Type 12: State Variable Analysis

For Concept, refer to Control Systems K-Notes, State Variable Analysis

Sample Problem 12:

A system is described by the following state and output equations

$$\frac{dx_1(t)}{dt} = -3x_1(t) + x_2(t) + 2u(t)$$

$$\frac{dx_2(t)}{dt} = -2x_2(t) + u(t)\,, \qquad y(t)=x_1(t)$$

when u(t) is the input and y(t) is the output. The system transfer function and the state-transition matrix of the above system is

(A) $\dfrac{(s+2)}{(s^2+5s-6)}$, $\begin{bmatrix} e^{-3t} & 0 \\ e^{-3t}+e^{-2t} & e^{-2t} \end{bmatrix}$

(B) $\dfrac{(s+3)}{(s^2+5s+6)}$, $\begin{bmatrix} e^{-3t} & e^{-2t}+e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$

(C) $\dfrac{(2s+5)}{(s^2+5s+6)}$, $\begin{bmatrix} e^{-3t} & e^{-2t}-e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$

(D) $\dfrac{(2s-5)}{(s^2+5s-6)}$, $\begin{bmatrix} e^{3t} & e^{-2t}-e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$

Solution: (C) is correct option

$$\text{T.F.} = C[sI-A]^{-1}B + D$$

where

$$A = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}, \quad B=\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad C=\begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$[sI-A] = \begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}$$

$$[sI-A]^{-1} = \frac{\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}}{(s+3)(s+2)}$$

$$\text{T.F.} = C[sI-A]^{-1}B = \frac{\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}}{(s+3)(s+2)}$$

$$\text{T.F.} = \frac{(2s+5)}{(s^2+5s+6)}$$

State-transition matrix

$$e^{At} = L^{-1}\left\{[sI - A]^{-1}\right\}$$

$$e^{At} = L^{-1}\begin{bmatrix} \dfrac{1}{s+3} & \dfrac{1}{(s+3)(s+2)} \\ 0 & \dfrac{1}{s+2} \end{bmatrix},$$

$$e^{At} = \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

Unsolved Problems:

Q.1 Given the system

$$\dot{X} = \begin{bmatrix} 0 & 0 & -20 \\ 1 & 0 & -24 \\ 0 & 1 & -9 \end{bmatrix} X + \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

The characteristic equation of the system is

(A) $s^3 + 20s^2 + 24s + 9 = 0$

(B) $s^3 + 9s^2 + 24s + 20 = 0$

(C) $s^3 + 24s^2 + 9s + 20 = 0$

(D) None of these

Q.2 For a system represented by state equation x(t) = Ax(t), response is $x(t) = \begin{bmatrix} e^{-2t} \\ -2e^{-2t} \end{bmatrix}$

When $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$ When $x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, system matrix 'A' is

(A) $\begin{bmatrix} 1 & 0 \\ -3 & -2 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ -5 & -3 \end{bmatrix}$

Q.3 The system is represented by state equations as follows.

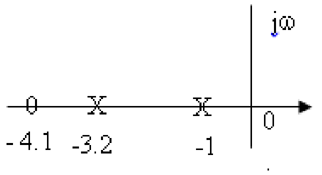
$$\dot{X}_1 = -4x_1 - x_2 + 3u$$

$$\dot{X}_2 = 2x_1 - 3x_2 + 5u$$

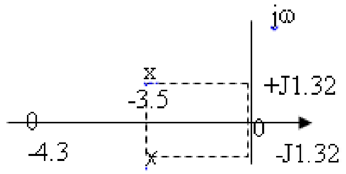
$$Y = x_1 + 2x_2$$

The poles and zeros locations are

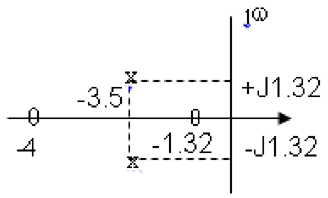
(A)



(B)



(C)



(D) None

Q.4 The dynamic model of a pendulum is given by $\frac{d^2\theta}{dt^2}+4\theta=T$, where Θ is the displacement in rad/sec and T is the applied torque in N-m. Its representation in the time scale state variable form $\dot{X}=\alpha X+\beta u$ can have the constants?

(A) $\alpha=\begin{bmatrix}0 & 1\\ -4 & 0\end{bmatrix},\ \beta=\begin{bmatrix}0\\1\end{bmatrix}$

(B) $\alpha=\begin{bmatrix}0 & 1\\ -4 & 0\end{bmatrix},\ \beta=\begin{bmatrix}1\\0\end{bmatrix}$

(C) $\alpha=\begin{bmatrix}0 & 0\\ 4 & 1\end{bmatrix},\ \beta=\begin{bmatrix}0\\1\end{bmatrix}$

(D) $\alpha=\begin{bmatrix}0 & 0\\ -4 & 1\end{bmatrix},\ \beta=\begin{bmatrix}1\\0\end{bmatrix}$

Q.5 Obtain the response y(t) of the following system

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\end{bmatrix}=\begin{bmatrix}-1 & -0.5\\ 1 & 0\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix}+\begin{bmatrix}0.5\\ 0\end{bmatrix}u\ ,\qquad \begin{bmatrix}X_1(0)\\ X_2(0)\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$y=[1\quad 0]\begin{bmatrix}X_1\\ X_2\end{bmatrix}$$

(A) $e^{-0.5t}\cos(0.5t)$ (B) $e^{-2t}\cos(0.5t)$ (C) $e^{-0.5t}\sin(0.5t)$ (D) $e^{-2t}\sin(0.5t)$