Chapter 1

Introduction to Signals and Systems

LEARNING OBJECTIVES

After reading this chapter, you will be able to understand:

- Classification of signals
- Basic operations on signals
- Amplitude scaling
- Time scaling
- Elementary signals
- Exponentially damped sinusoidal signals
- Systems and classification of systems
- Linear time invariant systems
- Interconnected systems
- Properties of continuous time LTI systems
- Invertibililty
- Discrete time LTI systems

Introduction

A signal is defined as a function of one or more variables that conveys information on the nature of a physical phenomenon.

When the function depends on a single variable, the signal is said to be one dimensional (Example: Speech signal), otherwise it is said to be multidimensional (Example: Image).

A system is defined as an entity that manipulates one or more signals to accomplish a function, there by yielding new signals, (Example: Communication system).

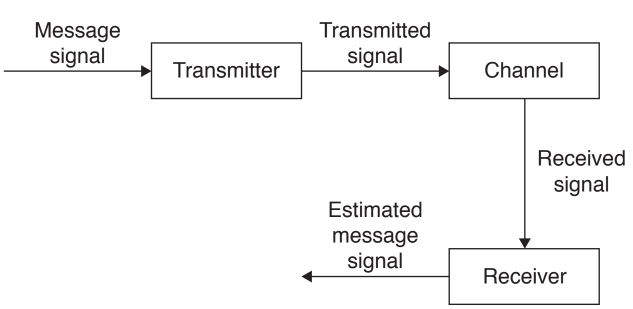


Figure 1 Elements of communication system

The transmitter changes the message signal into a form suitable for transmission over the channel.

The receiver process the channel output to produce an estimate of the message signal.

Classification of Signals

Continuous-time and discrete-time signals

A signal $x(t)$ is said to be a continuous-time signal if it is defined for all time t.

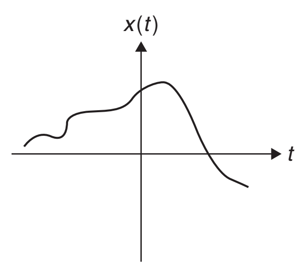


Figure 2 Continuous-signal

The above waveform represents an example of a continuous-time signal whose amplitude or value varies continuously with time.

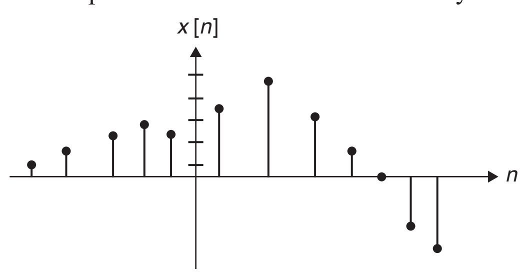


Figure 3 Representation of $x(t)$ as discrete time signal $x[n]$

A discrete time signal is defined only at discrete instants of time. Thus the independent variable has discrete values only, which are usually uniformly spaced. A discrete time signal is often derived from a continuous-time signal by sampling it at a uniform rate.

Let T_s denote the sampling period and n denotes an integer. Then sampling a continuous-time signal $x(t)$ at time $t = nT_s$ yields a sample with values $x(nT_s)$, we can write $x[n] = x(nT_s)$, $n = 0, \pm 1, \pm 2, \ldots$

Even and odd signals

A continuous-time signal $x(t)$ is said to be an even signal if $x(-t) = x(t)$ for all t. The signal $x(t)$ is said to be an odd signal if $x(-t) = -x(t)$ for all t. A discrete-time signal $x[n]$ is even if $x[-n] = x[n]$ and $x[n]$ is odd if $x[-n] = -x[n]$.

Examples of even signal $x(t) = \cos t$, odd signal $x(t) = \sin t$. An odd signal must necessarily be 0 at $t = 0$ or $n = 0$ since $x(-t) = -x(t)$, or $x[-n] = -x[n]$, so $x(0) = 0$ or $x[0] = 0$.

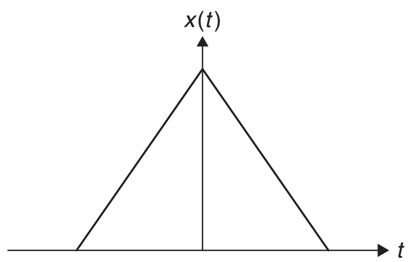


Figure 4 Even continuous-time signal

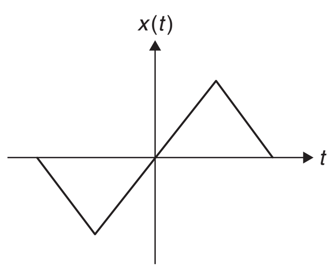


Figure 5 Odd continuous-time signal

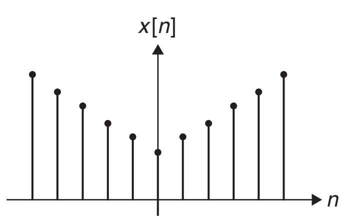


Figure 6 Even discrete-time signal

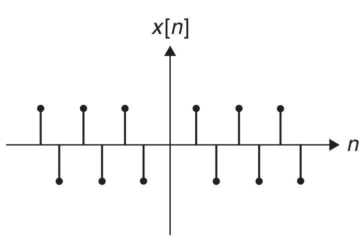


Figure 7 Odd discrete-time signal

Even signals are symmetric about vertical axis or time origin. Where as odd signals are anti-symmetrical about the time origin, (same in case of discrete time signals). Any signal $x(t)$ can be expressed as sum of even and odd components.

$$x(t) = x_e(t) + x_o(t)$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_o(t) = \frac{1}{2}[x(t) - x(t)]$$

In case of complex valued signal, a complex valued signal $x(t)$ is said to be conjugate symmetric if $x(-t) = x * (t)$. Where $x * (t)$ is complex conjugate

If $x(t) = a(t) + jb(t)$

$$x(-t) = a(-t) + jb(-t)$$

then,

$$x^*(t) = a(t) - jb(t)$$

By comparing above equations, A complex valued signal $x(t)$ is conjugate symmetric if its real part is even $[a(t) = a(-t)]$, and its imaginary part is odd $[b(-t) = -b(t)]$. (A similar remark applies for discrete signals too).

Example 1: The even and odd components of the signal $x(t) = e^{-2t} \sin t$ are?

Solution: By replacing t with $-t$, $x(-t) = e^{2t}$.

We have,

$$\sin(-t) = -e^{2t} \sin t.$$

Then,

$$x_e(t) = \frac{1}{2}[x(t) + (-t)] = \frac{1}{2}[e^{-2t} \sin t - e^{+2t} \sin t]$$

$$x_o(t) \ = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}[e^{-2t} \sin t + e^{2t} \sin t]$$

Now,

$$x_e(t) = \sin t \frac{1}{2}[e^{-2t} - e^{2t}] = -\sin h(2t) \sin t$$

$$x_o(t) = \sin t \frac{1}{2}[e^{-2t} + e^{2t}] = \cos h(2t) \sin t$$

Periodic and non-periodic signals

If $x(t) = x(t + T)$ for all t, where T is a positive constant, then $x(t)$ is said to be periodic.

If this condition satisfies for $T = T_0$, then it also satisfied for $T = 2T_0, 3T_0, 4T_0 \ldots.$

The smallest value of T that satisfies $x(t) = x(t + T)$ is called fundamental period of T and its reciprocal $f = \dfrac{1}{T_0}$ is called fundamental frequency.

Frequency measured in Hertz (Hz) and the Angular frequency measured in radians per second is defined by $\omega = 2\pi f = \frac{2\pi}{T_0}$.

A discrete time signal $x[n]$ is periodic with period N, (where N is a positive integer) if it is unchanged by a time shift of N.

$$x[n] = x[n + N] \text{ for integer } n, N.$$

And $x[n]$ is also periodic with period $2N$, $3N$.... The fundamental period N_0 is the smallest positive value of N for which of the above equation holds.

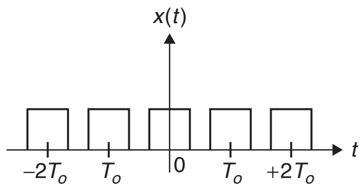


Figure 8 Continuous-time periodic signal with T_0.

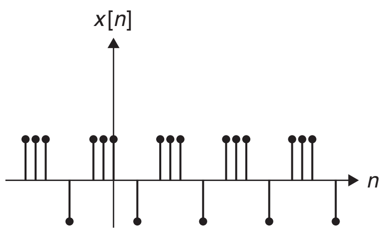


Figure 9 Discrete-time periodic signal with $N_0 = 4$

A signal $x(t)/x[n]$ that is not periodic will be referred to as an non-periodic signal.

Deterministic signals and random signals

The signal that can be completely specified by a mathematical equation is called deterministic signal, otherwise random signal.

Examples of deterministic signals.

$$x(t) = 2 - 2t\, u(t),\ x(t) = \sin \omega t,$$
$$x[n] = \cos \omega n,\ x[n] = a^n\, u[n]$$

A random signal is a signal about which there is uncertainty before it occurs. Such random signals are generally characterized by their mean, mean square values.

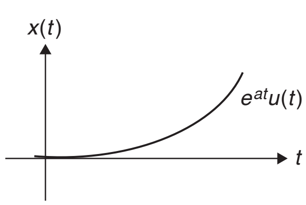


Figure 10 Deterministic signal

Figure 11 Random signal

Examples of random signals: the electrical noise generated in the amplifier, and interference component in television receiver.

Energy signals and power signals

Total energy of the continuous-time signal $x(t)$ is $E = \lim_{T\to\infty} \int_{-T/2}^{T/2} x^2(t)dt = \int_{-\infty}^{\infty} x^2(t)dt$

and its time average power

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt$$

The time averaged power of a periodic signal $x(t)$ of fundamental period T is given by $P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$.

In case of discrete-time signals the integrals are replaced by the corresponding sums.

$$E = \sum_{n=-\infty}^{\infty} x^2[n],$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x^2[n]$$

The average power of periodic signal $x(n)$ with fundamental period N is

$$P = \frac{1}{N}\sum_{n=0}^{N-1} x^2[n]$$

A signal is referred to as an energy signal if the total energy of the signal satisfies the condition $0 < E < \infty$.

A signal is said to be as a power signal if the average power of the signal satisfies the condition $0 < P < \infty$.

The energy and power clarification of signals are mutually exclusive, an energy signal has zero time averaged power where as a power signal has infinite energy.

It is of interest to note that periodic signals and random signals are usually viewed as power signals, where as signals that are both deterministic and non-periodic are usually viewed as energy signals.

Causal, non-causal, and anti-causal signals

A signal is said to be causal if it is defined only for $t > 0$.

Example: $u(t)$, $Ae^{bt}\, u(t)$, $\sin \omega t \cdot u(t)$.

A signal is said to be non-causal if it is defined for either $t \le 0$ or for both $t \le 0$ and $t > 0$.

Example: Ae^{bt}, for all t, sin ωt, for all t.

When a non-causal signal is defined only for $t \le 0$ it is called anti-causal signal.

Example: $Ae^{bt}\, u(-t)$, $\sin\omega t \cdot u(-t)$.

Example 2: What is the fundamental period of the discrete time signal $x[n] = e^{j(2\pi/6)n} + e^{j(3\pi/4)n}$?

Solution: The first exponential fundamental period $N_1 = m\left(\dfrac{2\pi}{2\pi/6}\right) = 6$ m will be integer for $m = 1$, so fundamental period $N_1 = 6$.

The second exponential, the fundamental period $N_2 = m\left(\dfrac{2\pi}{3\pi/4}\right) = m \cdot \dfrac{8}{3}$ will be integer for $m = 3$, so fundamental period, $N_2 = 8$.

The least common multiple for these fundamental periods is 24, i.e., the fundamental period of $x[n]$.

Example 3: Find the fundamental period (in sec) and fundamental frequency (in rad/sec) for the following signals.

- $v(t) = 20\sin 100t + 10\cos 300t + 5\sin\left(500t + \dfrac{\pi}{3}\right)$
- $v(t) = 3\cos 200t + 4\cos 300t + 6\sin 500t$

Solution:

- $v(t) = 20\sin 100t + 10\cos 300t + 5\sin\left(500t + \dfrac{\pi}{3}\right)$

  For the above signal, we can consider the individual time periods are $T_1 = \dfrac{2\pi}{100}$, $T_2 = \dfrac{2\pi}{300}$, and $T_3 = \dfrac{2\pi}{500}$.

  The LCM of T_1, T_2, T_3—least common multiple is $\dfrac{2\pi}{100} = T_0$ is the fundamental time period in seconds. $T_0 = \dfrac{2\pi}{100}$, the fundamental frequency is $\omega_0 = \dfrac{2\pi}{T_0} = 100$ rad/ sec.
- $v(t) = 3\cos 200t + 4\cos 300t + 6\sin 500t$

  For the above signal, we can consider the individual time periods are $T_1 = \dfrac{2\pi}{200}$, $T_2 = \dfrac{2\pi}{300}$, and $T_3 = \dfrac{2\pi}{500}$.

  The LCM of $(T_1, T_2, T_3), = \dfrac{2\pi}{100} = T_0$ is the fundamental period. [$\therefore$ Check $v(t + T_0) = v(t)$]. So fundamental frequency $(\omega_0) = \dfrac{2\pi}{T_0} = 100$ rad/ sec.

Basic Operations on Signals

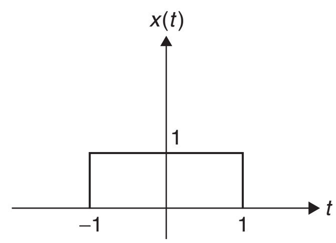
Operations Performed on Dependent Vvariables

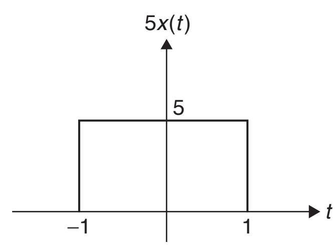
Amplitude scaling

$$y(t) = Cx(t),$$
$$y[n] = Cx[n].$$

The value of $y(t)$ is obtained by multiplying the corresponding value of $x(t)$ by the scalar C for each instant of time t.

A physical example of a device that performs amplitude scaling is an electronic amplifier.

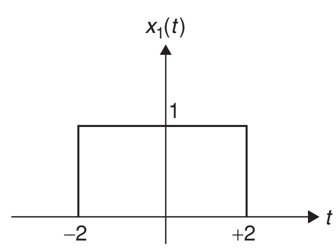


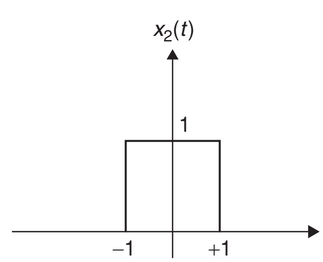


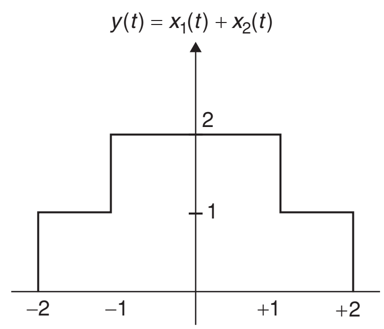
Addition

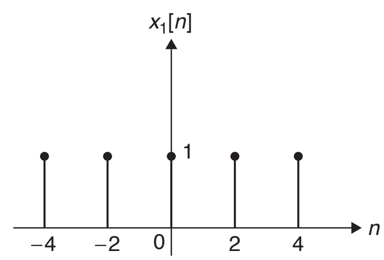
$$y(t) = x_1(t) + x_2(t)$$
$$y[n] = x_1[n] + x_2[n]$$

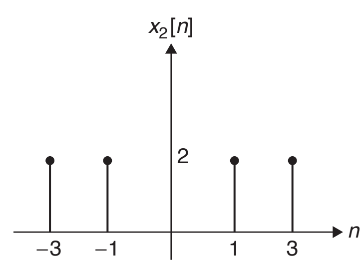
$x_1(t)$ and $x_2(t)$ are continuous-signals then $x_1(t) + x_2(t)$ denote addition of the two signals.

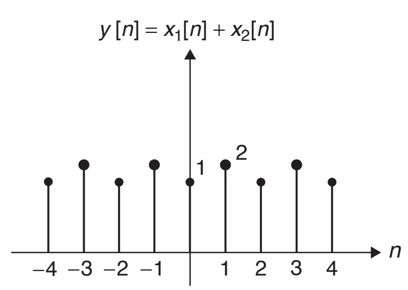












Multiplication

$$y[n] = x_1[n] \cdot x_2[n],$$

$$y(t) = x_1(t) \cdot x_2(t)$$

If $x_1(t)$ and $x_2(t)$ are two continuous-time signals, then their product $y(t) = x_1(t)\, x_2(t)$. The value of $y(t)$ at time 't' is given by the product of the corresponding values of $x_1(t)$ and $x_2(t)$.

For example in AM modulated signal, the message signal will be multiplied by the carrier signal; the resultant changes the amplitude of carrier as per message signal.

Differentiation

$$y(t) = \frac{d}{dt}x(t),$$

$$y[n] = x[n] - x[n-1]$$

The derivative of $x(t)$ with respect to time is defined as $y(t)$ = .

Example: The voltage developed across inductor is given by $v(t) = L\dfrac{di(t)}{dt}$.

Integration

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

$$y[n] = \sum_{m=-\infty}^{n} x[m]$$

The Integral of $x(t)$ with respect to time 't' is defined as $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$.

