

## Parallel Lines

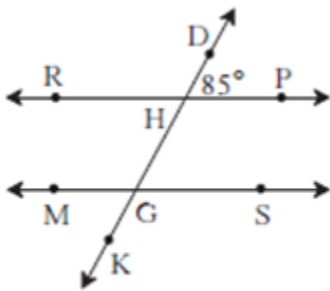
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### Practice set 2.1

Q. 1. In figure 2.5, line  $RP \parallel$  line  $MS$  and line  $DK$  is their transversal.  $\angle DHP = 85^\circ$

Find the measures of following angles.

- i.  $\angle RHD$  ii.  $\angle PHG$   
iii  $\angle HGS$  iv.  $\angle MGK$



**Fig. 2.5**

**Answer :** Given:  $RP \parallel$  line  $MS$  and line  $DK$  is their transversal.

(i)  $\angle DHP + \angle RHD = 180^\circ$  (linear pair angle) means that linear pair is a pair of adjacent, supplementary angle. Adjacent means next to each other, and supplementary means that measures of the two angles add up to equal  $180^\circ$ .

$$\angle DHP + \angle RHD = 180^\circ$$

$$85^\circ + \angle RHD = 180^\circ (\angle DHP = 85^\circ \text{ given})$$

$$\angle RHD = 180^\circ - 85^\circ$$

$$\angle RHD = 95^\circ$$

(ii)  $\angle RHD \cong \angle PHG$  (vertically opposite angles formed are congruent)

$$\text{So, } \angle PHG = 95^\circ$$

(iii) line  $RP \parallel$  line  $MS$  (given)

$\angle DHP \cong \angle HGS$  (corresponding angles) if two parallel line are cut by a transversal, then the pairs of corresponding angle are congruent.

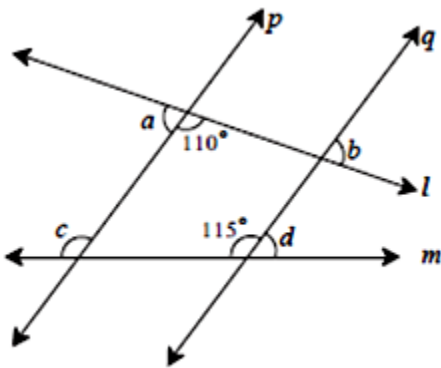
$$\angle DHP = 85^\circ \text{ (given)}$$

$$\text{So, } \angle HGS = 85^\circ$$

(iv)  $\angle HGS \cong \angle MKG$  (vertically opposite angles formed are congruent)

$$\text{So, } \angle MKG = 85^\circ$$

**Q. 2. In figure 2.6, line  $p \parallel$  line  $q$  and line  $l$  and line  $m$  are transversals. Measures of some angles are shown. Hence find the measures of  $\angle a$ ,  $\angle b$ ,  $\angle c$  &  $\angle d$ .**



**Fig. 2.6**

**Answer :** Given line  $P \parallel$  line  $Q$  and line  $L$  and  $M$  are transversal.

To find:  $\angle a$ ,  $\angle b$ ,  $\angle c$  &  $\angle d$ .

Construction: extend  $G$  and  $E$  in answer diagram.

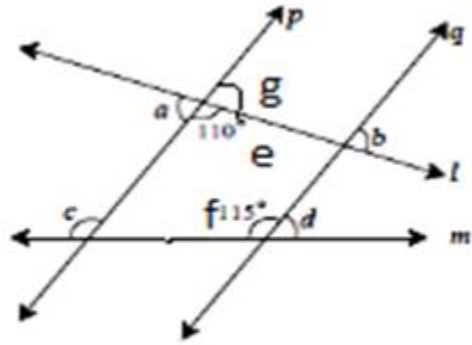
$\angle a + \angle e = 180^\circ$  (linear pair angle) means that linear pair is a pair of adjacent, supplementary angle.

Adjacent means next to each other, and supplementary means that measures of the two angles add up to equal  $180^\circ$ .

$$\angle a + 110^\circ = 180^\circ \text{ (given)}$$

$$\angle a = 180^\circ - 110^\circ$$

$$\angle a = 70^\circ$$



**Fig. 2.6**

$\angle a \cong \angle g$  (vertically opposite angles formed are congruent)

$\angle a = 70^\circ$  (prove above)

$\angle 70^\circ \cong \angle g$

Line P  $\parallel$  line Q and line L transversals (given)

$\angle g = \angle b$  (corresponding angles)

$\angle b = 70^\circ$

Line P  $\parallel$  line Q and line M is transversal (given)

$\angle c \cong \angle f$  (corresponding angles) if two parallel line are cut by a transversal, then the pairs of corresponding angle are congruent.

