CBSE Test Paper 04

Chapter 4 Determinants

1. If $\begin{vmatrix} 1 & 3 & 9 \\ 1 & x & x^2 \\ 4 & 16 & 64 \end{vmatrix} = 0 \Rightarrow$, then
   a. x = 2 or 6
   b. None of these
   c. x = 2
   d. x = 4 or 3
2. $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 4^2 & 3^2 & 2^2 \end{vmatrix}$ is equal to
   a. 1
   b. 2
   c. -2
   d. 0
3. The determinant $\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & q-r & r-p \end{vmatrix}$ is equal to
   a. 1
   b. 0
   c. –1
   d. None of these
4. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} =.$
   a. $2(a + b + c)^2$
   b. $-2(a + b + c)^3$

c. None of these

d. $4(a + b + c)^3$

5. If ω is non real cube root of unity, then $\begin{vmatrix} 2 & 2\omega & -\omega^2 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix}$ is equal to

a. -1
b. 0
c. None of these
d. 1

6. If A is a matrix of order 3×3, then $(A^2)^{-1}$ = ________.
7. ________ of an element a_{ij} of the determinant of matrix A is the determinant obtained by deleting i^{th} row and j^{th} column, and it is denoted by M_{ij}.
8. A square matrix is said to be singular if its determinant is ________.
9. If A is a non-singular matrix of order 3 and | adj A| = |A| then what is the value of k?
10. Find the inverse of the matrix (if it exists) given $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$.
11. Prove that $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$.
12. Without expanding, evaluate A= $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$.
13. Find the value of x, if:

i. $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

ii. $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

14. Without expanding, prove that $\Delta = \begin{bmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{bmatrix} = 0.$

15. Prove that $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$.

16. Using properties of determinants, prove that

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

17. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}, using A^{-1} solve the system of equations

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

18. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$,verify that $A^3 - 6A^2 + 9A + 4I = 0$ and hence find A^{-1}.

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Solution

1. d. x = 4 or 3

   Explanation: $\begin{vmatrix} 1 & 3 & 9 \\ 1 & x & x^2 \\ 4 & 16 & 64 \end{vmatrix} = 0$

   $\Rightarrow$ because, the value of the determinant is zero only when the two of its rows or column are identical, which is possible only when either x = 3 or x = 4.

2. c. -2

   Explanation: $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 4^2 & 3^2 & 2^2 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 16 & 9 & 4 \end{vmatrix}$

   Apply, $C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$

   $\begin{vmatrix} 0 & 0 & 1 \\ 2 & 1 & 2 \\ 12 & 5 & 4 \end{vmatrix} = 10 - 12 = -2$

3. b. 0

   Explanation: $\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & q-r & r-p \end{vmatrix}$

   Apply, $C_1 \to C_1 + C_2 + C_3$

   $\begin{vmatrix} 0 & b-c & c-a \\ 0 & y-z & z-x \\ 0 & q-r & r-p \end{vmatrix} = 0$ (Since C_1 =0)

4. c. None of these

   Explanation: $\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$,

   Apply, $R_1 \to R_1 + R_2 + R_3$,

$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$\Rightarrow (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Apply, $C_3 \to C_3 - C_1$, $C_2 \to C_2 - C_1$

$$\Delta = (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

= $(a + b + c)^3$

5. b. 0

   Explanation: Expanding along R_3

   $$= 1(2\omega + \omega^2) + 1(2 + \omega^2) = (2 + 2\omega + 2\omega^2 = 2(1 + \omega + \omega^2) = 2(0) = 0$$

6. $(A^{-1})^2$
7. Minor
8. 0
9. We know that, for a non-singular square matrix of order n

   $|adj (A)| = |A|^{n-1}$

   Here, the order of A is 3×3, therefore n = 3

   Hence, $|adj (A)| = |A|^2$ ...(i)

   But $|adj (A)| = |A|$ [given]....(ii)

   From Eqs. (i) and (ii), we get,

   k = 2

10. Let $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

    $\therefore |A| = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix} = 6 - (-8) = 6 + 8 = 14 \neq 0$

    $\therefore$ Matrix A is non-singular and hence A^{-1} exist.

    Now adj. A $= \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$ And $A^{-1} = \frac{1}{|A|} adj.\, A = \frac{1}{14}\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$

11. $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$

Operating $C_3 \to C_3 - C_1$

$= \begin{vmatrix} 2 & 7 & 63 \\ 3 & 8 & 72 \\ 5 & 9 & 81 \end{vmatrix}$

Taking 9 common from third column,

$= 9 \begin{vmatrix} 2 & 7 & 7 \\ 3 & 8 & 8 \\ 5 & 9 & 9 \end{vmatrix}$

$= 9 \times 0 = 0$ [two columns are identical]

12. A $= \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 6 \times 17 & 6 \times 3 & 6 \times 6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

$= 6 \begin{vmatrix} 17 & 3 & 6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 6 \times 0 = 0$ [R_1 and R_3 are identical]

13. i. Given: $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$\Rightarrow 2 - 20 = 2x^2 - 24$

