CHAPTER 5

Choice Under Uncertainty

So far we have assumed that prices, incomes, and other variables are known with certainty. However, many of the choices that people make involve considerable uncertainty. For example, most people borrow to finance large purchases, such as a house or a college education, and plan to pay for the purchase out of future income. But for most of us, future incomes are uncertain. Our earnings can go up or down; we can be promoted, demoted, or even lose our jobs. Or if we delay buying a house or investing in a college education, we risk having its price rise in real terms, making it less affordable. How should we take these uncertainties into account when making major consumption.or investment decisions?

Sometimes we must choose how much risk to bear. What, for example, should you do with your savings? Should you invest your money in something safe, such as a savings account, or something riskier but potentially more lucrative, such as the stock market? Another example is the choice of a job or even a career. Is it better to work for a large, stable company where job security is good but the chances for advancement are limited, or to join (or form) a new venture, which offers less job security but more opportunity for advancement?

To answer questions such as these, we must be able to quantify risk so we can compare the riskiness of alternative choices. We therefore begin this chapter by discussing measures of risk. Afterwards, we will examine people's preferences toward risk. (Most people find risk undesirable, but some people find it more undesirable than others.) Next, we will see how people can deal with risk. Sometimes risk can be reduced-by diversification, by buying insurance or by investing in additional information. In other situations (e.g., when investing in stocks or bonds), people must choose the amount of risk they wish to bear.

5.1 *Describing Risk*

To describe risk quantitatively, we need to know all the possible outcomes of a particular action and the likelihood that each outcome will occur.[1] Suppose, for example, that you are considering investing in a company that is exploring for offshore oil. If the exploration effort is successful, the company's stock will increase from $30 to $40 a share; if not, it will fall to $20 a share. Thus, there are two possible future outcomes, a $40 per share price and a $20 per share price.

Probability

Probability refers to the likelihood that an outcome will occur. In our example, the probability that the oil exploration project is successful might be ¼, and the probability that it is unsuccessful ¾. Probability is a difficult concept to formalize because its interpretation can depend on the nature of the uncertain events and on the beliefs of the people involved. One *objective* interpretation of probability relies on the frequency with which certain events tend to occur. Suppose we know that of the last 100 offshore oil explorations 25 have succeeded and 75 have failed. Then the probability of success of ¼ is objective because it is based directly on the frequency of similar experiences.

But what if there are no similar past experiences to help measure probability? In these cases objective measures of probability cannot be deduced, and a more *subjective* measure is needed. Subjective probability is the perception that an outcome will occur. This perception may be based on a person's judgment or experience, but not necessarily on the frequency with which a particular outcome has actually occurred in the past. When probabilities are subjectively determined, different people may attach different probabilities to different outcomes and thereby-make different choices. For example, if the search for oil were to take place in an area where no previous searches had ever occurred, I might attach a higher subjective probability than you to the chance that the. project will succeed because I know more about the project, or because I have a better understanding of the oil business and can therefore make better use of our common information. Either different information or different abilities to process the same information can explain why subjective probabilities vary among individuals.

[1] Some people distinguish between uncertainty and risk along the lines suggested by the economist Frank Knight some 60 years ago. *Uncertainty* can refer to situations in which many outcomes are possible but their likelihoods are unknown. *Risk* then refers to situations in which we can list all possible outcomes, and we know the likelihood that each outcome will occur. We will always refer to risky situations but will simplify the discussion by using uncertainty and risk interchangeably.

Whatever the interpretation of probability, it is used in calculating two important measures that help us describe and compare risky choices. One measure tells us the *expected value* and the other the *variability* of the possible outcomes.

Expected Value

The *expected value* associated with an uncertain situation is a weighted average of the *payoffs or* values associated with all possible outcomes, with the probabilities of each outcome used as weights. The expected value measures the *central tendency,* that is, the average payoff. Our offshore oil exploration example has two possible outcomes: Success yields a payoff of $40 per share, while Failure yields a payoff of $20 per share. Denoting "probability of" by Pr, the expected value in this case is given by

$$\text{Expected Value} = \Pr(\text{Success})(\$40/\text{share}) + \Pr(\text{Failure})(\$20/\text{share})$$
$$= (1/4)(\$40/\text{share}) + (3/4)(\$20/\text{share}) = \$25/\text{share}$$

More generally, if there are two possible outcomes having payoffs X_1 and X_2 and the probabilities of each outcome are given by Pr_1 and Pr_2, then the expected value is[2] $E(X) = Pr_1X_1 + Pr_2X_2$.

Variability

Suppose you are choosing between two part-time sales jobs that have the same expected income ($1500). The first job is based entirely on commission-the income earned depends on how much you sell. The second job is salaried. There are two equally likely incomes under the first job-$2000 for a good sales effort and $1000 for one that is only modestly successful. The second job pays $1510 most of the time, but you would earn $510 in severance pay if the company goes out of business. Table 5.1 summarizes these possible outcomes, their payoffs, and their probabilities.

TABLE 5.1 Income from Sales Jobs

	Outcome 1		Outcome 2	
	Probability	Income ($)	Probability	Income ($)
Job 1: Commission	.5	2000	.5	1000
Job 2: Fixed salary	.99	1510	.01	510

[2] When there are n possible outcomes, the expected value becomes $E(X) = Pr_1X_1 + Pr_2X_2 + \cdots + Pr_nX_n$.

TABLE 5.2 Deviations from Expected Income ($)

	Outcome 1	Deviation	Outcome 2	Deviation
Job1	$2000	$500	$1000	$500
Job 2	1510	10	510	990

Note that the two jobs have the same expected income because .5($2000) + .5($1000) = .99 ($1510) + .01 ($510) = $1500. But the *variability* of the possible payoffs is different for the two jobs. This variability can be measured by recognizing that large differences (whether positive or negative) between actual payoffs and the expected payoff, called *deviations,* signal greater risk. Table 5.2 gives the deviations of actual incomes from the expected income for the example of the two sales jobs.

In the first commission job, the average deviation is $500, which is obtained by weighting each deviation by the probability that each outcome occurs. Thus,

Average Deviation = .5($500) + .5($500) = $500

For the second fixed-salary job, the average deviation is:

Average Deviation = .99($10) + .01($990) = $19.80

The first job is thus substantially more risky than the second because its average deviation of $500 is much greater than the average deviation of $19.80 for the second job.

In practice one usually encounters two closely related but slightly different measures of variability. The *variance* is the average of the *squares* of the deviations of the payoffs associated with each outcome from their expected value. The *standard deviation* is the square root of the variance. Table 5.3 gives some of the relevant calculations for our example.

The average of the squared deviations under Job 1 is given by

Variance = 5($250,000) + 5($250,000) = $250,000

The standard deviation is therefore equal to the square root of $250,000, or $500. Similarly, the average of the squared deviations under Job 2 is given by

TABLE 5.3 Calculating Variance ($)

	Outcome 1	Deviation Squared	Outcome 2	Deviation Squared	Variance
Jobi	$2000	$250,000	$1000	$250,000	$250,000
Job 2	1510	100	510	980,100	9,900

$$\text{Variance} = .99(\$100) + .01(\$980{,}100) = \$9900$$

The standard deviation is the square root of $9,900, or $99.50. Whether we use variance or standard deviation to measure risk (it's really a matter of convenience-both provide the same ranking of risky choices), the second job is substantially less risky than the first. Both the variance and the standard deviation of the incomes earned are lower.[3]

The concept of variance applies equally well when there are many outcomes rather than just two. Suppose, for example, that the first job yields incomes ranging from $1000 to $2000 in increments of $100 that are all equally likely. The second job yields incomes from $1300 to $1700 (again in increments of $100) that are also all equally likely. Figure 5.1 shows the alternatives graphically. (If there had been only two outcomes that were equally probable, then the figure would be drawn as two vertical lines, each with a height of 0.5.)

You can see from Figure 5.1 that the first job is riskier than the second. The "spread" of possible payoffs for the first job is much greater than the spread of payoffs for the second. And the variance of the payoffs associated with the first job is greater than the variance associated with the second.