So,  $\angle f = 115^\circ$  (given)

Then,  $\angle c = 115^\circ$

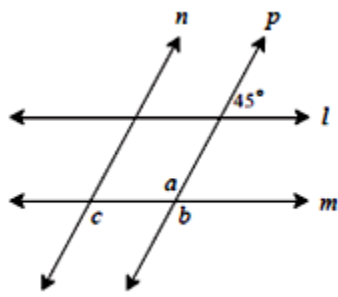
$f = 180^\circ$  (linear pair angle)

$\angle d + 115^\circ = 180^\circ$  (given)

$\angle d = 180^\circ - 115^\circ$

$\angle d = 65^\circ$

**Q. 3. In figure 2.7 line  $n \parallel$  line m and line  $n \parallel$  line p. Find  $\angle a$ ,  $\angle b$ ,  $\angle c$  from the given measure of an angle.**



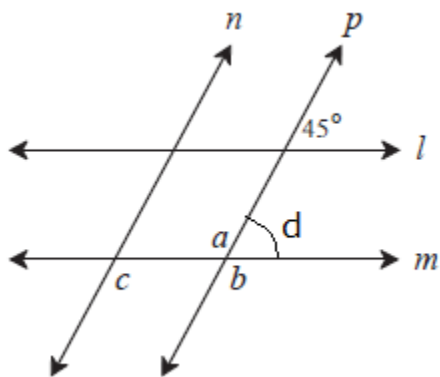
**Fig. 2.7**

**Answer :** Given line  $L \parallel$  line  $M$  and line  $P$  is transversal.

To find:  $\angle a$ ,  $\angle b$ ,  $\angle c$

Construction: extend  $E$  and  $D$  in answer diagram

$\angle e \cong \angle d$  (corresponding angle theorem) if two parallel line are cut by a transversal, then the pairs of corresponding angle are congruent.



$\therefore \angle d = 45^\circ$  (given above diagram)

$\angle a + \angle d = 180^\circ$  (linear pair angle)

$\angle a + 45^\circ = 180^\circ$

$\angle a = 180^\circ - 45^\circ$

$\angle a = 135^\circ$

$\angle a \cong \angle b$  (vertically opposite angles formed are congruent)

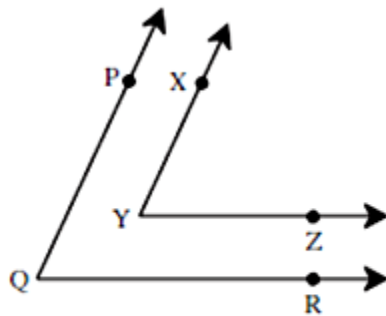
$$\therefore \angle b = 135^\circ$$

Line N  $\parallel$  line P and line M is transversal (given)

$\angle b \cong \angle c$  (corresponding angle theorem) if two parallel line are cut by a transversal, then the pairs of corresponding angle are congruent.

$$\therefore \angle c = 135^\circ.$$

**Q. 4. In figure 2.8, sides of  $\angle PQR$  and  $\angle XYZ$  are parallel to each other. prove that,  $\angle PQR \cong \angle XYZ$**

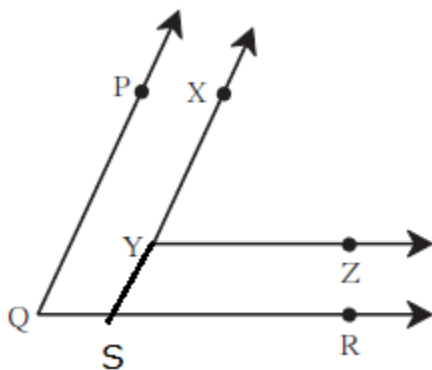


**Fig. 2.8**

**Answer :** Given  $\angle PQR$  AND  $\angle XYZ$  are parallel and also YZ and QR are parallel

TO find:  $\angle PQR \cong \angle XYZ$

Construction: extend sag XY such that Q-S-R.



