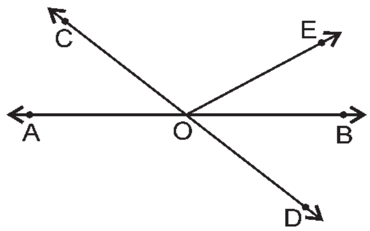
6 Lines and Angles

EXERCISE 6.1

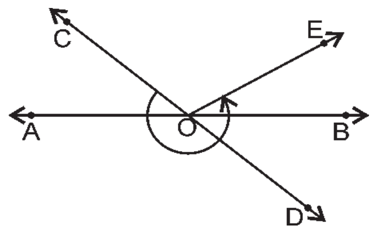
Q.1. *In the figure lines AB and CD intersect at O. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.*



Sol. Lines AB and CD intersect at O.

∠AOC + ∠BOE = 70° (Given) ...(1)

∠BOD = 40° (Given) ...(2)



Since, ∠AOC = ∠BOD

(Vertically opposite angles)

Therefore, ∠AOC = 40° [From (2)]

and 40° + ∠BOE = 70° [From (1)]

⇒ ∠BOE = 70° – 40° = 30°

Also, ∠AOC + ∠BOE + ∠COE = 180° (∵ AOB is a straight line)

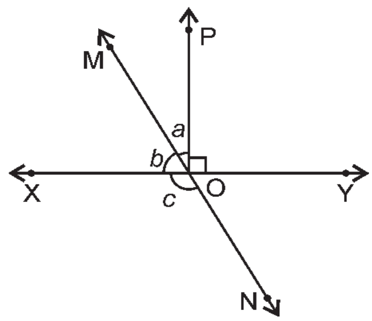
⇒ 70° + ∠COE = 180° [Form (1)]

⇒ ∠COE = 180° – 70° = 110°

Now, reflex ∠COE = 360° – 110° = 250°

Hence, **∠BOE = 30°** and **reflex ∠COE = 250°** **Ans.**

Q.2. *In the figure, lines XY and MN intersect at O. If ∠POY = 90° and a : b = 2 : 3, find c.*



Sol. In the figure, lines XY and MN intersect at O and ∠POY = 90°.

Also, given $a : b = 2 : 3$

Let $a = 2x$ and $b = 3x$.

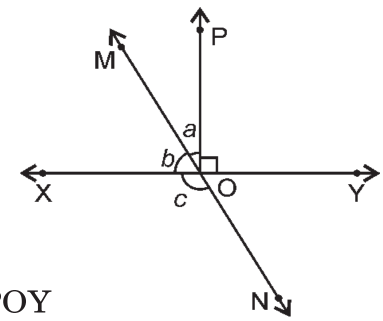
Since, ∠XOM + ∠POM + ∠POY = 180°

(Linear pair axiom)

⇒ $3x + 2x + 90° = 180°$

⇒ $5x = 180° - 90°$

⇒ $x = \frac{90°}{5} = 18°$



∴ ∠XOM = $b = 3x = 3 \times 18° = 54°$

and ∠POM = $a = 2x = 2 \times 18° = 36°$

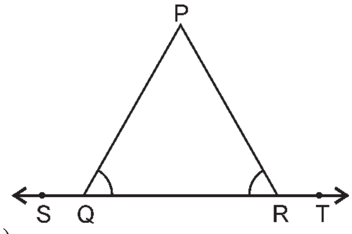
Now, ∠XON = c = ∠MOY = ∠POM + ∠POY

(Vertically opposite angles)

= 36° + 90° = 126°

Hence, **c = 126°** **Ans.**

Q.3. *In the figure, ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT.*

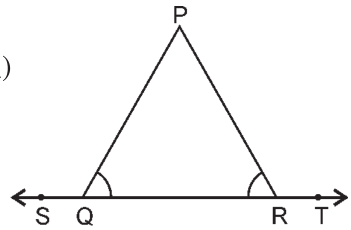


Sol. $\angle PQS + \angle PQR = 180°$ ...(1)

(Linear pair axiom)

$\angle PRQ + \angle PRT = 180°$ ...(2)

(Linear pair axiom)

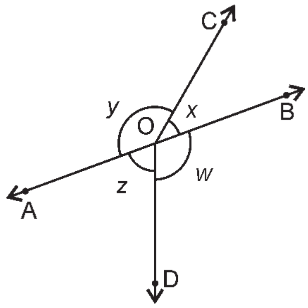


But, $\angle PQR = \angle PRQ$ (Given)

∴ From (1) and (2)

$\angle PQS = \angle PRT$ **Proved.**

Q.4. *In the figure, if x + y = w + z, then prove that AOB is a line.*



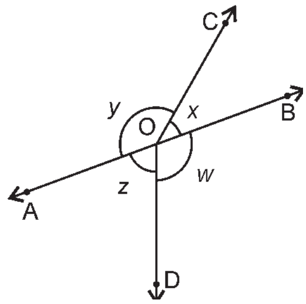
Sol. Assume AOB is a line.

