ANSWERS

EXERCISE 7.1

1. $-\frac{1}{2}\cos 2x$ **2.** $\frac{1}{3}\sin 3x$ **3.** $\frac{1}{2}e^{2x}$

4. $\frac{1}{3a}(ax+b)^3$ **5.** $-\frac{1}{2}\cos 2x-\frac{4}{3}e^{3x}$ **6.** $\frac{4}{3}e^{3x}+x+C$

7. $\frac{x^3}{3}-x+C$ **8.** $\frac{ax^3}{3}+\frac{bx^2}{2}+cx+C$ **9.** $\frac{2}{3}x^3+e^x+C$

10. $\frac{x^2}{2}+\log|x|-2x+C$ **11.** $\frac{x^2}{2}+5x+\frac{4}{x}+C$

12. $\frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{3}{2}}+8\sqrt{x}+C$ **13.** $\frac{x^3}{3}+x+C$

14. $\frac{2}{3}x^{\frac{3}{2}}-\frac{2}{5}x^{\frac{5}{2}}+C$ **15.** $\frac{6}{7}x^{\frac{7}{2}}+\frac{4}{5}x^{\frac{5}{2}}+2x^{\frac{3}{2}}+C$

16. $x^2-3\sin x+e^x+C$ **17.** $\frac{2}{3}x^3+3\cos x+\frac{10}{3}x^{\frac{3}{2}}+C$

18. $\tan x + \sec x + C$ **19.** $\tan x - x + C$

20. $2\tan x - 3\sec x + C$ **21.** C

22. A

EXERCISE 7.2

1. $\log(1+x^2)+C$ **2.** $\frac{1}{3}(\log|x|)^3+C$ **3.** $\log|1+\log x|+C$

4. $\cos(\cos x)+C$ **5.** $-\frac{1}{4a}\cos 2(ax+b)+C$

6. $\frac{2}{3a}(ax+b)^{\frac{3}{2}}+C$ **7.** $\frac{2}{5}(x+2)^{\frac{5}{2}}-\frac{4}{3}(x+2)^{\frac{3}{2}}+C$

8. $\frac{1}{6}(1+2x^2)^{\frac{3}{2}}+C$ **9.** $\frac{4}{3}(x^2+x+1)^{\frac{3}{2}}+C$ **10.** $2\log\left|\sqrt{x}-1\right|+C$

11. $\frac{2}{3}\sqrt{x+4}(x-8)+C$

12. $\frac{1}{7}(x^3-1)^{\frac{7}{3}}+\frac{1}{4}(x^3-1)^{\frac{4}{3}}+C$ **13.** $-\frac{1}{18(2+3x^3)^2}+C$

14. $\frac{(\log x)^{1-m}}{1-m}+C$ **15.** $-\frac{1}{8}\log|9-4x^2|+C$ **16.** $\frac{1}{2}e^{2x+3}+C$

17. $-\frac{1}{2e^{x^2}}+C$ **18.** $e^{\tan^{-1}x}+C$ **19.** $\log(e^x+e^{-x})+C$

20. $\frac{1}{2}\log(e^{2x}+e^{-2x})+C$ **21.** $\frac{1}{2}\tan(2x-3)-x+C$

22. $-\frac{1}{4}\tan(7-4x)+C$ **23.** $\frac{1}{2}(\sin^{-1}x)^2+C$

24. $\frac{1}{2}\log\left|2\sin x+3\cos x\right|+C$ **25.** $\frac{1}{(1-\tan x)}+C$

26. $2\sin\sqrt{x}+C$ **27.** $\frac{1}{3}(\sin 2x)^{\frac{3}{2}}+C$ **28.** $2\sqrt{1+\sin x}+C$

29. $\frac{1}{2}(\log\sin x)^2+C$ **30.** $-\log\left|1+\cos x\right|+C$ **31.** $\frac{1}{1+\cos x}+C$

32. $\frac{x}{2}-\frac{1}{2}\log\left|\cos x+\sin x\right|+C$ **33.** $\frac{x}{2}-\frac{1}{2}\log\left|\cos x-\sin x\right|+C$

34. $2\sqrt{\tan x}+C$ **35.** $\frac{1}{3}(1+\log x)^3+C$ **36.** $\frac{1}{3}(x+\log x)^3+C$

37. $-\frac{1}{4}\cos(\tan^{-1}x^4)+C$ **38.** D

39. B

EXERCISE 7.3

1. $\frac{x}{2}-\frac{1}{8}\sin(4x+10)+C$

2. $-\frac{1}{14}\cos 7x+\frac{1}{2}\cos x+C$

3. $\frac{1}{4}\left[\frac{1}{12}\sin 12x+x+\frac{1}{8}\sin 8x+\frac{1}{4}\sin 4x\right]+C$

4. $-\frac{1}{2}\cos(2x+1)+\frac{1}{6}\cos^3(2x+1)+C$

5. $\frac{1}{6}\cos^6 x-\frac{1}{4}\cos^4 x+C$

6. $\frac{1}{4}\left[\frac{1}{6}\cos 6x-\frac{1}{4}\cos 4x-\frac{1}{2}\cos 2x\right]+C$