In this particular example, all payoffs are equally likely, so the curves describing the payoffs under each job are flat. But in many cases, some payoffs

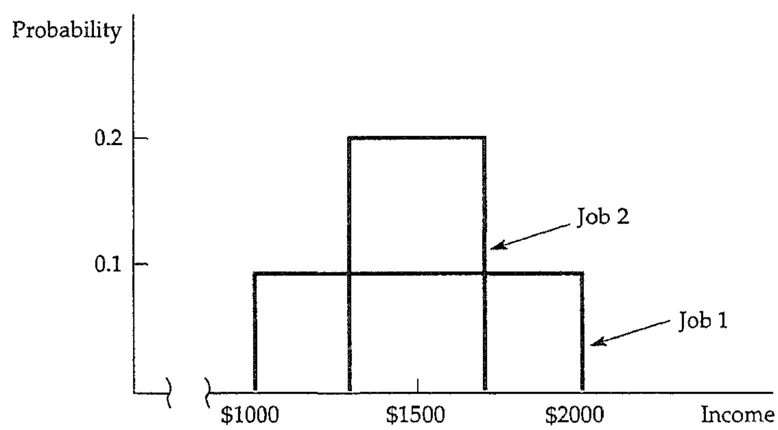

FIGURE 5.1 Outcome Probabilities for Two Jobs. The distribution of payoffs associated with Job 1 has a greater spread and a greater variance than the distribution of payoffs associated with Job 2. Both distributions are flat because all outcomes are equally likely.

[3] In general, when there are two outcomes with payoffs X_1 and X_2, each occurring with probability Pr_1 and Pr_2, and $E(X)$ is the expected value of the outcomes, the variance is given by $\sigma^2 = Pr_1[(X_1 - E(X))^2] + Pr_2[(X_2 - E(X))^2]$. The standard deviation, which is the square root of the variance, is written as σ.

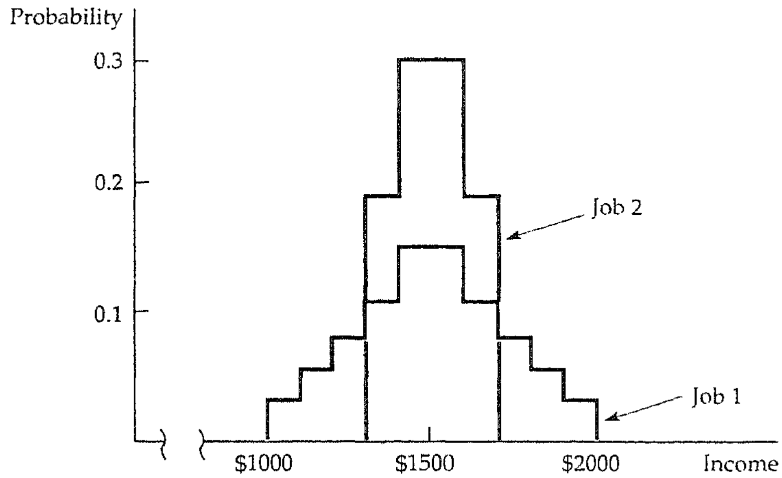

FIGURE 5.2 Unequal Probability Outcomes. The distribution of payoffs assodated with Job 1 has a greater spread and a greater variance than the distribution of payoffs associated with Job 2. Both distributions are peaked because the extreme payoffs are less likely than those near the middle of the distribution.

are more likely than others. Figure 5.2 shows a situation in which the more extreme payoffs are the least likely. Again, the salary from Job 1 has a greater variance. From this point on we will use the variance of payoffs to measure the .variability of risky situations.

Decision Making

Suppose you are choosing between the two sales jobs described in our original example. Which job would you take? If you dislike risk, you will take the second job. It offers the same expected income as the first but with less risk. But suppose we add $100 to each of the payoffs in the first job, so that the expected payoff increases from $1500 to $1600. Table 5.4 gives the new earning and the squared deviations.

TABLE 5.4 Incomes from Sales Jobs—Modified ($)

	Outcome 1	Deviation Squared	Outcome 2	Deviation Squared
Job1	$2100	$250,000	$1100	$250,000
Job2	1510	100	510	980,100

The jobs can then be described as follows:

Job 1:	Expected Income = \$1600	Variance = \$250,000
Job 2:	Expected Income = \$1500	Variance = \$9900

Job 1 offers a higher expected income but is substantially riskier than Job 2. Which job is preferred depends on you. An aggressive entrepreneur may opt for the higher expected income and higher variance, but a more conservative person might opt for the second. To see how people might decide between incomes that differ in both expected value and in riskiness, we need to develop our theory of consumer choice further,

5.2 Preferences Toward Risk

We used a job example to describe how people might evaluate risky outcomes, but the principles apply equally well to other choices. In this section we concentrate on consumer choices generally, and on the utility that consumers obtain from choosing among risky alternatives. To simplify things, we'll consider the consumption of a single commodity-the consumer's income, or more appropriately, the market basket that the income can buy. We assume that consumers know all probabilities, and (for much of this section) that payoffs are now measured in terms of utility rather than dollars.

Figure 5.3a shows how we can describe a woman's preferences toward risk. The curve *OE*, which gives her utility function, tells us the level of utility (on the vertical axis) that she can attain for each level of income (measured in thousands of dollars on the horizontal axis). The level of utility increases from 10 to 16 to 18 as income increases from \$10,000 to \$20,000 to \$30,000. But note that marginal utility is diminishing, falling from 10 when income increases from 0 to \$10,000, to 6 when income increases from \$10,000 to \$20,000, and "to 2 when income increases from \$20,000 to \$30,000.

Now suppose she has an income of \$15,000 and is considering a new but risky sales job that will either double her income to \$30,000 or cause it to fall to \$10,000. Each possibility has a probability of .5. As Figure 5.3a shows, the utility level associated with an income of \$10,000 is 10 (at point *A*), and the utility associated with a level of income of \$30,000 is 18 (at *E*). The risky job must be compared with the current job, for which the utility is 13 (at *B*).

To evaluate the new job, she can calculate the expected value of the resulting income. Because we are measuring value in terms of the woman's utility, we must calculate the *expected utility E(u)* she can obtain. The expected utility is the sum of the utilities associated with all possible outcomes, weighted by the probability that each outcome will occur. In this case, expected utility is

$$E(u) = (\tfrac{1}{2})u(\$10{,}000) + (\tfrac{1}{2})u(\$30{,}000) = 0.5(10) + 0.5(18) = 14$$