PQ  $\parallel$  XY (given)

PQ  $\parallel$  XS and QR is transversals (from construction)

$\angle PQR \cong \angle XSR$  (corresponding angle theorem) if two parallel line are cut by a transversal, then the pairs of corresponding angle are congruent) .....  
(1)

$YZ \parallel SK$  and  $XS$  is transversals (given)

$\angle XYZ \cong \angle XSR$  (corresponding angle theorem) if two parallel line are cut by a transversal, then the pairs of corresponding angle are congruent.)  
.....(2)

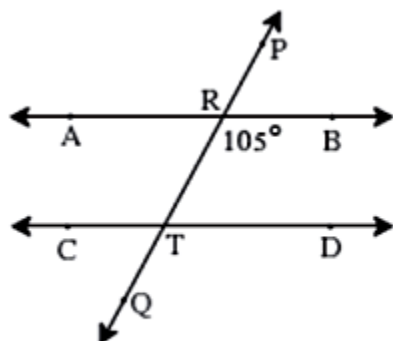
Equation (1) and (2) right side is equal

**So that,**  $\angle PQR \cong \angle XYZ$  hence proved.

**Q. 5. In figure 2.9, line  $AB \parallel$  line  $CD$  and line  $PQ$  is transversal. Measure of one of the angles is given.**

**Hence find the measures of the following angles.**

- i.  $\angle ART$  ii.  $\angle CTQ$   
iii.  $\angle DTQ$  iv.  $\angle PRB$



**Fig. 2.9**

**Answer :** Given  $AB \parallel$  line  $CD$  and line  $PQ$  is transversal .and  $\angle PRB = 105^\circ$  and  $\angle BRT = 105^\circ$

To find:  $\angle ART$ ,  $\angle CTQ$ ,  $\angle DTQ$ ,  $\angle PRB$

$\angle ART + \angle BRT = 180^\circ$  (linear pair angle) means that linear pair is a pair of adjacent, supplementary angle.

Adjacent means next to each other, and supplementary means that measures of the two angles add up to equal  $180^\circ$ .)

$$\angle ART + 105^\circ = 180^\circ (\angle BRT = 105^\circ \text{ given})$$

$$\angle ART = 180^\circ - 105^\circ$$

$$\angle ART = 75^\circ$$

$\angle ART \cong \angle PRQ$  (vertically opposite angles formed are congruent).

$$\text{So, } \angle PRB = 75^\circ (\because \angle ART = 75^\circ)$$

Line AB || line CD line PQ is transversal (given)

$\angle BRT \cong \angle DTQ$  (corresponding angle theorem) if two parallel line are cut by a transversal, then the pairs of corresponding angle are congruent).

$$\angle DTQ = 105 (\angle BRT \text{ is } 105^\circ)$$

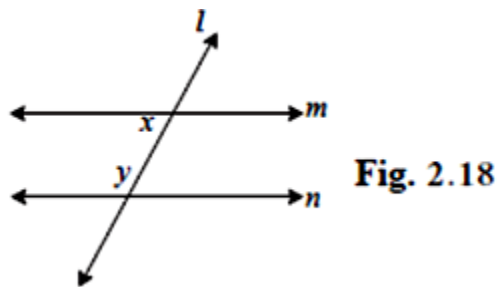
$$\text{So that, } \angle DTQ = 105^\circ$$

$\angle ART \cong \angle CTQ$  (corresponding angle theorem) if two parallel line are cut by a transversal, then the pairs of corresponding angle are congruent).

$$\text{So that, } \angle CTQ = 75^\circ (\text{because } \angle ART \text{ is } 75^\circ)$$

## Practice set 2.2

**Q. 1. In figure 2.18,  $y = 108^\circ$ . and  $x = 71^\circ$  Are the lines  $m$  and  $n$  parallel? Justify?**



**Answer :** Given  $x = 71^\circ$ ,  $y = 108^\circ$ .

To find: Are the lines  $m$  and  $n$  parallel or not?

$$x + y = 108^\circ + 71^\circ (\text{already given})$$

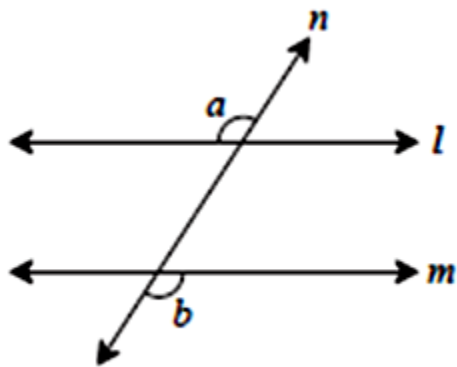
$$= 179^\circ$$

$$x + y \neq 180^\circ$$

They form a pair of interior angles which are not supplementary.

$\therefore$  Line M is not parallel to Line N.