7. $\frac{1}{2}\left[\frac{1}{4}\sin 4x-\frac{1}{12}\sin 12x\right]+C$

8. $2\tan\frac{x}{2}-x+C$

9. $x-\tan\frac{x}{2}+C$

10. $\frac{3x}{8}-\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+C$

11. $\frac{3x}{8}+\frac{1}{8}\sin 4x+\frac{1}{64}\sin 8x+C$

12. $x-\sin x+C$

13. $2(\sin x+x\cos\alpha)+C$

14. $-\frac{1}{\cos x+\sin x}+C$

15. $\frac{1}{6}\sec^3 2x-\frac{1}{2}\sec 2x+C$

16. $\frac{1}{3}\tan^3 x-\tan x+x+C$

17. $\sec x-\operatorname{cosec} x+C$

18. $\tan x+C$

19. $\log|\tan x|+\frac{1}{2}\tan^2 x+C$

20. $\log|\cos x+\sin x|+C$

21. $\frac{\pi x}{2}-\frac{x^2}{2}+C$

22. $\frac{1}{\sin(a-b)}\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|+C$

23. A

24. B

EXERCISE 7.4

1. $\tan^{-1}x^3+C$

2. $\frac{1}{2}\log\left|2x+\sqrt{1+4x^2}\right|+C$

3. $\log\left|\frac{1}{2-x+\sqrt{x^2-4x+5}}\right|+C$

4. $\frac{1}{5}\sin^{-1}\frac{5x}{3}+C$

5. $\frac{3}{2\sqrt{2}}\tan^{-1}\sqrt{2}\,x^2+C$

6. $\frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right|+C$

7. $\sqrt{x^2-1}-\log\left|x+\sqrt{x^2-1}\right|+C$

8. $\frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right|+C$

9. $\log\left|\tan x+\sqrt{\tan^2 x+4}\right|+C$

10. $\log\left|x+1+\sqrt{x^2+2x+2}\right|+C$

11. $\frac{1}{6}\tan^{-1}\left(\frac{3x+1}{2}\right)+C$

12. $\sin^{-1}\left(\frac{x+3}{4}\right)+C$

13. $\log\left|x-\frac{3}{2}+\sqrt{x^2-3x+2}\right|+C$

14. $\sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right)+C$

15. $\log\left|x-\frac{a+b}{2}+\sqrt{(x-a)(x-b)}\right|+C$

16. $2\sqrt{2x^2+x-3}+C$

17. $\sqrt{x^2-1}+2\log\left|x+\sqrt{x^2-1}\right|+C$

18. $\frac{5}{6}\log\left|3x^2+2x+1\right|-\frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+C$

19. $6\sqrt{x^2-9x+20}+34\log\left|x-\frac{9}{2}+\sqrt{x^2-9x+20}\right|+C$

20. $-\sqrt{4x-x^2}+4\sin^{-1}\left(\frac{x-2}{2}\right)+C$

21. $\sqrt{x^2+2x+3}+\log\left|x+1+\sqrt{x^2+2x+3}\right|+C$

22. $\frac{1}{2}\log\left|x^2-2x-5\right|+\frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right|+C$

23. $5\sqrt{x^2+4x+10}-7\log\left|x+2+\sqrt{x^2+4x+10}\right|+C$

24. B **25.** B

EXERCISE 7.5

1. $\log\frac{(x+2)^2}{|x+1|}+C$ **2.** $\frac{1}{6}\log\left|\frac{x-3}{x+3}\right|+C$

3. $\log|x-1|-5\log|x-2|+4\log|x-3|+C$

4. $\frac{1}{2}\log|x-1|-2\log|x-2|+\frac{3}{2}\log|x-3|+C$

5. $4\log|x+2|-2\log|x+1|+C$ **6.** $\frac{x}{2}+\log|x|-\frac{3}{4}\log|1-2x|+C$

7. $\frac{1}{2}\log|x-1|-\frac{1}{4}\log(x^2+1)+\frac{1}{2}\tan^{-1}x+C$

8. $\frac{2}{9}\log\left|\frac{x-1}{x+2}\right|-\frac{1}{3(x-1)}+C$ **9.** $\frac{1}{2}\log\left|\frac{x+1}{x-1}\right|-\frac{4}{x-1}+C$

10. $\frac{5}{2}\log|x+1|-\frac{1}{10}\log|x-1|-\frac{12}{5}\log|2x+3|+C$

11. $\frac{5}{3}\log|x+1|-\frac{5}{2}\log|x+2|+\frac{5}{6}\log|x-2|+C$

12. $\frac{x^2}{2}+\frac{1}{2}\log|x+1|+\frac{3}{2}\log|x-1|+C$

13. $-\log|x-1|+\frac{1}{2}\log(1+x^2)+\tan^{-1}x+C$

14. $3\log|x+2|+\frac{7}{x+2}+C$ **15.** $\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\tan^{-1}x+C$

16. $\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right|+C$ **17.** $\log\left|\frac{2-\sin x}{1-\sin x}\right|+C$

18. $x+\frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}-3\tan^{-1}\frac{x}{2}+C$ **19.** $\frac{1}{2}\log\left(\frac{x^2+1}{x^2+3}\right)+C$

20. $\frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right|+C$ **21.** $\log\left(\frac{e^x-1}{e^x}\right)+C$

22. B **23.** A

EXERCISE 7.6

1. $-x\cos x+\sin x+C$ **2.** $-\frac{x}{3}\cos 3x+\frac{1}{9}\sin 3x+C$

3. $e^x(x^2-2x+2)+C$ **4.** $\frac{x^2}{2}\log x-\frac{x^2}{4}+C$

5. $\frac{x^2}{2}\log 2x-\frac{x^2}{4}+C$ **6.** $\frac{x^3}{3}\log x-\frac{x^3}{9}+C$

7. $\frac{1}{4}(2x^2-1)\sin^{-1}x+\frac{x\sqrt{1-x^2}}{4}+C$ **8.** $\frac{x^2}{2}\tan^{-1}x-\frac{x}{2}+\frac{1}{2}\tan^{-1}x+C$

