

HCF & LCM

Prime Numbers

The numbers which cannot be broken into factors other than 1 and itself are known as **prime numbers**, e.g., 2, 3, 5, 7, 11, 13,....

It is important to note that

- 2 is the only even number which is prime.

Composite Numbers

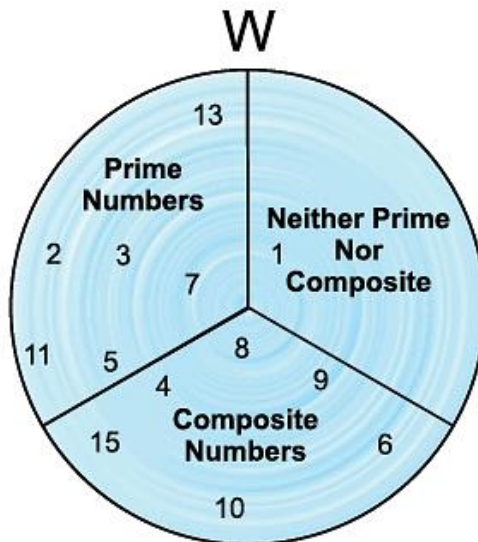
Numbers which can be expressed as product of factors other than 1 and itself are known as **composite numbers**, e.g., 4, 6, 8, 9, 10, 12, 14, 15,....

Note that composite numbers are not only even. The collection of composite numbers is a mixture of even numbers and odd numbers both.

4, 6, 8, 10, are even composite numbers

9, 15, 21, are odd composite numbers.

Since we cannot express 1 as a product of two different factors other than 1 and itself, we say that **1 is the only exceptional number which is neither prime nor composite.**



To determine whether a particular number is prime or composite, we don't have a set formula. For this, we have to perform actual division so many times in order to check whether we can divide the given number by another number or not. A Greek mathematician Eratosthenes developed a method, in the 3rd century B.C., to find prime numbers. His method for finding prime numbers is known as **Sieve method**. For numbers from 1 to 100, this method is explained below.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. First we write all the numbers from 1 to 100.
2. Then cross out first number 1, it is neither prime nor composite.
3. Next number is 2, it is the first prime number. We keep it and cross out all the multiples of 2.
4. Next number is 3, it is the next prime number. We keep it and cross out all the multiples of 3.
5. Next uncrossed number is 5, which is the next prime number. Keep it and cross out all the multiples of 5.
6. Next uncrossed number is 7, which is the next prime number. Keep it and cross out all the multiples of 7.

Continue keeping the next prime numbers and crossing out their multiples, until we are left with prime numbers only.

Thus, prime numbers from 1 to 100 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Twin Primes

Two Prime numbers which differ by 2 are called **twin primes**, e.g., (3, 5) (5, 7) (11, 13) (17, 19)

Such numbers will form a pair of consecutive odd numbers.

Prime Triplet

A set of three prime numbers differing by 2 form a **prime triplet**, e.g., (3, 5, 7)

(3, 5, 7) is the only prime triplet.

Coprime Numbers

- A pair of two numbers which have no common factor other than 1 are called **coprimes**, e.g., (2, 3) (3, 4) (4, 5) (3, 7) (4, 9)
- Coprime numbers need not be prime numbers themselves, e.g., (4, 9)

The only condition for two numbers to be coprimes is that there is not even a single number, other than 1, which can divide both of them.

Divisibility Rules

Divisibility rules help us to find whether a number divides another number completely without performing actual division.

- **Divisibility by 2**

A number is divisible by two if it has 0, 2, 4, 6 and 8 at its ones place.

For example, 240; 1,752; 2,014; 85,396; 53,178 are divisible by 2, because their ones digits are 0, 2, 4, 6 and 8 respectively.

- **Divisibility by 3**

A number is divisible by 3, if the sum of its digits is divisible by 3.

For example, 4,518 is divisible by 3, because

Sum of its digits = $4 + 5 + 1 + 8 = 18$, which is divisible by 3.