Example: The voltage across capacitor is given by

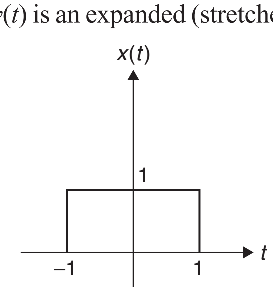
$$\vartheta(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau.$$

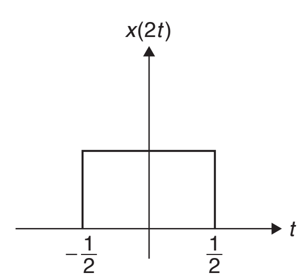
Operations Performed on the Independent variable

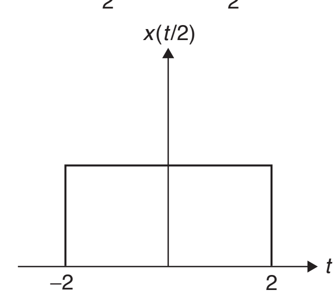
Time scaling

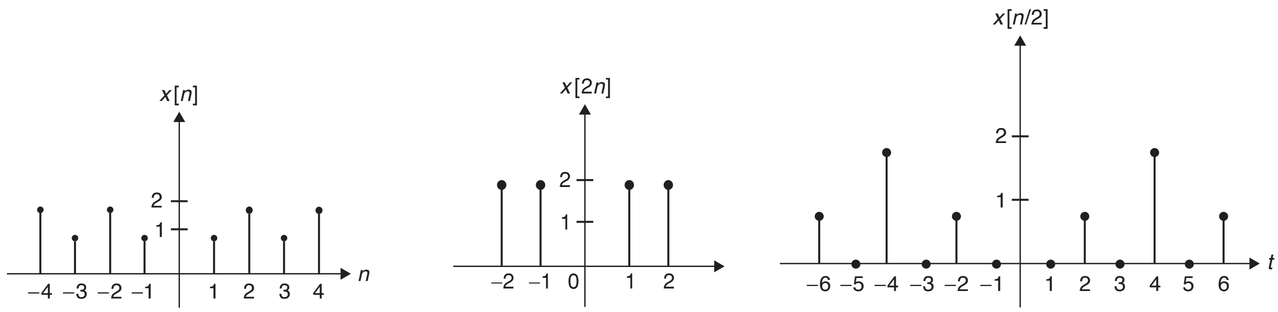
$$y(t) = x(\text{at})$$

If $a > 1$, the signal $y(t)$ is a compressed version of $x(t)$, if $0 < a < 1$ the signal $y(t)$ is an expanded (stretched) version of $x(t)$.





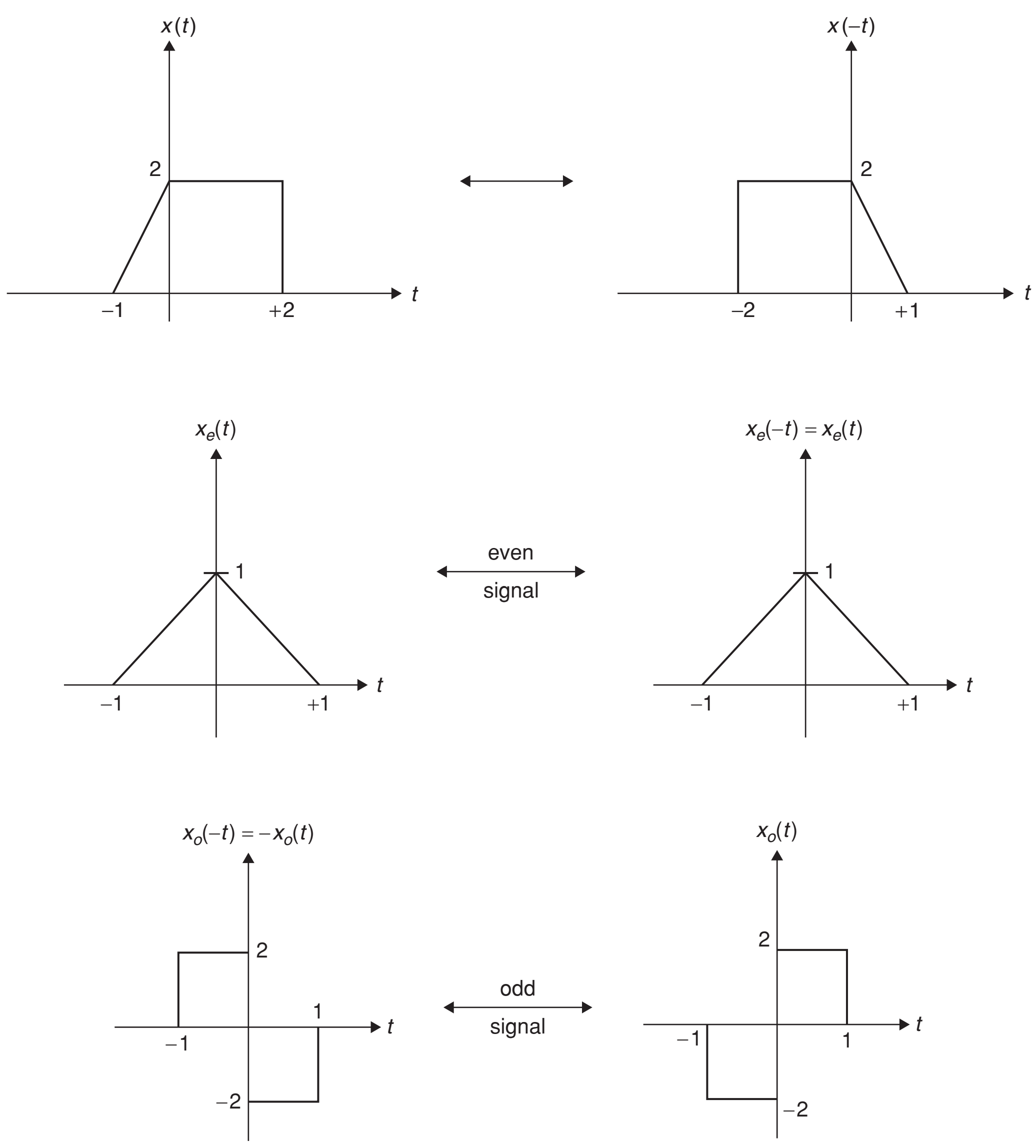




Reflection

$$y(t) = x(-t)$$

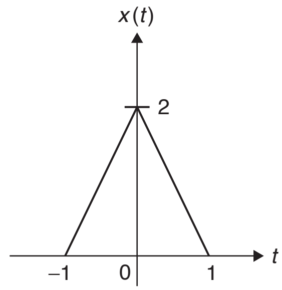
Even signal $[x(-t) = x(t)]$ is same as reflected signal odd signal $[x(-t) = -x(t)]$ is negative of its reflection. Reflected signal can be obtained by replacing t by $-t$.

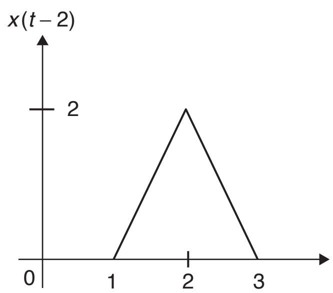


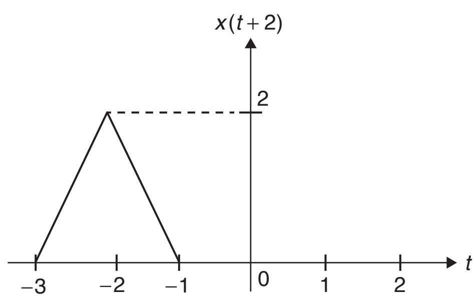
Time shifting

$$y(t) = x(t - t_0)$$

If $t_0 > 0$, the waveform of $y(t)$ is obtained by shifting $x(t)$ toward the right, relative to the time axis, if $t_0 < 0$, $x(t)$ is shifted to the left.







Precedence rule for time shifting and time scaling

If $y(t) = x(at - b)$, to obtain $y(t)$ from $x(t)$, the proper order is based on the fact that the scaling operation always replaces 't' by 'at', while time shifting operation always replaces 't' by '$t - b$'.

Hence time shifting operation is performed first on $x(t)$, resulting in an intermediate signal $\vartheta(t) = x(t - b)$.

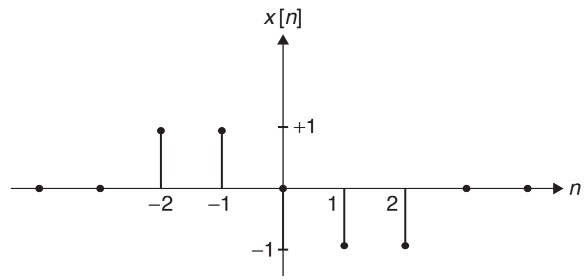
Next time scaling operation is performed on $\vartheta(t)$ resulting is desired output $y(t) = \vartheta(at) = x(at - b)$.

Incorrect way is first time scaling $[x(at)]$ and then applying time shifting results is $x[a(t - b)] \neq x[at - b]$. Similar approach we follow for discrete time signals.

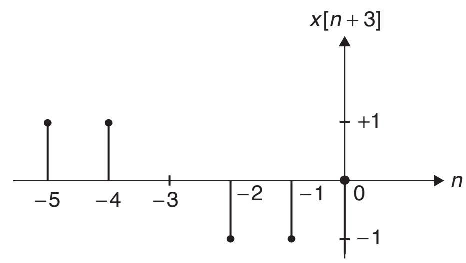
Example 4: The discrete time signal

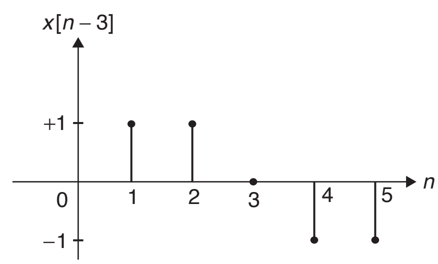
$$x[n] = \begin{cases} 1 & n = -1, \quad -2 \\ -1, & n = 1, \quad 2 \\ 0, & n = 0, \quad [n] > 2 \end{cases}$$

Then $x[n + 3]$, and $x[n - 3]$ are?

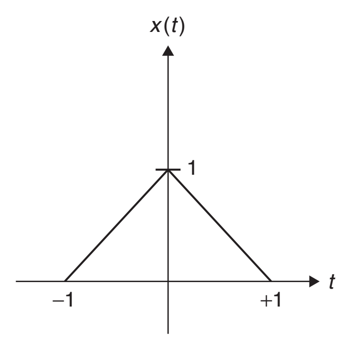


Solution:

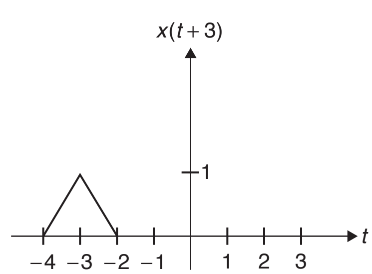


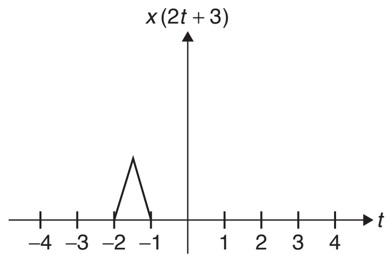


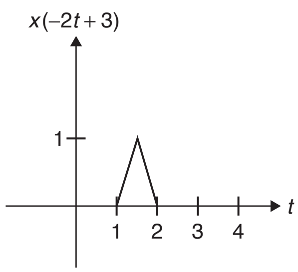
Example 5: A triangular pulse $x(t)$ is depicted in figure sketch $y(t) = x(-2t + 3) + x(2t + 3)$.

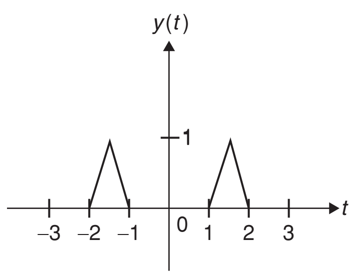


Solution:

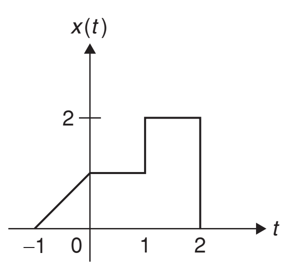




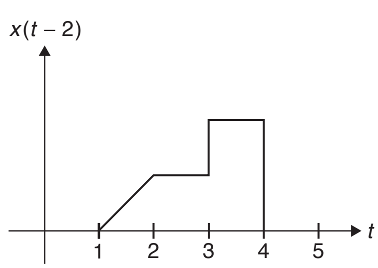




Example 6: For the signal $x(t)$ depicted in figure, draw the signals $x(t-2)$, $x(2t+1)$ $x(3-t)$, $x(-t-1)$, $[x(-2t)+x(2t)]$?

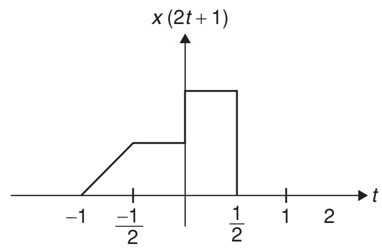


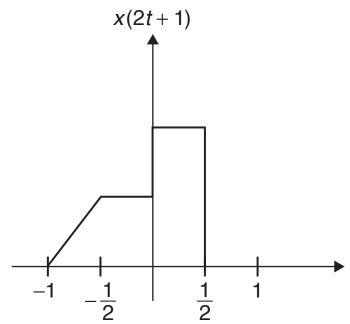
Solution:



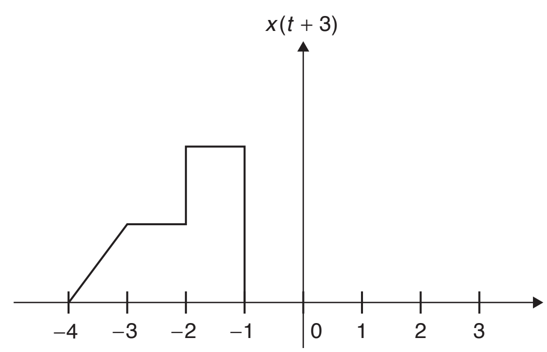
$x(t-2)$ is right shift by 2. To find out $x(2t+1)$, scaling by 2 replaces t by $2t$, similarly shifting by + 1 replaces t by $t+1$.

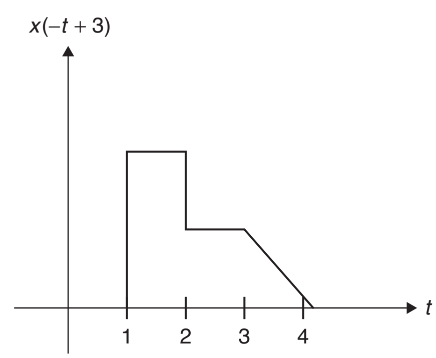
To get $x(2t+1)$, first we need to perform shifting $x(t+1)$ then scaling $x(2t+1)$ but first scaling results in $x(2t)$, and then shifting results in $x(2(t+1)) = x(2t+2)$ which is not desired.



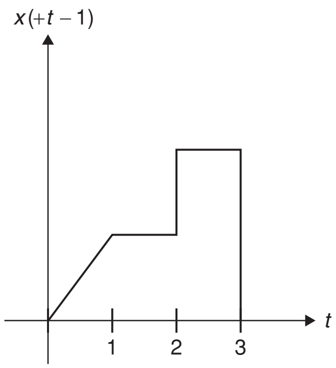


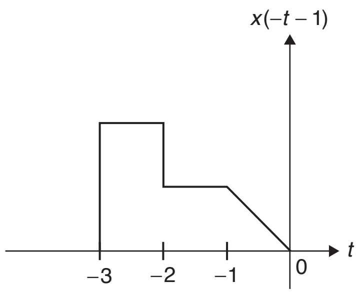
$x(3-t)$, first shifting $x(t+3)$, then scaling by -1, i.e., $x(-t+3)$.

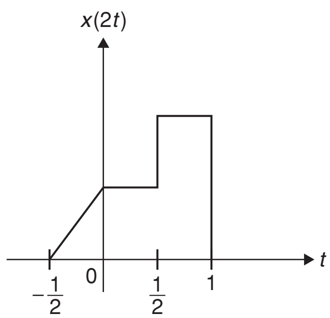


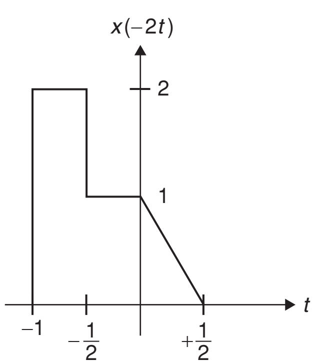


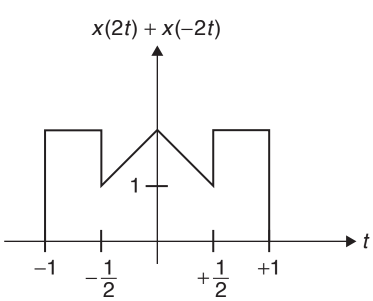
For $x(-t-1)$ first shifting $x(t-1)$ then scaling $x(-t-1)$.