9. $(2x^2-1)\frac{\cos^{-1}x}{4}-\frac{x}{4}\sqrt{1-x^2}+C$

10. $\left(\sin^{-1}x\right)^2x+2\sqrt{1-x^2}\sin^{-1}x-2x+C$

11. $-\sqrt{1-x^2}\cos^{-1}x+x+C$ **12.** $x\tan x+\log|\cos x|+C$

13. $x\tan^{-1}x-\frac{1}{2}\log(1+x^2)+C$ **14.** $\frac{x^2}{2}(\log x)^2-\frac{x^2}{2}\log x+\frac{x^2}{4}+C$

15. $\left(\frac{x^3}{3}+x\right)\log x-\frac{x^3}{9}-x+C$ **16.** $e^x\sin x+C$

17. $\frac{e^x}{1+x}+C$ **18.** $e^x\tan\frac{x}{2}+C$

19. $\frac{e^x}{x}+C$ **20.** $\frac{e^x}{(x-1)^2}+C$

21. $\frac{e^{2x}}{5}(2\sin x-\cos x)+C$ **22.** $2x\tan^{-1}x-\log(1+x^2)+C$

23. A **24.** B

EXERCISE 7.7

1. $\frac{1}{2}x\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+C$ **2.** $\frac{1}{4}\sin^{-1}2x+\frac{1}{2}x\sqrt{1-4x^2}+C$

3. $\frac{(x+2)}{2}\sqrt{x^2+4x+6}+\log\left|x+2+\sqrt{x^2+4x+6}\right|+C$

4. $\frac{(x+2)}{2}\sqrt{x^2+4x+1}-\frac{3}{2}\log\left|x+2+\sqrt{x^2+4x+1}\right|+C$

5. $\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+\frac{x+2}{2}\sqrt{1-4x-x^2}+C$

6. $\frac{(x+2)}{2}\sqrt{x^2+4x-5}-\frac{9}{2}\log\left|x+2+\sqrt{x^2+4x-5}\right|+C$

7. $\frac{(2x-3)}{4}\sqrt{1+3x-x^2}+\frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right)+C$

8. $\frac{2x+3}{4}\sqrt{x^2+3x}-\frac{9}{8}\log\left|x+\frac{3}{2}+\sqrt{x^2+3x}\right|+C$

9. $\frac{x}{6}\sqrt{x^2+9}+\frac{3}{2}\log\left|x+\sqrt{x^2+9}\right|+C$

10. A **11.** D

EXERCISE 7.8

1. $\frac{1}{2}(b^2-a^2)$ **2.** $\frac{35}{2}$ **3.** $\frac{19}{3}$

4. $\frac{27}{2}$ **5.** $e-\frac{1}{e}$ **6.** $\frac{15+e^8}{2}$

EXERCISE 7.9

1. 2 **2.** $\log\frac{3}{2}$ **3.** $\frac{64}{3}$

4. $\frac{1}{2}$ **5.** 0 **6.** $e^4\,(e-1)$

7. $\frac{1}{2}\log 2$ **8.** $\log\left(\frac{\sqrt{2}-1}{2-\sqrt{3}}\right)$ **9.** $\frac{\pi}{2}$

10. $\frac{\pi}{4}$ **11.** $\frac{1}{2}\log\frac{3}{2}$ **12.** $\frac{\pi}{4}$

13. $\frac{1}{2}\log 2$ **14.** $\frac{1}{5}\log 6+\frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$

15. $\frac{1}{2}(e-1)$ **16.** $5-\frac{5}{2}\left(9\log\frac{5}{4}-\log\frac{3}{2}\right)$

17. $\frac{\pi^4}{1024}+\frac{\pi}{2}+2$ **18.** 0 **19.** $3\log 2+\frac{3\pi}{8}$

20. $1+\frac{4}{\pi}-\frac{2\sqrt{2}}{\pi}$ **21.** D **22.** C

EXERCISE 7.10

1. $\frac{1}{2}\log 2$ **2.** $\frac{64}{231}$ **3.** $\frac{\pi}{2}-\log 2$

4. $\frac{16\sqrt{2}}{15}(\sqrt{2}+1)$ **5.** $\frac{\pi}{4}$ **6.** $\frac{1}{\sqrt{17}}\log\frac{21+5\sqrt{17}}{4}$

7. $\frac{\pi}{8}$ **8.** $\frac{e^2(e^2-2)}{4}$ **9.** A

10. B

EXERCISE 7.11

1. $\frac{\pi}{4}$ **2.** $\frac{\pi}{4}$ **3.** $\frac{\pi}{4}$ **4.** $\frac{\pi}{4}$

5. 29 **6.** 9 **7.** $\frac{1}{(n+1)(n+2)}$

8. $\frac{\pi}{8}\log 2$ **9.** $\frac{16\sqrt{2}}{15}$ **10.** $\frac{\pi}{2}\log\frac{1}{2}$ **11.** $\frac{\pi}{2}$

12. π **13.** 0 **14.** 0 **15.** 0

16. $-\pi \log 2$ **17.** $\frac{a}{2}$ **18.** 5 **20.** C

21. C

Miscellaneous Exercise on Chapter 7

1. $\frac{1}{2}\log\left|\frac{x^2}{1-x^2}\right|+C$ **2.** $\frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}}-(x+b)^{\frac{3}{2}}\right]+C$

3. $-\frac{2}{a}\sqrt{\frac{(a-x)}{x}}+C$ **4.** $-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+C$

5. $2\sqrt{x}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log(1+x^{\frac{1}{6}})+C$