Consider 72,019.

Sum of its digits = $7 + 2 + 0 + 1 + 9 = 19$, which is not divisible by 3. So 72,019 is not divisible by 3.

- **Divisibility by 4**

A number is divisible by 4 if the number formed by its digits in tens and ones places is divisible by 4

For example, 7,80,456 is divisible by 4, because the number formed by its tens and ones digits is 56, which is divisible by 4.

7,52,963 is not divisible by 4, because the number formed by its tens and ones digits is 63, which is not divisible by 4.

- **Divisibility by 5**

A number is divisible by 5, if it has 0 or 5 at its ones place

For example, 8,975 is divisible by 5, because it has 5 in its ones place.

Also, 75,293 is not divisible by 5, because its ones digit is neither 0 nor 5.

- **Divisibility by 6**

6 is a multiple of 2 and 3.

If we write some multiples of 2, 3 and 6, we have

Multiples of 2 are 2, 4, **6**, 8, 10, **12**, 14, 16, **18**, 20, 22, **24**, 26,...

Multiples of 3 are 3, **6**, 9, **12**, 15, **18**, 21, **24**, 27,....

Multiples of 6 are **6**, **12**, **18**, **24**,

We find that 6, 12, 18, 24, come in both the lists of the multiples of 2 and 3.

These numbers make the table of 6. So we can say that

If a number is divisible by two coprime numbers, it is divisible by their product also.

Therefore, the numbers which are multiples of 6 are multiples of 2 and 3 both.

Thus, **a number is divisible by 6 if it is divisible by 2 and 3 both.**

- **Divisibility by 8**

a number is divisible by 8, if the number formed by its digits in hundreds tens and ones places is divisible by 8.

For example, 72,15,976 is divisible by 8, because the number formed by its hundreds, tens and ones digits is 976, which is divisible by 8. 7,41,138 is not divisible by 8 because the number formed by its hundreds tens and ones digits is 138, which is not divisible by 8.

- **Divisibility by 9**

A number is divisible by 9, if the sum of its digits is divisible by 9.

For example, a number 90,981 is divisible by 9,

because Sum of its digits = $9 + 0 + 9 + 8 + 1 = 27$, which is divisible by 9. Consider 54,731.

Sum of its digits = $5 + 4 + 7 + 3 + 1 = 20$, which is not divisible by 9. So, 54,731 is not divisible by 9.

- **Divisibility by 10**

A number is divisible 10, if it has 0 at its ones place

For example, 98,370 is divisible by 10, because it has 0 in its units place.

53,948 is not divisible by 10, because its ones digit is not 0.

- **Divisibility by 11**

A number is divisible by 11, if the difference of the sum of its digits in odd places and the sum of its digits in even places (starting from unit's place) is either 0 or divisible by 11.

Consider the following numbers.

4,961, 61,831, 96,251

In 4,961, the sum of the digits in odd places is 10 ($1 + 9 = 10$) and the sum

of digits in even places is 10 ($6 + 4 = 10$)

Difference of the two sums = $10 - 10 = 0$.

Hence, 4,981 is divisible by 11.

In 61,831, the sum of the digits in odd places is 15 ($1 + 8 + 6 = 15$) and the sum of the digits in even place is 4 ($3 + 1 = 4$).

Difference of the two sums = $15 - 4 = 11$, which is divisible by 11. Hence, 61,831 is divisible by 11.

In 96,251, sum of the digits in odd places is 12 ($1 + 2 + 9 = 12$) and sum of the digits in even places is 11 ($5 + 6 = 11$).

Difference of the two sums = $12 - 11 = 1$, which is not divisible by 11.

Hence, 96,251 is not divisible by 11.

Factors and Multiples

When a number divides another number exactly, then the divisor is called a **factor** of the dividend.

The **dividend** is called a **multiple** of the divisor.

For example, 42 is divisible by 7, so 7 is a factor of 42 and 42 is a multiple of 7.