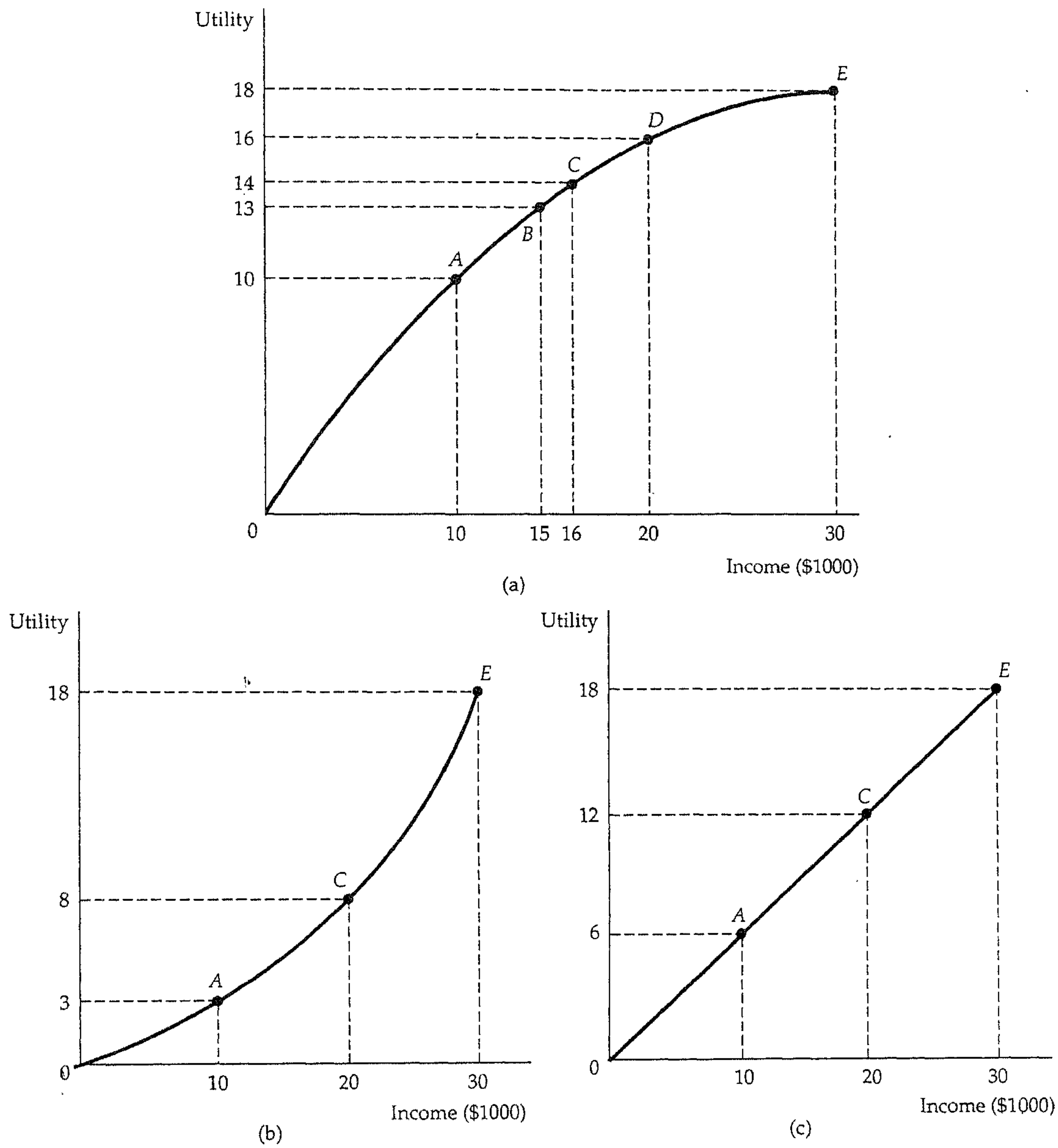

FIGURE 5.3 Risk Aversion. People differ in their preferences toward risk. In (a) a consumer's marginal utility diminishes as income increases. The consumer is risk averse because she would prefer a certain income of $20/000 (with a utility of 16) to a gamble with a .5 probability of $10,000'and a .5 probability of $30/000 (and expected utility of 14). In (b) the consumer is risk loving, because she would prefer the same gamble (with expected utility of 10.5) to the certain income (with a utility of 8). Finally, in (c) the consumer is risk neutral and is indifferent between certain events and uncertain events with the same expected income.

The new risky job is thus preferred to the original job because the expected utility of 14 is greater than the original utility of 13.

The old job involved no risk-it guaranteed an-income of $15,000 and a utility level of 13. The new job is risky, but it offers both a higher expected income ($20,000) and, more important, a higher expected utility. If the woman wished to increase her expected utility, she would take the risky job,

Different Preferences Toward Risk

People differ in their willingness to bear risk. Some are risk averse, some risk loving, and some risk neutral. A person who prefers a certain given income to a risky job with the same expected income is described as being *risk averse.* (Such a person has a diminishing marginal utility of income.) Risk aversion is the most common attitude toward risk. To see that most people are risk averse most of the time, note the vast number of risks that people insure against. Most people not only buy life insurance, health insurance, and car insurance, but also seek occupations with relatively stable wages.

Figure 5.3a applies to a woman who is risk averse. Suppose she can have a certain income of $20,000, or a job yielding an income of $30,000 with probability .5 and an income of $10,000 with probability .5 (so that the expected income is $20,000). As we saw, the expected utility of the uncertain income is 14, an average of the utility at point *A* (10) and the utility at *E* (18), and is shown by *C*. Now we can compare the expected utility associated with the risky job to the utility generated if $20,000 were earned without risk. This utility level, 16, is given by *D* in Figure 5.3a. It is clearly greater than the expected utility associated with the risky job.

A person who is *risk neutral* is indifferent between a certain income and an uncertain income with the same expected value. In Figure 5.3c the utility associated with a job generating .an income of either $10,000 or $30,000 with equal probability is 12, as is the utility of receiving a certain income of $20,000.[4]

Figure 5.3b shows the third possibility-*risk loving.* In this case the expected utility of an uncertain income that will be either $10,000 with probability .5 or $30,000 with probability .5 is *higher* than the utility associated with a certain income of $20,000. Numerically,

$$E(u) = .5u(\$10{,}000) + .5u(\$30{,}000) = .5(3) + .5(18) = 10.5 > u(\$20{,}000) = 8$$

The primary evidence for risk loving is that many people enjoy gambling. Some criminologists might describe criminals as risk lovers, especially when a

[4] When people are risk neutral, the marginal utility of income is constant, so the income they earn can be used as an indicator of well-being. A government policy that doubled people's incomes would then also double their utility. At the same time, government policies that alter the risks that people face, without changing their expected incomes, would not affect their well-being. Risk neutrality allows one to avoid the complications that might be associated with the effects of governmental actions on the riskiness of outcomes.

robbery is committed that has a high prospect of apprehension and punishment. These special cases aside, few people are risk loving, at least with respect to major purchases or large amounts of income or wealth.[5]

The *risk premium* is the amount of money that a risk-averse person would pay to avoid taking a risk. The magnitude of the risk premium depends in general on the risky alternatives that the person faces. To determine the risk premium, we have reproduced the utility function of Figure 5.3a in Figure 5.4. Recall that an expected utility of 14 is achieved by a woman who is going to take a risky job with an expected income of $20,000. This is shown graphically by drawing a horizontal line to the vertical axis from point *F*, which bisects straight line *AE* (thus representing an average of $10,000 and $30,000). But the utility level of 14 can also be achieved if the woman has a *certain* income of $16,000, as shown by dropping a vertical line from point *C*. Thus, the risk premium of $4000, given by line segment *CF*, is the amount of income ($20,000 minus $16,000) she would give up to leave her indifferent between the risky job and the safe one.

How risk averse a person is depends on the nature of the risk and on the person's income. Generally, risk-averse people prefer risks involving a smaller

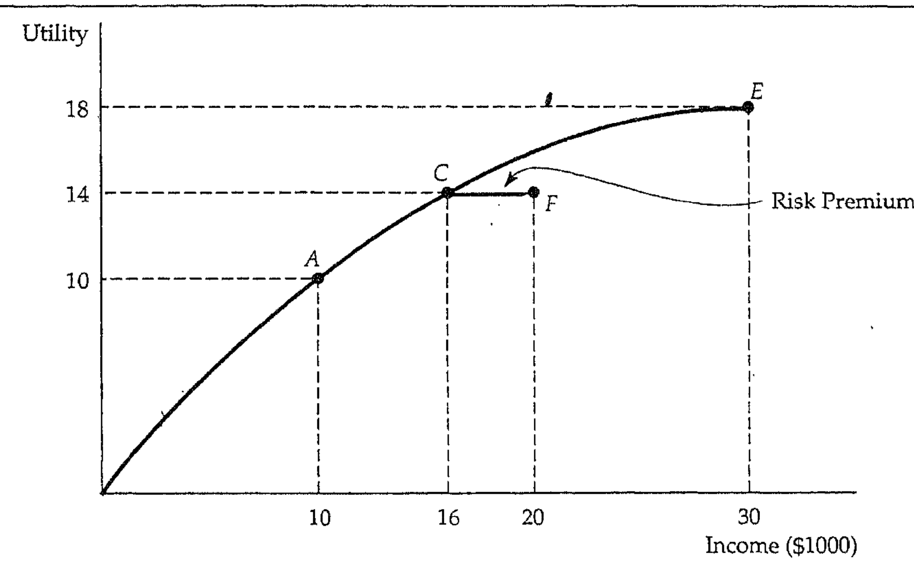

FIGURE 5.4 Risk Premium. The risk premium, CF, measures me amount of income an individual would give up to leave her indifferent between a risky choice and a certain one. Here, the risk premium is $4000 because a certain income of $16,000 gives her the same expected utility (14) as the uncertain income that has an expected value of $20,000.