6. $-\frac{1}{2}\log|x+1|+\frac{1}{4}\log(x^2+9)+\frac{3}{2}\tan^{-1}\frac{x}{3}+C$

7. $\sin a \log|\sin(x-a)|+x\cos a+C$ **8.** $\frac{x^3}{3}+C$

9. $\sin^{-1}\left(\frac{\sin x}{2}\right)+C$ **10.** $-\frac{1}{2}\sin 2x+C$

11. $\frac{1}{\sin(a-b)}\log\left|\frac{\cos(x+b)}{\cos(x+a)}\right|+C$ **12.** $\frac{1}{4}\sin^{-1}(x^4)+C$

13. $\log\left(\frac{1+e^x}{2+e^x}\right)+C$ **14.** $\frac{1}{3}\tan^{-1}x-\frac{1}{6}\tan^{-1}\frac{x}{2}+C$

15. $-\frac{1}{4}\cos^4 x+C$ **16.** $\frac{1}{4}\log(x^4+1)+C$

17. $\frac{[f(ax+b)]^{n+1}}{a(n+1)}+C$ **18.** $\frac{-2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}}+C$

19. $\frac{2(2x-1)}{\pi}\sin^{-1}\sqrt{x}+\frac{2\sqrt{x-x^2}}{\pi}-x+C$

20. $-2\sqrt{1-x}+\cos^{-1}\sqrt{x}+\sqrt{x-x^2}+C$

21. $e^x \tan x + C$

22. $-2\log|x+1|-\frac{1}{x+1}+3\log|x+2|+C$

23. $\frac{1}{2}\left[x\cos^{-1}x-\sqrt{1-x^2}\right]+C$

24. $-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right]+C$

25. $e^{\frac{\pi}{2}}$

26. $\frac{\pi}{8}$

27. $\frac{\pi}{6}$

28. $2\sin^{-1}\frac{(\sqrt{3}-1)}{2}$

29. $\frac{4\sqrt{2}}{3}$

30. $\frac{1}{40}\log 9$

31. $\frac{\pi}{2}-1$

32. $\frac{\pi}{2}(\pi-2)$

33. $\frac{19}{2}$

40. $\frac{1}{3}\left(e^2-\frac{1}{e}\right)$

41. A

42. B

43. D

44. B

EXERCISE 8.1

1. $\frac{14}{3}$

2. $16-4\sqrt{2}$

3. $\frac{32-8\sqrt{2}}{3}$

4. 12π

5. 6π

6. $\frac{\pi}{3}$

7. $\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$

8. $(4)^{\frac{2}{3}}$

9. $\frac{1}{3}$

10. $\frac{9}{8}$

11. $8\sqrt{3}$

12. A

13. B

EXERCISE 8.2

1. $\frac{\sqrt{2}}{6}+\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$ **2.** $\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$

3. $\frac{21}{2}$ **4.** 4 **5.** 8

6. B **7.** B

Miscellaneous Exercise on Chapter 8

1. (i) $\frac{7}{3}$ (ii) 624.8

2. $\frac{1}{6}$ **3.** $\frac{7}{3}$ **4.** 9 **5.** 4

6. $\frac{8}{3}\frac{a^2}{m^3}$ **7.** 27 **8.** $\frac{3}{2}(\pi-2)$

9. $\frac{ab}{4}(\pi-2)$ **10.** $\frac{5}{6}$ **11.** 2 **12.** $\frac{1}{3}$

13. 7 **14.** $\frac{7}{2}$ **15.** $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)+\frac{1}{3\sqrt{2}}$

16. D **17.** C **18.** C **19.** B

EXERCISE 9.1

1. Order 4; Degree not defined **2.** Order 1; Degree 1

3. Order 2; Degree 1 **4.** Order 2; Degree not defined

5. Order 2; Degree 1 **6.** Order 3; Degree 2

7. Order 3; Degree 1 **8.** Order 1; Degree 1

9. Order 2; Degree 1 **10.** Order 2; Degree 1

11. D **12.** A

EXERCISE 9.2

11. D **12.** D

EXERCISE 9.3

1. $y'' = 0$
2. $xy\,y'' + x\,(y')^2 - y\,y' = 0$
3. $y'' - y' - 6y = 0$
4. $y'' - 4y' + 4y = 0$
5. $y'' - 2y' + 2y = 0$
6. $2xyy' + x^2 = y^2$
7. $xy' - 2y = 0$
8. $xyy'' + x(y')^2 - yy' = 0$
9. $xyy'' + x(y')^2 - yy' = 0$
10. $(x^2 - 9)\,(y')^2 + x^2 = 0$
11. B
12. C

EXERCISE 9.4

1. $y = 2\tan\frac{x}{2} - x + C$
2. $y = 2\sin(x + C)$
3. $y = 1 + Ae^{-x}$
4. $\tan x \tan y = C$
5. $y = \log(e^x + e^{-x}) + C$
6. $\tan^{-1} y = x + \frac{x^3}{3} + C$
7. $y = e^{cx}$
8. $x^{-4} + y^{-4} = C$
9. $y = x\sin^{-1}x + \sqrt{1 - x^2} + C$
10. $\tan y = C(1 - e^x)$
11. $y = \frac{1}{4}\log\left[(x+1)^2(x^2+1)^3\right] - \frac{1}{2}\tan^{-1}x + 1$
12. $y = \frac{1}{2}\log\left(\frac{x^2-1}{x^2}\right) - \frac{1}{2}\log\frac{3}{4}$
13. $\cos\left(\frac{y-2}{x}\right) = a$
14. $y = \sec x$
15. $2y - 1 = e^x(\sin x - \cos x)$
16. $y - x + 2 = \log(x^2(y+2)^2)$
17. $y^2 - x^2 = 4$
18. $(x + 4)^2 = y + 3$
19. $(63t + 27)^{\frac{1}{3}}$
20. 6.93%
21. Rs 1648
22. $\frac{2\log 2}{\log\left(\frac{11}{10}\right)}$
23. A