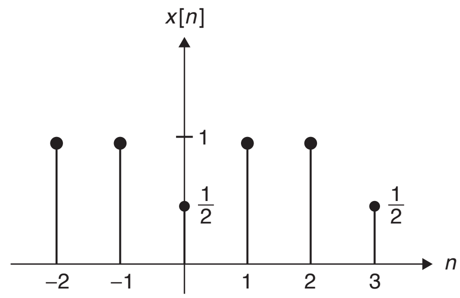




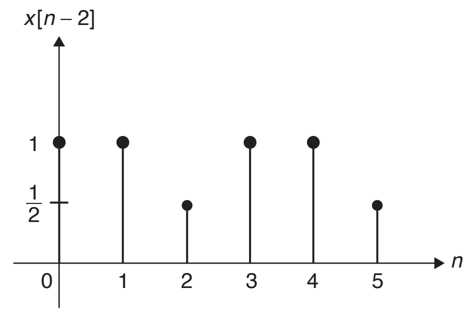


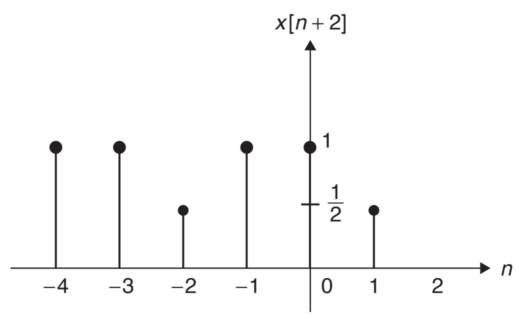


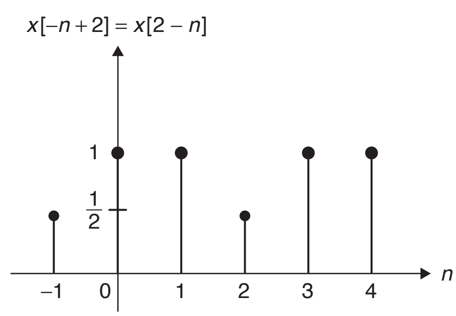
Example 7: For the discrete-time signal shown in figure, draw the following signals $x[n-2]$, $x[2-n]$ $x[3n+2]$, $x[n/2]$, $x[n]+x[-n]$.

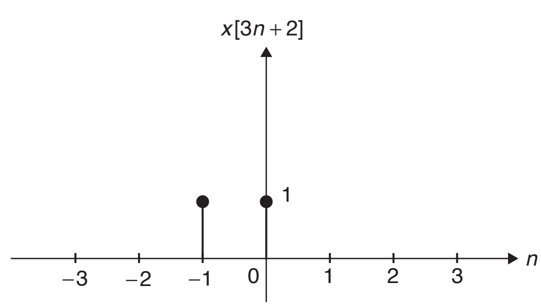


Solution:





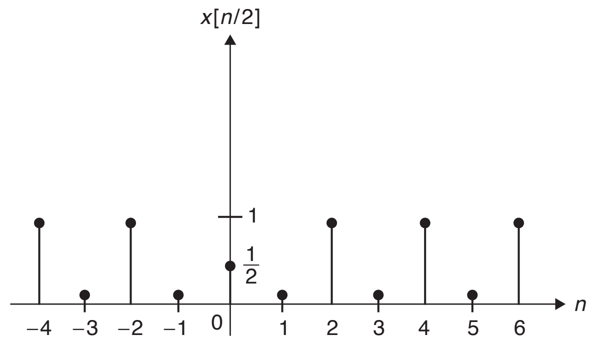




$x[3n + 2]$ will be defined in the range of $-2 \le 3n + 2 \le 3, -4 \le 3n \le 1, -4/3 \le n < 0/3$ and $-1 \le n \le 0$

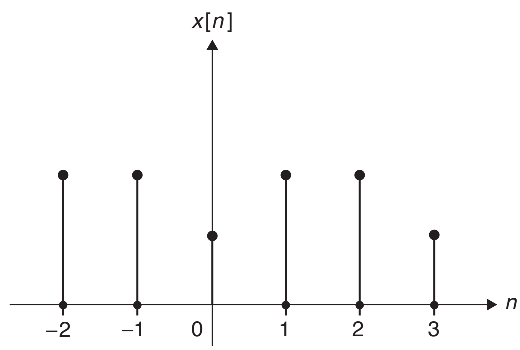
$$x(3n + 2), x(3 \times 0 + 2) = x(2) = 1$$

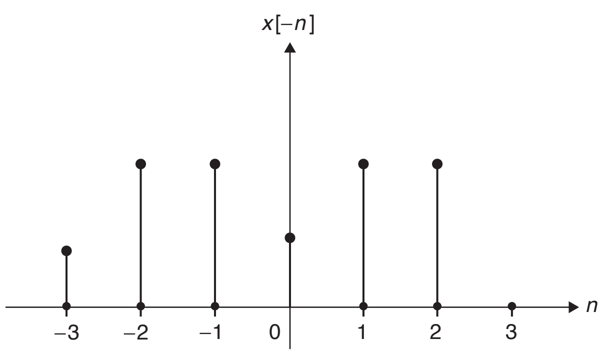
$$x(3x - 1 + 2) = x(-1) = 1$$

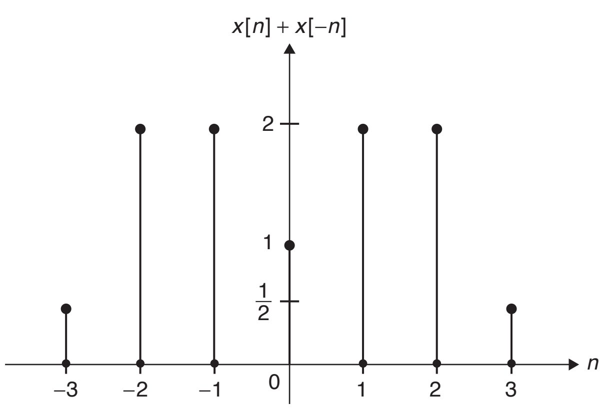


For $x[n/2]$, the range is $-2 \le n/2 \le 3 \Rightarrow -4 \le n \le 6$. Now, $x[n/2]$ for $n = 4$ is $x[-4/2] = x[-2] = 1$.

For $n = -3$, $x[-3/2] = 0$. Which is not defined and so on….







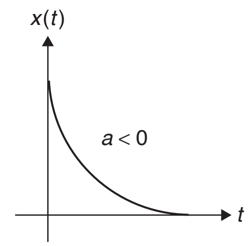
Elementary Signals

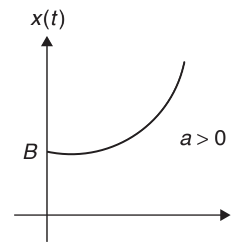
Exponential Signals

$$x(t) = Be^{at}$$

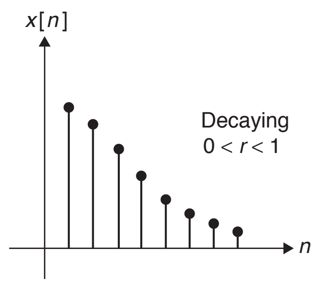
a, B are real parameters, B is the amplitude of the exponential signal measured at $t = 0$, when $a < 0$, it is decaying exponential.

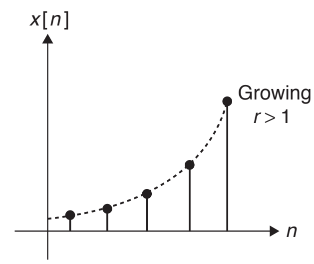
When $a > 0$, it is growing exponential





In discrete time, real exponential signal $x[n] = Br^n, r = e^{\alpha}$,





For $r < 0$, the discrete time exponential $x[n]$ assumes alternating signs for the n, r^n is positive for n even and negative for n odd.

Sinusoidal Signals

$$x(t) = A\cos(\omega t + \phi)$$

A sinusoidal signal is an example of periodic signal $x(t + T) = x(t)$, where $T = \dfrac{2\pi}{\omega}$

Consider discrete-time signal,

$$x[n] = A\cos(\Omega n + \phi).$$

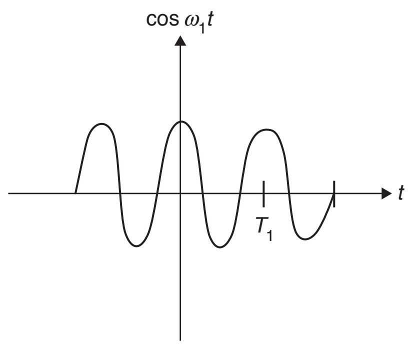
It is periodic signal

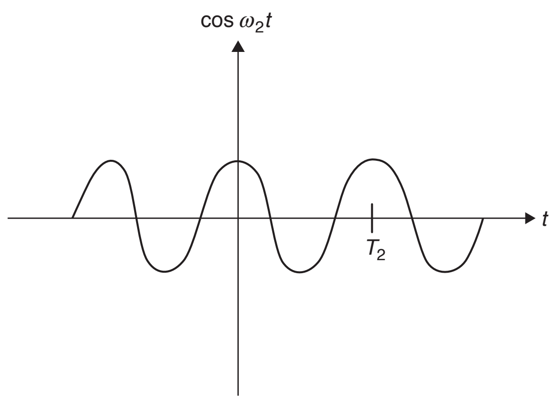
$$x[n + N] = x[n],$$

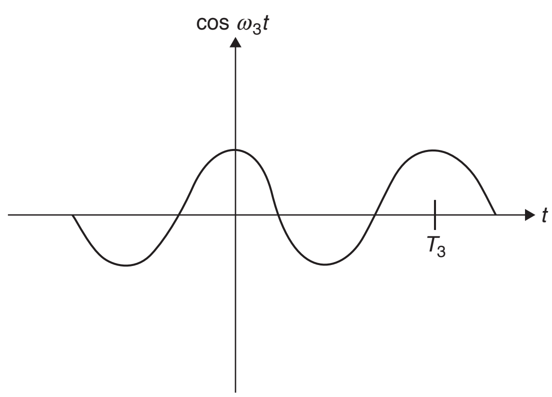
where $N = \dfrac{2\pi m}{\Omega}$, m, N integers.

The angular frequency Ω must be a rational multiple of 2π. Relation between sinusoidal and complex exponential signals is $e^{j\theta} = \cos\theta + j\sin\theta$. $x(t) = e^{j\omega_0 t}$ is a complex exponential signal. An important property of this signal is that it is periodic, $e^{j\omega_0 t} = e^{j\omega_0(t+T)} = e^{j\omega_0 t}e^{j\omega_0 T}$ for periodicity $e^{j\omega_0 T} = 1$.

This is true for the fundamental period $T_0 = \dfrac{2\pi}{|\omega_0|}$. Thus the signals $e^{j\omega_0 t}$ and $e^{-j\omega_0 t}$ have the same fundamental period. $e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t$ (from Euler's relations) we can observe from the relation $T_0 = \frac{2\pi}{|\omega_0|}$, that fundamental period T_0 is inversely proportional to the fundamental frequency ω_0.







For the above 3 sinusoidal, $\omega_1 > \omega_2 > \omega_3$, which implies that $T_1 < T_2 < T_3$.

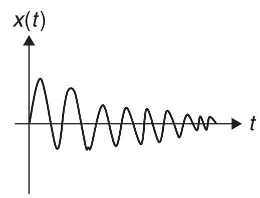
In case of discrete-time complex exponentials $e^{j\omega_0 n}$ is periodic only if, for some integers M, N the fundamental period will be $N = M\left(\frac{2\pi}{\omega_0}\right)$, for integers M, N, assuming M, N do not have any factors in common.

Exponentially Damped Sinusoidal Signals

The multiplication of a sinusoidal signal by a real valued decaying exponential signal results in a new signal referred to as an exponentially damped sinusoidal signal.

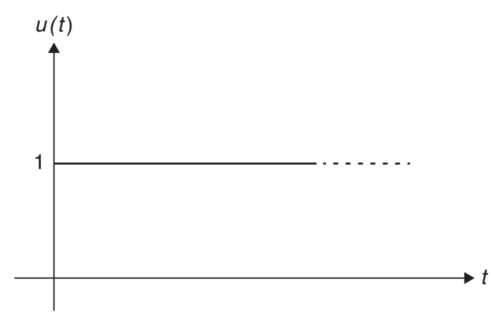
The discrete-time version of the exponentially damped sinusoidal signal is $x[n] = Ar^n \sin[\Omega n + \phi]$. For the signal to decay exponentially with time, the parameter r should lie in the range $0 < |r| < 1$.

For continuous-time, $x(t) = Ae^{-\alpha t}\sin(\omega t + \phi)\ \alpha > 0$.

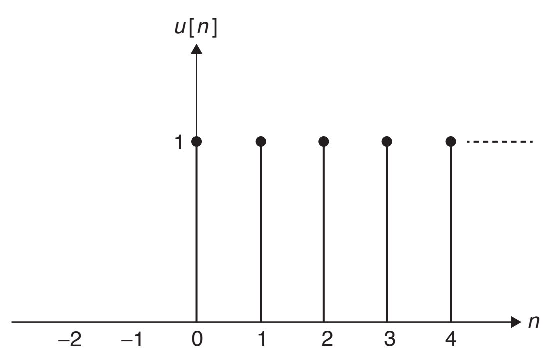


Step Function

$$
\begin{aligned}
u(t) &= 1,\ t > 0 \\
&= 0,\ t < 0 \\
&= \frac{1}{2},\ t = 0
\end{aligned}
$$



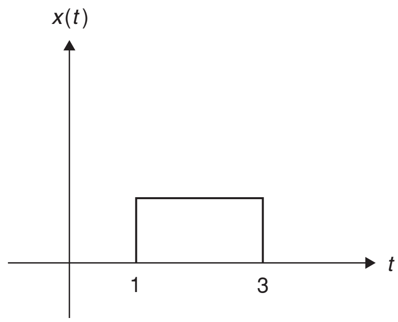
For discrete-time version $u[n] = 1,\ n \geq 0 = 0,\ n < 0$

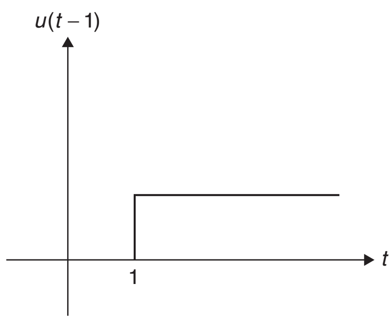


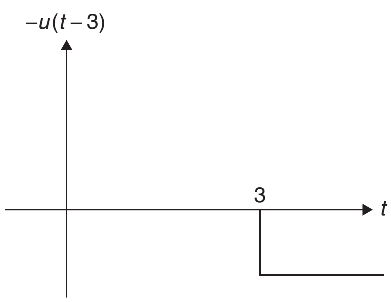
The unit step function $u(t)$ is a particularly simple signal to apply as a test signal, the unit step function is useful because the output of a system due to a step input reveals a great deal about how quickly the system responds to an abrupt change in the input signal. A similar remark applies to $u[n]$ in the context of discrete-time system.

If we want a signal to start at $t = 0$ (so that it has value of zero for $t < 0$) we need to multiply the signal by $u(t)$.

Consider a rectangular pulse from $t = 1$ to $t = 3$, it can be derived from delayed unit impulse functions as shown here:

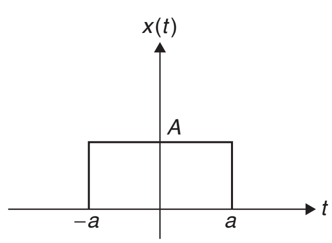






$$x(t) = u(t-1) - u(t-3)$$

Rectangular Pulse



$$x(t) = A,\ 0 \le |t| \le a$$
$$= 0,\ |t| > a$$

It can be written as $x(t) = A\,u(t+a) - A\,u(t-a)$.

Impulse Function

In discrete time

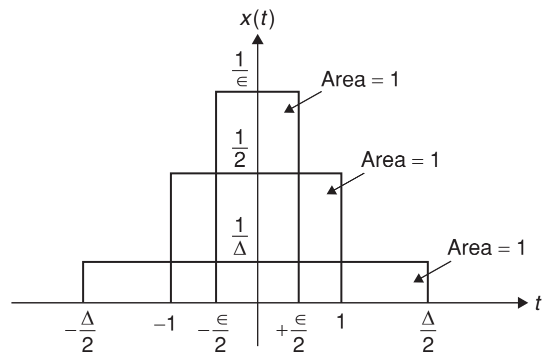
$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \ne 0 \end{cases}$$

The continuous-time version of the unit impulse is defined by $\delta(t) = 0$ for $t \ne 0$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1.$$

The impulse $\delta(t)$ is also referred to as the Dirac delta function.

We can visualize an impulse function as a tall, narrow, rectangular pulse of unit area, the width of this rectangular public is very small $\in \to 0$, and consequently its height is very large $\frac{1}{\in} \to \infty$



Evolution of a rectangular pulse of unit area into an Impulse of unit strength

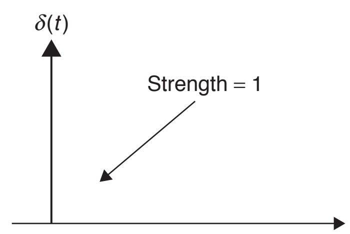


Figure 11 Unit impulse

The impulse $\delta(t)$ and unit step function $u(t)$ are related to each other as $\delta(t) = \frac{d}{dt}[u(t)]$ (or) $\delta[n] = u[n] - u[n-1]$.