**Q. 2.** In figure 2.19, if  $\angle a \cong \angle b$  then prove that line L  $\parallel$  line M.



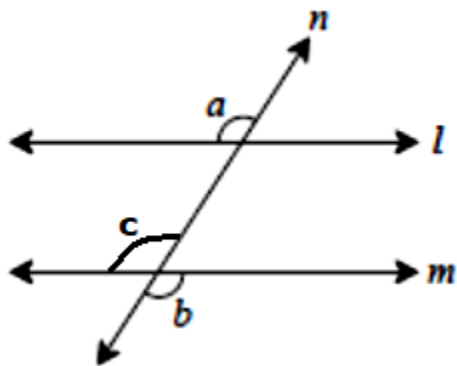
**Fig. 2.19**

**Answer :**

$\angle a \cong \angle b$

To find: line L  $\parallel$  line M

Construction: extend C in figure.



$\angle a \cong \angle b$  (given)

$\angle b \cong \angle c$  (vertically opposite angle)

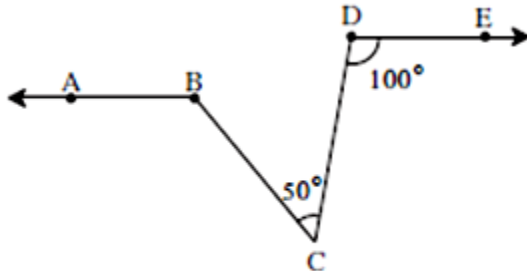
$\angle a \cong \angle c$  (if whenever an element A is related to an element B and B is related to an element C then A is also related to c that is called transitivity)

But they form a pair of corresponding angle that are congruent.



$\therefore$  line L  $\parallel$  line M (hence proved).

**Q. 4.** In figure 2.21, if ray BA  $\parallel$  ray DE,  $\angle c = 50^\circ$  and  $\angle D = 100^\circ$ . Find the measure of  $\angle ABC$ .



**Fig. 2.21**

(Hint: Draw a line passing through point C and parallel to line AB.)

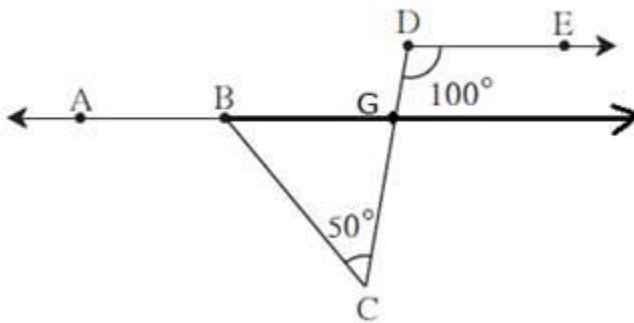
**Answer :** Given: ray  $\angle$  BA  $\parallel$   $\angle$  DE and  $\angle c = 50^\circ$ ,  $\angle D = 100^\circ$ .

To find:  $\angle$  ABC.

**Construction:** Extend AB such that A-B-F-G.

BA  $\parallel$  DE (given)

AG  $\parallel$  DE (construction) and DC is transversal.



$\angle D \cong \angle GFC$  ((corresponding angle theorem) if two parallel line are cut by a transversal, then the pairs of corresponding angle are congruent).

$\angle d = 100^\circ$  (given)

So that  $\angle GFC = 100^\circ$

$\angle GFC + \angle BFC = 180$ (linear pair angle

$$\angle GFC = 100 \text{ (proved above)}$$

$$\angle GFC + \angle BFC = 180^\circ$$

$$\angle 100 + \angle BFC = 180^\circ$$

$$\therefore \angle BFC = 180 - 100$$

$$\angle BFC = 80^\circ$$

In  $\Delta BFC$

$$\angle BFC + \angle c + \angle FBC = 180^\circ \text{ (sum of angle of } \Delta \text{)}$$

$$\angle 80^\circ + 50^\circ + \angle FBC = 180^\circ \text{ (already given value above } \angle BFC \text{ and } \angle c \text{)}$$

$$\angle FBC = 180^\circ - 130^\circ$$

$$\angle FBC = 50^\circ$$

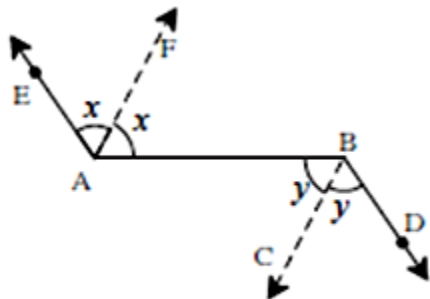
$$\angle ABC + \angle FBC = 180^\circ$$

$$\angle ABC + 50^\circ = 180^\circ$$

$$\angle ABC = 180 - 50^\circ$$

$$\angle ABC = 130^\circ$$

**Q. 5.** In figure 2.22, ray  $AE \parallel$  ray  $BD$ , ray  $AF$  is the bisector of  $\angle EAB$  and ray  $BC$  is the bisector of  $\angle ABD$ . Prove that line  $AF \parallel$  line  $BC$ .