EXERCISE 9.5

1. $(x-y)^2 = Cx\,e^{\frac{-y}{x}}$
2. $y = x\log|x| + Cx$

3. $\tan^{-1}\left(\frac{y}{x}\right)=\frac{1}{2}\log(x^2+y^2)+C$ **4.** $x^2 + y^2 = Cx$

5. $\frac{1}{2\sqrt{2}}\log\left|\frac{x+\sqrt{2}y}{x-\sqrt{2}y}\right|=\log|x|+C$ **6.** $y+\sqrt{x^2+y^2}=Cx^2$

7. $xy\cos\left|\frac{y}{x}\right| = C$ **8.** $x\left[1-\cos\left(\frac{y}{x}\right)\right]=C\sin\left(\frac{y}{x}\right)$

9. $cy = \log\left|\frac{y}{x}\right|-1$ **10.** $ye^{\frac{x}{y}}+x=C$

11. $\log(x^2 + y^2) + 2\tan^{-1}\frac{y}{x} = \frac{\pi}{2}+\log 2$

12. $y + 2x = 3x^2 y$ **13.** $\cot\left(\frac{y}{x}\right)=\log|ex|$

14. $\cos\left(\frac{y}{x}\right)=\log|ex|$ **15.** $y=\frac{2x}{1-\log|x|}\ (x\neq 0,\ x\neq e)$

16. C **17.** D

EXERCISE 9.6

1. $y = \frac{1}{5}(2\sin x - \cos x) + C\,e^{-2x}$ **2.** $y = e^{-2x} + Ce^{-3x}$

3. $xy=\frac{x^4}{4}+C$ **4.** $y(\sec x + \tan x) = \sec x + \tan x - x + C$

5. $y = (\tan x - 1) + Ce^{-\tan x}$ **6.** $y=\frac{x^2}{16}(4\log|x|-1)+Cx^{-2}$

7. $y\log x=\frac{-2}{x}(1+\log|x|)+C$ **8.** $y=(1+x)^{-1}\ \log|\sin x|+C(1+x^2)^{-1}$

9. $y=\frac{1}{x}-\cot x+\frac{C}{x\sin x}$ **10.** $(x + y + 1) = C\,e^y$

11. $x=\frac{y^2}{3}+\frac{C}{y}$ **12.** $x = 3y^2 + Cy$

13. $y = \cos x - 2\cos^2 x$ **14.** $y(1 + x^2) = \tan^{-1} x - \frac{\pi}{4}$

15. $y = 4\sin^3 x - 2\sin^2 x$ **16.** $x + y + 1 = e^x$

17. $y = 4 - x - 2e^x$ **18.** C **19.** D

Miscellaneous Exercise on Chapter 9

1. (i) Order 2; Degree 1 (ii) Order 1; Degree 3

(iii) Order 4; Degree not defined

3. $y' = \frac{2y^2 - x^2}{4xy}$ **5.** $(x + yy')^2 = (x - y)^2 (1 + (y')^2)$

6. $\sin^{-1} y + \sin^{-1} x = C$ **8.** $\cos y = \frac{\sec x}{\sqrt{2}}$

9. $\tan^{-1} y + \tan^{-1}(e^x) = \frac{\pi}{2}$ **10.** $e^{\frac{x}{y}} = y + C$

11. $\log|x - y| = x + y + 1$ **12.** $ye^{2\sqrt{x}} = (2\sqrt{x} + C)$

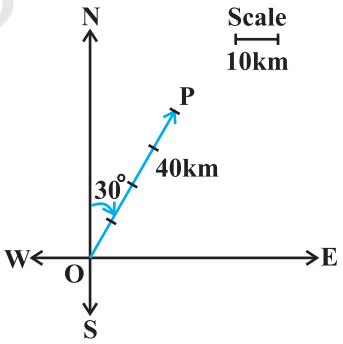
13. $y\sin x = 2x^2 - \frac{\pi^2}{2}$ $(\sin x \neq 0)$ **14.** $y = \log\left|\frac{2x+1}{x+1}\right|$, $x \neq -1$

15. 31250 **16.** C

17. C **18.** C

EXERCISE 10.1

1. In the adjoining figure, the vector $\overrightarrow{OP}$ represents the required displacement.



2. (i) scalar (ii) vector (iii) scalar (iv) scalar (v) scalar (vi) vector

3. (i) scalar (ii) scalar (iii) vector (iv) vector (v) scalar

4. (i) Vectors $\vec{a}$ and $\vec{b}$ are coinitial

(ii) Vectors $\vec{b}$ and $\vec{d}$ are equal

(iii) Vectors $\vec{a}$ and $\vec{c}$ are collinear but not equal

5. (i) True (ii) False (iii) False (iv) False

EXERCISE 10.2

1. $|\vec{a}|=\sqrt{3}, |\vec{b}|=\sqrt{62}, |\vec{c}|=1$

2. An infinite number of possible answers.

3. An infinite number of possible answers.

4. $x = 2, y = 3$

5. -7 and 6; $-7\hat{i}$ and $6\hat{j}$

6. $-4\hat{j}-\hat{k}$

7. $\frac{1}{\sqrt{6}}\hat{i}+\frac{1}{\sqrt{6}}\hat{j}+\frac{2}{\sqrt{6}}\hat{k}$