Conversely $u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = \int_{o}^{\infty} \delta(t-\tau)d\tau$ (or)

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k] = \sum_{k=-\infty}^{n} \delta[k]$$

The properties of impulse function:

1. $\delta(-t) = \delta(t)$
2. $x(t)\delta(t) = x(0)\ \delta(t),\ x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$
3. $\int_{-\infty}^{\infty} x(t)\delta(t)dt = x(0), \int_{-\infty}^{\infty} x(t)\ \delta(t-t_0)dt = x(t_0)$
4. $\int_{-\infty}^{\infty} x(\lambda)\delta(t-\lambda)d\lambda = x(t)$

5. $\delta(at) = \frac{1}{a}\delta(t),\ a > 0$

6. $\int_{-\infty}^{\infty} \delta'(t)dt = 0$

7. $\int_{-\infty}^{\infty} f(t)\delta'(t - t_0)dt = \frac{d}{dt} f(t)|_{t=t_0}$

8. $\int_{-\infty}^{\infty} f(t)\delta''(t - t_0)dt = \frac{d^n}{dt^n} f(t)|_{t=t_0}$

9. $x[n]\ \delta[n] = x[0]\ \delta\ [n], x[n]\delta[n - n_0] = x[n_0]\delta[n - n_0]$

10. $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$

Example 8: $\frac{\omega^2 + 1}{\omega^2 + 9}\delta(\omega - 1) = \frac{1^2 + 1}{1^2 + 9}(\omega - 1) = \frac{1}{5}\delta(\omega - 1).$

Solution:

$$\sin\left(t - \frac{\pi}{2}\right)\delta(t) = \sin\left(0 - \frac{\pi}{2}\right)\delta(t) = -\delta(t)$$

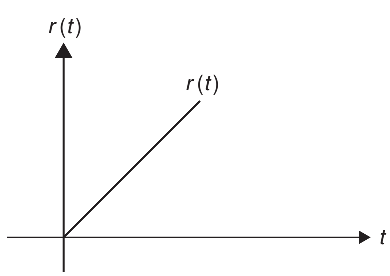
$$\int_{-\infty}^{\infty} \delta(t-4)\cos\left(\frac{\pi t}{4}\right)dt = \cos\left(\frac{\pi}{4}.4\right) = \cos\pi = -1$$

$$\int_{-\infty}^{\infty} e^{-2(x-t)}\delta(2-t)dt = e^{-2(x-2)}$$

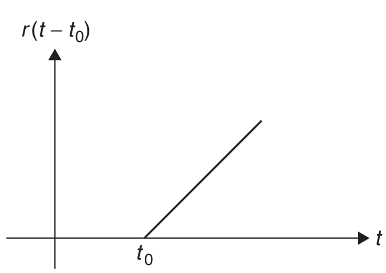
Ramp Function

The impulse function $\delta(t)$ is the derivative of the step function $u(t)$ with respect to time. By the same token, the integral of the step function $u(t)$ is a ramp function of unit slope.

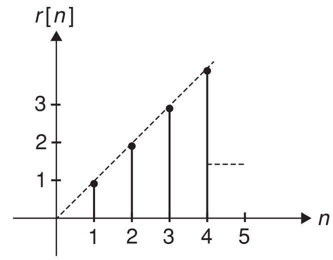
$$r(t) = t,\ t \geq 0,\ 0,\ t < 0 \qquad \text{or} \qquad r(t) = t\,u(t)$$



$$r(t - t_0) = t - t_0,\ t \geq t_0$$
$$= 0,\ t < t_0$$



The discrete-time version of the ramp function is defined by

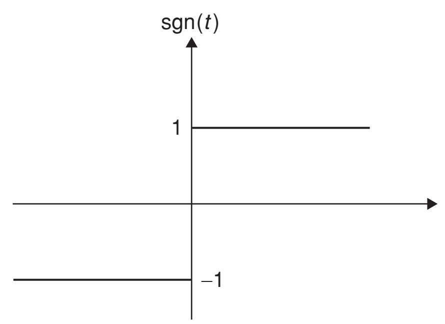


$$r[n] = n,\ n \geq 0$$
$$= 0\ n \leq 0$$
$$\text{(or) } r[n] = n\ u\ [n]$$

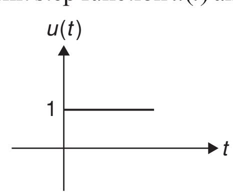
Signum Function

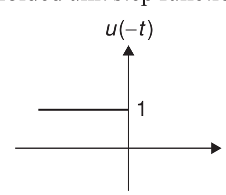
The signum function is defined as

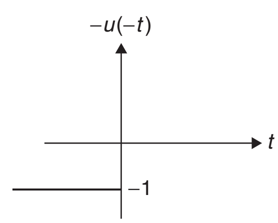
$$\text{sgn}(t) = \begin{cases} -1; & t < 0 \\ 0; & t = 0 \\ +1; & t > 0 \end{cases}$$

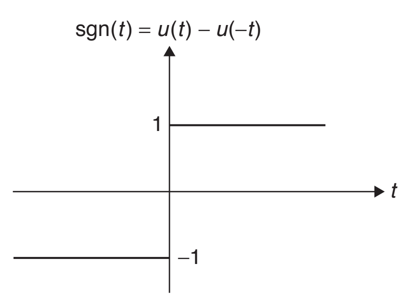


The representation is shown in below figure, it can be derived from unit step function $u(t)$ and folded unit step function $u(-t)$.









$\text{sgn}(t) = u(t) - u(-t) = u(t) - [1 - u(t)] = 2u(t) - 1$. In discrete-time

$$\text{sgn}[n] = \begin{cases} -1, & n < 0 \\ 0, & n = 0 \\ +1, & n > 0 \end{cases}$$

$$\text{sgn}[n] = u[n] - u[-n] = 2u[n] - 1$$

Relation between elementary signals:

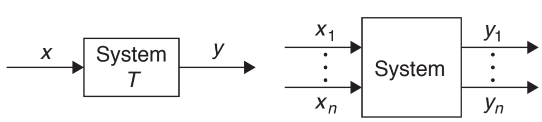
1. $\frac{d}{dt}u(t) = \delta(t)$, or $u(t) = \int_0^{\infty} \delta(t-\sigma)d\sigma$
2. $x(t)u(t) = 0$ for $t < 0 = x(t)$ for $t \geq 0$
3. $\frac{d}{dt}r(t) = u(t)$ (or) $\int_{-\infty}^{t} u(t)dt = r(t)$
4. $\frac{d}{dt}r(t-t_0) = u(t-t_0)$ or $\int_{-\infty}^{t} u(t-t_0)dt = r(t-t_0)$
5. $x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$
6. $x[n]\delta[n - n_0] = x[n_0]\delta[n - n_0]$
7. $u(kt - t_0) = u(t - t_0/k)$, $k \neq 0$.

Systems and Classification of Systems

A system is a mathematical model of a physical process that relates the input signal to the output signal.

Let x and y be the input and output signals, respectively, of a system. Then the system is viewed as a transform of x into y.

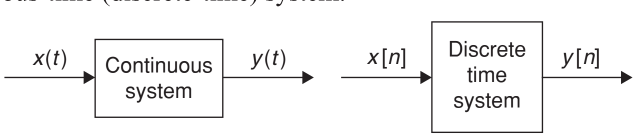
$$y = T\{x\}$$



System with single or multiple input and output signals.

Continuous-time and Discrete-time Systems

If the input and output signals x and y are continuous-time (discrete-time) signals, then the system is called a continuous-time (discrete-time) system.



Where $x(t)/x[n]$ is the input and $y(t)/y[n]$ is the output. (or) we can represent by the notation $x(t) \rightarrow y(t)$ or $x[n] \rightarrow y[n]$.

System with Memory and without Memory

A system is said to be memory less if the output at any time depends on only the input at that same time, otherwise the system is said to be system with memory. Memory less example: $y(t) = kx(t)$, $u[n] = kx[n]$.

The input output relationship of a resistor will be $v(t) = Ri(t)$. Where current $i(t)$ is input and $v(t)$ voltage is output. The output at time $t = t_0$ depends on only input at that instant.

A system is said to possess memory if its output signal depends on past or future values of the input signals. The example for continuous-time system with memory is capacitor, if input is taken to be the current and the output is the voltage.

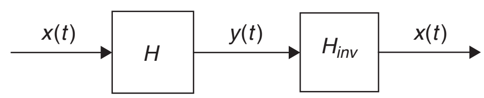
$$V(t) = \frac{1}{C}\int_{-\infty}^{t} i(T)dT \text{ where } C \text{ is capacitance}$$

$y(t) = x^2(t)$ is memory less system, but $y(t) = x(t^2)$ is system with memory,

With Memory example: $y[n] = \sum_{k=-\infty}^{n} x[k]$, $y[n] = x[n-1]$

Invertible and Non-invertible Systems

A system is said to be invertible if distinct inputs leads to distinct outputs or a system is said to be invertible if the input of the system can be recovered from the output. When several different inputs result in the same output (as in rectifier), it is impossible to obtain the input from output, and the system is non invertible.



The system that achieves the inverse operation of obtaining $x(t)$ from $y(t)$ is the inverse system (H_{inv}) for H. For example if H is an ideal integrator, then its inverse system (H_{inv}) is an ideal differentiator.

$$\begin{aligned} x(t) &= H_{\text{inv}}\{y(t)\} \\ &= H_{\text{inv}}H\{x(t)\} \end{aligned}$$

$H_{\text{inv}}H = I$, I is identity operator,

Cascading a system with its inverse system results an identity system. The output of a system described by the identity operator is exactly equal to the input. The property of invertibility is of particular importance in the design of communication systems.

When a transmitted signal propagates through a communication channel, it gets distorted owing to non-ideal frequency response, and finite band width of channel. To compensate this distortion equalizer network will be used which is the inverse of channel.

Invertible system: $y(t) = x(t - z)$, its inverse system is $x(t) = y(t + z)$.

$$y(t) = \frac{1}{L}\int_{-\infty}^{t} x(\tau)d\tau; \text{ inverse system } x(t) = L\frac{d}{dt}y(t).$$

Noninvertible system: $y(t) = x^2(t)$, the condition for invariability is different inputs should produce different output but for this case $-x(t)$ and $+x(t)$ both will lead to same output so non invertible system.

Causal and Non-causal Systems

If the output of system at present time depends on only the present input $x(t)$ or past values of the input, but not on its future values, then the system is causal.

In other words a casual (also known as physical or non anticipative) system is one for which the output at any instant t_0 depends only on the value of input $x(t)$ for $t \le t_0$. In causal system the output cannot start before the input is applied. A system that violates the conditions of causality is called a non-causal (or anticipative) system. The RC circuit is a causal system, since the capacitor voltage responds only to present and past values of the source voltage. $y(t) = x(-t)$ is non-causal because when $t > 0$, consider for $t = +2$, $y(2) = x(-2)$ it seems like a causal system but for $t < 0$, consider for $t = -2$, $y(-2) = x(2)$ have output depends on future value, the causal input output relation should valid for all times in a causal system. So the above system is a non-causal system. $y[n] = x[n]\cos[n + 1]$ is a causal system, here $\cos[n + 1]$ is a constant that varies with n, but output $y[n]$ depends on $x[n]$ (present input) only, so system is causal.

Example of causal system:

$$y[n] = \sum_{k=-\infty}^{n} x[k],\ y(t) = t.x(t)\, y[n] = x[-n]u[n]$$

Example of non-causal system:

$$y[n] = x[n + 1],$$

$$y[n] = \frac{1}{2M+1}\sum_{k=-M}^{M} x[n-k],\ y(t) = x(t+2) - x(t-2)$$

All memory less systems are causal but not vice versa.

Linear Systems and Non-linear Systems

A system is said to be a linear system if it satisfies the following two conditions.

a. Additivity: given that $y_1 = T\{x_1\}$, and $y_2 = T\{x_2\}$, then $T\{x_1 + x_2\} = y_1 + y_2$ for any signals x_1 and x_2.

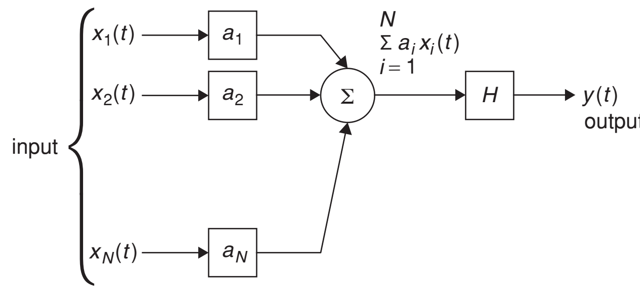
b. Homogeneity (or scaling): given that $T\{x\} = y$ then $T\{ax\} = ay$, for any signal x and y, any scalar a.

The above conditions can be combined as $T\{a_1x_1 + a_2x_2\} = a_1y_1 + a_2y_2$. Any system that doesn't satisfy the above condition is called non-linear system. Example for non-linear system:

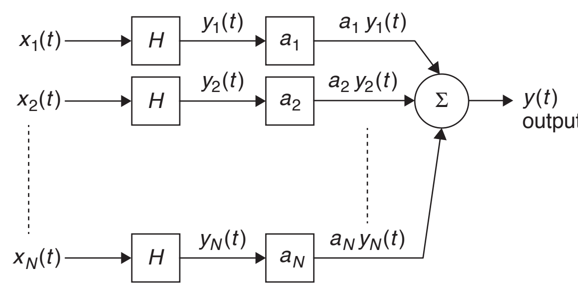
$$y = x^2,\ y = \sin`x'.$$

From the condition 'b', we can say that a zero input yields zero output for linear systems.

When a system violates either the principle of super position or the property of homogeneity, the system is said to be 'non-linear'.



The combined operation of amplitude scaling and summation, then precedes the system operator H for multiple inputs to produce final output $y(t)$.



The system operator H precedes the amplitude scaling for each input; the resulting outputs are summed to produce the overall output $y(t)$.

If these two configurations produce the same output $y(t)$, then the system operator H is linear.

Consider the system output $y(t)$ and input $x(t) = \sum_{i=1}^{N} a_i x_i(t)$.

$$\text{Then}\quad y(t) = H\left\{\sum_{i=1}^{N} a_i x_i(t)\right\} = \sum_{i=1}^{N} a_i H\{x_i(t)\} = \sum_{i=1}^{N} a_i y_i(t).$$

Now, the system will be linear system.

Consider a system by input–output relation as $y(t) = tx(t)$ if $x(t)$ is expressed as weighted sum $x(t) = \sum_{i=1}^{N} a_i x_i(t)$.

$$\text{Then } y(t) = t\cdot\sum_{i=1}^{N} a_i x_i(t) = \sum_{i=1}^{N} a_i t x_i(t)$$

Now, $y_i(t) = t \cdot x_i(t)$ is output due to each input acting individually then $y(t) = \sum_{i=1}^{N} a_i y_i(t)$ so the system is linear.

Consider discrete time system $y[n] = x[n]\,x[n - 1]$. Then $y_1[n] = x_1[n]\,x_1[n - 1]$, $y_2[n] = x_2[n]\,x_2[n - 1]$. If input is $a_1 x_1[n] + a_2 x_2[n]$.

Then $y[n] = (a_1\, x_1[n] + a_2\, x_2[n])\,(a_1\, x_1[n - 1] + a_2 x_2[n - 1]) \neq a_1 y_1[n] + a_2 y_2[n]$, so system is non-linear.

Consider system $y(t) = x^2(t)$, $y_1(t) = x_1^2(t)$, $y_2(t) = x_2^2(t)$. If $x_3(t) = a_1x_1(t) + a_2x_2(t)$ for linear system $y_3(t) = a_1\, y_1(t) + a_2y_2(t)$, but $y_3(t) = x_3^2(t) = [a_1x_1(t) + a_2x_2(t)]^2 \neq a_1x_1^2(t) + a_2 x_2^2(t)$, so system is non-linear.

Consider the system If $y[n] = \mathrm{Re}\{x[n]\}$, assume $x[n] = p[n] + jq[n]$, then $y[n] = p[n]$.

For linearity, the system must satisfy both additively and homogeneity properties and that the signals, as well as any scaling constants, are allowed to be complex.

To check homogeneity property $ay[n] = T\{ax[n]\}$. Here consider scaling constant $a = j$,

$$x_1[n] = jx[n] = jp[n] - q[n]$$

$$y_1[n] = \text{Re}\{x_1[n]\} = -q[n] \neq jy[n]$$

The system violates homogeneity property hence the system is not linear.

Consider the system $y[n] = 5x[n] + 7$, we can see that the system violates zero – in/zero – out property of linear systems. For zero input $x[n] = 0$, $y[n] = 0$ for linear systems, here $y[n] = 7$, so the system is non-linear.

$$\frac{dy(t)}{dt} + 4y(t) = x(t) \text{ is also a linear system,}$$

$$y(t)\frac{dy(t)}{dt} + 4y(t) = x(t) \text{ is non-linear system}$$

Time-Invariant and Time-varying Systems

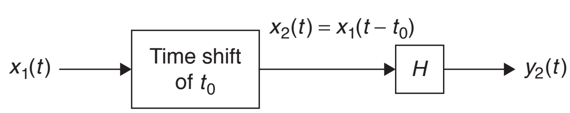
A system is called time-invariant if a time shift (delay or advance) in the input signal causes the same time shift in the output signal.

$$T\{x(t - T)\} = y(t - T)(\text{for CT})$$

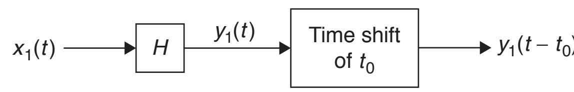
$$T\{x[n - k]) = y(n - k) \text{ (for DT)}$$

A system which does not satisfy above condition is called time-varying system.

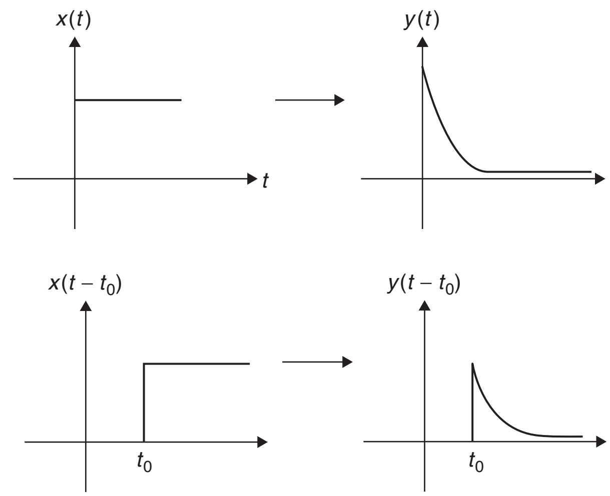
This implies that a time-invariant system responds identically no matter when input signal is applied. In other way, the characteristics of a time-invariant system do not change with time.



Time shift preceding operator H.



Time shift following operator H. If $y_2(t) = y_1(t - t_0)$ then the system is time-invariant.