**Fig. 2.22**

**Answer :** Given ray  $AE \parallel$  ray  $BD$ . ray  $AF$  is bisector of  $\angle EAB$  and ray  $BC$  is the bisector of  $\angle ABD$ .

To find: line  $AF \parallel$  line  $BC$

$$\angle EAB = 2x \text{ (ray AF bisector } \angle EAB)$$

When a line, shape, or angle into two exactly equal parts is called bisector.

$$\angle ABD = 2y \text{ (ray BC bisector } \angle ABD)$$

Ray AE  $\parallel$  ray BD and AB is transversal.

$\angle EAD \cong \angle ABD$  (alternate angle) two angle formed when a line crosses two other lines, that lie on opposite side of the transversal line and on opposite relative sides of the other lines. If the two lines crossed are parallel, the alternate angles are equal.)

$$2x = 2y$$

$$x = y$$

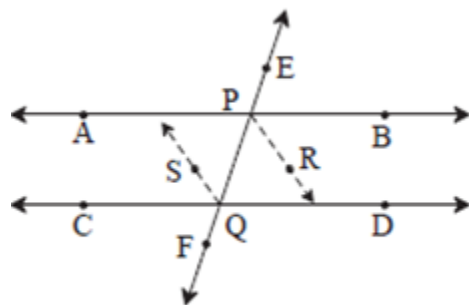
$$\angle FAB \cong \angle ABC$$

But they form a pair of alternate angle that are congruent.

$\therefore$  line AF  $\parallel$  line BC (hence proved)

**Q. 6. A transversal EF of line AB and line CD intersects the lines at point P and Q respectively. Ray PR and ray QS are parallel and bisectors of  $\angle BPQ$  and  $\angle PQC$  respectively.**

**Prove that line AB  $\parallel$  line CD.**



**Fig. 2.23**

**Answer :** ray PR  $\parallel$  SQ and PQ is transversal (given)

To find: AB  $\parallel$  CD

$\angle RPQ \cong \angle PQS$  (alternate angle) two angle formed when a line crosses two other lines, that lie on opposite side of the transversal line and on opposite relative sides of the other lines. If the two lines crossed are parallel, the alternate angles are equal.)

$$X = y$$

$$\angle BPQ = 2x \text{ (ray PR bisect } \angle BPQ)$$

$$\angle PQC = 2y \text{ (ray SQ bisect } \angle PQC)$$

When a line, shape, or angle into two exactly equal parts is called bisector.

$$X = y$$

$$2x = 2y \text{ (multiply 2 on both side)}$$

$$\angle BPQ = \angle PQC$$

But they form a pair of alternate angles that are congruent.

$\therefore AB \parallel CD$  (hence proved)

## Problem set 2

**Q. 1 A. Select the correct alternative and fill in the blank in the following statements.**

**If a transversal intersects two parallel lines then the sum of interior angles on the same side of the transversal is .....**

- A.  $0^\circ$
- B.  $90^\circ$
- C.  $180^\circ$
- D.  $360^\circ$

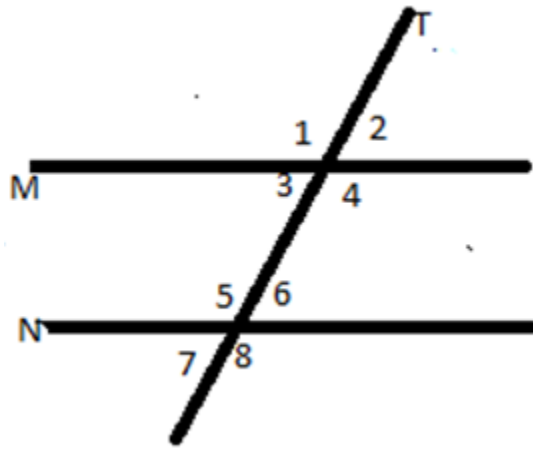
**Answer :** When a transversal intersects two parallel lines, then sum of the interior angles, formed on the same side of the transversal is  $180^\circ$ .