[5] People may be averse to some risks and act like risk lovers with respect to others. This issue was treated by Milton Friedman and Leonard J. Savage in "The Utility Analysis of Choices Involving Risk," *Journal of Political Economy* (1948): 279-304.

variability of outcomes. We saw that when there are two outcomes, *an* income of \$10,000 and an income of \$30,000, the risk premium is \$4000. Now consider a second risky job, involving a .5 probability of receiving an income of \$40,000 and a utility level of 20 and a .5 probability of getting an income of \$0. The expected income is again \$20,000, but the expected utility is only 10:

$$\text{Expected utility} = .5u(\$0) + .5u(\$40{,}000) = 0 + .5(20) = 10$$

Since the utility of having a certain income of \$20/000 is 16, the woman loses 6 units of utility if she is required to accept the job. The risk premium in this case is equal to \$10,000 because the utility of a certain income of \$10,000 is 10. She can afford to give up \$10,000 of her \$20,000 expected income to have the certain income of \$10,000 and will have the same level of expected utility. Thus the greater the variability, the more a person is willing to pay to avoid the risky situation.

EXAMPLE 5.1 BUSINESS EXECUTIVES AND THE CHOICE OF RISK

Are business executives more risk loving than most people? When they are presented with alternative strategies, some risky, some safe, which do they choose? In one study, 464 business executives were asked to respond to a questionnaire that described risky situations that the executive might face as the vice-president of a hypothetical company.[6] In the in-basket were four risky items, each of which had a given probability of a favorable and an unfavorable outcome. The payoffs and probabilities were chosen so that each item had the same expected value. In increasing order of the risk involved (as measured by the difference between the favorable and unfavorable outcomes), the four items were (1) a lawsuit involving a patent violation, (2) a customer threat concerning the supplying of a competitor, (3) a union dispute, and (4) a joint venture with a competitor. The executives were asked a series of questions to learn how much they were willing to take or avoid risks. Thus, in some situations executives could opt to delay a choice, to collect information, to bargain, or to delegate a decision, so as to avoid taking risks or to modify the risks that they would take later.

The study found that executives vary substantially in their preferences toward risk. Roughly 20 percent of those answering indicated that they were relatively neutral toward risk, while 40 percent opted for the more risky alternatives, and 20 percent were clearly risk averse (20 percent did not respond). More important, executives (including those who chose risky alternatives) made substantial efforts to reduce or eliminate risk, usually by delaying decisions and by collecting more information.

In general, risk can arise where the expected gain is either positive (e.g., a chance for a large reward versus a small one) or negative (e.g., a chance for a

[6] This example is based on Kenneth R. MacCrimmon and Donald A. Wehrung, "The Risk In-Basket," *Journal of Business 57* (1984): 367-387.

large loss or for no loss). The study found that executives differ in their preferences toward risk, depending on whether the risk involved gains or losses. In general, those executives who liked risky situations did so when losses were involved. (Perhaps they were willing to gamble against a large loss in the hope of breaking even.) However, when the risks involved gains, the same executives were more conservative, opting for the less risky alternatives.[7]

EXAMPLE 5.2 DETERRING CRIME

Fines may deter certain types of crimes, such as speeding, double-parking, tax evasion, and air polluting, better than incarceration.[8] The party choosing to violate the law in these ways has good information and can reasonably be assumed to be behaving rationally.

Other things being equal, the greater the fine, the more a potential criminal will be discouraged from engaging in the crime. For example, if it were costless to catch criminals and if the crime imposed a calculable cost of $1000 on society, we might choose to catch all violators and impose a fine of $1000 on each. That would discourage people whose benefit from engaging in the activity was less than the fine.

In practice, however, it is very costly to catch lawbreakers. Therefore, we save on administrative costs by imposing relatively high fines/but allocating resources so that the probability of a violator's being apprehended is much less than one. Thus the size of the fine that needs to be imposed to discourage criminal behavior depends on the risk preferences of the potential violators. The more risk averse a person is, the smaller the fine that must be imposed to discourage him or her, as the following example demonstrates.

Suppose that a city wants to deter people from double-parking. By double-parking, a typical resident saves $5 in terms of his own time available to engage in activities that are more pleasant than searching for a parking space. If the driver is risk neutral and if it were costless to catch violators, a fine of just over $5, say, $5.01, would need to be assessed every time he double-parked. This would ensure that the net benefit of double-parking to the driver (the $5 benefit less the $5.01 fine) would be less than zero, so that he would choose to obey the law. In fact, all potential violators whose benefit was less than or

[7] It is interesting that some people treat the risk of a small gain in income very differently from the risk of a small loss. Prospect theory, developed by psychologists Daniel Kahneman and Amos Tversky, helps to explain this phenomenon. See "Rational Choice and the Framing of Decisions," *Journal of Business* 59 (1986): S251-S278, and "Prospect Theory: An Analysis of Decision Under Risk," *Econometrica* 47 (1979): 263-292.

[8] This discussion builds indirectly on Gary S. Becker, "Crime and Punishment: An Economic Approach," *Journal of Political Economy* (Mar./Apr. 1968): 169-217. See also Mitchell Polinsky and Steven Shavell, "The Optimal Tradeoff Between the Probability and the Magnitude of Fines," *American Economic Review* 69 (Dec. 1979): 880-891.

equal to $5 would be discouraged; while a few whose benefit was greater than $5 would violate the law (they might have to double-park in an emergency).

Extensive monitoring is expensive but fortunately may not be necessary The same deterrence effect can be obtained by assessing a fine of $50 and catching only one in ten violators (or perhaps a fine of $500 with a one-in-100 chance of being caught). In each case the expected penalty is $5 ([$50][.1] or [$500][.01]). A policy of a high fine and a low probability of catching a violator is likely to reduce enforcement costs.

The fines to be assessed need not be large. If drivers were substantially risk averse, a much lower fine could be used because they would be willing to forgo the activity in part because of the risk associated with the enforcement process. In the previous example, a $25 fine with a .1 probability of catching the violator might discourage most people from violating the law.

5.3 *Reducing Risk*

Sometimes consumers choose risky alternatives that suggest risk-loving rather than risk-averse behavior, as the recent growth in state lotteries shows. Nonetheless, in the face of a broad variety of risky situations, consumers are generally risk averse. In this section we describe three ways by which consumers commonly reduce risks: diversification, insurance, and obtaining more information about choices and payoffs.

Diversification

Suppose that you plan to take a part-time job selling appliances on a commission basis. You can decide to sell only air conditioners or only heaters, or you can spend half your time selling each. Of course, you can't be sure how hot or cold the weather will be next year. How should you apportion your time to minimize the risk involved in the sales job?

The answer is that risk can be minimized by *diversification-by* allocating your time toward selling two or more products (whose sales are not closely related), rather than a single product. Suppose that there is a fifty-fifty chance that it will be a relatively hot year, and a fifty-fifty chance that it will be cold. Table 5.5 gives the earnings that you can make selling air conditioners and heaters.

If you sell only air conditioners or only heaters, your actual income will be either $12,000 or $30,000 but your expected income will be $21,000 [.5($30,000) +. .5($12/000)]. But suppose you diversify by dividing your time evenly between the two products. Then your income will certainly be $21,000, whatever the weather. If the weather is hot, you will earn $15,000

TABLE 5.5 Income from Sales of Equipment

	Hot Weather	Cold Weather
Air conditioner sales	$30,000	$12,000
Heater sales	12,000	30/000

from air conditioner sales and $6000 from heater sales; if it is cold, you will earn $6000 from air conditioner sales and $15/000 from heater sales. Hence, by diversifying you eliminate all risk.

Diversification is not always this easy. In our example heater and air conditioner sales were inversely related-whenever the sales of one were strong, the sales of the other were weak. But the principle of diversification is a general one. As long as you can allocate your effort or your investment funds toward a variety of activities whose outcomes are not closely related, you can eliminate some risk..

Insurance

We have seen that risk-averse people will be willing to give up income to avoid risk. In fact, if the cost of insurance is equal to the expected loss (e.g., a policy with an expected loss of $1000 will cost $1000), risk-averse people will want to buy enough insurance to allow them to fully recover from any financial losses they might suffer.

The reasoning is implicit in our discussion of risk aversion. Buying insurance assures a person of having the same income whether or not there is a loss. Because the insurance cost is equal to the expected loss, this certain income is equal to the expected income from the risky situation. For a risk-averse consumer, the guarantee of the same income whatever the outcome generates more utility than would be the case if that person had a high income when there was no loss and a low income when a loss occurred.