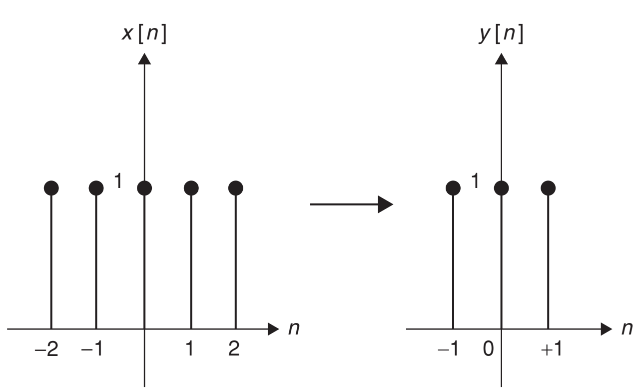
If the system is time-invariant, then the delayed output $y(t - t_0)$ can also be obtained by first delaying the input $x(t)$ before applying it to the system.

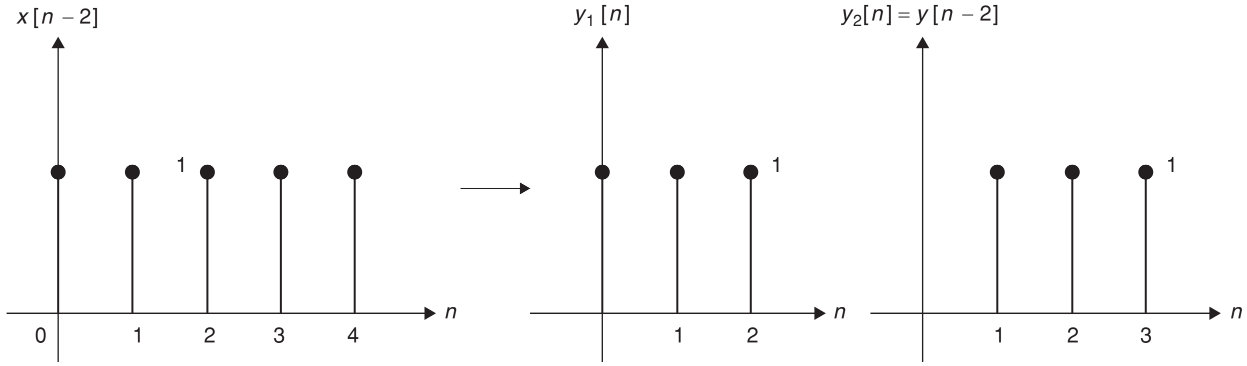
Consider a system specified by $y(t) = e^{-2t}x(t - 1)$ for the delayed input $x(t - t_0)$, output $y_1(t) = e^{-2t}x(t - t_0 - 1)$ but delayed output $y_2(t) = y(t - t_0) = e^{-2(t-t_0)}\, x(t - t_0 - 1)$ $y_1(t) \neq y_2(t)$, so the system is time varying.

Consider another system $y(t) = (\sin t)\, x(t + 1)$ for delayed input $x(t - t_0)$, output $y_1(t) = \sin t \cdot x(t - t_0 + 2)$ the delayed output. Now, $y_2(t) = y(t - t_0) = \sin(t - t_0)\, x(t - t_0 + 2)$ $y_1(t) \neq y_2(t)$. So the system is time varying.

Consider a discrete-time system $y[n] = \sin(x[n])$, for delayed, input $x[n - n_0]$, output $y_1[n] = \sin(x[n - n_0])$. When output is delayed, $y_2[n] = y[n]–n_0] = \sin(x[n - n_0])$ here $y_1[n] = y_2[n]$, so the system is time-invariant.

Consider a discrete-time system $y[n] = x[2n]$, for delayed input $x[n - n_0]$, output $y_1[n] = x[2n - n_0]$. When output is delayed, $y_2[n] = y[n - n_0] = x[2(n - n_0)]$ $y_1[n] \neq y_2\,[n]$ so the system is time invariant.





So, the above system is time variant, $y_2[n] \neq y_1[n]$. If the system is linear and also time-invariant, then it is called a linear time-invariant (LTI) system.

Stable Systems

A system is bounded input bounded output (BIBO) stable if for any bounded input x defined by $|x| \leq K_1$. The corresponding output y is also bounded defined by $|y| \leq K_2$, where K_1 and K_2 are finite real constants.

Consider the system $y[n] = \frac{1}{3}(x[n] + x[n-2] + x[n-3])$.

If input is finite value, then $|x[n]| \leq k < \infty$ for all n,

$$\text{Then } |y[n]| = \left|\frac{1}{3}(x[n] + [n-2] + x[n-3])\right|$$

$$\leq \frac{1}{3}(|x[n]| + |x[n-2]| + |x[n-3]|)$$

$$\leq \frac{1}{3}(k_1 + k_1 + k_1) = k_1 < \infty$$

The output is also finite for finite input, so system is BIBO stable.

Consider a continuous-time system $y(t) = a^t x(t)$. If input is finite value $|x(t)| \leq k_1 < \infty$ for all n. Then $|y(t)| = |a^t x(t)| = |a^t|\,|x(t)|$.

With $a > 1$, the multiplying factor a^t diverges for increasing t, so system is unstable for $a > 1$, for $a < 1$, the system will be stable.

Consider the continuous-time system $y(t) = t \cdot x(t)$. Even for bounded input $|x(t)| \leq k < \infty$ the output is unbounded $|y(t)| = |t|\,|x(t)| = |t|k$. So the system is unstable.

Consider the continuous-time system $y(t) = e^{2x(t)}$, when $x(t)$ is finite. $|x(t)| < k < \infty - k < x(t) < k$, then $y(t) = e^{2x(t)}$ i.e., $e^{-2k} < |y(t)| < e^{2k} y(t)$ is also finite. So the system is stable.

Linear Time Invariant Systems

Impulse Response

The Impulse response $h(t)$ of a continuous-time LTI system (represented by T) is defined to be the response of the system, when the input is $\delta(t)$, that is $h(t) = T\{\delta(t)\}$.

In continuous-time case, a true impulse signal having zero width and infinite amplitude cannot physically be generated and is usually approximated by a pulse of large amplitude and brief duration. Thus impulse response may be interpreted as the system behavior in response to a high energy input of extremely brief duration.

Response to an Arbitrary Input $x(t)$

Given the impulse response $h(t)$ we determine the output $y(t)$ due to an arbitrary input signal $x(t)$, by expressing the input as a weighted super position of time-shifted impulses. By linearity and time invariance, the output signal must be a weighted super position of time-shifted impulse responses, this weighted super position is termed as the convolution integral for continuous-time system, and the convolution sum for discrete-time system.

$$y[t] = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Input $x(t)$ can be expressed as

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

Output

$$y(t) = T\{x(t)\}$$

$$= T\left\{\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau\}d\tau\right\}$$

$$= \int_{-\infty}^{\infty} x(\tau)\; T\{\delta(t-\tau)\}d\tau$$

Since the system is time invariant.

$$T\{\delta(t-\tau)\} = h(t-\tau) \; (\because \text{ impulse response}).$$

So

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau.$$

Convolution Integral

The convolution of two signals $x(t)$, $h(t)$ is denoted by

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

The output of any LTI system is the convolution of the input and the Impulse response of the system.

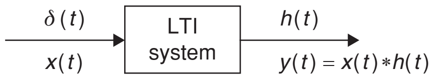


Figure 12 Continuous-time LTI system.

Properties of convolution integral:

1. Commutative $x(t) * h(t) = h(t) * x(t)$
2. Associative $\{x(t) * h_1(t)\} * h_2(t) = x(t) * \{h_1(t) * h_2(t)\}$
3. Distributive $x(t) * \{h_1(t) + h_2(t)\} = x(t) * h_1(t) + x(t) * h_2(t)$
4. $y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$
5. $x_1(t) * x_2(t) = c(t), x_1(t - T_1) * x_2(t_2 - T_2) = c(t - T_1 - T_2)$
6. Convolution with an impulse $x(t) * \delta(t) = x(t), x(t - t_1) * \delta(t - t_2) = x(t - t_1 - t_2)$
7. The width property: if $x_1(t)$ is defined for T_1 duration and $x_2(t)$ is defined for T_2 duration, then duration (width) of $x_1(t) * x_2(t)$ is $T_1 + T_2$.
8. Causality: If $h(t)$ and $x(t)$ are causal systems then

$$y(t) = x(t) * h(t) = \int_{0-}^{t} x(\tau)h(t-\tau)d\tau, t \geq 0$$
$$= 0, t < 0$$

or
$$y(t) = \int_{0-}^{t} h(\tau)x(t-\tau)d\tau, t \geq 0$$
$$= 0, t < 0$$

Step Response

The step response $s(t)$ of a continuous-time LTI system is defined to be the response of the system when input is $u(t)$: unit step response $s(t) = T\{u(t)\}$ $s(t)$ can be easily determinate by

$$s(t) = h(t) * u(t)$$

$$= \int_{-\infty}^{\infty} h(\tau)u(t-\tau)d\tau = \int_{-\infty}^{t} h(\tau)d\tau = \int_{0}^{\infty} h(t-\tau)d\tau$$

or
$$h(t) = \frac{d}{dt}s(t).$$

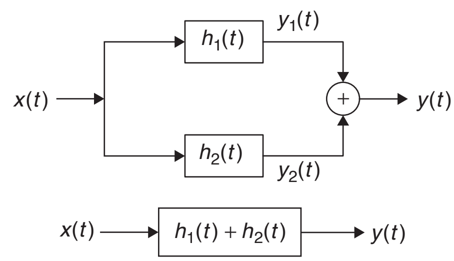
Table 1 *Convolution table*

Sl. No.	$x_1(t)$	$x_2(t)$	$x_1(t) * x_2(t) = x_2(t) * x_1(t)$
1	$x(t)$	$\delta(t-T)$	$x(t-T)$
2	$u(t)$	$u(t)$	$t \cdot u(t)$
3	$e^{at}\,u(t)$	$e^{bt}\,u(t)$	$\frac{e^{at}-e^{bt}}{a-b}u(t), a \neq b$
4	$e^{at}\,u(t)$	$e^{at}\,u(t)$	$te^{at}\,u(t)$
5	$e^{at}\,u(-t)$	$e^{bt}\,u(-t)$	$\frac{e^{at}-e^{bt}}{b-a}u(-t)$

Interconnected Systems

A larger, more complex system can often be viewed as the interconnection of several smaller subsystems, each of which is easier to characterize. Knowing the characterizations of these subsystems, it becomes simpler to analyze such large systems

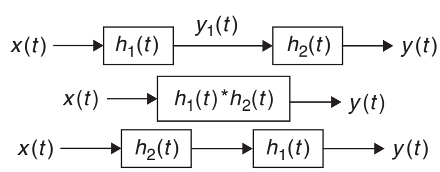
Consider two continuous-time LTI systems are in parallel,



$$y_1(t) = h_1(t) * x(t), y_2(t) = h_2(t) * x(t)$$
$$y(t) = y_1(t) + y_2(t) = h_1(t) * x(t) + h_2(t) * x(t)$$
$$= [h_1(t) + h_2(t)] * x(t)$$

This is another interpretation of distributive property of convolution.

Consider two continuous-time LTI systems in cascade



$$y_1(t) = x(t) * h_1(t)$$
$$y(t) = y_1(t) * h_2(t)$$
$$= x(t) * h_1(t) * h_2(t)$$
$$= x(t) * \{h_1(t) * h_2(t)\}$$

This is another interpretation of associate property and commutative property of convolution.

Properties of Continuous-time LTI Systems

Systems with or without Memory

A system is memory less if its output at any time depends only on the value of the input at that same time.

$$\text{Output: } y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau.$$

If $y(t)$ need to be dependent only present input $x(t)$ then $h(\tau)\, x(t-\tau) = kx(t)$, this is possible when $h(\tau)$ is defined for only $\tau = 0$ i.e., $h(t) = 0$ for $t \neq 0$.

The corresponding Impulse response $h(t)$ of memory less system is simply $h(t) = k\delta(t)$. Therefore, if $h(t_0) \neq 0$ for $t_0 \neq 0$, the continuous-time LTI system has memory.

Causality The output of a causal system depends only on the present and past values of input to the system.

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

$y(t)$ should not depend on future values, i.e., $h(t) = 0$ for $t < 0$.

For a causal continuous-time LTI system, we have $h(t) = 0,\ t < 0$.

The output of causal LTI system is

$$y(t) = \int_{0}^{t} h(\tau)x(t-\tau)d\tau = \int_{0}^{t} x(\tau)h(t-\tau)d\tau$$

Stability A system is bounded input-bounded output (BIBO) stable if the output is guaranteed to be bounded for every bounded input.

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

$|y(t)| < \infty$, for $|x(t)| < \infty$ then

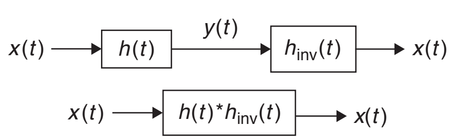
$$|y(t)| = \int_{-\infty}^{\infty} |\,h(\tau)x(t-\tau)\,|\,d\tau$$

$$= \int_{-\infty}^{\infty} |\,h(\tau)\,|\,d\tau < \infty \text{ as } |\,x(t)\,| < \infty$$

A continuous-time LTI system is BIBO stable if its impulse response is absolutely integrable.

$$\text{i.e.,} \int_{-\infty}^{\infty} |\,h(\tau)\,|\,d\tau < \infty$$

Invertibililty If an LTI system is invertible, then it has an LTI inverse system, when the inverse system is connected in series with original system, it produces an output equal to the input to the first system.



That is $x(t) = x(t) * \{h(t) * h_{\text{inv}}(t)\}$. This will be valued only when $h(t) * h_{\text{inv}}(t) = \delta(t)$

Discrete-time LTI Systems

1. Impulse response $h[n] = T\{\delta[n]\}$
2. Response to an arbitrary input $x[n]$ is

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

3. Convolution $y[n] = x[n] * h[n]$

$$= \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

4. Properties of convolution sum $x[n] * h[n] = h[n] * x[n]$ is commutative. $\{x[n] * h_1[n]\} * h_2[n] = x[n] * \{h_1[n] * h_2[n]\}$ is associative. $x[n] * \{h_1[n] + h_2[n]\} = x[n] * h_1[n] + x[n] * h_2[n]$ is distributive.
5. Shifting property $x_1[n] * x_2[n] = c[n]$, then $x_1[n-n_1] * x_2[n-n_2] = c[n-n_1-n_2]$.
6. Convolution with impulse $x[n]*\delta[n] = x[n]$, $x[n-n_1] * \delta[n-n_2] = x[n-n_1-n_2]$.
7. The width property: if $x_1[n]$ and $x_2[n]$ have finite widths of w_1 and w_2, respectively then the width of $x_1[n]*x_2[n]$ is $w_1 + w_2$. The width of a signal is one less than the number of elements.
8. Causal system output: if $x[n]$ and $h[n]$ are causal, then

$$y[n] = \sum_{k=0}^{\infty} x[k]h[n-k].$$

9. Convolution sum table:

Sl. No.	$x_1[n]$	$x_2[n]$	$x_1[n]*x_2[n]$
1	$\delta[n-k]$	$x[n]$	$x[n-k]$
2	$u[n]$	$u[n]$	$(n+1)\cdot u[n]$
3	$a^n u[n]$	$b^n u[n]$	$\left[\dfrac{a^{n+1}-b^{n+1}}{a-b}\right]u[n],\ a \neq b$
4	$u[n]$	$nu[n]$	$\dfrac{n(n+1)}{2}u[n]$
5	$a^n u[n]$	$a^n u[n]$	$(n+1)a^n u[n]$

10. When two LTI Discrete time systems with impulse response $h_1[n]$ and $h_2[n]$ respectively are connected in parallel, the composite parallel system impulse response is $h_1[n] + h_2[n]$, if these system are connected in cascade (in any order), the impulse response of the composite system is

$$h_1[n] * h_2[n] = h_2[n] * h_1[n].$$

11. Step response

$$s[n] = h[n] * u[n] = \sum_{k=0}^{\infty} h[n-k] = \sum_{k=-\infty}^{n} h[k],$$

$$h[n] = s[n] - s[n-1]$$

12. If $h[n] \neq 0$, for $n \neq 0$, the discrete-time LTI system has memory (or) $h[n] = 0$, for $n \neq 0$ system is memory less.
13. Causality condition is $h[n] = 0$ for $n < 0$, causal system output

$$y[n] = \sum_{k=0}^{\infty} h[k]x[n-k] = \sum_{k=0}^{\infty} x[k]h[n-k].$$

14. A DT, LTI system is BIBO stable if $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$.
15. Invertibility: $h[n] * h_{\text{inv}}[n] = \delta[n]$.

Table *The steps involved in calculation of convolution.*

Continuous-time Convolution	Discrete-time Convolution
$y(t) = x(t) * h(t) = h(t) *$ $x(t) = \int_{-\infty}^{+\infty} h(\tau)\times(t-\tau)d\tau$ $= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$	$y[n] = x[n] * h[n] = h[n] * x[n]$ $= \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$ $= \sum_{k=-\infty}^{+\infty} h[k]\times[n-k]$
$x(t) \to x(\tau)$ or $h(t) \to h(\tau)$	$h[n] \to h[k]$ or $x[n] \to x[k]$
Folding or flipping $x(-\tau)$ or $h(-\tau)$	Folding or flipping $x[-k]$ or $h[-k]$
Shifting $x(t-\tau)$ or $h(t-\tau)$	Shifting $x[n-k]$ (or) $h[n-k]$
Multiplication $x[t-\tau]$ $h[\tau]$ or $h[t-\tau]\times[\tau]$	Multiplication $h[k]$ $x[n-k]$ or $x[k]$ $h[n-k]$
Integration	Summation

Solved Examples

Example 1: Consider the following signals $x(t) = \sin\pi t + \cos 2\pi t$.

$$y(t) = \sin 2t + \sin 5t,\ z(t) = \sin t + \cos\pi t$$

Periodic signals are?

