9.4 Observational astrophysics

Astronomical magnitudes

Apparent magnitude	$m_1 - m_2 = -2.5\log_{10}\frac{F_1}{F_2}$	(9.27)	m_i F_i	apparent magnitude of object i energy flux from object i
Distance modulus[a]	$m - M = 5\log_{10} D - 5$ $= -5\log_{10} p - 5$	(9.28) (9.29)	M $m-M$ D p	absolute magnitude distance modulus distance to object (parsec) annual parallax (arcsec)
Luminosity–magnitude relation	$M_{\text{bol}} = 4.75 - 2.5\log_{10}\frac{L}{L_\odot}$ $L \simeq 3.04 \times 10^{(28-0.4M_{\text{bol}})}$	(9.30) (9.31)	M_{bol} L $L_\odot$	bolometric absolute magnitude luminosity (W) solar luminosity (3.826×10^{26} W)
Flux–magnitude relation	$F_{\text{bol}} \simeq 2.559 \times 10^{-(8+0.4m_{\text{bol}})}$	(9.32)	F_{bol} m_{bol}	bolometric flux (W m^{-2}) bolometric apparent magnitude
Bolometric correction	$BC = m_{\text{bol}} - m_{\text{V}}$ $= M_{\text{bol}} - M_{\text{V}}$	(9.33) (9.34)	BC m_{V} M_{V}	bolometric correction V-band apparent magnitude V-band absolute magnitude
Colour index[b]	$B - V = m_{\text{B}} - m_{\text{V}}$ $U - B = m_{\text{U}} - m_{\text{B}}$	(9.35) (9.36)	$B-V$ $U-B$	observed $B-V$ colour index observed $U-B$ colour index
Colour excess[c]	$E = (B-V) - (B-V)_0$	(9.37)	E $(B-V)_0$	$B-V$ colour excess intrinsic $B-V$ colour index

[a] Neglecting extinction.
[b] Using the UBV magnitude system. The bands are centred around 365 nm (U), 440 nm (B), and 550 nm (V).
[c] The $U-B$ colour excess is defined similarly.

Photometric wavelengths

Mean wavelength	$\lambda_0 = \frac{\int \lambda R(\lambda)\,\mathrm{d}\lambda}{\int R(\lambda)\,\mathrm{d}\lambda}$	(9.38)	λ_0 λ R	mean wavelength wavelength system spectral response
Isophotal wavelength	$F(\lambda_{\text{i}}) = \frac{\int F(\lambda)R(\lambda)\,\mathrm{d}\lambda}{\int R(\lambda)\,\mathrm{d}\lambda}$	(9.39)	$F(\lambda)$ λ_{i}	flux density of source (in terms of wavelength) isophotal wavelength
Effective wavelength	$\lambda_{\text{eff}} = \frac{\int \lambda F(\lambda)R(\lambda)\,\mathrm{d}\lambda}{\int F(\lambda)R(\lambda)\,\mathrm{d}\lambda}$	(9.40)	λ_{eff}	effective wavelength

Planetary bodies

Bode's law[a]	$$D_{\rm AU} = \frac{4+3\times 2^n}{10} \quad (9.41)$$	$D_{\rm AU}$ planetary orbital radius (AU) n index: Mercury $=-\infty$, Venus $=0$, Earth $=1$, Mars $=2$, Ceres $=3$, Jupiter$=4$, ...
Roche limit	$$R \gtrsim \left(\frac{100M}{9\pi\rho}\right)^{1/3} \quad (9.42)$$ $$\gtrsim 2.46R_0 \quad \text{(if densities equal)} \quad (9.43)$$	R satellite orbital radius M central mass ρ satellite density R_0 central body radius
Synodic period[b]	$$\frac{1}{S} = \left|\frac{1}{P} - \frac{1}{P_\oplus}\right| \quad (9.44)$$	S synodic period P planetary orbital period $P_\oplus$ Earth's orbital period

[a]Also known as the "Titius–Bode rule." Note that the asteroid Ceres is counted as a planet in this scheme. The relationship breaks down for Neptune and Pluto.
[b]Of a planet.

Distance indicators

Hubble law	$$v = H_0 d \quad (9.45)$$	v cosmological recession velocity H_0 Hubble parameter (present epoch) d (proper) distance
Annual parallax	$$D_{\rm pc} = p^{-1} \quad (9.46)$$	$D_{\rm pc}$ distance (parsec) p annual parallax ($\pm p$ arcsec from mean)
Cepheid variables[a]	$$\log_{10}\frac{\langle L\rangle}{L_\odot} \simeq 1.15\log_{10}P_{\rm d} + 2.47 \quad (9.47)$$ $$M_{\rm V} \simeq -2.76\log_{10}P_{\rm d} - 1.40 \quad (9.48)$$	$\langle L\rangle$ mean cepheid luminosity $L_\odot$ Solar luminosity $P_{\rm d}$ pulsation period (days) $M_{\rm V}$ absolute visual magnitude
Tully–Fisher relation[b]	$$M_{\rm I} \simeq -7.68\log_{10}\left(\frac{2v_{\rm rot}}{\sin i}\right) - 2.58 \quad (9.49)$$	$M_{\rm I}$ I-band absolute magnitude $v_{\rm rot}$ observed maximum rotation velocity ($\rm km\,s^{-1}$) i galactic inclination (90° when edge-on)
Einstein rings	$$\theta^2 = \frac{4GM}{c^2}\left(\frac{d_{\rm s}-d_{\rm l}}{d_{\rm s}d_{\rm l}}\right) \quad (9.50)$$	θ ring angular radius M lens mass $d_{\rm s}$ distance from observer to source $d_{\rm l}$ distance from observer to lens
Sunyaev–Zel'dovich effect[c]	$$\frac{\Delta T}{T} = -2\int \frac{n_{\rm e}kT_{\rm e}\sigma_{\rm T}}{m_{\rm e}c^2}\,{\rm d}l \quad (9.51)$$	T apparent CMBR temperature dl path element through cloud R cloud radius $n_{\rm e}$ electron number density k Boltzmann constant
... for a homogeneous sphere	$$\frac{\Delta T}{T} = -\frac{4Rn_{\rm e}kT_{\rm e}\sigma_{\rm T}}{m_{\rm e}c^2} \quad (9.52)$$	$T_{\rm e}$ electron temperature $\sigma_{\rm T}$ Thomson cross section $m_{\rm e}$ electron mass c speed of light

[a]Period–luminosity relation for classical Cepheids. Uncertainty in $M_{\rm V}$ is ± 0.27 (Madore & Freedman, 1991, Publications of the Astronomical Society of the Pacific, **103**, 933).
[b]Galaxy rotation velocity–magnitude relation in the infrared I waveband, centred at 0.90 µm. The coefficients depend on waveband and galaxy type (see Giovanelli *et al.*, 1997, The Astronomical Journal, **113**, 1).
[c]Scattering of the cosmic microwave background radiation (CMBR) by a cloud of electrons, seen as a temperature decrement, ΔT, in the Rayleigh–Jeans limit ($\lambda \gg 1$ mm).