**Q. 1 B. Select the correct alternative and fill in the blank in the following statements.**

**The number of angles formed by a transversal of two lines is .....**

- A. 2
- B. 4
- C. 8
- D. 16

**Answer :** When two parallel lines cut by a third line, the third line is called the transversal. 8 angles are formed when parallel lines M and N are cut by a transversal line T.



**Q. 1 C. Select the correct alternative and fill in the blank in the following statements.**

A transversal intersects two parallel lines. If the measure of one of the angles is  $40^\circ$  then the measure of its corresponding angle is .....

- A.  $40^\circ$
- B.  $140^\circ$
- C.  $50^\circ$
- D.  $180^\circ$

**Answer :** A transversal intersects two parallel lines so; corresponding angle is equal so that corresponding angle is also  $40^\circ$

**Q. 1 D. Select the correct alternative and fill in the blank in the following statements.**

In  $\triangle ABC$   $\angle A = 76^\circ$ ,  $\angle B = 48^\circ$ ,  $\angle C = \dots$

- A.  $66^\circ$
- B.  $56^\circ$
- C.  $124^\circ$
- D.  $28^\circ$

**Answer :**  $\angle A + \angle B + \angle C = 180^\circ$  (the sum of the measures of the interior angles of a triangle is  $180^\circ$ )

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle 76 + \angle 48 + \angle C = 180^\circ$$

$$\angle 124 + \angle c = 180^\circ$$

$$\angle C = 180 - 124$$

$$\angle C = 56^\circ$$

**Q. 1 E. Select the correct alternative and fill in the blank in the following statements.**

**Two parallel lines are intersected by a transversal. If measure of one of the alternate interior angles is  $75^\circ$  then the measure of the other angle is .....**

- A.  $105^\circ$**
- B.  $15^\circ$**
- C.  $75^\circ$**
- D.  $45^\circ$**

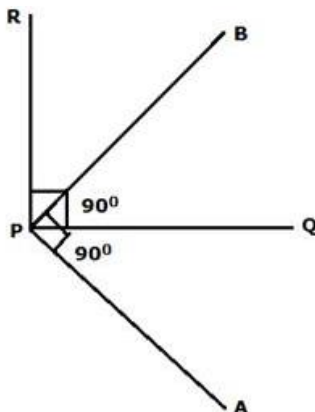
**Answer :** If measure of one of the alternate interior angles is  $75^\circ$  then the measure of the other angle is  $75^\circ$  because two parallel are intersected by transversal.

**Q. 2. Ray PQ and ray PR are perpendicular to each other. Points B and A are in the interior and exterior of  $\angle QPR$  respectively. Ray PB and ray PA are perpendicular to each other.**

**Draw a figure showing all these rays and write –**

- i. A pair of complementary angles**
- ii. A pair of supplementary angles.**
- iii. A pair of congruent angles.**
- iii. A pair of congruent angles.**

**Answer :** The figure is attached below:



(i)  $\angle RPB + \angle BPQ = 90^\circ$

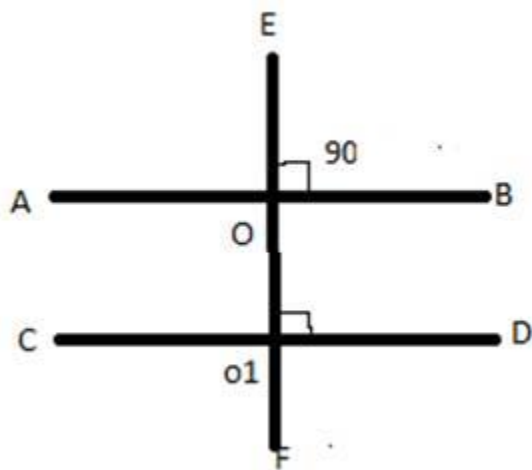
(ii)  $\angle RPQ + \angle BPA = 180^\circ$

(iii)  $\angle RPQ = \angle BPA$  (congruent angle)

(iv)  $\angle RPB = \angle QPA$  (congruent angle)

**Q. 3. Prove that, if a line is perpendicular to one of the two parallel lines, then it is perpendicular to the other line**

**Answer :** The diagram is given below:



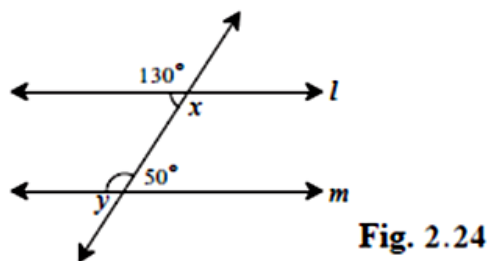
To find: transversal line will be perpendicular to other parallel line.