8. $\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}$

9. $\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k}$

10. $\frac{40}{\sqrt{30}}\hat{i}-\frac{8}{\sqrt{30}}\hat{j}+\frac{16}{\sqrt{30}}\hat{k}$

12. $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

13. $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

15. (i) $-\frac{1}{3}\hat{i}+\frac{4}{3}\hat{j}+\frac{1}{3}\hat{k}$ (ii) $-3\hat{i}+3\hat{k}$

16. $3\hat{i}+2\hat{j}+\hat{k}$

18. (C)

19. (B), (C), (D)

EXERCISE 10.3

1. $\frac{\pi}{4}$

2. $\cos^{-1}\left(\frac{5}{7}\right)$

3. 0

4. $\frac{60}{\sqrt{114}}$

6. $\frac{16\sqrt{2}}{3\sqrt{7}}, \frac{2\sqrt{2}}{3\sqrt{7}}$

7. $6|\vec{a}|^2+11\vec{a}.\vec{b}-35|\vec{b}|^2$

8. $|\vec{a}|=1, |\vec{b}|=1$

9. $\sqrt{13}$

10. 8

12. Vector $\vec{b}$ can be any vector **13.** $\frac{-3}{2}$

14. Take any two non-zero perpendicular vectors $\vec{a}$ and $\vec{b}$

15. $\cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$ **18.** (D)

EXERCISE 10.4

1. $19\sqrt{2}$ **2.** $\pm\frac{2}{3}\hat{i} \mp \frac{2}{3}\hat{j} \mp \frac{1}{3}\hat{k}$ **3.** $\frac{\pi}{3}; \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}$

5. $3, \frac{27}{2}$ **6.** Either $|\vec{a}|=0$ or $|\vec{b}|=0$

8. No; take any two nonzero collinear vectors

9. $\frac{\sqrt{61}}{2}$ **10.** $15\sqrt{2}$ **11.** (B) **12.** (C)

Miscellaneous Exercise on Chapter 10

1. $\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

2. $x_2 - x_1, y_2 - y_1, z_2 - z_1; \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

3. $\frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

4. No; take $\vec{a}$, $\vec{b}$ and $\vec{c}$ to represent the sides of a triangle.

5. $\pm\frac{1}{\sqrt{3}}$ **6.** $\frac{3}{2}\sqrt{10}\,\hat{i} + \frac{\sqrt{10}}{2}\hat{j}$ **7.** $\frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}$

8. 2 : 3 **9.** $3\vec{a} + 5\vec{b}$ **10.** $\frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$; $11\sqrt{5}$

12. $\frac{1}{3}(160\hat{i} - 5\hat{j} + 70\hat{k})$ **13.** $\lambda = 1$ **16.** (B)

17. (D) **18.** (C) **19.** (B)

EXERCISE 11.1

1. $0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ **2.** $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$ **3.** $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$

5. $\frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{17}$; $\frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$; $\frac{4}{\sqrt{42}}, \frac{5}{\sqrt{42}}, \frac{-1}{\sqrt{42}}$

EXERCISE 11.2

4. $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$, where λ is a real number

5. $\vec{r} = 2\hat{i} - \hat{j} + 4\hat{k} + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and cartesian form is

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$$

6. $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$

7. $\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

8. Vector equation of the line: $\vec{r} = \lambda(5\hat{i} - 2\hat{j} + 3\hat{k})$;

Cartesian equation of the line: $\frac{x}{5} = \frac{y}{-2} = \frac{z}{3}$

9. Vector equation of the line: $\vec{r} = 3\hat{i} - 2\hat{j} - 5\hat{k} + \lambda(11\hat{k})$

Cartesian equation of the line: $\frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{11}$

10. (i) $\theta = \cos^{-1}\left(\frac{19}{21}\right)$ (ii) $\theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$

11. (i) $\theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$ (ii) $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

12. $p = \frac{70}{11}$ **14.** $\frac{3\sqrt{2}}{2}$ **15.** $2\sqrt{29}$

16. $\frac{3}{\sqrt{19}}$ **17.** $\frac{8}{\sqrt{29}}$

EXERCISE 11.3

1. (a) $0, 0, 1; 2$ (b) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}$

(c) $\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}; \frac{5}{\sqrt{14}}$ (d) $0, 1, 0; \frac{8}{5}$

2. $\vec{r} \cdot \left(\frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{70}} \right) = 7$

3. (a) $x + y - z = 2$ (b) $2x + 3y - 4z = 1$

(c) $(s - 2t)x + (3 - t)y + (2s + t)z = 15$

4. (a) $\left(\frac{24}{29}, \frac{36}{29}, \frac{48}{29} \right)$ (b) $\left(0, \frac{18}{25}, \frac{24}{25} \right)$

(c) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right)$ (d) $\left(0, \frac{-8}{5}, 0 \right)$

5. (a) $[\vec{r} - (\hat{i} - 2\hat{k})] \cdot (\hat{i} + \hat{j} - \hat{k}) = 0; \quad x + y - z = 3$

(b) $[\vec{r} - (\hat{i} + 4\hat{j} + 6\hat{k})] \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0; \quad x - 2y + z + 1 = 0$

6. (a) The points are collinear. There will be infinite number of planes passing through the given points.