To clarify this argument, suppose a homeowner faces a 10 percent probability that his house will be burglarized and he will suffer a $10,000 loss. Let's assume he has $50,000 worth of property. Table 5.6 shows his wealth with two possibilities-to insure or not to insure.

TABLE 5.6 The Decision to Insure

Insurance	Burglary (Pr = .1)	No Burglary (Pr = .9)	Expected Wealth
No	$40,000	$50,000	$49,000
Yes	49,000	49,000	49,000

The decision to purchase insurance does not alter his expected wealth. It does, however, smooth it out over both possible outcomes. This is what generates a higher level of expected utility for the homeowner. Why? We know that the marginal utility in both the no-loss and loss situations is the same for the man who buys insurance (because his wealth is the same). But when there is no insurance, the marginal utility in'the event of a loss is higher than if no loss occurs (recall that with risk aversion there is diminishing marginal utility). Therefore, a transfer of wealth from the no-loss to the loss situation must increase total, utility. And this transfer of wealth is exactly what the purchase of insurance accomplishes.

Consumers usually buy insurance from companies that specialize in selling it. In general, insurance companies are profit-maximizing firms that offer insurance because they know that when they pool policies, they face relatively little risk. The ability to avoid risk by operating on a large scale is based on the *law of large numbers,* which tells us that although single events may be random and largely unpredictable, the average outcome of many similar events can be predicted. For example, I may not be able to predict whether a coin toss will come out heads or tails, but I know that when many coins are flipped, approximately half will turn up heads and half tails. Similarly, if I am selling automobile insurance, I cannot predict whether a particular driver will have an accident, but I can be reasonably sure, judging from past experience, about how many accidents a large group of drivers will have.

By operating on a large scale, insurance companies can assure themselves that over a large enough number of events, the total premiums paid in will be equal to the total amount of money paid out. To return to our burglary example, a man knows that there is a 10 percent probability that his house will be burgled; if it is, he will suffer a $10,000 loss. Prior to facing this risk, he calculates the expected loss to be $1000 (.10 X $10,000), but there is substantial risk involved, .since there is a 10 percent probability of a large loss. Now suppose 100 people are similarly situated and all of them buy burglary insurance from an insurance company. Because they are all similarly situated, the insurance company charges each of them a premium of $1000 for the insurance. This $1000 premium generates an insurance fund of $100,000 from which losses can be paid. The insurance company can rely on the law of large numbers, which tells it that the expected loss over the 100 individuals is likely to be very close to $1000 each. Therefore, the total payout will be close to $100,000, and the company need not worry about losing more than that.

Insurance companies typically charge premiums above the expected loss because they need to cover their administrative costs. As a result, many people choose to self-insure rather than buy from an insurance company. One way to avoid risk is to self-insure by diversifying. For example, self-insurance against the risks associated with investing usually takes the form of diversifying one's portfolio, say, by buying a mutual fund. Self-insurance against other risks can be achieved by spending money. For example, a person can seIf-insure against the risk of loss by putting money into a fund to cover future loss. Or one may self-insure against the loss of future earnings by putting funds into an individual retirement account.

EXAMPLE 5.3 THE VALUE OF TITLE INSURANCE WHEN BUYING A HOME

Suppose a family is buying its first home. The family knows that to close the sale of the house they will need a deed that gives them the clear "title" to the house. Without such a clear title, there is always a chance that the seller of the house is not its true owner. Of course, the seller could be engaging in fraud but is more likely to be unaware of the exact nature of his or her ownership rights. For example, the owner may have borrowed heavily, using the house as "collateral" for the loan. Or the property might carry with it a legal requirement that limits the use to which it may be put.

Suppose the family is willing to pay $150,000 for the house but believes there is a one in ten chance that careful research will show that the current seller does not own the property. The property would then be worth only $50,000. If there were no insurance available, a risk-neutral family would bid at most $140,000 for the property (.9[$150,000] + .1[$50,000]). However, a family that expects to tie up most of their assets in their house would most likely be risk averse and would therefore bid substantially less to buy the house, say, $120,000.

In situations such as this, it is clearly in the interest of the seller to assure the buyer that there is no risk of a lack of full ownership. The seller does this by purchasing "title insurance." The title insurance company researches the history of the property, checks to-see whether any legal liabilities are attached to it, and generally assures itself that. there is no ownership problem. The insurance company then agrees to bear any remaining risk that might exist.[9]

Because the title insurance company is a specialist in such insurance and can collect the relevant information relatively easily, the cost of title insurance is often less than the expected value of the loss involved. A fee of $1,000 for title insurance is not unusual, and the expected loss can be substantially higher. Clearly, it is in the interest of the sellers of homes to provide such insurance, because all but the most risk-loving buyers will pay substantially more for the house when it is insured than when it is not. In fact, most states require sellers to provide title insurance before the sale can be complete.

[9] Because such risks are also of concern to mortgage lenders, they usually require new buyers to have title insurance before they will issue a mortgage.

The Value of Information

The decision a consumer makes when outcomes are uncertain is based on limited information. If more information were available, the consumer could make better predictions and reduce risk. Because information is a valuable commodity, people will pay for it The *value of complete information* is the difference between the expected value of a choice when there is complete information and the expected value when information is incomplete.

To see how valuable information can be, suppose you are a store manager and must decide how many suits to order for the fall season. If you order 100 suits, your cost is $180 per suit, but if you order only 50 suits, your cost increases to$200. You know you will be selling the suits for $300 each, but you are not sure what total sales will be. All suits not sold can be returned, but for only half of what you paid for them. Without additional information, you will act on your belief that there is a .5 probability that 100 suits will be sold and a .5 probability that sales will be 50. Table 5.7 gives the profit that you would earn in each of the two cases.

Without additional information, you would choose to buy 100 suits if you were risk neutral, taking the chance that your profit might be either $12,000 or $1500. But if you were risk averse, you might buy 50 suits because then you would know for sure that your income would be $5000.

With complete information, you can make the correct suit order whatever the sales might be. If sales were going to be 50 and you ordered 50 suits, your profit would be $5000. If, on the other hand, sales were going to be 100 and you ordered 100 suits, your profit would be $12,000. Since both outcomes are equally likely, your expected profit with complete information would be $8500. The value of information is computed as

	Expected value with complete information:	$8500
Less:	Expected value with uncertainty (buy 100 suits):	−$6750
	Value of complete information	$1750

Thus, it is worth paying up to $1750 to obtain an accurate prediction of sales. Even though forecasting is inevitably imperfect, it may be worth investing in a marketing study that provides a better forecast of next year's sales.

	Sales of 50	Sales of 100	Expected Profit
1. Buy 50 suits	$5000	$5000	$5000
2. Buy 100 suits	1500	12,000	6750

EXAMPLE 5.4 THE VALUE OF INFORMATION IN THE DAIRY INDUSTRY

Historically, the U.S. dairy industry has allocated its advertising expenditures more or less uniformly throughout the year.[10] But per-capita consumption of milk declined by 24 percent between 1955 and 1980, which stirred milk producers to look for a new sales strategy to encourage milk consumption. One strategy would be to increase advertising expenditures and to continue to advertise at a uniform rate throughout the year. A second strategy is to invest in market research to obtain more information about the seasonal demand for milk, and then reallocate expenditures, so that advertising is most intense when the demand for milk is greatest.

Research into milk demand shows that sales follow a seasonal pattern, with demand the greatest during the spring and lowest during summer and early fall. The price elasticity of milk demand is negative but small and the income elasticity positive and large. Most important is that milk advertising has the most effect on sales when consumers have the strongest preference for milk (March, April, and May), and the least when preferences are weakest (August, September, and October).

In this case, the cost of obtaining the seasonal information about milk demand is relatively low, and the value of the information is substantial. To estimate this value, we can compare the actual sales of milk during 1972-1980 with what the sales would have been had the advertising expenditures been made in proportion to the strength of the seasonal demand. In the latter case, 30 percent of the advertising budget would be allocated in the first quarter of the year, and only 20 percent in the third quarter.

Making these calculations for the New York metropolitan area shows that the value of information-the value of .the additional milk sales-was $4,046,557. This means a 9 percent increase in the profit to producers.