$AB \parallel CD$  and  $EF$  is transversal both.

So,  $\angle EOB = \angle O_1D = 90^\circ$

So that transversal line will be perpendicular to other parallel line also. (hence proved)

**Q. 4. In figure 2.24, measures of some angles are shown. Using the measures find the measures of  $\angle x$  and  $\angle y$  hence show that line  $l \parallel$  line  $m$ .**



**Answer :** Given: value of  $\angle x$  and  $\angle y$

To find: line  $l \parallel$  line  $m$

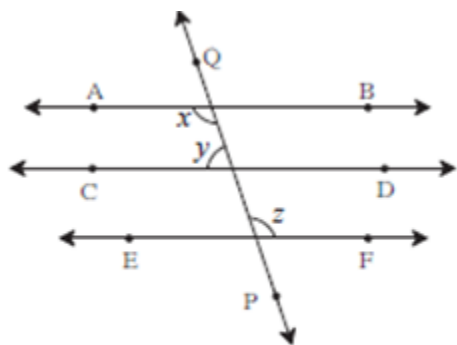
$\angle y = 180 - 50^\circ$  (linear pair angle) means that linear pair is a pair of adjacent, supplementary angle. Adjacent means next to each other, and supplementary means that measures of the two angles add up to equal  $180^\circ$ .)

$$\angle y = 130^\circ$$

Transversal line making same angle to both line.

So, that line  $L \parallel$  line  $M$ .

**Q. 5.** Line  $AB \parallel CD \parallel$  Line  $EF$  and line  $QP$  is their transversal. If  $Y: z = 3:7$  then find the measure of  $\angle x$ . (See figure 2.25.)



**Fig. 2.25**

**Answer :** Given line  $AB \parallel CD \parallel$  Line  $EF$  and line  $QP$  is their transversal.

To find:  $\angle X$ .

$AB \parallel CD \parallel EF$  (linear pair angle) (linear pair angle) means that linear pair is a pair of adjacent, supplementary angle. Adjacent means next to each other, and supplementary means that measures of the two angles add up to equal  $180^\circ$ .

$$X = z \text{ (alternate interior angles) } \dots\dots\dots (1)$$

Alternate interior angles are a pair of angles on the inner side of each of those two lines but on opposite side of the transversal.

$$Y: z = 3:7 \text{ (given)}$$

Let the common ratio between  $y$  and  $z$  be a



$X + y = 180^\circ$  (co-interior angles on the same side of the transversal)

$Z + y = 180^\circ$  (using equation 1)

$$7a + 3a = 180^\circ$$

$$10a = 180^\circ$$

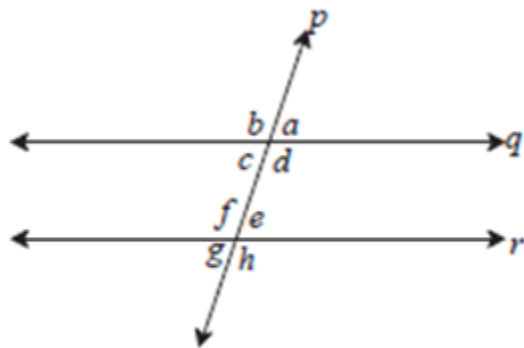
$$A = 18^\circ$$

$$\therefore x = 7a$$

$$X = 7 \times 18^\circ$$

$$X = 126^\circ$$

**Q. 6.** In figure 2.26, if line Q  $\parallel$  line R and p is their transversal and if  $a = 80^\circ$  find the values of f and g.