(A) $x(t)$ and $z(t)$ (B) $x(t)$ and $y(t)$
(C) $y(t)$ and $z(t)$ (D) Only $y(t)$

Solution: (B)

$x(t)$ is periodic with period $T = 2$ sec and $y(t)$ is periodic with period $T = 2\pi$ sec. For $z(t)$ fundamental time period can't be determined.

Example 2: Consider the following signals with $z > 0$.

(a) $e^{-zt}\,u(t)$ (b) $e^{zt}u(t)$ (c) $tu(t)$

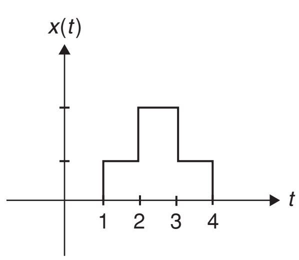
The energy signals are?

(A) a and b (B) b only
(C) a only (D) all

Solution: (C)

All integrable signals are energy signals. 'a' is only integrable signal and 'b', 'c' are not integrable signals.

Example 3: Consider the voltage waveform $x(t)$

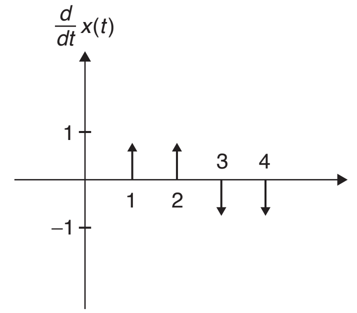


The equation for $x(t)$ is?

(A) $u(t-1) - u(t-2) - u(t-3) - u(t-4)$
(B) $u(t-1) + u(t-2) - u(t-3) - u(t-4)$
(C) $u(t-1) + u(t-2) + u(t-3) - u(t-4)$
(D) $u(t-1) - u(t-2) + u(t-3) + u(t-4)$

Solution: (B)

Differentiate $x(t)$

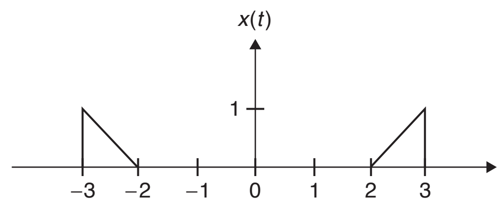


$$\frac{d}{dt}x(t) = \delta(t-1) + \delta(t-2) - \delta(t-3) - \delta(t-4)$$

Now integrate, then

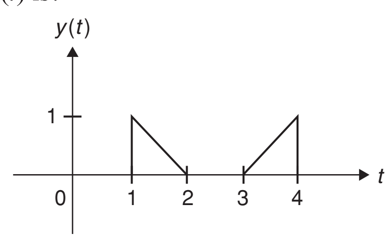
$$x(t) = u(t-1) + u(t-2) - u(t-3) - u(t-4)$$

Example 4: The signal $x(t)$ is depicted as

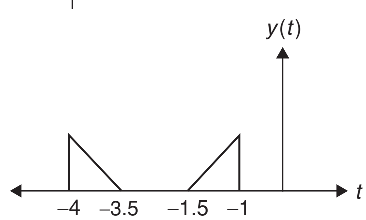


The signal $y(t)$ is related to $x(t)$ as $y(t) = x(2t + 5)$, the sketch of $y(t)$ is?

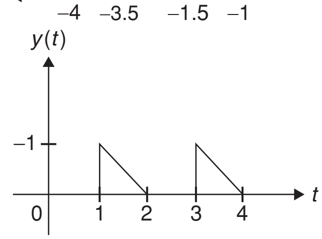
(A)



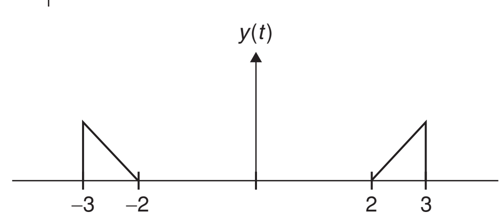
(B)



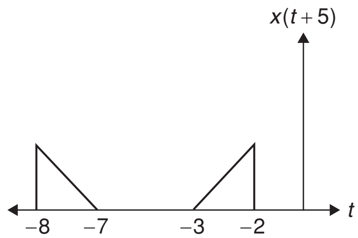
(C)

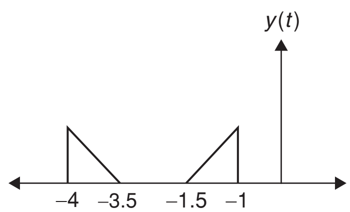


(D)



Solution: (B)





Example 5: The even part of a function $x(n) = u(n) + u(-n)$ is?

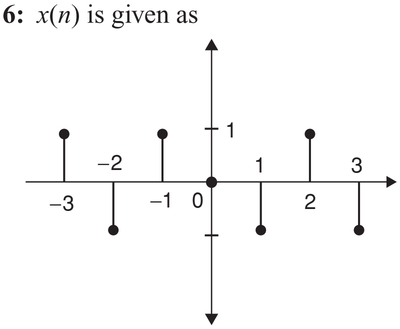
(A) $u(n) - u(-n)$ (B) $u(n) + u(-n)$
(C) $u(-n) - u(n)$ (D) $2u(n)$

Solution: (B)

$$\text{Even part of } x(n) = \frac{x(n)+x(-n)}{2}$$
$$= \frac{u(n)+u(-n)+u(-n)+u(n)}{2}$$

$x_1(n) = u(n) + u(-n)$

Example 6: $x(n)$ is given as



Choose the correct option for $y(n) = x(1-n) - x(n-1)$?

(A) {− 2, 2, − 2, 0, 2, − 2, 2}
(B) {− 1, 1, − 1, 0, 2, 2, 2}
(C) {2, − 2, 2, 0, 2, − 2, 2}
(D) {− 2, 2, − 2, 0, − 2, 2, − 2}

Solution: (A)

$$x(n-1) = \{1,-1,\underset{\uparrow}{1},0,-1,1,-1\}$$
$$x(1-n) = \{1,1-\underset{\uparrow}{1},0,1,-1,1\}$$
$$x(1-n) - x(n-1) = \{-2,2,-\underset{\uparrow}{2},0,2,-2,2\}$$

Example 7: A discrete time signal

$$x(n) = \cos\left(\frac{\pi}{7}n\right) + \sin\left(\frac{\pi n}{8} + \frac{1}{2}\right), \text{ the period of } x(n)?$$

(A) Periodic with period 112
(B) Periodic with period 223
(C) Periodic with period 334
(D) Not periodic

Solution: (A)

$N_1 = 14, N_2 = 16$ $N = \text{LCM}(14, 16) = 112$

So $x(n)$ is periodic with period 112.

Example 8: $x(n) = \{1,2,-\underset{\uparrow}{3},2,1\}, h(n) = \{4,-\underset{\uparrow}{2},1\}$

Find $x[n] * h(n)$?

(A) $\{4, 6, -15, \underset{\uparrow}{16}, -3, 0, 1\}$

(B) $\{\underset{\uparrow}{4}, 6, -15, 16, -3, 0, 1\}$

(C) $\{4, 6, 15, -\underset{\uparrow}{16}, 3, 0, 1\}$

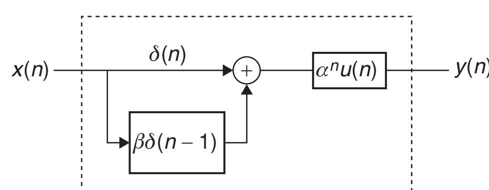
(D) $\{\underset{\uparrow}{4}, 6, 15, -16, 3, 0, 1\}$

Solution: (A)

$x(n) * h(n) = h(n+2) + 2h(n+1) - 3h(n) + 2h(n-1) + h(n-2)$

4	−2	1				
	8	−4	2			
		−12	6	−3		
			8	−4	2	
				4	−2	1
4	6	−15	16	−3	0	1

Example 9:

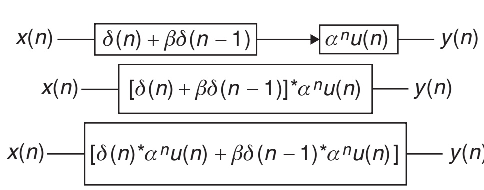


What is the transfer function $h(n)$

(A) $\alpha^{n+1}u(n) + \beta\alpha^{n-1}u(n-1) = h(n)$
(B) $\alpha^n u(n-1) + \beta\alpha^{n-1}u(n) = h(n)$
(C) $\alpha^n u(n) + \beta\alpha^n u(n-1) = h(n)$
(D) $\alpha^n u(n) + \beta\alpha^{n-1}u(n-1) = h(n)$

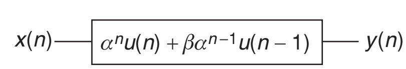
Solution: (D)

Blocks are parallel, addition blocks are cascaded convolution so



From basic formulae

$$x(t) * \delta(t-t_0) = x(t-t_0)$$
$$x(t) \cdot \delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$
$$\int_{-\infty}^{\infty} x(t) * \delta(t-t_0)\,dt = x(t_0)$$



Example 10: The result of $u(t) * h(t)$, when $h(t) = \begin{cases} e^{at}, t < 0 \\ e^{-bt}, t > 0 \end{cases}$

(A) $\frac{1}{a}e^{at} + \frac{1}{a}(1-e^{at})u(t) + \frac{1}{b}(1-e^{-bt})u(t)$

(B) $\frac{1}{a}e^{at} - \frac{1}{a}(1-e^{at})u(t) + \frac{1}{b}(1-e^{bt})u(t)$

(C) $\frac{1}{a}e^{at} - \frac{1}{a}(1-e^{at})u(t) + \frac{1}{b}(1-e^{bt})u(t)$

(D) $\frac{1}{a}e^{-at} + \frac{1}{a}(1-e^{-at})u(t) + \frac{1}{b}(1-e^{bt})u(t)$

Solution: (A)

For $t < 0$, $\int_{-\infty}^{0} e^{at}\,dt = \frac{e^{at}}{a}$

For $t \geq 0$ $\int_{-\infty}^{0} e^{at}\,dt + \int_{-\infty}^{t} e^{-bt}\,dt$

$= \frac{1}{a} + \frac{1}{b} - \frac{1}{b}e^{-bt} = \frac{1}{a} + \frac{1}{b}(1-e^{-bt})$

$= u(t) * h(t) = \frac{e^{at}}{a}(1-u(t)) + \left(\frac{1}{a} + \frac{1}{b}(1-e^{bt})\right)u(t)$

$= \frac{1}{a}e^{at} + \frac{1}{a}(1-e^{at})u(t) + \frac{1}{b}(1-e^{-bt})u(t)$

Example 11: The discrete time input $x(n)$ and output $y(n)$ relation ship as $y(n) = \sqrt{nx(n)}$, then properties of the system are

(A) Linear, time in variant, causal, stable
(B) Non-linear, time in variant, causal, stable
(C) Non-linear, time variant, causal, unstable
(D) Linear, time in variant, causal, unstable

Solution: (C)

$$y_1(n) \to \sqrt{nx_1(n)}$$

$$y_2(n) \to \sqrt{nx_2(n)}$$

$$y_1(n) + y_2(n) \to \sqrt{n(x_1(n) + x_2(n)}$$

$$\neq \sqrt{nx_1(n)} + \sqrt{nx_2(n)}$$

It is not additive so it is non-linear

$$y(n) \to \sqrt{nx(n)}$$

$$y(n-n_0) \to \sqrt{(n-n_0)\times(n-n_0)} \neq \sqrt{nx(n-n_0)}$$

So it is time variant.

At any discrete time $n = n_0$, the response depends only on the excitation at that time. So 'causal' if the excitation is a constant the response is unbounded as 'n' approaches infinity so unstable.

Example 12: A continuous-time linear system with input $x(t)$ and output $y(t)$ yields the following input out pairs:

$$x(t) = e^{-jt} \leftrightarrow y(t) = e^{j3t}$$
$$x(t) = e^{jt} \leftrightarrow y(t) = e^{-j3t}$$

If $x(t) = \sin(t-1)$.

The corresponding $y(t)$ is

(A) $3\sin(t-1)$
(B) $\sin(3t-1)$
(C) $\sin(3t-3)$
(D) $\sin(t-3)$

Solution: (B)

$$\sin(t-1) = \frac{e^{j(t-1)} - e^{-j(t-1)}}{2j} = \frac{e^{-j}e^{jt} - e^{j}e^{-jt}}{2j}$$

Since system is linear

$$y(t) = \frac{e^{-j}e^{j3t} - e^{j}e^{-j3t}}{2j}$$

$$= \frac{e^{j(3t-1)} - e^{-j(3t-1)}}{2j} = \sin(3t-1)$$

Example 13: The rise of cosine pulse $x(t)$ is defined as

$$x(t) = \begin{cases} 1+\cos\omega t, & \frac{-\pi}{\omega} \leq t \leq \frac{\pi}{\omega}. \\ 0, & \text{otherwise} \end{cases}$$

The total energy of $x(t)$ is?

(A) $\frac{\pi}{\omega}$
(B) $\frac{2\pi}{\omega}$
(C) $\frac{3\pi}{\omega}$
(D) $\frac{4\pi}{\omega}$

Solution: (C)

$$\text{Energy} = \int_{-\frac{\pi}{\omega}}^{\frac{\pi}{\omega}} (1+\cos\omega t)^2\,dt$$

$$= 2\int_{0}^{\frac{\pi}{\omega}} \left(\frac{1}{2}\cos 2\omega t + 2\cos\omega t + \frac{3}{2}\right)dt$$

$$= 2\left(\frac{3}{2}\right)\left(\frac{\pi}{\omega}\right) = \frac{3\pi}{\omega}$$

Example 14: A signal $v(n)$ is defined by

$$v(n) = \begin{cases} 1; & n = 1 \\ -1; & n = -1 \\ 0; & n = 0 \text{ and } |n| > 1 \end{cases}$$

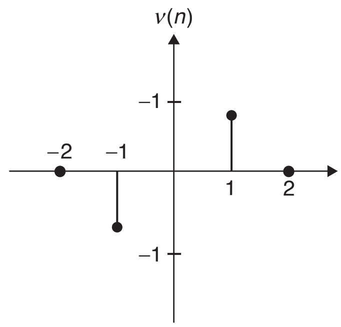
Which is the value of the composite signal defined as $v[n] + v[-n]$?

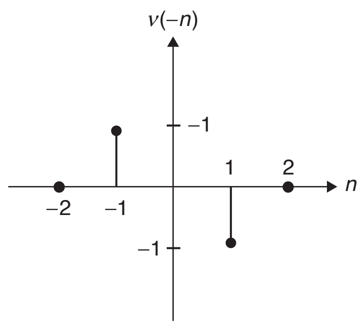
(A) 0 for all integer values of 'n'
(B) '2' for all integer value of 'n;'
(C) 1 for all integer values of 'n'
(D) –1 for all integer values of 'n'.

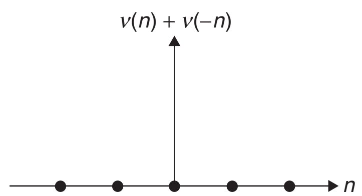
Solution: (A)

$$v(n) + v(-n) = 0$$

For all integer values of n







Example 15: The impulse response of a LTI system is given as $h(n) = \left(\frac{-1}{4}\right)^n u(n)$ the step response is

(A) $\frac{1}{4}\left[5 + \left(\frac{-1}{5}\right)^n\right]$ (B) $\frac{1}{5}\left[4 + \left(\frac{-1}{4}\right)^n\right]$

(C) $\frac{1}{4}\left[5 - \left(\frac{-1}{4}\right)^n\right]$ (D) $\frac{1}{5}\left[4 - \left(\frac{-1}{5}\right)^n\right]$

Solution: (B)

For $n < 0$, $s(n) = 0$

For $n \geq 0$, $s(n) = \sum_{k=0}^{n} \left(\frac{-1}{4}\right)^k$

$$= \frac{1 - \left(\frac{-1}{4}\right)^{n+1}}{1 + \frac{1}{4}}$$

$$= \frac{1}{5}\left(4 + \left(\frac{-1}{4}\right)^n\right)$$

Exercises

Practice Problems 1

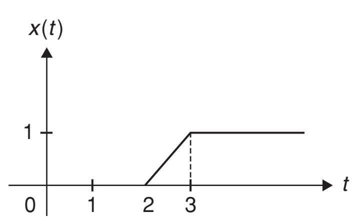
Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. The period of signal $x(t) = \sin t + \cos\sqrt{2}\, t$ is

   (A) $\pi/\sqrt{2}$ (B) 2π

   (C) 3π (D) not periodic

2. Consider the following signal $x(t)$



   The function that describe the $x(t)$ is?

   (A) $(t-2)\,u(t-2) + (t-3)\,u(t-3)$

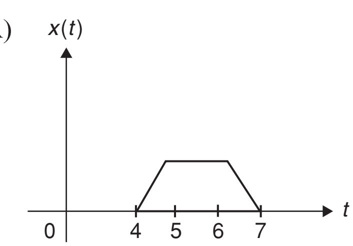
   (B) $(t-2)\,u(t-2) - (t-3)\,u(t-3)$

   (C) $(t-2)\,u(t-2) + (t+3)\,u(t+3)$

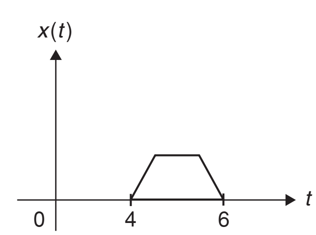
   (D) $(t-2)\,u(t-2) - (t+3)\,u(t+3)$

3. A signal is described by $x(t) = r(t-4) - r(t-5) - r(t-6) + r(t-7)$, where $r(t)$ is a unit ramp function starting at $t = 0$. The signal $x(t)$ is represented as?