*5.4 *The Demand for Risky Assets*

Most people are risk averse. Given a choice, they prefer a fixed monthly income to one that is as large on average but that fluctuates randomly from month to month. Yet many of these same people will invest all or part of their savings in stocks, bonds, and other assets that carry some risk. Why do

[10] This example is based on Henry Kinnucan and Olan D. Forker, "Seasonality in **the** Consumer Response to Milk Advertising with Implications for Milk Promotion Policy," *American Journal of* Agri *cultural Economics* 68 (1986): 562-571.

risk-averse people invest in the stock market, and thereby risk losing part or all of their investment?[11] How do people decide how much risk to bear when making investments and planning for the future? To answer these questions, we need to examine the demand for risky assets.

Assets

An *asset* is something that provides a monetary flow to its owner. For example, apartments in an apartment building can be rented out, providing a flow of rental income to the owner of the building. Another example is a savings account in a bank that pays interest (usually every day or every month). Typically, these interest payments are reinvested in the account.

The monetary flow that one receives from owning an asset can take the form of an explicit payment, such as the rental income from an apartment building: Every month the landlord receives rent checks from the tenants. Another explicit payment is the dividend on shares of common stock: Every three months the owner of a share of General Motors stock receives a quarterly dividend payment.

But sometimes the monetary flow from ownership of an asset is implicit; it takes the form of an increase or decrease in the price or value of the asset. (An increase in the value of an asset is a *capital gain,* a decrease a *capital loss.)* For example, as the population of a city grows, the value of an apartment building may increase. The owner of the building will then earn a capital gain beyond the rental income he or she receives. Although the capital gain is *unrealized* until the building is sold because no money is actually received until then, there is an implicit monetary flow because the building *could* be sold at any time. The monetary flow from owning General Motors stock is also partly implicit. The price of the stock changes from day to day, and each time it changes, the owner of the stock gains or loses.

A *risky asset* provides a monetary flow that is at least in part random. In other words, the monetary flow is not known with certainty in advance. A share of General Motors stock is an obvious example of a risky asset-one cannot know whether the price of the stock will rise or fall over time, and one cannot even be sure that the company will continue to pay the same (or any) dividend per share. But although people often associate risk with the stock market, most other assets are also risky.

The apartment building is one example of this. One cannot know how much land values will rise or fall, whether the building will be fully rented all the time, or even whether the tenants will pay their rent promptly. Corporate bonds are another example-the corporation that issued the bonds could go

[11] Most Americans have at least some money invested in stocks or other risky assets, but often the investment is made indirectly. For example, many people who hold full-time jobs have shares in a pension fund, funded in part by their own salary contributions, and in part by contributions made by their employers. Usually these pension funds are invested partly in the stock market.

bankrupt and fail to pay bond owners their interest and principal. Even long-term U.S. government bonds that mature in 10 or 20 years are risky. Although it is highly unlikely that the federal government will go bankrupt/ the rate of inflation could unexpectedly increase and make future interest payments and the eventual repayment of .principal worth less in real terms, and thereby reduce the value of the bonds.

In contrast to a risky asset, we call an asset *riskless* (or risk free) if it pays a monetary flow that is known with certainty. Short-term U.S. government bonds-called Treasury bills-are a riskless, or almost riskless, asset. Because these bonds mature in a few months, there is very little risk from an unexpected increase in the rate of inflation. And one can be reasonably confident that the U.S. government will not default on the bond (i.e., refuse to pay back the holder when the bond comes due). Other examples of riskless or almost riskless assets include passbook savings accounts in a bank and short-term certificates of deposit.

Asset Returns

People buy and hold assets because of the monetary flows they provide. To compare assets with each other, it helps to think of this monetary flow relative to the asset's price or value. The *return* on an asset is the total monetary flow it yields as a fraction of its price. For example, a bond worth $1000 today that pays out $100 this year (and every year) has a return of 10 percent.[12] If an apartment building was worth $10 million last year, increased in value to $11 million this year, and also provided a net (of expenses) rental income of $0.5 million, it would have yielded a return of 15 percent over the past year. Or if a share of General Motors stock had been worth $80 at the beginning of the year, fell to $72 by the end of the year, and paid a dividend of $4, it would have yielded a return of -5 percent (the dividend yield of 5 percent less the capital loss of 10 percent).

When people invest their savings in stocks,bonds, land, or other assets, they usually hope to earn a return that exceeds the rate of inflation, so that by delaying consumption, they can buy more in the future than they could by spending all their income now. As a result, we often express the return on an asset in real (inflation-adjusted) terms.The *real return* on an asset is its simple (or nominal) return *less* the rate of inflation. For example, if the annual rate of inflation had been 5 percent, the bond, the apartment building, and the share of GM stock described above would have yielded real returns of 5 percent, 10 percent, and -10 percent, respectively.

[12] The price of a bond often changes during a year. If the bond appreciated (depreciated) in value during the year, its return would be greater than (less than) 10 percent. Also, the definition of return given above should not be confused with the "internal rate of return" sometimes used to compare monetary flows occurring over some time. We discuss other return measures in Chapter 15, when we deal with present discounted values.

TABLE 5.8 Investments—Risk and Return (1926–1991)

	Real Rate of Return (%)	Risk (standard deviation, %)
Common stocks	8.8	21.2
Long-term corporate bonds	2.4	8.5
U.S. Treasury bills	0.5	3.4

Since most assets are risky, an investor cannot know in advance what returns they will yield over the coming year. For example, the apartment building might have depreciated in value instead of appreciating, and the price of GM stock might have risen instead of falling. However, we can still compare assets by looking at their *expected returns.* The expected return on an asset is just the expected value of its return (i.e., the return that it should earn on average). In some years the *actual* return that an asset earns may be much higher than its expected return, and in some years much lower, but over a long period the average return should be close to the expected return.

Different assets have different expected returns. For example. Table 5.8 shows that the expected real return on a U.S. Treasury bill has been less than 1 percent, while the real return for a representative stock on the New York Stock Exchange has been almost 9 percent.[13] Why would anyone buy a Treasury bill when the expected return on stocks is so much higher? The answer is that the demand for an asset depends not just on its expected return, but also on its *risk* Although stocks have a higher expected return than Treasury bills, they also carry much more risk. One measure of risk, the standard deviation of the real return, is equal to 21.2 percent for common stocks, but only 8.3 percent for corporate bonds, and 3.4 percent for U.S. Treasury bills. Clearly, the higher the expected return on investment, the greater the risk involved. As a result, a risk-averse investor must balance expected return against risk. We examine this trade-off in more detail below.

The Trade-off Between Risk and Return

Suppose a woman has to invest her savings in two assets-Treasury bills, which are almost risk free, and a representative group of stocks![14] She has to

[13] The expected real return for the New York Stock Exchange Index, an average of all stocks traded on the exchange, is about 9 percent. For some stocks the expected return is higher, and for some it is lower.

[14] The easiest way to invest in a representative group of stocks is to buy shares in a *mutual fund.* A mutual fund invests in many stocks, so that by buying the funds, one effectively buys a portfolio of many stocks.

decide how much of her savings to invest in each of these two assets-she might invest only in Treasury bills, only in stocks, or in some combination of the two. As we will see, this is analogous to the consumer's problem of allocating a budget between purchases of food and clothing.

Denote the risk-free return on the Treasury bill by R_f. Because the return is risk free, the expected and actual returns are the same. Also, let the expected return from investing in the stock market be R_m, and the *actual* return be r_m. The actual return is risky. At the time of the investment decision, we know the set of possible outcomes and the likelihood of each, but we do not know what particular outcome will occur. The risky asset will have a higher expected return than the risk-free asset ($R_m > R_f$). Otherwise, risk-averse investors would buy only Treasury bills and no stocks would be sold.

To determine how much money the investor should put in each asset, let's set b equal to the fraction of her savings placed in the stock market, and $(1 - b)$ the fraction used to purchase Treasury bills. The expected return on her total portfolio, R_p, is a weighted average of the expected return on the two assets:[15]

$$R_p = bR_m + (1-b)R_f \qquad (5.1)$$

Suppose, for example, that Treasury bills pay 4 percent ($R_f = .04$), the stock market's expected return is 12 percent ($R_m = .12$), and $b = ½$. Then $R_p = 8$ percent. How risky is this portfolio? One measure of its riskiness is the variance of the portfolio's return. Let's describe the variance of the risky stock market investment by σ_m^2 and the standard deviation by σ_m. With some algebra, we can show that the standard deviation of the portfolio (with one risky and one risk-free asset) is the fraction of the portfolio invested in the risky asset times the standard deviation of that asset:[16]

$$\sigma_p = b\sigma_m \qquad (5.2)$$

The Investor's Choice Problem

We have still not determined how this investor should choose this fraction b. To do this, we must first show that she faces a risk-return trade-off analogous to the budget line of a consumer. To see what this trade-off is, note that equation (5.1) for the expected return on the portfolio can be rewritten as

$$R_p = R_f + b(R_m - R_f)$$

Now, from equation (5.2) we see that $b = \sigma_p/\sigma_m$, so that

[15] The expected value of the sum of two variables is the sum of the expected values, so $R_p = E[br_m] + E[(1 - b)R_f] = bE[r_m] + (1 - b)R_f = bR_m\ (1 - b)R_f$.