**Fig. 2.26**

**Answer :** Given line Q  $\parallel$  line R and p is their transversal. and  $a = 80^\circ$

To find: values of F and G.

$$\angle a = 80^\circ$$

$$\angle a = \angle c = 80^\circ \text{ (vertically opposite angle)}$$

$\angle c + \angle f = 180^\circ$  (angle made on the same side of parallel lines are supplementary means their sum is  $180^\circ$ )

$$\angle 80^\circ + \angle f = 180^\circ$$

$$\angle f = 180 - 80$$

$$\angle f = 100^\circ$$

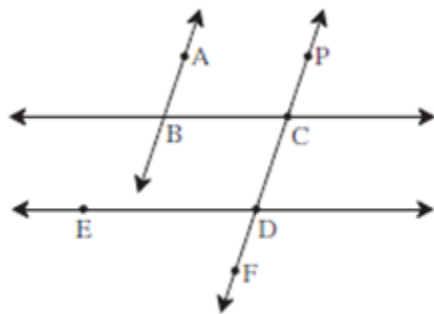
$\angle f + \angle g = 180^\circ$  (linear pair angle) means that linear pair is a pair of adjacent, supplementary angle. Adjacent means next to each other, and supplementary means that measures of the two angles add up to equal 180.)

$$\angle 100 + \angle g = 180$$

$$\angle g = 180 - 100$$

$$\angle g = 80^\circ$$

**Q. 7.** In figure 2.27, if line AB  $\parallel$  line CF and line BC  $\parallel$  line ED then prove that  $\angle ABC = \angle FDE$



**Fig. 2.27**

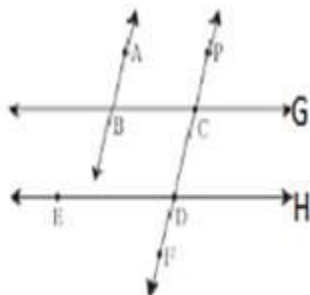
**Answer :** Given: Line AB  $\parallel$  CF and line BC  $\parallel$  ED

To find:  $\angle ABC = \angle FDE$

**Construction:** G and h in diagram.

AB  $\parallel$  CD and BC is transversal both

So,  $\angle ABC = \angle PCG$  (linear pair angle)



**Fig. 2.27**

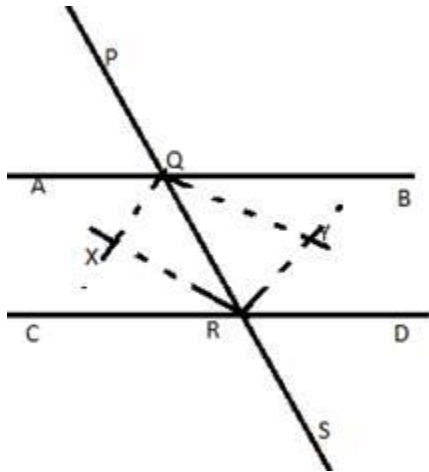
$BC \parallel ED$  and  $PF$  is transversal line to both

So,  $\angle ECG = \angle CDH$

$\angle CD = \angle FDE$  (Both angle is opposite)

$\therefore \angle ABC = \angle FDE$  (hence proved)

**Q. 8.** In figure 2.28, line  $PS$  is a transversal of parallel line  $AB$  and line  $CD$ . If Ray  $QX$ , ray  $QY$ , ray  $RX$ , ray  $RY$  are angle bisectors, then prove that  $\square QXRY$  is a rectangle.



**Answer :** Given:  $PS$  is transversal of parallel line  $AB$  and line  $CD$ .

To find:  $QRY$  is rectangle.

$$\angle AQR + \angle CRQ = 180^\circ$$

$$\frac{\angle AQR}{2} + \frac{\angle CRQ}{2} = \frac{180^\circ}{2} \quad (\text{divide by } 2)$$

$$\angle XQR + \angle XRQ = 90^\circ$$

$$\left[ \frac{\angle AQR}{2} = \angle XQR \text{ and } \frac{\angle CRQ}{2} = \angle XRQ \right] \quad (\text{QX and RX are bisector})$$

In  $\triangle XQR$

$$\angle XQR + \angle XRQ + \angle QXR = 180^\circ$$

$$90^\circ + \angle QXR = 180^\circ \quad (\angle XQR + \angle XRQ = 180^\circ \text{ proved above})$$

$$\angle QXR = 180^\circ - 90^\circ$$

$$\angle QXR = 90^\circ$$

Similarly,  $\angle QYR = 90^\circ$

$$\angle AQR + \angle BQR = 180 \text{ (straight line)}$$

$$\frac{\angle AQR}{2} + \frac{\angle BQR}{2} = \frac{180^\circ}{2} \text{ (divide by 2)}$$

$$\angle XQR + \angle YQR = 90^\circ \text{ (QX and QY are bisector } \angle)$$

$$\angle XQY = 90^\circ$$

Similarly,  $\angle XRY = 90^\circ$

If any quadrilateral has all the angle  $90^\circ$  it is a rectangle, so that QXRY is rectangle.