Short Answer Type Questions

Q.1. Find the equation of the circle whose centre is (2, -5) and which passes through the point (3, 2).

Sol. Let, the equation of the circle be

$(x-h)^2+(y-k)^2=r^2$ **(i)**

[centre = (h, k) radius = r]

Given, (h, k) = (2, -5)

$(x-2)^2+(y+5)^2=r^2$ **(ii)**

Since the circle passes through the point (3,2).

$(3-2)^2+(2+5)^2=r^2$

$\Rightarrow r^2=1+49$

$\Rightarrow r^2=50$

$\Rightarrow r=5\sqrt{2}$

Now $(x-2)^2+(y+5)^2=\left(5\sqrt{2}\right)^2$

$\Rightarrow x^2-4x+4+y^2+10y+25=50$

$\Rightarrow 2^2+y^2-4x+10y-21=0$

Q.2. Find the equation of the circle whose centre is (2, -3) and which passes through the intersection of the lines $3x+2y=11$ and $2x+3y=4$.

Sol. Find the point of intersection of the lines. Then, solve as Q no.1.

Q.3. If one end of a diameter of circle $x^2+y^2-4x-6y+11=0$ is (3,4) then find the co-ordinate of the other end of the diameter. [DDE]

Sol. Given equation of the circle is

$x^2+y^2-4x-6y+11=0$

$\Rightarrow (x^2-4x+4)+(y^2-6y+9)-4-9+11=0$

$\Rightarrow (x-2)^2+(y-3)^2=\left(\sqrt{2}\right)^2$

∴ Centre of the circle, (h, k) = (2, 3).

Also, one end of the diameter has co-ordinates (a, b).

$\therefore \frac{3+a}{2} = 2$ and $\frac{4+b}{2} = 2$

$\Rightarrow a = 1$

$\Rightarrow b = 2$

∴ The co-ordinates of the other end of the diameter are (1, 2).

Q.4. Find the equation of the circle having centre (3, -4) and touching the line $\mathbf{5x + 12y - 19 = 0}$.

Sol. Given,

Centre of the circle, $(h, k) = (3, -4)$ and radius of the circle, r = perpendicular distance of the line $5x + 12y - 19 = 0$ from $(3, -4)$

$$= \left|\frac{5 \cdot 3 + 12 \cdot (-4) - 19}{\sqrt{5^2 + 12^2}}\right|$$

$$= \left|\frac{15 - 48 - 19}{\sqrt{169}}\right|$$

$$= \frac{52}{13}$$

$$= 4$$

∴ Equation of the circle is $(x - h)^2 + (y - k)^2 = r^2$

$\Rightarrow (x - h)^2 + (y - k)^2 = r^2$

$\Rightarrow (x - 3)^2 + (y + 4)^2 = 4^2$

$\Rightarrow x^2 - 6x + 9 + y^2 + 8y + 16 = 16$

$\Rightarrow x^2 + y^2 - 6x + 8y + 9 = 0$

Q.5. Find the area of the circle having centre at (1,2) and passing through (4,6).

Sol. Find the value of radius, r like Q no.1 and then put the value in the formula for area of a circle $\text{i.e.} \pi r^2$.

Q.6. Find the equation of the circle passing through (0, 0) and making intercepts 5 and 7 on co-ordinate axes. [KVS 2017]

Sol. 'C' is the centre of the circle with coordinates $\left(\frac{5}{2}, \frac{7}{2}\right)$, since AB is the diameter and C is the midpoint of AB.

Radius, $OC = \sqrt{\left(\frac{5}{2} - 0\right)^2 + \left(\frac{7}{2} - 0\right)^2}$

$= \sqrt{\frac{25+49}{4}}$

$= \frac{\sqrt{74}}{2}$

∴ Equation of the circle is

$\left(x - \frac{5}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \left(\frac{\sqrt{74}}{2}\right)^2$

$\Rightarrow x^2 - 5x + \frac{25}{4} + y^2 - 7y + \frac{49}{4} = \frac{74}{4}$

$\Rightarrow x^2 + y^2 - 5x - 7y + \frac{25+49-74}{4} = 0$

$\Rightarrow x^2 + y^2 - 5x - 7y = 0$

Q.7. Find the equation of a circle whose diameter are $2x - 3y + 12 = 0$ and $x + 4y - 5 = 0$ and area is 154 sq. units.

Sol. The given diameter are

$2x - 3y + 12 = 0$ …(1)

$x + 4y - 5 = 0$ …(2)

$(1) - (2) \times 2 \Rightarrow -11y + 22 = 0$

$\Rightarrow y = 2$

$(1) \qquad \Rightarrow 2x + 6 = 0$

$\Rightarrow x = -3$

∴ Centre, C = (-3, 2).