Therefore, $x + y = 180°$ ...(1)

[Linear pair axiom]

$w + z = 180°$ ...(2)

[Linear pair axiom]



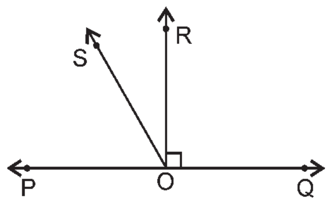
Now, from (1) and (2)

$x + y = w + z$

Hence, our assumption is correct, AOB is a line **Proved.**

Q.5. *In the figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that*

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$

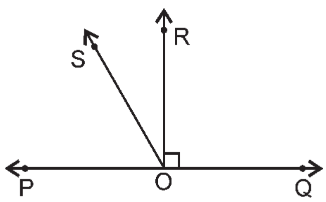


Sol. $\angle ROS = \angle ROP - \angle POS$ ...(1)

and $\angle ROS = \angle QOS - \angle QOR$ ...(2)

Adding (1) and (2),

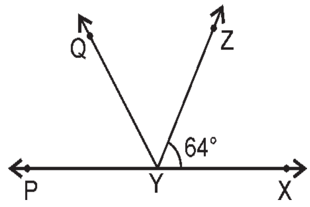
$\angle ROS + \angle ROS = \angle QOS - \angle QOR + \angle ROP - \angle POS$



$\Rightarrow$ $2\angle ROS = \angle QOS - \angle POS$ ($\because$ $\angle QOR = \angle ROP = 90°$)

$\Rightarrow$ $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$ **Proved.**

Q.6. *It is given that ∠XYZ = 64° and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ∠ZYP, find ∠XYQ and reflex ∠QYP.*



Sol. From figure,

$\angle XYZ = 64°$ (Given)

Now, $\angle ZYP + \angle XYZ = 180°$

(Linear pair axiom)

$\Rightarrow \angle ZYP + 64° = 180°$

$\Rightarrow \angle ZYP = 180° - 64° - 116°$

Also, given that ray YQ bisects ∠ZYP.

But, $\angle ZYP = \angle QYP \; \angle QYZ = 116°$

Therefore, $\angle QYP = 58°$ and $\angle QYZ = 58°$

Also, $\angle XYQ = \angle XYZ + \angle QYZ$

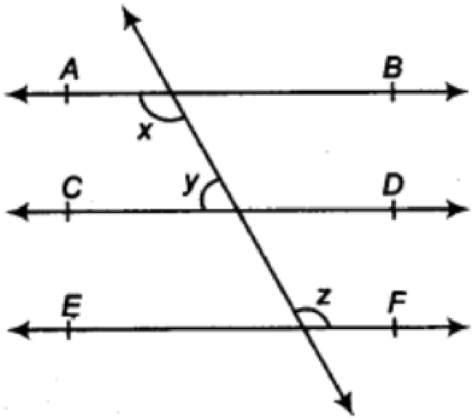
$\Rightarrow \angle XYQ = 64° + 58° = 122°$

and reflex $\angle QYP = 360° - \angle QYP = 360° - 58° = 302°$ ($\because \angle QYP = 58°$)

Hence, **∠XYQ = 122° and reflex ∠QYP = 302° Ans.**

EXERCISE 6.2

Q.1 In figure, if AB || CD, CD || EF and y : z = 3 : 7, find x.



Solution:

AB || CD, and CD || EF [Given]

∴ AB || EF

∴ x = z [Alternate interior angles] ....(1)

Again, AB || CD

⇒ x + y = 180° [Co-interior angles]

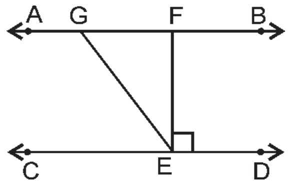
⇒ z + y = 180° ... (2) [By (1)]

But y : z = 3 : 7

$z = \frac{7}{3} y = \frac{7}{3}(180° - z)$ [By (2)]

⇒ 10z = 7 x 180°

Q.2. *In the figure, if AB || CD, EF ⊥ CD and ∠GED = 126°. Find ∠ AGE, ∠GEF and ∠FGE.*



