Basic Mathematics(Pre-requisite)

Concepts of Differentiation and Integration

Calculus

Calculus is basically a way of calculating rate of changes (similar to slopes, but called derivatives in calculus), areas, volumes, and surface areas (for starters).

It's easy to calculate these kinds of things with algebra and geometry if the shapes you're interested in are simple. For example, if you have a straight line you can calculate the slope easily. But if you want to know the slope at an arbitrary point (any random point) on the graph of some function like x-squared or some other polynomial, then you would need to use calculus. In this case, calculus gives you a way of "zooming in" on the point you're interested in to find the slope exactly at the point. This called a derivative.

If you have a cube or a sphere, you can calculate the volume and surface area easily. If you have an odd shape, you need to use calculus. You use calculus to make a infinite number or really small slices of the object you're interested in, determine to sizes of the slices, and then add all those sizes up. This process is called integration. It turns out that integration is the reverse of derivation (finding a derivative).

In summary, calculus is a tool that lets you do calculation with complicated curves shapes, etc. that you would normally not be able to do with just algebra and geometry.

Differentiation and Integration

Differentiation is the process of obtaining the derived function $f'(x)$ from the function $f(x)$, where $f'(x)$ is the derivative of f at x.

The derivatives of certain common functions are given in the Table of derivatives,

Table of derivatives :

$f(x)$	$f'(x)$
x^n	nx^{n--1}
$\sin x$	$\cos x$
$\cos x$	$--\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$--\mathrm{cosec}^2 x$

sec x	sec x tan x
cosec x	−-(cosec x)(cot x)
ln x	1x1x
e^x	e^x

Many other functions can be differentiated using the following rules of differentiation:

(i) If $h(x) = k f(x)$ for all x, where k is a constant, then $h'(x) = k f'(x)$.

(ii) If $h(\mathrm{x}) = f(x) + g(x)$ for all x, then $h'(x) = f'(x) + g'(x)$.

(iii) The product rule: If $h(x) = f(x)g(x)$ for all x, then $h'(\mathrm{x}) = f(x)g'(\mathrm{x}) + f'(x)+g'(x)$.

(iv) The reciprocal rule: If $h(x) = 1/f(x)$ and $f(x) \neq 0$ for all x, then

$$h'\left(x\right) = -\frac{f'(x)}{(f(x))^2}$$

(v) The quotient rule: If $h(x) = f(x)/g(x)$ and $g(x) \neq 0$ for all x, then

$$h'\left(x\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

(vi) The chain rule: If $h(x) = (f \circ g)(x) = f(g(x))$ for all x, then $h'(x) = f'(g(x))g'(x)$.

Example:

$$f\left(x\right) = x^n$$
$$f'\left(x\right) = \frac{d}{dx} f\left(x\right) = \frac{d}{dx}\left(x^n\right)$$
$$\Rightarrow f'\left(x\right) = nx^{n-1}$$

Integration is the process of finding an anti-derivative of a given function *f*. 'Integrate *f*' means 'find an anti-derivative of *f*'. Such an anti-derivative may be called an indefinite integral of *f* and be denoted by $\int f(x)dx$.

The term 'integration' is also used for any method of evaluating a definite integral.

$$\int_a^b f\left(x\right) dx$$

The definite integral can be evaluated if an anti-derivative Φ of *f* can be found, because then its value is Φ(*b*) − Φ(*a*). (This is provided that *a* and *b* both belong to an interval in which *f* is continuous.)

However, for many functions *f*, there is no anti-derivative expressible in terms of elementary functions, and other methods for evaluating the definite integral have to be sought, one such being so-called numerical integration.

Example:

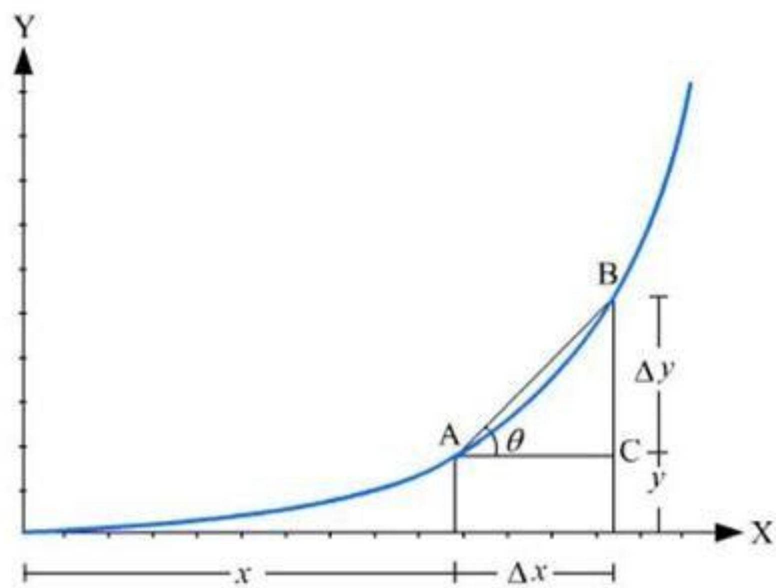
$$f\left(x\right) = x^n$$

$$\Rightarrow \int f\left(x\right)dx = \int x^n dx$$

$$\Rightarrow \int f\left(x\right)dx = \frac{x^{n+1}}{n+1} + \text{constant}$$

Differential and Integral Calculus

Differential calculus



Let *x* and *y* be two quantities interrelated in such a way that for each value of *x* there is one and only one value of *y*.

The graph represents the y versus x curve. Any point in the graph gives an unique values of x and y. Let us consider the point A on the graph. We shall increase x by a small amount Δx, and the corresponding change in y be Δy.

Thus, when x change by Δx, y change by Δy and the rate of change of y with respect to x is equal to $\frac{\Delta y}{\Delta x}$

In the triangle ABC, coordinate of A is (x, y); coordinate of B is $(x + \Delta x, y + \Delta y)$

The rate $\frac{\Delta y}{\Delta x}$ can be written as,

$$\frac{\Delta y}{\Delta x} = \frac{BC}{AC} = \tan\theta = \text{slope of the line AB}$$

But this cannot be the precise definition of the rate because the rate also varies between the point A and B. So, we must take very small change in x. That is Δx is nearly equal to zero. As we make Δx smaller and smaller the slope $\tan\theta$ of the line AB approaches the slope of the tangent at A. This slope of the tangent at A gives the rate of change of y with respect to x at A.

This rate is denoted by $\frac{dy}{dx}$

and,

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

Question:

Find the slope of the curve $y = 1 + x^2$ at $x = 5$.

Solution:

$y = 1 + x^2$

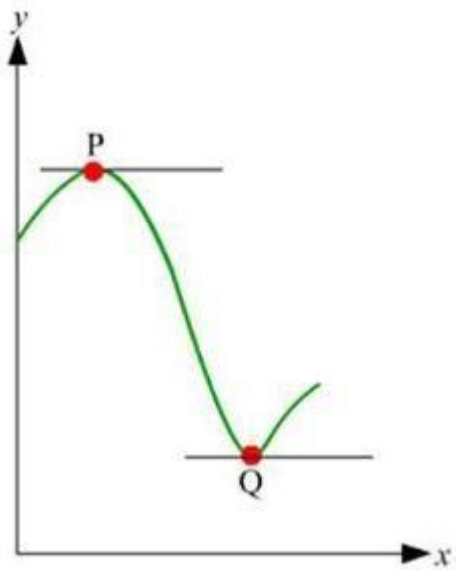
We know slope is given by, $\frac{dy}{dx}$

$$\therefore \frac{dy}{dx}=\frac{d}{dx}(1+x^2)=0+2x=2x$$

So, at $x = 5$ the slope is = 2 × 5 = 10

Maxima and Minima

Let x and y be two quantities interrelated in manner as shown in the graph below:



At the points P and Q the tangents to the curve is parallel to the x-axis.

Hence, its slope tanθtanθ = 0.

But we know that the slope of the curve at any point equals the rate of change $\frac{dy}{dx}$ at the point.

Thus, at maximum (at P) or at minimum (at Q),

$$\frac{dy}{dx}=0$$

Just before the maximum the slope is positive, at the maximum it is zero and just after the maximum it is negative. This implies, $\frac{dy}{dx}$ decrease at a maximum.

$$\therefore \frac{d}{dx}\left(\frac{dy}{dx}\right) < 0 \text{ at maximum}$$

$$\text{Or, } \frac{d^2y}{dx^2} < 0 \text{ at maximum}$$

Similarly, at the minimum:

$$\frac{d^2y}{dx^2} < 0 \text{ at maximum}$$

Question:

A particle is thrown vertically upwards. At what time does the particle reach the maximum height? Find it using differential calculus.

Solution:

The equation of motion of the particle is given by.

$$y = ut = \frac{1}{2}gt^2$$

Where, y is the vertical displacement, u is the initial velocity, g is the acceleration due to gravity, t is the time.