Consider, $105 = 7 \times 15$.

Here also, 7 and 15 are factors of 105 and 105 is a multiple of both 7 and 15.

- **Even Numbers:** Numbers divisible by 2 are called even numbers.

For example, 2, 4, 6, 8, are even numbers.

- **Odd Numbers:** Numbers not divisible by 2 are called odd numbers

For example, 1, 3, 5, 7, 9, are odd numbers.

Common Factors

Observe the following:

(a) All the factors of 15 are 1, 3, 5 and 15.

All the factors of 20 are 1, 2, 4, 5, 10 and 20.

Thus, common factors of 15 and 20 are 1 and 5.

(b) All the factors of 32 are 1, 2, 4, 8, 16, 32.

All the factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

Thus, common factors of 32 and 48 are 1, 2, 4, 8, 16.

Tips: does not have to factors So, it is not a prime number. Also, does not have more than to factors. So, it is not a composite number. Hence, is neither a prime nor a composite number

Highest Common Factor H.C.F

In the example a given above, common factors of 15 and 20 are 1 and 5. Out of these two common factors, 5 is the highest greatest common factor. Therefore, we call 5 as

the highest common factor of 15 and 20.

Again in example by, common factors of 32 and 48 are 1, 2, 4, 8 and 16. Out of these, 16 is the highest greatest common factor. Therefore, 16 is the highest common factor of 32 and 48.

Hence, the H.C.F. highest common factor of two or more than two numbers is the highest greatest among all their common factors.

Highest common factor H.C.F. is also called **Greatest common divisor (G.C.D.)**.

Example 1: Find all the common factors of:

(a) 24 and 40

(b) 20 and 64

(a) $24 = 1 \times 24$, $24 = 2 \times 12$,

$24 = 3 \times 8$, $24 = 4 \times 6$

Therefore, all the factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.

Also, $40 = 1 \times 40$, $40 = 2 \times 20$, $40 = 4 \times 10$, $40 = 5 \times 8$.

Therefore, all the factors of 40 are 1, 2, 4, 5, 8, 10, 20 and 40.

Thus, common factors of 24 and 40 are 1, 2, 4 and 8.

(b) $20 = 1 \times 20$, $20 = 2 \times 10$, $20 = 4 \times 5$

Therefore, all the factors of 20 are 1, 2, 4, 5, 10 and 20.

Also, $64 = 1 \times 64$, $64 = 2 \times 32$, $64 = 4 \times 16$, $64 = 8 \times 8$.

Therefore, all the factors of 64 are 1, 2, 4, 8, 16, 32 and 64.

Thus, common factors of 20 and 64 are 1, 2 and 4.

Example 2: Find the H.C.F. of 18, 45 and 63.

$18 = 1 \times 18$, $18 = 2 \times 9$, $18 = 3 \times 6$

All the factors of 18 are 1, 2, 3, 6, 9 and 18.

Also, $45 = 1 \times 45$, $45 = 3 \times 15$, $45 = 5 \times 9$.

All the factors of 45 are 1, 3, 5, 9, 15 and 45.

Also, $63 = 1 \times 63$, $63 = 3 \times 21$, $63 = 7 \times 9$.

All the factors of 63 are 1, 3, 7, 9, 21 and 63.

Common factors of 18, 45 and 63 are 1, 3 and 9.

Thus, H.C.F. of 18, 45 and 63 is 9.

Example 3: Find the H.C.F. of 15 and 28.

$15 = 1 \times 15$, $15 = 3 \times 5$.

All the factors of 15 are 1, 3, 5 and 15.

Also, $28 = 28 \times 1$, $28 = 2 \times 14$, $28 = 4 \times 7$

All the factors of 28 are 1, 2, 4, 7, 14 and 28.

Common factor of 15 and 28 is 1.

Thus, H.C.F. of 15 and 28 is 1.

We call 15 and 28 are co-prime.