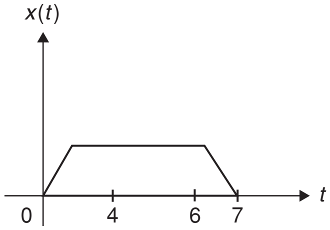
   (A)



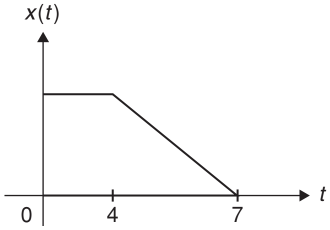
   (B)



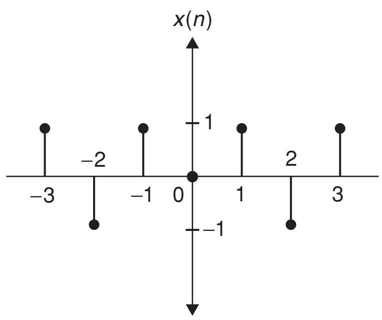
(C)



(D)

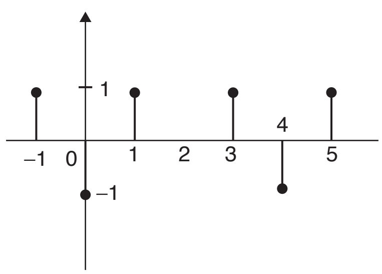


4. $x(n)$ is given as

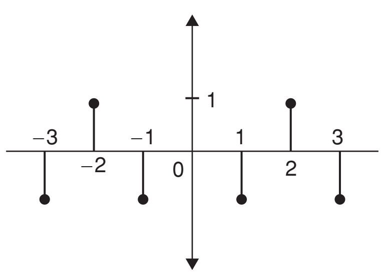


Choose the sketch for the signal $x(2 - n)$?

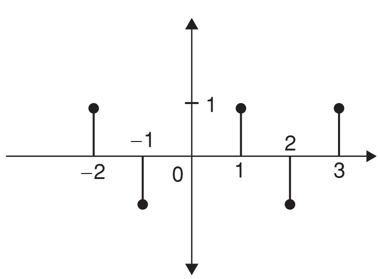
(A)



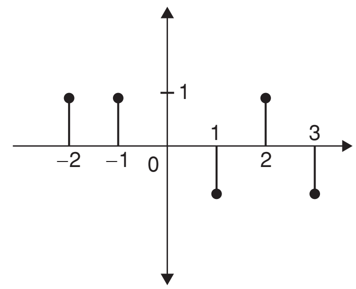
(B)



(C)



(D)

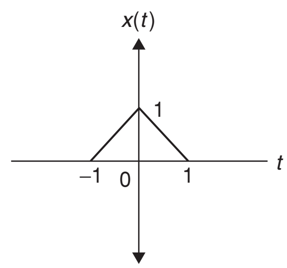


5. A discrete time signal $x(n)$ defined as $x(n) = e^{-j\left(\frac{n}{4}+\pi\right)}$ the period of signal $x[n]$ is?

(A) Periodic with $N = 8$ (B) Periodic with $N = 16$

(C) Periodic with $N = 4$ (D) None of the above

6. The signal $x(t)$ is

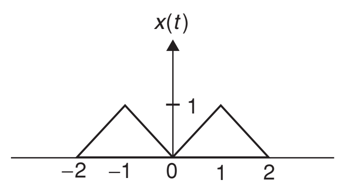


$x(t)$ is applied to a differentiator defined by $y(t) = \dfrac{dx(t)}{dt}$, The total energy of $y(t)$ is?

(A) 2 (B) 1

(C) 3 (D) 4

7. The signal $x(t)$ is



The total energy of $x(t)$ is?

(A) 4/3 (B) 3/4

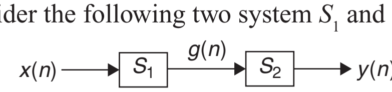
(C) 1 (D) 5/4

8. The sinusoidal signal $x(t) = 3\cos(100t + \pi/4)$ is passed through a square law device defined by the input, output relation $y(t) = x^2(t)$, the DC component in the signal is

(A) 9 (B) 9/2

(C) 9/4 (D) 9/8

9. Consider the following two system S_1 and S_2 as shown



S_1: causal LTI $g(n) = g(n-1) + x(n)$

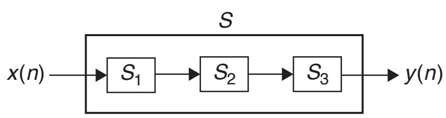
S_2: causal LTI $y(n) = ay(n-1) + bg(n)$

The difference equation for cascaded system is $y(n) = -\frac{1}{4}y(n-2) + \frac{5}{4}y(n-1) + \frac{1}{2}x(n)$ the values of a and b are?

(A) $\frac{1}{4}, \frac{1}{2}$ (B) $\frac{1}{2}, \frac{1}{2}$

(C) $\frac{1}{2}, \frac{1}{4}$ (D) $\frac{1}{4}, \frac{1}{4}$

10. Three discrete time systems S_1, S_2 and S_3 are connected in cascade to form a new system 'S' as



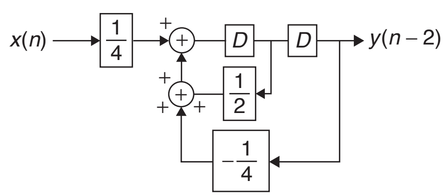
Consider the following statements.

(1) If S_1, S_2 and S_3 are linear, then 'S' is linear.
(2) If S_1, S_2 and S_3 are nonlinear, then 'S' is nonlinear.
(3) If S_1, S_2 and S_3 are causal, then 'S' is causal.
(4) If S_1, S_2 and S_3 are time invariant, then 'S' is the time invariant.

True statements are?

(A) 1, 2, 3 (B) 2, 3, 4
(C) 1, 3, 4 (D) All

11. The system shown as



(A) Stable and causal
(B) Stable but not causal
(C) Causal but unstable
(D) Unstable but not causal

12. The following signal

$$y(n) = \sum_{k=n-n_0}^{n+n_0} x[k] \text{ is}$$

(A) Causal (B) Non-causal
(C) Both (D) Anti-causal

13. $x(n) = \{1, 2, 0, 2, 1\}$, $h(n) = x(n)$ find $x(n) * h(n)$?

(A) $\{1, 4, 4, 4, \underset{\uparrow}{10}, 4, 4, 4, 1\}$ (B) $\{\underset{\uparrow}{1}, 4, 4, 4, 10, 4, 4, 4, 1\}$

(C) $\{1, 2, 3, 4, \underset{\uparrow}{10}, 4, 3, 2, 1\}$ (D) $\{\underset{\uparrow}{1}, 2, 3, 4, 10, 4, 3, 2, 1\}$

14. A Discrete time LTI system with input $u[n]$ produces output $\delta(n)$ then output due to the input $nu(n)$

(A) $u(n)$ (B) $u(n-1)$
(C) $u(n) - u(n-1)$ (D) $u(n) + u(n-1)$

15. A discrete time system has impulse response $h(n) = a^n u(n+2)|a| < 1$ which one of the following statement is correct.

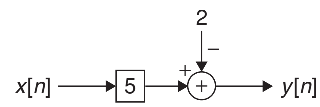
The system is

(A) Stable, causal and memory less
(B) Stable, non-causal and has memory
(C) Stable, non-causal and memory less
(D) Unstable, non-causal and memory less

16. Which one of the following systems having an impulse response, given is stable and causal LTI system?

(A) $h_1(t) = 1$ (B) $h_2(t) = u(t)$
(C) $h_3(t) = \frac{u(t+1)}{t+1}$ (D) $h_4(t) = e^{-2t} u(t)$

17. The discrete time input $x(n)$ and output $y(n)$ relationship as



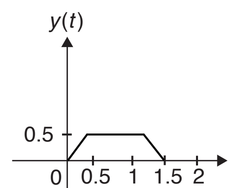
Determine system properties?

(A) Non-linear, time invariant, causal, stable
(B) Non-linear, time variant, causal, unstable
(C) Linear, time in variant, causal, stable
(D) Non-linear, time in variant, non-causal, stable

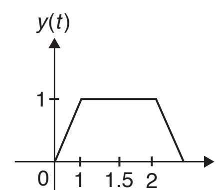
18. Suppose that $x(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & \text{elsewhere} \end{cases}$

$h(t) = x(2t)$ then $y(t) = x(t) * h(t)$ is

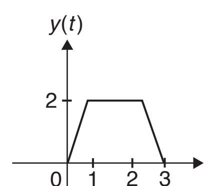
(A)



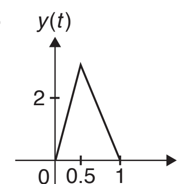
(B)



(C)



(D)



19. Consider the following systems $y_1(t) = x(t+4)$, $y_2(t) = \int_{-\infty}^{t} x(dT)\, dT$, $y_3(t) = 2\frac{d}{dt}x(t)$

The non invertible system is

(A) $y_1(t)$ and $y_2(t)$ (B) $y_1(t)$ and $y_3(t)$
(C) Only $y_3(t)$ (D) All the above

20. If the signals $f_1(t), f_2(t)$ are orthogonal over the interval (t_1, t_2), then

(A) $\int_{t_1}^{t_2} f_1(t) f_2(t) dt = \frac{\pi}{2}$ (B) $\int_{t_1}^{t_2} f_1(t) f_2(t) dt = 0$

(C) $\int_{t_2}^{t_1} f_1(t) * f_2(t) dt = 0$ (D) $\int_{t_1}^{t_2} f_1(t) f_2(t) dt = 1$

21. What are the even and odd components of the signal $f(t) = e^{jt}$

(A) $\cos t, j\sin t$ (B) $\cos t, -j\sin t$
(C) $\sin t, j\cos t$ (D) $-\cos t, j\sin t$

22. A continuous-time LTI system has 'step response' $s(t)$ and impulse response $h(t)$ are related by

(A) $h(t) = \int s(t)dt$ (B) $h(t) = \frac{ds(t)}{dt}$

(C) $h(t) = s(t) + s(t - T)$ (D) Not related

23. $x(t) * u(t - t_0) =$

(A) $\int_{-\infty}^{\infty} x(t)dt$ (B) $\int_{-\infty}^{t_0} x(t)dt$

(C) $\int_{-\infty}^{t_0} x(\tau - t_0)d\tau$ (D) $\int_{-\infty}^{t-t_0} x(\tau)d\tau$

24. The range of value of a and b for stability of an LTI system $h[n] = (-3a)^n;\ n \geq 0$
$= b^{-n},\ n < 0$

(A) $0 < |a| < \frac{1}{3};\quad 0 < |b| < 2$

(B) $0 < |a| < \frac{1}{3};\quad 0 < |b| < 1$

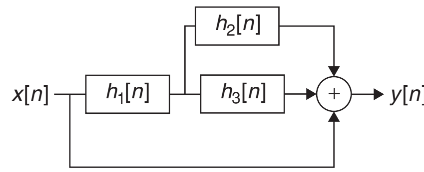
(C) $0 < |a| < \frac{1}{3};\quad 0 < |b| < 3$

(D) $0 < |a| < \frac{1}{2};\quad 0 < |b| < 2$

25. The system $y(t) = 2\sin t + 3\cos t$ is

(A) Non-linear, time variant, with memory, casual
(B) Non-linear, time invariant, memory less, casual
(C) Linear, time invariant, with memory, causal
(D) Linear, time variant, with memory, non-causal

26. The equivalent impulse response for this system



(A) $(h_1[n] + h_3[n]) * (h_1[n] + h_2[n]) + 1$
(B) $h_1[n] * (h_3[n] + h_2[n] + 1)$
(C) $h_1[n] * (h_3[n] + h_2[n]) + 1$
(D) $(h_3[n] + h_1[n]) * h_2[n] + 1$

27. The system $h[n] = \delta[n] + \delta[n + 2] + \delta[n - 2]$

(A) Causal and stable
(B) Causal but not stable
(C) Not causal but stable
(D) Neither causal nor stable

28. Given $x[n]$ defined in range $-6 \leq n \leq 3$, $y[n]$ in range $-2 \leq n \leq 7$ then the range of their convolution?

(A) $-6 \leq n \leq 7$ (B) $-2 \leq n \leq 3$
(C) $-8 \leq n \leq 7$ (D) $-8 \leq n \leq 10$

29. Response of a system to a complex input $x(t) = e^{j5t}$ is given as $y(t) = 5te^{j5t}$ then the system is

(A) Not LTI (B) LTI
(C) May be LTI (D) Not predictable

30. Impulse response of two LTI systems is given $h_1(t) = e^{-(5-4j)t}u(t)$, $h_2(t) = e^{-3t}\cos 4t \cdot u(t)$ the stable system is

(A) Only $h_1(t)$ (B) Only $h_2(t)$
(C) Both $h_1(t)$ and $h_2(t)$ (D) None of these.

Practice Problems 2

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. Determine the fundamental period of these signals.

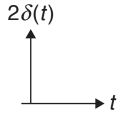
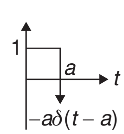
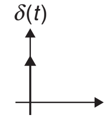
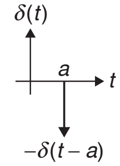
(1) $\cos\frac{\pi}{4}t + \sin\frac{\pi}{3}t$

(2) $\sin\sqrt{2}t + \cos 2t$

(A) $12, \sqrt{2}$ (B) $12, \frac{1}{\sqrt{2}}$

(C) 24, non-periodic (D) 24π, non-periodic

2. Match the first order derivatives of group (1) to group (2)

Group (1)		Group (2)	
P.	$u(t)$	**1.**	$2\delta(t)$ 
Q.	$u(t) - u(t - a)\quad a > 0$	**2.**	 1, a, t, $-a\delta(t - a)$
R.	$t(u(t) - u(t - a))\quad a > 0$	**3.**	$\delta(t)$ 
S.	Sgn(t)	**4.**	$\delta(t)$  a, t, $-\delta(t - a)$

(A) P – 1, Q – 4, R – 3, S –2
(B) P – 3, Q – 4, R – 2, S – 1
(C) P – 3, Q – 2, R – 4, S – 1
(D) P – 3, Q – 1, R – 2, S – 4

3. Periodic signals are

(A) $f[n + mN] = f[n]$ (B) $f(t - T) = f(t)$
(C) $f(t + T) = f(t)$ (D) A, B and C

4. Which is correct regarding power, and energy signals respectively?

(A) 0 < P < °, E = °; 0 < E < °, P = 0
(B) 0 < P < °, E = 0; 0 < E < °, P = °
(C) 0 < P < °, E = 0; 0 < E < °, P = 0
(D) 0 < P < °, E = °; 0 < E < °, P = °

5. Find the wrong statement

(A) $\int_{-\infty}^{\infty} \delta(t) = 1$

(B) $\delta(-t) = \delta(t)$

(C) $\sin\left(t - \frac{\pi}{2}\right)\delta(t) = -1$

(D) $\cos t \cdot \delta(t - \pi) = -\delta(t - \pi)$

6. Discrete time unit impulse can be obtained as

(A) $\delta[n] = \frac{du[n]}{dn}$

(B) $\delta[n] = u[n] - u[n-1]$

(C) A or B

(D) $\delta[n] = \int_{-\infty}^{\infty} u[n]dn$

7. For unit impulse signal of zero duration, the magnitude is

(A) 1

(B) ∘

(C) 0

(D) t

8. Match the following

	Group (1)		Group (2)
P.	$\int_{-\infty}^{\infty} \delta(t)dt$	**1.**	0
Q.	$\int_{-\infty}^{\infty} x(\lambda) \cdot \delta(t-\lambda)d\lambda$	**2.**	1
R.	$\int_{-\infty}^{\infty} x(t)\delta(t-\lambda)dt$	**3.**	$\frac{1}{\lvert a \rvert}\delta(t)$
S.	$\delta(at)$	**4.**	$\lvert a \rvert \delta(t)$
		5.	$x(t)$
		6.	$x(\lambda)$

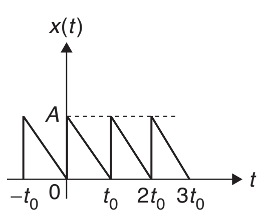
(A) P – 2, Q – 6, R – 5, S – 4

(B) P – 2, Q – 5, R – 6, S – 3

(C) P – 1, Q – 5, R – 6, S – 4

(D) P – 1, Q – 6, R – 5, S – 3

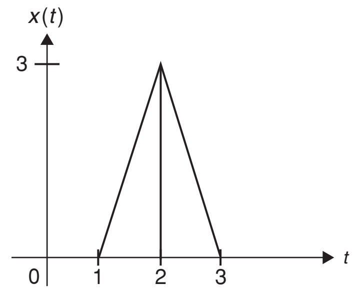
9. Find the value of $x(0)$ for the following sawtooth wave shown



(A) A (B) A/2

(C) Undefined (D) 0

10. The energy of signal is



(A) 3 joules (B) 9 joules

(C) 6 joules (D) 12 joules

11. Find the mutually orthogonal functions below.

(1) $\sin m\omega_0 t$, $\cos m\omega_0 t$ (2) $\sin m\omega_0 t$, $\sin n\omega_0 t$

(3) $\sin m\omega_0 t$, $\tan n\omega_0 t$ (4) $\cos n\omega_0 t$, $\cos m\omega_0 t$

(5) $\tan n\omega_0 t$, $\sec m\omega_0 t$

(A) 1, 2, 4 (B) 1, 2, 4, 5

(C) 2, 4 (D) 1, 3, 5

12. Two complex functions $f_1(t)$ and $f_2(t)$ are orthogonal over the interval (t_1, t_2) if

(A) $\int_{t_1}^{t_2} f_1(t) f_2^*(t)dt = 0$ (B) $\int_{t_1}^{t_2} f_1^*(t) f_2(t)dt = 0$

(C) Both A and B (D) $\int_{t_1}^{t_2} f_1(t) * f_2(t)dt = 0$

13. Which of the following is not correct about the signals $e^{j\omega t}$, $e^{-j\omega t}$?

(1) Both oscillate at same frequency ω

(2) One oscillate at 'ω' frequency, other at '$-\omega$'.

