delta x}{x}\right)}{\frac{\Delta x}{x}}$$

$$= \frac{1}{x} \quad \left[\text{since } \lim_{h \to 0} \frac{\log_e(1 + h)}{h} = 1\right]$$

Thus, $\frac{d}{dx}\log_e x = \frac{1}{x}$.