Tips: Two numbers are said to be co-prime, if their H.C.F. is 1

Properties of H.C.F

1. The H.C.F. of two or more numbers cannot be greater than any one of them.
2. If a number is a factor of another number, then their H.C.F. is the smaller number.

Common Multiples

Let us try to find out the multiples of 8 and 12.

Multiples of 8 are 8, 16, 24, 32, 40, 48,

Multiples of 12 are 12, 24, 36, 48, 60, 72,

Thus, common multiples of 8 and 12 are 24, 48,

Again, find the common multiples of 10 and 15.

Multiples of 10 are 10, 20, 30, 40, 50, 60,

Multiples of 15 are 15, 30, 45, 60, 75,

Thus, common multiples of 10 and 15 are 30, 60,

Lowest Common Multiple L.C.M

In the examples given above, common multiples of 8 and 12 are 24, 48, and common multiples of 10 and 15 are 30, 60,

The lowest common multiple of 8 and 12 is 24.

And the lowest common multiple of 10 and 15 is 30.

Hence, the lowest common multiple L.C.M. of two or more numbers is the least smallest of their common multiples.

Example 4: Find the first two common multiples of:

(a) 4 and 10

(b) 5, 6 and 15.

(a) Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40,

Multiples of 10 are 10, 20, 30, 40,

Thus, the first two common multiples of 4 and 10 are 20 and 40.

(b) Multiples of 5 are 5, 10, 15, 20, 25, 30, 35, 40, 45, 60, 65,

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60,

Multiples of 15 are 15, 30, 45, 60, 75,

Thus, the first two common multiples of 5, 6 and 15 are 30 and 60.

Example 5: Find the L.C.M. of 12 and 18.

Multiples of 12 are 12, 24, 36, 48, 60, 72,

Multiples of 18 are 18, 36, 54, 72, 90,

Thus, common multiples of 12 and 18 are 36, 72,

Therefore, L.C.M. of 12 and 18 is 36.

Properties of L.C.M

1. The L.C.M. of two co-prime numbers is their product.
2. If one number is the multiple of the other, then their L.C.M. is the greater number.
3. The L.C.M. of two or more numbers cannot be less than any one of them.

Exponential Notation or Index Notation

When a number is multiplied with itself several times, we express the product in the form given below; called the **exponential** or **index notation**.

Product form	Exponential form	Read as
3×3	3^2	3 raised to the power 2.
$4 \times 4 \times 4$	4^3	4 raised to the power 3.
$6 \times 6 \times 6 \times 6$	6^4	6 raised to the power 4.
$10 \times 10 \times 10 \times 10 \times 10 \times 10$	10^6	10 raised to the power 6.

In 3^2 , 3 is called the **base** and 2 is called the exponent or index or power.

Similarly in 6^4 , 6 is the base and 4 is the exponent index.

Observe the following examples:

$$2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$$

$$5 \times 5 \times 5 \times 5 \times 8 \times 8 \times 8 = 5^4 \times 8^3$$

$$10 \times 10 \times 10 \times 10 \times 10 \times 17 \times 17 \times 17 \times 17 = 10^5 \times 17^4$$

Example 6: Write in the exponential form:

(a) $5 \times 5 \times 5 \times 5 \times 5 \times 5$

(b) $8 \times 8 \times 8 \times 8 \times 9 \times 9$

(a) $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$

(b) $8 \times 8 \times 8 \times 8 \times 9 \times 9 = 8^4 \times 9^2$.

Example 7: Write in the product form:

(a) 8^7

(b) $6^3 \times 11^4$

(a) $8^7 = 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$

(b) $6^3 \times 11^4 = 6 \times 6 \times 6 \times 11 \times 11 \times 11 \times 11$.

Tips: In index notation, we cannot interchange base and exponent, i.e., 2^3 is not equal to 3^2 .

Prime Factorisation

Observe the following:

$$54 = 1 \times 54, 54 = 2 \times 27, 54 = 3 \times 18, 54 = 6 \times 9, 54 = 2 \times 3 \times 9,$$

$$54 = 2 \times 3 \times 3 \times 3$$

We see that in $54 = 2 \times 3 \times 3 \times 3$, all the factors are prime.