[16] To see why, note from footnote 3 that we can write the variance of the portfolio return as $\sigma_p^2 = E[br_m + (1 - b)R_f - R_p]^2$. Substituting equation (5.1) for the expected return on the portfolio, R_p, we have

$$\sigma_p^2 = E[br_m + (1 - b)R_f - bR_m - (1 - b)R_f]^2 = E[b(r_m - R_m)]^2 = b^2\sigma_m^2.$$

Because the standard deviation of a random variable is the square root of its variance, $\sigma_p = b\sigma_m$.

$$R_p = R_f + \frac{(R_m - R_f)}{\sigma_m}\sigma_p \tag{5.3}$$

This equation is a *budget line* because it describes the trade-off between risk (σ_p) and expected return (R_p). Note that it is the equation for a straight line; R_m, R_f, and σ_m are constants, so the slope $(R_m - R_f)/\sigma_m$, is a constant, as is the intercept R_f. The equation says that *the expected return on the portfolio R_p increases as the standard deviation of that return σ_p increases.* We call the slope of this budget line, $(R_m - R_f)/\sigma_m$, the *price of risk* because it tells us how much extra risk an investor must incur to enjoy a higher expected return.

The budget line is drawn in Figure 5.5. As the figure shows, if the investor wants no risk, she can invest all her funds in Treasury bills (b = 0), and earn

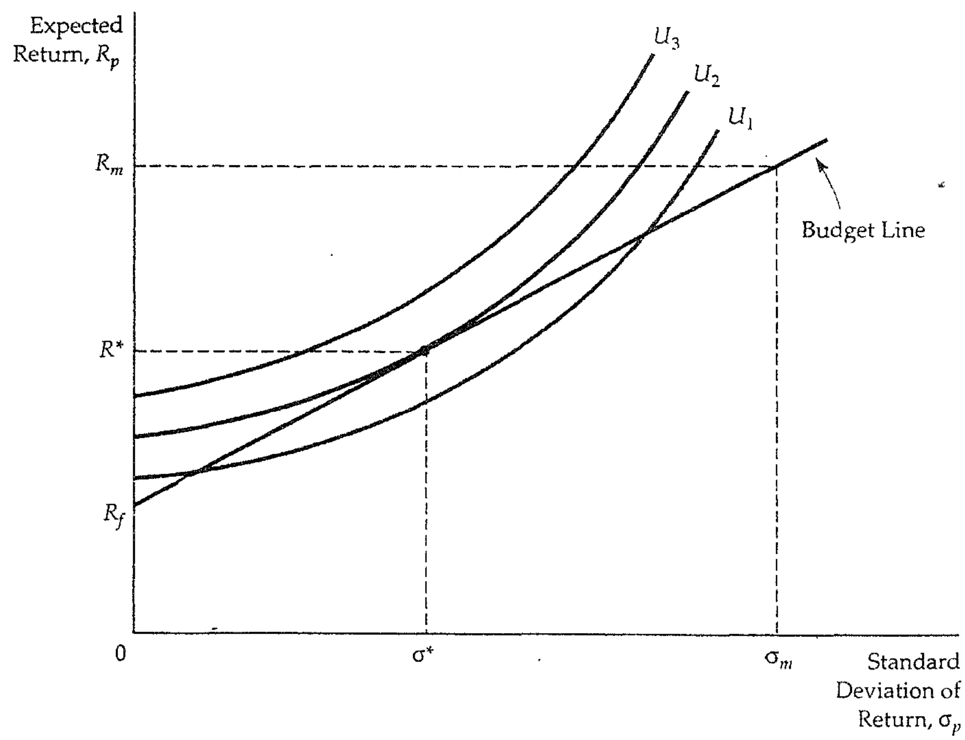

FIGURE 5.5 Choosing Between Risk and Return. An investor is dividing her funds between two assets. Treasury bills, which are risk free, and stocks. The budget line describes the trade-off between the expected return and the riskiness of that return, as measured by its standard deviation. The slope of the budget line is $(R_m - R_f)/\sigma_m$ which is the price of risk. Three indifference curves are shown; each curve shows combinations of risk and return that leave an investor equally satisfied. The curves are upward-sloping because a risk-averse investor will require a higher expected return if she is to bear a greater amount of risk. The utility-maximizing investment portfolio is at the point where indifference curve U_2 is tangent to the budget line.

an expected return R_f. To receive a higher expected return, she must incur some risk. For example, she could invest all her funds in stocks (b=1), and earn an expected return R_m, but incur a standard deviation σ_m. Or she might invest some fraction of her funds in each type of asset, earn an expected return somewhere between R_f and R_m, and face a standard deviation less than σ_m, but greater than zero.

Figure 5.5 also shows the solution to the investor's problem. Three indifference curves are drawn in the figure. Each curve describes combinations of risk and return that leave the investor equally satisfied. (The curves are upward-sloping because risk is undesirable, so with a greater amount of risk, it takes a greater expected return to make the investor equally well-off.) The curve U_3 yields the greatest amount of satisfaction, and U_1 the least amount. (For a given amount of risk, the investor earns a higher expected return on U_3 than on U_2, and a higher expected return on U_2 than on U_1.) Of the three indifference curves, the investor would prefer to be on U_3 but this is not feasi-

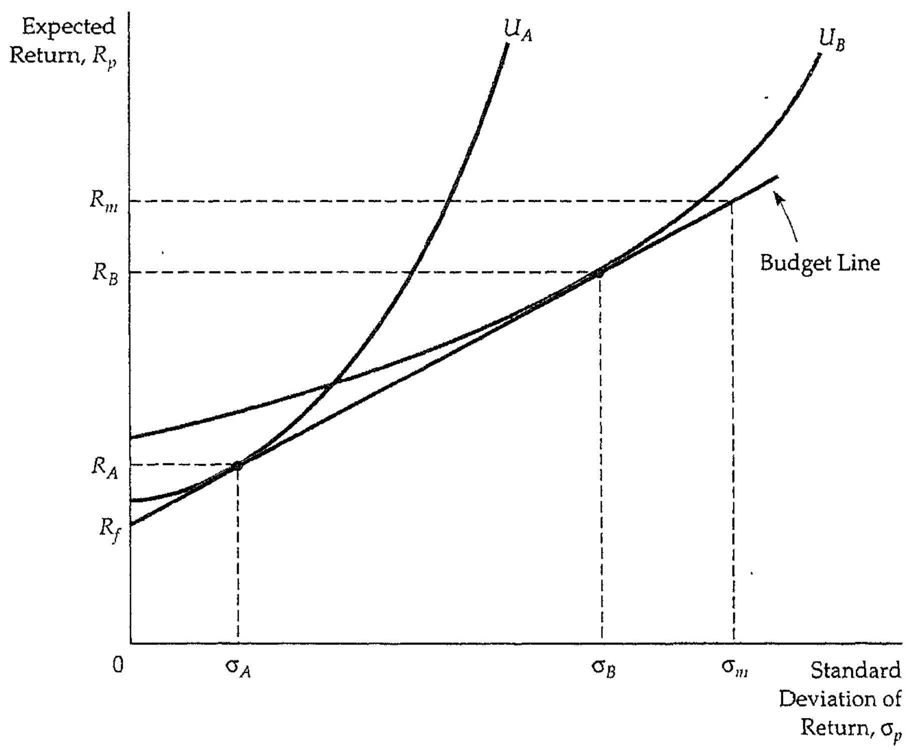

FIGURE 5.6 The Choices of Two Different Investors. Investor *A* is very risk averse. His portfolio will consist mostly of the risk-free asset, so his expected return R_A will be only slightly greater than the risk-free return, but the risk σ_A will be small. Investor *B* is less risk averse. She will invest a large fraction of her funds in stocks. The expected return on her portfolio R_B will be larger, but the return will also be riskier.

ble because it does not touch the budget line. Curve U_1 is feasible, but the investor can do better. Like the consumer choosing quantities of food and clothing, our investor does best by choosing a combination of risk and return at the point where an indifference curve (in this case U_2) is tangent to the budget line. At that point, the investor's return has an expected value R^* and a standard deviation σ^*.