Sol. In the given figure, AB || CD, EF ⊥ CD and ∠GED = 126°

∠AGE = ∠LGE (Alternate angle)

∴ ∠AGE = 126°

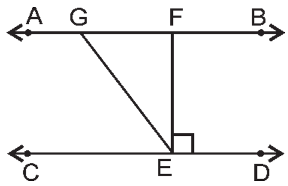
Now, ∠GEF = ∠GED – ∠DEF

= 126° – 90° = 36° (∵ ∠DEF = 90°)

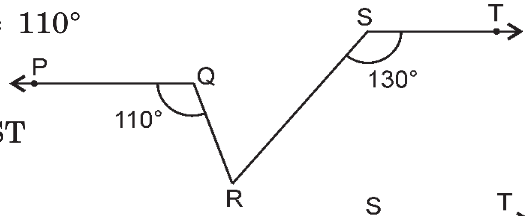
Also, ∠AGE + ∠FGE = 180° (Linear pair axiom)

⇒126° + FGE = 180°

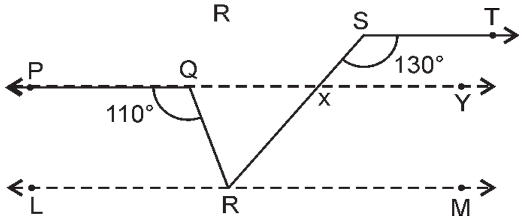
⇒ ∠FGE = 180° – 126° = 54°



Q.3. *In the figure, if PQ || ST, ∠PQR = 110° and ∠ RST = 130°, find ∠QRS.*



Sol. Extend PQ to Y and draw LM || ST through R.



∠TSX = ∠QXS [Alternate angles]

⇒ ∠QXS = 130°

∠QXS + ∠RXQ = 180° [Linear pair axiom]

⇒ ∠RXQ = 180° – 130° = 50° ...(1)

∠PQR = ∠QRM [Alternate angles]

⇒ ∠QRM = 110° ...(2)

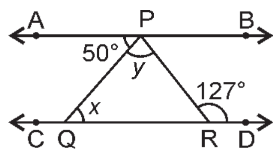
∠RXQ = ∠XRM [Alternate angles]

⇒ ∠XRM = 50° [By (1)]

∠QRS = ∠QRM – ∠XRM

= 110° – 50° = **60°** **Ans.**

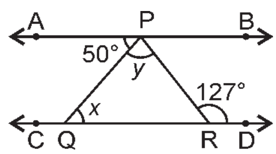
Q.4. *In the figure, if AB || CD, ∠APQ = 50° and ∠PRD = 127°, find x* and *y*.



Sol. In the given figure, AB || CD, ∠APQ = 50° and ∠PRD = 127°

∠APQ + ∠PQC = 180°

[Pair of consecutive interior angles are supplementary]



⇒ 50° + ∠PQC = 180°

⇒ ∠PQC = 180° – 50° = 130°

Now, ∠PQC + ∠PQR = 180° [Linear pair axiom]

⇒ 130° + x = 180°

⇒ x = 180° – 130° = 50°

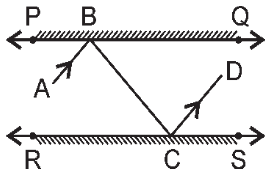
Also, $x + y$ = 127° [Exterior angle of a triangle is equal to the sum of the two interior opposite angles]

⇒ 50° + y = 127°

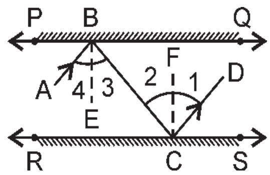
⇒ y = 127° – 50° = 77°

Hence, ***x* = 50° and *y* = 77°** **Ans.**

Q.5. *In the figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.*



Sol. At point B, draw BE ⊥ PQ and at point C, draw CF ⊥ RS.



$\angle 1 = \angle 2$ ...(i)

(Angle of incidence is equal to angle of reflection)

$\angle 3 = \angle 4$ ...(ii) [Same reason]

Also, $\angle 2 = \angle 3$ ... (iii) [Alternate angles]

$\Rightarrow \angle 1 = \angle 4$ [From (i), (ii), and (iii)]

$\Rightarrow 2\angle 1 = 2\angle 4$

$\Rightarrow \angle 1 + \angle 1 = \angle 4 + \angle 4$

$\Rightarrow \angle 1 + \angle 2 = \angle 3 + \angle 4$ [From (i) and (ii)]

$\Rightarrow \angle BCD = \angle ABC$

Hence, AB || CD. [Alternate angles are equal] **Proved.**