Thus, prime factorisation of 54 is $2 \times 3 \times 3 \times 3$.

Similarly, prime factorisation of 24 is $2 \times 2 \times 2 \times 3$.

Prime factorisation of 45 is $3 \times 3 \times 5$.

When a number is expressed as a product of prime numbers, we call it prime factorisation

To get the prime factorisation of a number, we divide the given number by the prime numbers 2, 3, 5, 7, 11, etc., so long as the quotient is divisible by that number.

Factor Tree

The prime factorisation of a number can be shown diagrammatically, known as **factor tree**.

Example 8: Express 56 as the product of prime factors.

We proceed as follows:

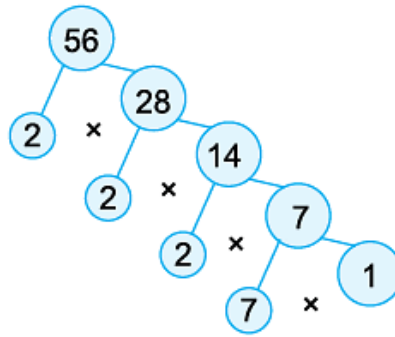
Divide 56 by 2

Divide 28 by 2

Divide 14 by 2

Divide 7 by 7

2	56
2	28
2	14
7	7
	1

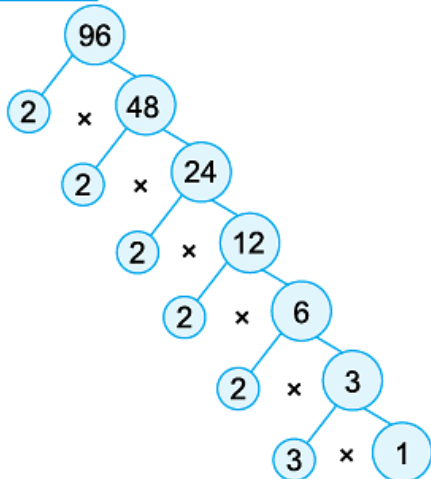


(Factor tree)

Hence, the prime factorisation of 56 is
 $= 2 \times 2 \times 2 \times 7 = 2^3 \times 7$.

Example 9: Draw a factor tree to find the prime factorisation of 96.

2	96
2	48
2	24
2	12
2	6
3	3
	1



(Factor tree)

Thus, the prime factorisation of 96 is
 $= 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$.

H.C.F by Prime Factorisation Method

To find the H.C.F. of two or more numbers, we express each one of them as the product of prime factors. Then, the product of terms containing least powers of common factors is their H.C.F.

Example 10: Find the H.C.F. of 28 and 72 by prime factorisation method.

Resolving each of the given numbers into prime factors, we get

$$28 = 2 \times 2 \times 7 = 2^2 \times 7$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$$

2	28	2	72
2	14	2	36
7	7	2	18
	1	3	9
		3	3
			1

H.C.F. of 28 and 72 = Product of common prime factors

= Product of terms with smallest powers of common factors

$$= 2^2 = 4$$

Thus, H.C.F. of 28 and 72 = 4.

Example 11: Find the H.C.F. of 14, 84 and 105.

Resolving the given numbers into prime factors, we get.

$$14 = 2 \times 7$$

$$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$$

$$105 = 3 \times 5 \times 7$$

2	14	2	84	3	105
7	7	2	42	5	35
	1	3	21	7	7
		7	7		1
			1		

H.C.F. of 14, 84 and 105 = Product of common prime factors

= Product of terms with smallest powers of common factors.

$$= 7^1 = 7$$

Thus, the H.C.F. of 14, 84 and 105 = 7.

H.C.F by Long Division Method

To find the HCF of two or three given numbers, divide the greater number by the smaller one. Then divide the divisor by the remainder. Go on repeating the process

of dividing the preceding divisor by the remainder, till zero remainder is obtained. The last divisor is the HCF of the given numbers.