People differ in their attitudes toward risk. This is illustrated in Figure 5.6, which shows how two different investors choose their portfolios. Investor A is very risk averse. His indifference curve U_A *is* tangent to the budget line at a point of low risk, so he will invest almost all his funds in Treasury bills and earn an expected return R_A just slightly larger than the risk-free return R_f. Investor B is less risk averse. She will invest most of her funds in stocks, and the return on her portfolio will have a higher expected value R_B but also a higher standard deviation σ_B.

In Chapters 3 and 4, we simplified the problem of consumer choice by assuming the consumer had only two goods to choose from, food and clothing. In the same spirit, we have simplified the investor's choice between only Treasury bills and stocks. However, the basic principles would be the same if we had more assets (e.g., corporate bonds, land, different types of stocks, etc.). Every investor faces a trade-off between risk and return.[17] How much extra risk an investor is willing to bear to earn a higher expected return depends on how risk averse that investor is. Less risk-averse investors tend to include a larger fraction of risky assets in their portfolios.

Summary

1. Consumers and managers frequently make decisions in which there is uncertainty about the future. This uncertainty is characterized by the term risk, when each of the possible outcomes and its probability of occurrence is known.

2. Consumers and investors are concerned about the expected value and the variability of uncertain outcomes. The expected value is a measure of the central tendency of the value of the risky outcomes. The variability is frequently measured by the variance of outcomes, which is the average of the squares of the deviations of each possible outcome from its expected value.

3. Facing uncertain choices, consumers maximize their expected utility, an average of the utility associated with each outcome, with the associated probabilities serving as weights.

[17] Although we have not discussed this point, what matters is "systematic" or nondiversifiable risk, since investors can eliminate "nonsystematic" risk by holding a well-diversified portfolio (e.g., via a mutual fund). We discuss systematic versus nonsystematic risk in Chapter 15, For a more detailed treatment, see a standard text on finance. A good one is Richard Brealey and Stewart Myers, *Principles of Corporate Finance,* 4th Edition (New York: McGraw-Hill, 1991).

4. A person who would prefer a certain return of a given amount to a risky investment whose expected return is the same amount is risk averse. The maximum amount of money that a risk-averse person would pay to avoid taking a risk is the risk premium.
5. A person who is indifferent between a risky investment and the certain receipt of the expected return on that investment is risk neutral.
6. A risk-loving consumer would prefer a risky investment with a given expected return to the certain receipt of that expected sum.
7. Risk can be reduced by (a) diversification, (b) purchasing insurance, and (c) obtaining additional information.
8. The law *of large numbers* enables insurance companies to provide actuarially fair insurance for which the premium paid equals the expected value of the loss being insured against.
9. Consumer theory can be applied to decisions to invest in risky assets. The budget line reflects the price of risk, and consumers' indifference curves reflect their attitudes toward risk.

Questions for Review

1. What does it mean to say that a person is risk averse? Why are some people likely to be risk averse, while others are risk lovers?
2. Why is the variance a better measure of variability than the range?
3. What does it mean for consumers to maximize expected utility? Can you think of a case where a person might not maximize expected utility?
4. Why do people want to fully insure against uncertain situations when insurance is actuarially fair?
5. Why is an insurance company likely to behave as if it is risk neutral even if its managers are risk-averse individuals? (Hint: How many projects are insured by an entire insurance company? How many does each individual manager deal with?)
6. When is it worth paying to obtain more information to reduce uncertainty?
7. How does the diversification of an investor's portfolio avoid risk?
8. Why do some investors put a large portion of their portfolios into risky assets, while others invest largely in risk-free alternatives? (Hint: Do the two investors receive exactly the same return on average? Why?)

Exercises

1. Consider a lottery with three possible outcomes: $100 will be received with probability .1, $50 with probability .2, and $10 with probability .7.
   a. What is the expected value of the lottery?
   b. What is the variance of the outcomes of the lottery?
   c. What would a risk-neutral person pay to play the lottery?
2. Suppose you have invested in a new computer company whose profitability depends on (1) whether the U.S. Congress passes a tariff that raises the cost of Japanese computers, and (2)

whether the U.S. economy grows slowly or quickly. What are the four mutually exclusive states of the world that you should be concerned about?

3. Richard is deciding whether to buy a state lottery ticket. Each ticket costs $1, and the probability of the following winning payoffs is given as follows:

Probability	Return
.5	$0.00
.25	$1.00
.2	$2.00
.05	$7.50

a. What is the expected value of Richard's payoff if he buys a lottery ticket? What is the variance?
b. Richard's nickname is "No-risk Rick." He is an extremely risk-averse individual. Would he buy the ticket?
c. Suppose Richard was offered insurance against losing any money. If he buys 1,000 lottery tickets, how much would he be willing to pay to insure his gamble?
d. In the long run, given the price of the lottery ticket and the probability/return table, what do you think the state would do about the lottery?

4. Suppose an investor is concerned about a business choice in which there are three prospects, whose probability and returns are given below:

Probability	Return
.0.2	$100
'0.4	50
0.4	-25

What is the expected value of the uncertain investment? What is the variance?

5. You are an insurance agent who has-to write a policy for a new client named Sam. His company, Society for Creative Alternatives to Mayonnaise (SCAM), is working on a low-fat, low-cholesterol mayonnaise substitute for the sandwich condiment industry. The sandwich industry will pay top dollar to whoever invents such a mayonnaise substitute first. Sam's SCAM seems like a very risky proposition to you. You have calculated his possible returns table as follows:

Probability	Return	
-999	-$1,000,000	<he fails)
.001	$1,000,000,000	(he succeeds and sells the formula)

a. What is the expected return of his project? What is the variance?
b. What is the most Sam is willing to pay for insurance? Assume Sam is risk neutral.
c. Suppose you found out that the Japanese are on the verge of introducing their own mayonnaise substitute next month. Sam does not know this and has just turned down your final offer of $1000 for the insurance. If Sam tells you his SCAM is only six months away from perfecting his mayonnaise substitute *and* knowing what you know about the Japanese, would you raise or lower your policy premium on any subsequent proposal to Sam? Based on his information, would Sam accept?

6. Suppose that Natasha's utility function is given by $u(I)$ = y5, where I represents annual income in thousands of dollars.
a. Is Natasha risk loving, risk neutral, or risk averse? Explain.
b. Suppose that Natasha is currently earning an income of $10,000 (I = 10) and can earn that income next year with certainty. She is offered a chance to take a new job that offers a .5 probability of earning $16,000, and a .5 probability of earning $5000. Should she take the new job?
c. In (b), would Natasha be willing to buy insurance to protect against the variable income associated with the new job? If so/how much would she be willing to pay for that insurance? (Hint: What is the risk premium?)

7. Draw a utility function over income $u(I)$ that has the property that a man is a risk lover when his income is low but a risk averter when his income is high. Can you explain why such a utility function might reasonably describe a person's tastes?

8. A city is considering how much to spend monitoring parking meters. The following information is available to the city manager:
i. Hiring each meter-monitor costs $10,000 per year.

ii. With one monitoring person hired, the probability of a driver getting a ticket each time he or she parks illegally is equal to .25.
iii. With two monitors hired, the probability of getting a ticket is .5, with three monitors the probability is .75, and with four the probability is equal to 1.
iv. The current fine for overtime parking with two metering persons hired is $20.

a. Assume first that all drivers are risk neutral. What parking fine would you levy and how many meter monitors would you hire (1, 2, 3, or 4) to achieve the current level of deterrence against illegal parking at the minimum cost?
b. Now assume that drivers are very risk averse. How would your answer to (a) change?
c. (For discussion) What if drivers could insure themselves against the risk of parking fines? Would it make good public policy to allow such insurance to be available?