$\Rightarrow -18 = 2x^2 - 24$

$\Rightarrow 2x^2 = -18 + 24$

$\Rightarrow 2x^2 = 6$

$\Rightarrow x^2 = 3$

$\Rightarrow x = \pm\sqrt{3}$

ii. $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$\Rightarrow 10 - 12 = 5x - 6x$

$\Rightarrow -2 = -x$

$\Rightarrow x = 2$

14. $R_1 \to R_1 + R_2$

$\Delta = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

$$\Delta = (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

$= 0$ [R_1 and R_2 are identical]

15. According to the question,we are supposed to prove that,

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$$

Let LHS = $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$

Therefore,On applying ,$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$,we get,

LHS = $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & p-1 \\ 0 & 3 & 3p-2 \end{vmatrix}$

Therefore,on expanding along C_1, we get

LHS = $1 \times \begin{vmatrix} 1 & p-1 \\ 3 & 3p-2 \end{vmatrix}$

= 1 [(3p - 2) - (3p - 3)]

= 3p - 2 - 3p + 3 = 1 = RHS

16. To prove $\begin{vmatrix} a^2 & b & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$ = $4a^2b^2c^2$

Let LHS = $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$

Therefore, on taking a common from C_1, b from C_2 and c from C_3, we get,

LHS = abc$\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$

Therefore,on applying $C_1 \to C_1 + C_2 - C_3$, we get,

LHS = abc$\begin{vmatrix} 0 & c & a+c \\ 2b & b & a \\ 2b & b+c & c \end{vmatrix}$

Now, on applying $R_2 \to R_2$ - R_3, we get,

LHS = abc$\begin{vmatrix} 0 & c & a+c \\ 0 & -c & a-c \\ 2b & b+c & c \end{vmatrix}$

Therefore,on expanding along C_1, we get,

LHS = abc [2b {c (a - c) + c(a + c)}]

= $2ab^2c)$ (2ac)

= $4a^2b^2c^2$ = RHS

17. $|A| = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$(A) = -1 \neq 0$

$A^{-1} exists$

Here, $A_{11} = 0, A_{12} = 2, A_{13} = 1$

$A_{21} = -1, A_{22} = -9, A_{23} = -5$

$A_{31} = 2, A_{32} = 23, A_{33} = 13$

$A^{-1} = \frac{1}{|A|}(adjA)$

$= \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$= \begin{bmatrix} 0 & -1 & 2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

The given system of equation can be written is Ax = B

X= $A^{-1}B$

$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ 3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 5 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 11 \\ -5 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 0 & -1 & -1 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

x = 1

y = 2

z = 3

18. Given: $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$$\therefore A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

Now $A^3 = A^2.A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

L.H.S. $= A^3 - 6A^2 + 9A - 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6 \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$+9 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix}$$

$$+\begin{bmatrix}18 & -9 & 9\\ -9 & 18 & -9\\ 9 & -9 & 18\end{bmatrix}-\begin{bmatrix}4 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 4\end{bmatrix}$$

$$=\begin{bmatrix}22-36 & -21+30 & 21-30\\ -21+30 & 22-36 & -21+30\\ 21-30 & -21+30 & 22-36\end{bmatrix}+\begin{bmatrix}18-4 & -9-0 & 9-0\\ -9-0 & 18-4 & -9-0\\ 9-0 & -9-0 & 18-4\end{bmatrix}$$

$$=\begin{bmatrix}-14 & 9 & -9\\ 9 & -14 & 9\\ -9 & 9 & 14\end{bmatrix}+\begin{bmatrix}-14 & 9 & -9\\ 9 & -14 & 9\\ -9 & 9 & 14\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=0$$ = R.H.S.

Now, to find A^{-1}, multiplying $A^3-6A^2+9A-4I^{-1}=0.A^{-1}$by A^{-1}

$\Rightarrow A^3A^{-1}-6A^2A^{-1}+9AA^{-1}-4I.A^{-1}=0A^{-1}$

$\Rightarrow A^2-6A+9I-4A^{-1}=0$

$\Rightarrow 4A^{-1}=A^2-6A+9I$

$$\Rightarrow 4A^{-1}=\begin{bmatrix}6 & -5 & 5\\ -5 & 6 & -5\\ 5 & -5 & 6\end{bmatrix}-6\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}+9\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow 4A^{-1}=\begin{bmatrix}6 & -5 & 5\\ -5 & 6 & -5\\ 5 & -5 & 6\end{bmatrix}-\begin{bmatrix}12 & -6 & 6\\ -6 & 12 & -6\\ 6 & -6 & 12\end{bmatrix}+\begin{bmatrix}9 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 9\end{bmatrix}$$

$$\Rightarrow 4A^{-1}=\begin{bmatrix}6-12+9 & -5+6+0 & 5-6+0\\ -5+6+0 & 6-12+9 & -5+6+0\\ 5-6+0 & -5+6+0 & 6-12+9\end{bmatrix}=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$$

$$\Rightarrow A^{-1}=\frac{1}{4}\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$$