(3) Two phases rotating in opposite directions.

(4) When added, they yield a real function.

(A) 2, 4 (B) 1, 3, 4

(C) 4 (D) 2

14. If $y(t)$ is the out put of a CT–LTI system with input $x(t)$, the output of the system if the input is $\frac{dx(t)}{dt}$.

(A) $-ty(t)$ (B) $ty(t)$

(C) $\frac{d}{dt} y(t)$ (D) $\equiv y(t)dt$

15. If $h(t)$ is the impulse response of casual, linear, time invariant continuous, system. Then the output of the system for input $x(t)$ is

(A) $\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$ (B) $\int_{0}^{t} x(\tau)h(t-\tau)d\tau$

(C) $\int_{-\infty}^{0} x(\tau)h(t-\tau)d\tau$ (D) $\int_{0}^{\infty} x(\tau)h(t-\tau)d\tau$

16. Which of the following system is time-invariant?

(A) $y[n] = x[n] - x[n-1]$ (B) $y[n] = nx[n]$

(C) $y[n] = x[-n]$ (D) $y[n] = n\, x^2[n]$

17. Which of the following system is time-variant?
(A) $y[n] = x[n] - x[n-1]$
(B) $y[n] = x[n] - 6x[n-1]$
(C) $y[n] = a^{x[n]}$
(D) $y[n] = nx^2[n]$

18. Which of the following system is linear?
(A) $y[n] = x^2[n]$ (B) $y[n] = Bx[n] + C$
(C) $y[n] = nx^2[n]$ (D) $y[n] = nx[n]$

19. Which of the following system is casual?
(A) $y[n] = x[n] + 3x[n+4]$
(B) $y[n] = x[n] - x[n-1]$
(C) $y[n] = x[n^2]$
(D) $y[n] = x[2n]$

20. Pick up the non-casual system?
(A) $y[n] = \sum_{m=-\infty}^{n} x[n]$ (B) $y[n] = ax[n]$
(C) $y[n] = x[-n]$ (D) $y[n] = nx[n]$

21. The response of the LTI system whose input $x[n] = \{-\underset{\uparrow}{1}, 2, 3, 1\}$ impulse response
$h[n] = \{1, 2, 1, -1\}$
(A) $\{-\underset{\uparrow}{1}, 0, 6, 10, 3, -2, -1\}$
(B) $\{1, \underset{\uparrow}{4}, 8, 8, 3, -2, -1\}$
(C) $\{-1, \underset{\uparrow}{0}, 6, 10, 3, -2, -1\}$
(D) $\{1, 4, \underset{\uparrow}{8}, 8, 3, -2, -1\}$

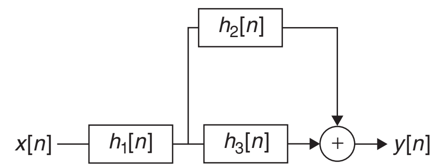
22. The discrete time system
$y[n] = x[n-3] - 4x[n-7]$ is a
(A) Dynamic system
(B) Time varying system
(C) Memory less system
(D) Non-linear system

23. Which of the following system is causal?
(A) $h[n] = n\left(\frac{1}{2}\right)^{n+1} u[n+1]$
(B) $y[n] = x^2[n] - x^2(n+1)$
(C) $y[n] = x[-n] + x[2n+1]$
(D) $h[n] = n\left(\frac{1}{2}\right)^{n+1} u[n]$

24. The system $y[n] = \sqrt{x[n]} + 3\sqrt{x[n+3]}$
(A) Non-linear, time variant, memory, causal, stable
(B) Linear, time invariant, memory less, causal unstable
(C) Non-linear, time invariant, memory less, non-causal, unstable
(D) Non-linear, time invariant, memory, non-causal, stable

25. $x(t) = u(t)$ is
(A) Power signal with power = –1
(B) Energy signal with energy = 1
(C) Power signal with power = 1
(D) Energy signal with energy = –1

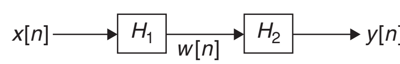
26. The equivalent impulse response of the system



(A) $(h_1[n] + h_3[n]) * (h_1[n] + h_2[n])$
(B) $(h_1[n] * h_3[n]) + h_1[n] * h_2[n]$
(C) $(h_1[n] + h_3[n]) * h_2[n]$
(D) $(h_1[n] + h_2[n]) * h_3[n]$

27. The period of the $x[n] = \cos\frac{\pi n}{4}\cos\frac{\pi n}{3}$
(A) 12
(B) 24
(C) 6
(D) Non-periodic

28. Consider the cascade of following two systems H_1 and H_2



H_1 is given by $w[n] = \frac{1}{4}w(n-1) + x[n]$
Now, H_2 is given by $y[n] = a\,y[n-1] + b\,w[n]$
The difference equation for cascaded system is $y[n] + \frac{1}{4}y\ [n-2] - \frac{5}{4}y(n-1) = x[n]$
Then the value of 'a' is
(A) 1
(B) 1/3
(C) 1/4
(D) 1/2

29. The value of b is
(A) 1
(B) 1/2
(C) 1/3
(D) 1/4

30. Impulse response of
$y[n] = x[n] + 2x[n-1] - 4 \times [n-2]$
(A) $\delta[n] - 2\delta[n-1]$
(B) $\delta[n] + 2\delta[n-1]$
(C) $\delta[n] + 2\delta[n-1] + 4\delta[n-2]$
(D) $\delta[n] + 2\delta[n-1] - 4\delta[n-2]$

Previous Years' Questions

1. A continuous-time system is described by $y(t) = e^{-|x(t)|}$, where $y(t)$ is the output and $x(t)$ is the input, $y(t)$ is bounded **[2006]**

(A) Only when $x(t)$ is bounded
(B) Only when $x(t)$ is non-negative
(C) Only for $t \ge 0$ if $x(t)$ is bounded for $t \ge 0$
(D) Even when $x(t)$ is not bounded

2. $y[n]$ denotes the output and $x[n]$ denotes the input of a discrete-time system given by the difference equation $y[n] - 0.8y[n-1] = x[n] + 1.25x[n+1]$. Its right-sided impulse response is **[2006]**

(A) Causal (B) Unbounded
(C) Periodic (D) Non-negative

3. Let a signal $a_1\sin(\omega_1 t + \phi_1)$ be applied to a stable linear time invariant system. Let the corresponding steady state output be represented as $a_2F(\omega_2 t + \phi_2)$. Then which of the following statements is true? **[2007]**

(A) F is not necessarily a 'sine' or 'cosine' function but must be periodic with $\omega_1 = \omega_2$.
(B) F must be a 'sine' or 'cosine' function with $a_1 = a_2$.
(C) F must be a 'sine' function with $\omega_1 = \omega_2$ and $\phi_1 = \phi_2$.
(D) F must be a 'sine' or 'cosine' function with $\omega_1 = \omega_2$.

4. If $u(t)$, $r(t)$ denote the unit step and unit ramp functions respectively and $u(t) * r(t)$ their convolution, then the function $u(t+1) * r(t-2)$ is given by **[2007]**

(A) $(1/2)\,(t-1)\,(t-2)$ (B) $(1/2)\,(t-1)\,(t-2)$
(C) $(1/2)\,(t-1)^2\,u(t-1)$ (D) None of these

5. The integral $\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} \sin(t-\tau)\cos\tau\, d\tau$ equals **[2007]**

(A) $\sin t \cos t$ (B) 0
(C) $(1/2)\cos t$ (D) $(1/2)\sin t$

6. A signal $e^{-\alpha t}\sin(\omega t)$ is the input to a linear time invariant system. Given K and ϕ are constants, the output of the system will be of the form $Ke^{-\beta t}\sin(\upsilon t + \phi)$ where **[2008]**

(A) β need not be equal to a but α equal to ω
(B) υ need not be equal to ω but β equal to $\propto$
(C) β equal to $\propto$ and υ equal to ω
(D) β need not be equal to $\propto$ and υ need not be equal to ω

7. The impulse response of a causal linear time invariant system is given as $h(t)$. Now consider the following two statements

Statement (I): Principle of superposition holds.
Statement (II): $h(t) = 0$ for $t < 0$.
Which one of the following statements is correct? **[2008]**

(A) Statement (I) is correct and statement (II) is wrong.
(B) Statement (II) is correct and statement (I) is wrong.
(C) Both Statement (I) and statement (II) are wrong.
(D) Both Statement (I) and statement (II) are correct.

8. A system with input $x(t)$ and output $y(t)$ is defined by the input–output relation

$$y(t) = \int_{-\infty}^{-2t} x(t)\,dt$$

The system will be **[2008]**

(A) Causal, time-invariant and unstable.
(B) Causal, time-invariant and stable.
(C) Non-causal, time-invariant and unstable.
(D) Non-causal, time-variant and unstable.

9. Let $x(t)$ be a periodic signal with time period T. Let $y(t) = x(t-t_0) + x(t+t_0)$ for some t_0. The fourier series coefficients of $y(t)$ are denoted by b. If $b_k = 0$ for all odd k, then t_0 can be equal to **[2008]**

(A) $\dfrac{T}{8}$ (B) $\dfrac{T}{4}$
(C) $\dfrac{T}{2}$ (D) $2T$

10. A linear time invariant system with an impulse response $h(t)$ produces output $y(t)$ when input $x(t)$ is applied. When the input $x(t-\tau)$ is applied to a system with impulse response $h(t-\tau)$, the output will be **[2009]**

(A) $y(t)$ (B) $y(2(t-\tau))$
(C) $y(t-\tau)$ (D) $y(t-2\tau)$

11. A cascade of 3 linear time invariant systems is causal and unstable. From this, we conclude that **[2009]**

(A) Each system in the cascade is individually causal and unstable.
(B) At least one system is unstable and at least one system is causal
(C) At least one system is causal and all systems are unstable.
(D) The majority are unstable and the majority are causal.

12. The period of the signal $x(t) = 8\sin\left\{0\cdot 8\pi t + \dfrac{\pi}{4}\right\}$ is **[2010]**

(A) $0.4\pi s$ (B) $0.8\pi s$
(C) $1.25\, s$ (D) $2.5 s$

13. The system represented by the input–output relationship $y(t) = \displaystyle\int_{-\infty}^{5t} x(\tau)\,d\tau,\ t > 0$ is **[2010]**

(A) Linear and causal
(B) Linear but non-causal
(C) Causal but non-linear
(D) Neither linear non-causal

14. At $t = 0$, the function $f(t) = \frac{\sin t}{t}$ has **[2010]**

(A) A minimum (B) A discontinuity
(C) A point of inflection (D) A maximum

15. Given two continuous-time signals $x(t) = e^{-t}$ and $y(t) = e^{-2t}$ which exist for $t > 0$, the convolution $z(t) = x(t) * y(t)$ is **[2011]**

(A) $e^{-t} - e^{2t}$ (B) e^{-3t}
(C) e^{+t} (D) $e^{-1} + e^{-2t}$

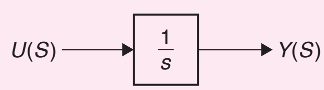
16. The response $h(t)$ of a linear time invariant system to an impulse $\delta(t)$, under initially relax condition is $h(t) = e^{-t} + e^{-2t}$. The response of this system for a unit step input $u(t)$ is

(A) $u(t) + e^{-t} + e^{-2t}$ (B) $(e^{-t} + e^{-2t})\, u(t)$
(C) $(1.5 - e^{-t} - 0.5e^{-2t})$ (D) $e^{-t}\,\delta(t) + e^{-2t}\, u(t)$

17. The input $x(t)$ and output $y(t)$ of a system are related as $y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\,d\tau$. The system is **[2012]**

(A) Time-invariant and stable
(B) Stable and not time-invariant
(C) Time-invariant and not stable
(D) Not time-invariant and not stable

18. Assuming zero initial condition, the response $y(t)$ of the system given below to a unit step input $u(t)$ is **[2013]**



(A) $u(t)$ (B) $tu(t)$
(C) $\frac{t^2}{2}u(t)$ (D) $e^{-t}u(t)$

19. The impulse response of a system is $h(t) = tu(t)$. For an input $u(t - 1)$, the output is **[2013]**

(A) $\frac{t^2}{2}u(t)$ (B) $\frac{t(t-1)}{2}u(t-1)$
(C) $\frac{(t-1)^2}{2}u(t-1)$ (D) $\frac{t^2-1}{2}u(t-1)$

20. Which one of the following statements is NOT TRUE for a continuous-time causal and stable LTI system? **[2013]**

(A) All the poles of the system must lie on the left side of the $j\omega$ axis.
(B) Zeros of the system can lie anywhere in the s-plane.
(C) All the poles must lie within $|s| = 1$.
(D) All the roots of the characteristic equation must be located on the left side of the $j\omega$ axis.

21. Two systems with impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade. Then the overall impulse response of the cascaded system is given by **[2013]**

(A) Product of $h_1(t)$ and $h_2(t)$
(B) Sum of $h_1(t)$ and $h_2(t)$
(C) Convolution of $h_1(t)$ and $h_2(t)$
(D) Subtraction of $h_2(t)$ from $h_1(t)$

22. The impulse response of a continuous-time system is given by $h(t) = \delta(t - 1) + \delta(t - 3)$. The value of the step response at $t = 2$ is **[2013]**

(A) 0 (B) 1
(C) 2 (D) 3

23. The response $y(t)$ to a unit step input is **[2013]**

(A) $\frac{1}{2} - \frac{1}{2}e^{-2t}$ (B) $1 - \frac{1}{2}e^{-2t} - \frac{1}{2}e^{-t}$
(C) $e^{-2t} - e^{-t}$ (D) $1 - e^{-t}$

24. $x(t)$ is non-zero only for $T_x < t < T'_x$, and similarly, $y(t)$ is non-zero only $T_y < t < T'_y$. Let $z(t)$ be convolution of $x(t)$ and $y(t)$. Which one of the following statements is TRUE? **[2014]**

(A) $z(t)$ can be non-zero over an unbounded interval.
(B) $z(t)$ is non-zero for $t < T_x + T_y$.
(C) $z(t)$ is zero-outside of $T_x + T_y < t < T'_x + T'_y$.
(D) $z(t)$ is non-zero for $t > T'_x + T'_y$.

25. A discrete system is represented by the difference equation

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} a & a-1 \\ a+1 & a \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

It has initial conditions $X_1(0) = 1$; $X_2(0) = 0$. The pole locations of the system for a $a = 1$, are **[2014]**

(A) $1 \pm j0$ (B) $-1 \pm j0$
(C) $\pm 1 + j0$ (D) $0 \pm j1$

26. Consider an LTI system with impulse response $h(t) = e^{-5t}\, u(t)$. If the output of the system is $y(t) = e^{-3t}\, u(t) - e^{-5t}\, u(t)$ then the input, $x(t)$, is given by **[2014]**

(A) $e^{-3t}\, u(t)$ (B) $2e^{-3t}\, u(t)$
(C) $e^{-5t}\, u(t)$ (D) $2e^{-5t}\, u(t)$

27. Consider a continuous-time system with input $x(t)$ and output $y(t)$ given by

$y(t) = x(t)\cos(t)$

This system is **[2016]**

(A) linear and time-invariant
(B) non-linear and time-invariant
(C) linear and time-varying
(D) non-linear and time-varying

28. The value of $\int_{-\infty}^{+\infty} e^{-t}\delta(2t-2)\,dt, where\,\delta(t)$, is the Dirac delta function, is **[2016]**

(A) $\frac{1}{2e}$ (B) $\frac{2}{e}$
(C) $\frac{1}{e^2}$ (D) $\frac{1}{2e^2}$

29. Consider a causal LTI system characterized by differential equation $\frac{dy(t)}{dt}+\frac{1}{6}y(t) = 3x(t)$. The response of the system to the input $x(t) = 3e^{-\frac{t}{3}}u(t)$, where u(t) denotes the unit step function, is ________. **[2016]**

(A) $9e^{-\frac{t}{3}}u(t)$

(B) $9e^{-\frac{t}{6}}u(t)$

(C) $9e^{-\frac{t}{3}}u(t)-6e^{-\frac{t}{6}}u(t)$

(D) $54e^{-\frac{t}{6}}u(t)-54e^{-\frac{t}{3}}u(t)$

Answer Keys

Exercises

Practice Problems 1

1. D	**2.** B	**3.** A	**4.** A	**5.** D	**6.** A	**7.** A	**8.** B	**9.** A	**10.** C
11. A	**12.** B	**13.** B	**14.** B	**15.** B	**16.** D	**17.** A	**18.** A	**19.** C	**20.** B
21. A	**22.** B	**23.** D	**24.** B	**25.** B	**26.** C	**27.** C	**28.** D	**29.** D	**30.** C

Practice Problems 2

1. C	**2.** B	**3.** D	**4.** A	**5.** C	**6.** B	**7.** B	**8.** B	**9.** B	**10.** C
11. A	**12.** C	**13.** D	**14.** C	**15.** B	**16.** A	**17.** D	**18.** D	**19.** B	**20.** C
21. A	**22.** A	**23.** D	**24.** D	**25.** C	**26.** B	**27.** B	**28.** A	**29.** A	**30.** D

Previous Years' Questions

1. D	**2.** A	**3.** D	**4.** C	**5.** D	**6.** A	**7.** D	**8.** D	**9.** B	**10.** D
11. B	**12.** D	**13.** B	**14.** B	**15.** A	**16.** C	**17.** D	**18.** B	**19.** C	**20.** C
21. C	**22.** B	**23.** B	**24.** C	**25.** A	**26.** B	**27.** C	**28.** A	**29.** D	