Hence, the equation of the circle (with radius r) is $(x+3)^2 + (y-2)^2 = r^2$ …(3)

Also, $\pi r^2 = 154$

$\Rightarrow \frac{22}{7} \times r^2 = 154$

$\Rightarrow r^2 = 49$

$\Rightarrow r = 7$

∴ Putting the value of r in equation (3)

$(x+3)^2 + (y-2)^2 = 7^2$

$\Rightarrow x^2 + 6x + 9 + y^2 - 4y + 4 = 49$

$\Rightarrow x^2 + y^2 + 6x - 4y - 36 = 0$, is the required equation of the circle.

Q.8. If $lx + my = 1$ touches the circle $x^2 + y^2 = a^{-2}$, then prove that the point (l, m) lies on the circle $x^2 + y^2 = a^{-2}$.

Sol. Since the line $lx + my - 1 = 0$ touches the circle $x^2 + y^2 = a^2$ (radius = a), so the perpendicular distance of the centre (0, 0) from the line is

$$r = \frac{|0 \cdot l + 0 \cdot m - 1|}{\sqrt{l^2 + m^2}}$$

$$\Rightarrow a = \frac{|-1|}{\sqrt{l^2 + m^2}}$$

$\Rightarrow l^2 + m^2 = a^{-2}$

Since (l, m) satisfies $x^2 + y^2 = a^{-2}$, so (l, m) lies on the circle $x^2 + y^2 = a^{-2}$.

Q.9. If the eccentricity of the hyperbola is $\sqrt{2}$. Then, find the general equation of hyperbola. [DDE]

Sol. Given, eccentricity $e = \sqrt{2}$

$$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{2}$$

$\Rightarrow \sqrt{a^2 + b^2} = \sqrt{2a}$

$\Rightarrow a^2 + b^2 = 2a^2$

$\Rightarrow a^2 = b^2$

∴ Equation of the hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1 \text{ or } \frac{y^2}{a^2} - \frac{x^2}{a^2} = 1$$

$\Rightarrow x^2 - y^2 = a^2$

$\Rightarrow y^2 - x^2 = a^2$

Q.10. Write the equation of directrix of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ such that $a > b$.

Sol. The given equation of the ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \qquad a > b$

∴ Equation of the directrices $x = \pm\frac{a}{e}$.

[The equation of directrices of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is also $x = \pm\frac{a}{e}$.]

Q.11. Find the equation for the ellipse that satisfies the given condition: major axis on the x-axis and passes through the points (4, 3) and (6, 2). [DDE]

Sol. Let the equation of the ellipse with major axis on x-axis be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \; (1) \qquad\qquad (a > b)$$

Since the ellipse passes through (4, 3) and (6, 2).

$$\therefore (1) \Rightarrow \frac{16}{a^2} + \frac{9}{b^2} = 1 \qquad (2)$$

$$\text{and} \frac{36}{a^2} + \frac{4}{b^2} = 1 \qquad (3)$$

$$(2) \times 4 - (3) \times 9 \Rightarrow \frac{64}{a^2} - \frac{324}{a^2} = 4 - 9$$

$$\Rightarrow -\frac{260}{a^2} = -5$$

$$\Rightarrow a^2 = 52$$

$$(2) \Rightarrow \frac{9}{b^2} = 1 - \frac{16}{52}$$

$$\Rightarrow \frac{9}{b^2} = \frac{36}{52}$$

$$\Rightarrow b^2 = 13$$

$$\therefore (1) \Rightarrow \frac{x^2}{52} + \frac{y^2}{13} = 1, \text{is the required equation of the ellipse.}$$

Q.12. If the line $y = mx + 1$ is tangent to the parabola $y^2 = 4x$, then find the value of m.

Sol. Given equation of the line is

$y = mx + 1$ …(1)

And equation of the parabola is

$y^2 = 4x$ …(2)

Substituting the value of y from (i) in (ii), we get $(mx + 1)^2 = 4x$

$\Rightarrow m^2x^2 + 2mx + 1 = 4x$

$\Rightarrow m^2x^2 + (2m - 4)x + 1 = 0.$

For line (i) to be a tangent to (ii), discriminant should be zero.