Example 12: Find the H.C.F. of 144, 180 and 324.

We first find the HCF of any two numbers (144 and 180).

Since, last divisor is 36, so 36 is the HCF of 144 and 180.

Now, we find the HCF of 36 and 324.

Hence, HCF of 144, 180 and 324 is 36.

$$\begin{array}{r}
 144 \overline{) 180} \quad 1 \\
 \underline{-144} \\
 36 \overline{) 144} \quad 4 \\
 \underline{-144} \\
 0 \\
 36 \overline{) 324} \quad 9 \\
 \underline{-324} \\
 0
 \end{array}$$

L.C.M by Prime Factorisation Method

To find the L.C.M. of two or more numbers, we express each one of them as the product of prime factors. Then the product of terms with highest powers of all the factors gives their L.C.M.

Example 13: Find the L.C.M. of 15 and 40, using prime factorisation method.

Resolving the given numbers into prime factors, we get

3	15
5	5
	1

2	40
2	20
2	10
5	5
	1

So, $15 = 3 \times 5$

Also, $40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5$.

The different prime factors of 15 and 40 are 2, 3 and 5.

L.C.M. of 15 and 40 = Product of terms containing highest powers of 2, 3 and 5
 $= 2^3 \times 3 \times 5 = 8 \times 3 \times 5 = 120$.

Example 14: Find the L.C.M. of 16, 48 and 64.

Resolving each of the given numbers into prime factors, we get

2	16
2	8
2	4
2	2
	1

2	48
2	24
2	12
2	6
3	3
	1

2	64
2	32
2	16
2	8
2	4
2	2
	1

$$16 = 2 \times 2 \times 2 \times 2 = 2^4 \quad 48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3 \quad 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$$

The different prime factors of 16, 48 and 64 are 2 and 3.

Thus, L.C.M. of 16, 48 and 64

= Product of terms containing highest powers of 2 and 3.

$$= 2^6 \times 3$$

$$= 64 \times 3 = 192.$$

L.C.M. by Long Division Method

We proceed as below:

1. Arrange the given numbers in a line, in an order.
2. Divide by a number which exactly divides at least two of the given numbers and carry forward the numbers which are not divisible.
3. Repeat the process till none of the two given numbers are divisible by the same number.
4. The product of all the divisors and the numbers left undivided is the required LCM.

Example 15: Find the LCM of 18, 36 and 42 by long division method.

So, LCM of 18, 36 and 42

$$= 2 \times 3 \times 3 \times 2 \times 7 = 252$$

Hence, LCM of 18, 36 and 42 = 252.

2	18	36	42
3	9	18	21
3	3	6	7
	1	2	7

Relationship Between H.C.F. and L.C.M, of Two Given Numbers

For two given numbers, we have.

- Product of two numbers = Product of their H.C.F. and L.C.M.

(First number $\times$ Second number = H.C.F. $\times$ L.C.M)

- $$\text{L.C.M.} = \frac{\text{first number} \times \text{second number}}{\text{H.C.F.}}$$
- $$\text{H.C.F.} = \frac{\text{first number} \times \text{second number}}{\text{L.C.M.}}$$

Example 16: The H.C.F. of two numbers is 52 and their L.C.M is 312. If one of the numbers is 104, find the other.

It is given here that:

H.C.F. = 52, L.C.M. = 312 and one number = 104

The other number

$$= \frac{\text{H.C.F.} \times \text{L.C.M.}}{\text{given number}} = \frac{52 \times 312}{104} = 156.$$

Hence, the other number is 156.

Example 17: The product of two numbers is 867 and their HCF is 17. Find their L.C.M.

We know that:

$$\text{LCM} = \frac{\text{Product of given numbers}}{\text{Their H.C.F.}} = \frac{867}{17} = 51.$$

Hence, the LCM of given numbers is 51.