MATHEMATICS

DPP
DAILY PRACTICE PROBLEMS

DPP No. 32

Total Marks : 29

Max. Time : 31 min.

Topics : Method of Differentiation, Complex Number, Continuity & Derivability, Application of Derivatives, Sequence & Series, Function

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Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3,4	(3 marks, 3 min.)	[12, 12]
Multiple choice objective (no negative marking) Q.5	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q.6,7,8	(4 marks, 5 min.)	[12, 15]

1. Let $y = \tan^{-1}\left(\dfrac{2\cos(3x^2-2)+5\sin(3x^2-2)}{5\cos(3x^2-2)-2\sin(3x^2-2)}\right)$, then $\dfrac{dy}{dx} =$

   (A) $6x - 2$ (B) $6x$ (C) $5x$ (D) $\dfrac{6x}{x^2+1}$

2. If $y = at^2 + 2bt + c$ and $t = ax^2 + 2bx + c$, then $\dfrac{d^3y}{dx^3}$ equals

   (A) $24a^2\,(at + b)$ (B) $24a\,(ax + b)^2$ (C) $24a\,(at + b)^2$ (D) $24a^2\,(ax + b)$

3. The complex number $z = x + iy$ for which $\log_{1/2}|z-2| > \log_{1/2}|z|$, are given by:

   (A) $\text{Re}\,(z) \le 1$ (B) $\text{Im}\,(z) \le 1$ (C) $\text{Re}\,(z) > 1$ (D) $\text{Im}\,(z) > 1$

4. If $g(x) = \dfrac{2h(x)+|h(x)|}{2h(x)-|h(x)|}$ where $h(x) = \sin x - \sin^n x$, $n \in R^+$, the set of positive real numbers, and

   $$f(x) = \begin{cases} [g(x)], & x \in \left(0, \dfrac{\pi}{2}\right) \cup \left(\dfrac{\pi}{2}, \pi\right) \\[2mm] 3, & x = \dfrac{\pi}{2} \end{cases}$$
   where [.] denotes greatest integer function. Then

   (A) $f(x)$ is continuous and differentiable at $x = \dfrac{\pi}{2}$, when $0 < n < 1$

   (B) $f(x)$ is continuous and differentiable at $x = \dfrac{\pi}{2}$, when $n > 1$

   (C) $f(x)$ is continuous but not differentiable at $x = \dfrac{\pi}{2}$, when $0 < n < 1$

   (D) $f(x)$ is continuous but not differentiable at $x = \dfrac{\pi}{2}$, when $n > 1$

5. For the series $S = 1 + \dfrac{1}{(1+3)}(1+2)^2 + \dfrac{1}{(1+3+5)}(1+2+3)^2 + \dfrac{1}{(1+3+5+7)}(1+2+3+4)^2 + \ldots$

   (A) 7th term is 16   (B) 7th term is 18

   (C) sum of first 10th terms is $\dfrac{505}{4}$ (D) sum of first 10th term is $\dfrac{405}{4}$

6. Let $f(x) = \dfrac{1}{1-x}$, $g(x) = fofofofofofof(x)$ and $h(x) = \tan^{-1}(g(-x^2 - x))$, then find $\displaystyle\lim_{n\to\infty}\sum_{r=1}^{n} h(r)$.

7. Prove that in the curve $y = a\,\ell n\,(x^2 - a^2)$, sum of the tangent and subtangent varies as the product of the coordinates of the point of contact.

8. If the equation $a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x = 0$, (where n is natural number) has a positive root α, prove that the equation $na_0 x^{n-1} + (n-1)a_1 x^{n-2} + \cdots + a_{n-1} = 0$ also has a positive root smaller than α.

Answers Key

1. (B)   **2.** (D)   **3.** (C)   **4.** (B)

5. (A)(C) **6.** $\dfrac{\pi}{4}$