$D = 0$

$\Rightarrow (2m - 4)^2 - 4m^2 \cdot 1 = 0$

$\Rightarrow 4m^2 - 16m + 16 - 4m^2 = 0$

$\Rightarrow 4m^2 - 16m + 16 - 4m^2 = 0$

$\Rightarrow m = 1$

$\therefore m = 1$

Q.13. Find the co-ordinates of points on parabola $y^2 = 8x$ whose focal distance is 4. [DDE]

Sol. Given equation of the parabola is $y^2 = 8x$

$\Rightarrow y^2 = 4 \cdot 2 \cdot x$

$\therefore a = 2, \text{focus} = (2,0)$

Let $P(x, y)$ be any point on the parabola, then $PS = 4$

$\Rightarrow \sqrt{(x-2)^2 + (y-0)^2} = 4$

$\Rightarrow x^2 - 4x + 48x = 16$

$\Rightarrow x^2 + 4x - 12 = 0$

$\Rightarrow (x + 6)(x - 2) = 0$

$\therefore x = -6, x = 2 \ \Rightarrow y^2 = 8 \cdot 2$

$\Rightarrow y^2 = 8(-6)$ $\qquad$ $y^2 = 16 \Rightarrow y = \pm 4$

$\Rightarrow y = \sqrt{-48}$

Not possible

∴ The point P is (2, 4), (2, -4).

Q.14. Find the equation of the set of all points such that the difference of their distance from (4,0) and (-4, 0) is always equal to 2 units. Write the name of the curve. [DDE]

Sol. Let P(x, y) denote all such points that the difference of their distance from A(4,0) and B(-4,0) is equal to 2.

$\therefore PA - PB = 2$

$\Rightarrow \sqrt{(x-4)^2 + (y-0)^2} - \sqrt{(x+4)^2 + (y-0)^2} = 2$

$\Rightarrow \sqrt{(x-4)^2 + y^2} = 2 + \sqrt{(x+4)^2 + y^2}$

$\Rightarrow (x-4)^2 + y^2 = 4 + 4\sqrt{(x+4)^2 + y^2} + (x+4)^2 + y^2$

$\Rightarrow x^2 - 8x + 16 + y^2 = 4 + 4\sqrt{(x+4)^2 + y^2} + x^2 + 8x + 16 + y^2$

$\Rightarrow -4 - 16x = 4\sqrt{(x+4)^2 + y^2}$

$\Rightarrow -4(1 + 4x) = 4\sqrt{(x+4)^2 + y^2}$

$\Rightarrow 1 + 8x + 16x^2 = x^2 + 8x + 16 + y^2$

$\Rightarrow 1 + 8x + 16x^2 = x^2 + 8x + 16 + y^2$

$\Rightarrow 15x^2 - y^2 = 15$

$\Rightarrow \frac{x^2}{1} - \frac{y^2}{15} = 1,$ Which is the equation of a hyperbola

Q.15. Find the equation of the set of all points the sum of whose distance from the points $A(3, 0)$ and $B(9, 0)$ is 12 units. Write the name of the curve.

Sol. Solve as Q no.6.

Q.16. Find the equation of the ellipse referred to its axes as the axes of co-ordinates with latus rectum of length 4 and distance between foci is $4\sqrt{2}$.

Sol. Let the equation of the ellipse be

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ ... (i)

Length of latus rectum $= \frac{2b^2}{a} = 4$

$\Rightarrow b^2 = 2a$

Distance between the foci $= 4\sqrt{2}$

$\Rightarrow 2ae = 4\sqrt{2}$

$\Rightarrow ae = 2\sqrt{2}$

Since $b^2 = a^2(1 - e^2)$

$\Rightarrow 2a = a^2 - 8$

$\Rightarrow a^2 - 2a - 8 = 0$

$\Rightarrow (a-4)(a+2) = 0$

$\Rightarrow a = 4$ or $a = -2$

$(ii) \Rightarrow b^2 = 2 \cdot 4$ $[\therefore \text{a cannot be negative}]$

$\Rightarrow b^2 = 8$

$\therefore (i) \Rightarrow \frac{x^2}{16} + \frac{y^2}{8} = 1,$ is the required equation of the ellipse.

Q.17. Find the equation of the hyperbola whose conjugate axis is 5 and distance between the foci is 13.

Sol. Let the equation of the ellipse be

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ (1)$ $[a > b]$

Length of conjugate axis = 5

$\Rightarrow 2b = 5$

$\Rightarrow b = \frac{5}{2}$

And distance between the foci = 13

$\Rightarrow 2c = 13$

$\Rightarrow c = \frac{13}{2}$

Since $c^2 = a^2 + b^2$

$\Rightarrow a^2 = c^2 - b^2$

$= \frac{169}{4} - \frac{25}{4}$

$= \frac{144}{4}$

$\Rightarrow a^2 = 36$

$\Rightarrow a = 6$

$\therefore (1) \Rightarrow \frac{x^2}{36} + \frac{y^2}{25} = 1$, is the required equation of the ellipse.

Q.18. If the parabola $y^2 = 4ax$ passes through the point (3, 2), find the length of its latus rectum.

Sol. Given equation of the parabola is

$y^2 = 4ax$

Since it passes through (3, 2)

$4 = 4a \cdot 3$

$a = \frac{1}{3}$

∴ Length of the latus rectum = $4a$

$= \frac{4}{3}$