(b) $2x + 3y - 3z = 5$

7. $\frac{5}{2}, 5, -5$ **8.** $y = 3$ **9.** $7x - 5y + 4z - 8 = 0$

10. $\vec{r} \cdot (38\hat{i} + 68\hat{j} + 3\hat{k}) = 153$ **11.** $x - z + 2 = 0$

12. $\cos^{-1} \frac{15}{\sqrt{731}}$

13. (a) $\cos^{-1}\left(\frac{2}{5}\right)$ (b) The planes are perpendicular

(c) The planes are parallel (d) The planes are parallel

(e) 45°

14. (a) $\frac{3}{13}$ (b) $\frac{13}{3}$

(c) 3 (d) 2

Miscellaneous Exercise on Chapter 11

3. $90°$ **4.** $\frac{x}{1}=\frac{y}{0}=\frac{z}{0}$ **5.** 0^{o}

6. $k=\frac{-10}{7}$ **7.** $\vec{r}=\hat{i}+2\,\hat{j}+3\,\hat{k}+\lambda\,(\hat{i}+2\,\hat{j}-5\,\hat{k})$

8. $x+y+z=a+b+c$ **9.** 9

10. $\left(0,\frac{17}{2},\frac{-13}{2}\right)$ **11.** $\left(\frac{17}{3},0,\frac{23}{3}\right)$ **12.** $(1,-2,7)$

13. $7x-8y+3z+25=0$ **14.** $p=\frac{3}{2}$ or $\frac{11}{6}$ or $\frac{7}{3}$

15. $y-3z+6=0$ **16.** $x+2y-3z-14=0$

17. $33x+45y+50z-41=0$ **18.** 13

19. $\vec{r}=\hat{i}+2\,\hat{j}+3\hat{k}+\lambda\,(-3\hat{i}+5\,\hat{j}+4\hat{k})$

20. $\vec{r}=\hat{i}+2\,\hat{j}-4\hat{k}+\lambda(2\,\hat{i}+3\hat{j}+6\hat{k})$ **22.** D

23. B

EXERCISE 12.1

1. Maximum Z = 16 at (0, 4)

2. Minimum Z = – 12 at (4, 0)

3. Maximum Z = $\frac{235}{19}$ at $\left(\frac{20}{19},\frac{45}{19}\right)$

4. Minimum Z = 7 at $\left(\frac{3}{2},\frac{1}{2}\right)$

5. Maximum Z = 18 at (4, 3)

6. Minimum Z = 6 at all the points on the line segment joining the points (6, 0) and (0, 3).

7. Minimum Z = 300 at (60, 0);

Maximum Z = 600 at all the points on the line segment joining the points (120, 0) and (60, 30).

8. Minimum Z = 100 at all the points on the line segment joining the points (0, 50) and (20, 40);

Maximum Z = 400 at (0, 200)

9. Z has no maximum value

10. No feasible region, hence no maximum value of Z.

EXERCISE 12.2

1. Minimum cost = Rs 160 at all points lying on segment joining $\frac{8}{3}, 0$ and $2, \frac{1}{2}$.

2. Maximum number of cakes = 30 of kind one and 10 cakes of another kind.

3. (i) 4 tennis rackets and 12 cricket bats

(ii) Maximum profit = Rs 200

4. 3 packages of nuts and 3 packages of bolts; Maximum profit = Rs 73.50.

5. 30 packages of screws A and 20 packages of screws B; Maximum profit = Rs 410

6. 4 Pedestal lamps and 4 wooden shades; Maximum profit = Rs 32

7. 8 Souvenir of types A and 20 of Souvenir of type B; Maximum profit = Rs 160.

8. 200 units of desktop model and 50 units of portable model; Maximum profit = Rs 1150000.

9. Minimise $Z = 4x + 6y$

subject to $3x + 6y \geq 80$, $4x + 3y \geq 100$, $x \geq 0$ and $y \geq 0$, where x and y denote the number of units of food F_1 and food F_2 respectively; Minimum cost = Rs 104

10. 100 kg of fertiliser F_1 and 80 kg of fertiliser F_2; Minimum cost = Rs 1000

11. (D)

Miscellaneous Exercise on Chapter 12

1. 40 packets of food P and 15 packets of food Q; Maximum amount of vitamin A = 285 units.

2. 3 bags of brand P and 6 bags of brand Q; Minimum cost of the mixture = Rs 1950

3. Least cost of the mixture is Rs 112 (2 kg of Food X and 4 kg of food Y).

5. 40 tickets of executive class and 160 tickets of economy class; Maximum profit = Rs 136000.

6. From A : 10,50, 40 units; From B: 50,0,0 units to D, E and F respectively and minimum cost = Rs 510

7. From A: 500, 3000 and 3500 litres; From B: 4000, 0, 0 litres to D, E and F respectively; Minimum cost = Rs 4400

8. 40 bags of brand P and 100 bags of brand Q; Minimum amount of nitrogen = 470 kg.

9. 140 bags of brand P and 50 bags of brand Q; Maximum amount of nitrogen = 595 kg.

10. 800 dolls of type A and 400 dolls of type B; Maximum profit = Rs 16000

EXERCISE 13.1

1. $P(E|F)=\frac{2}{3}, P(F|E)=\frac{1}{3}$ **2.** $P(A|B)=\frac{16}{25}$

3. (i) 0.32 (ii) 0.64 (iii) 0.98

4. $\frac{11}{26}$

5. (i) $\frac{4}{11}$ (ii) $\frac{4}{5}$ (iii) $\frac{2}{3}$

6. (i) $\frac{1}{2}$ (ii) $\frac{3}{7}$ (iii) $\frac{6}{7}$

7. (i) 1 (ii) 0

8. $\frac{1}{6}$ **9.** 1 **10.** (a) $\frac{1}{3}$, (b) $\frac{1}{9}$

11. (i) $\frac{1}{2}, \frac{1}{3}$ (ii) $\frac{1}{2}, \frac{2}{3}$ (iii) $\frac{3}{4}, \frac{1}{4}$

12. (i) $\frac{1}{2}$ (ii) $\frac{1}{3}$ **13.** $\frac{5}{9}$

14. $\frac{1}{15}$ **15.** 0 **16.** C **17.** D

EXERCISE 13.2

1. $\frac{3}{25}$ **2.** $\frac{25}{102}$ **3.** $\frac{44}{91}$

4. A and B are independent **5.** A and B are not independent

6. E and F are not independent

7. (i) $p = \frac{1}{10}$ (ii) $p = \frac{1}{5}$

8. (i) 0.12 (ii) 0.58 (iii) 0.3 (iv) 0.4

9. $\frac{3}{8}$ **10.** A and B are not independent

11. (i) 0.18 (ii) 0.12 (iii) 0.72 (iv) 0.28

12. $\frac{7}{8}$ **13.** (i) $\frac{16}{81}$, (ii) $\frac{20}{81}$, (iii) $\frac{40}{81}$

14. (i) $\frac{2}{3}$, (ii) $\frac{1}{2}$ **15.** (i), (ii) **16.** (a) $\frac{1}{5}$, (b) $\frac{1}{3}$, (c) $\frac{1}{2}$

17. D **18.** B

EXERCISE 13.3

1. $\frac{1}{2}$ **2.** $\frac{2}{3}$ **3.** $\frac{9}{13}$ **4.** $\frac{12}{13}$

5. $\frac{22}{133}$ **6.** $\frac{4}{9}$ **7.** $\frac{1}{52}$ **8.** $\frac{1}{4}$

9. $\frac{2}{9}$ **10.** $\frac{8}{11}$ **11.** $\frac{5}{34}$ **12.** $\frac{11}{50}$

13. A **14.** C

EXERCISE 13.4

1. (ii), (iii) and (iv) **2.** X = 0, 1, 2; yes **3.** X = 6, 4, 2, 0

4. (i)

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(ii)

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(iii)

X	0	1	2	3	4
P(X)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

5. (i)

X	0	1	2
P(X)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(ii)

X	0	1
P(X)	$\frac{25}{36}$	$\frac{11}{36}$

6.

X	0	1	2	3	4
P(X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

7.

X	0	1	2
P(X)	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

8. (i) $k = \frac{1}{10}$ (ii) $P(X < 3) = \frac{3}{10}$ (iii) $P(X > 6) = \frac{17}{100}$

(iv) $P(0 < X < 3) = \frac{3}{10}$

9. (a) $k = \frac{1}{6}$ (b) $P(X < 2) = \frac{1}{2}, P(X \le 2) = 1, P(X \ge 2) = \frac{1}{2}$

10. 1.5 **11.** $\frac{1}{3}$ **12.** $\frac{14}{3}$

13. Var(X) = 5.833, S.D = 2.415

14.

X	14	15	16	17	18	19	20	21
P(X)	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Mean = 17.53, Var(X) = 4.78 and S.D(X) = 2.19

15. E(X) = 0.7 and Var (X) = 0.21 **16.** B **17.** D

EXERCISE 13.5

1. (i) $\frac{3}{32}$ (ii) $\frac{7}{64}$ (iii) $\frac{63}{64}$

2. $\frac{25}{216}$ **3.** $\left(\frac{29}{20}\right)\left(\frac{19}{20}\right)^9$

4. (i) $\frac{1}{1024}$ (ii) $\frac{45}{512}$ (iii) $\frac{243}{1024}$

5. (i) $(0.95)^5$ (ii) $(0.95)^4 \times 1.2$ (iii) $1 - (0.95)^4 \times 1.2$
(iv) $1 - (0.95)^5$

6. $\left(\frac{9}{10}\right)^4$ **7.** $\left(\frac{1}{2}\right)^{20}\left[{}^{20}C_{12} + {}^{20}C_{13} + \ldots + {}^{20}C_{20}\right]$

9. $\frac{11}{243}$

10. (a) $1-\left(\frac{99}{100}\right)^{50}$ (b) $\frac{1}{2}\left(\frac{99}{100}\right)^{49}$ (c) $1-\frac{149}{100}\left(\frac{99}{100}\right)^{49}$

11. $\frac{7}{12}\left(\frac{5}{6}\right)^5$ **12.** $\frac{35}{18}\left(\frac{5}{6}\right)^4$ **13.** $\frac{22\times 9^3}{10^{11}}$

14. C **15.** A

Miscellaneous Exercise on Chapter 13

1. (i) 1 (ii) 0

2. (i) $\frac{1}{3}$ (ii) $\frac{1}{2}$

3. $\frac{20}{21}$

4. $1-\sum_{r=7}^{10} {}^{10}C_r (0.9)^r (0.1)^{10-r}$

5. (i) $\left(\frac{2}{5}\right)^6$ (ii) $7\left(\frac{2}{5}\right)^4$ (iii) $1-\left(\frac{2}{5}\right)^6$ (iv) $\frac{864}{3125}$

6. $\frac{5^{10}}{2\times 6^{9}}$ **7.** $\frac{625}{23328}$ **8.** $\frac{2}{7}$

9. $\frac{31}{9}\left(\frac{2}{3}\right)^{4}$ **10.** $n \geq 4$ **11.** $\frac{-91}{54}$

12. $\frac{1}{15}, \frac{2}{5}, \frac{8}{15}$ **13.** $\frac{14}{29}$ **14.** $\frac{3}{16}$

15. (i) 0.5 (ii) 0.05 **16.** $\frac{16}{31}$

17. A **18.** C **19.** B