The vertical displacement is a function of time. Therefore, it will be maximum when $\frac{dy}{dt} = 0.$

$$y = ut - \frac{1}{2}gt^2$$

$$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}\left(ut - \frac{1}{2}gt^2\right)$$

$$\Rightarrow \frac{dy}{dt} = u - gt$$

$$\because \frac{dy}{dt} = 0$$

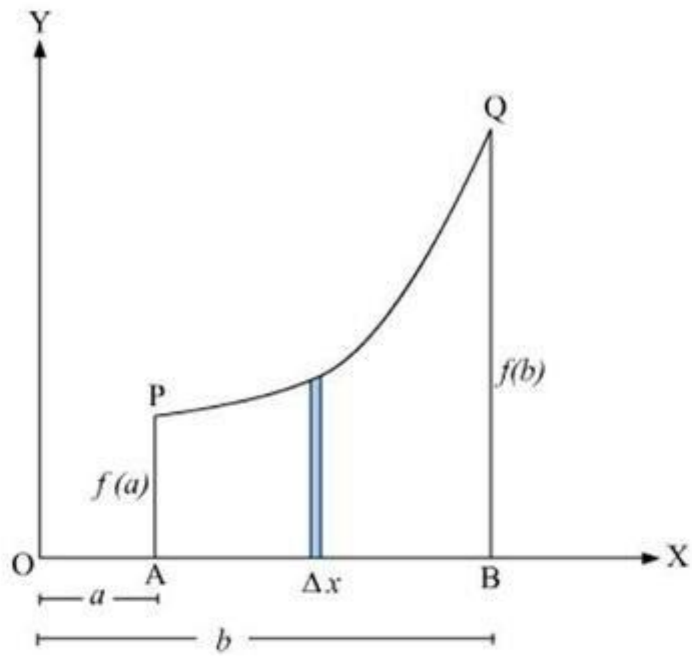
$$\Rightarrow u - gt = 0$$

$$\Rightarrow t = \frac{u}{g}$$

Thus, the particle reaches maximum height at the time, $t = \frac{u}{g}$

Integral calculus

In the graph we have a curve PQ representing the relation between *x* and *y*. The equation of the curve is, *y* = *f*(*x*)



We shall find out the area under this curve. That is we need the area of APQB. To find this we shall first consider a very thin rectangle touching the curve and standing on the x-axis. The width of the rectangle is so that both the edges of the side near the curve actually touch the curve almost at the same point which means that Δ*x* is so small that it tends to zero.

Area of this thin rectangle is = *f*(*x*) Δ*x*

We shall take *n* such rectangles and fill the area. The area of APQB in the sum of the area of the rectangles. This may be written as

$$S = \lim_{\Delta x \to 0} \sum_{i=1}^{n} f(x_i)\Delta x$$

This quantity is also denoted as,

$$S = \int_a^b f(x)\,dx$$

Question:

The equation of motion of a particle is given by, $x = 1 + t^3$. Calculate the distance travelled by the particle from time $t = 0$ s to $t = 5$ s.

Solution:

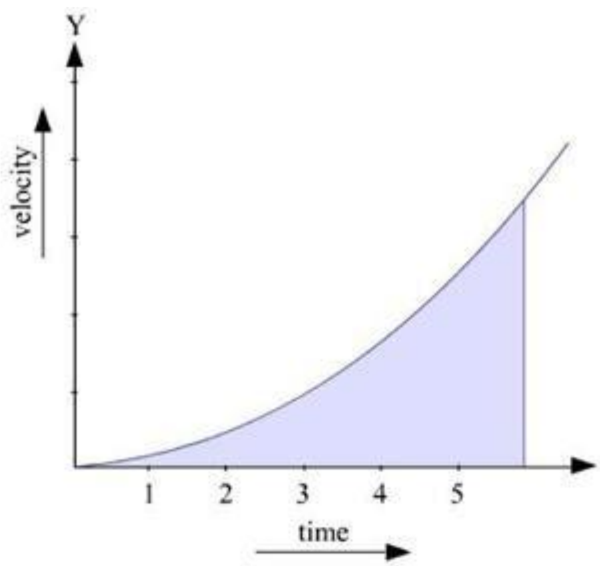
The motion equation is given by,

$x = 1 + t^3$

The velocity at any instance is,

$$v = \frac{dx}{dt} = \frac{d}{dx}(1+t^3) = 3t^2$$

The velocity time graph is:



The are under this graph is the distance covered.

The area can be found by integrating the curve from $t = 0$s to $t = 5$s

$$S = \int_0^5 (3t^2)\, dt$$
$$\Rightarrow S = 3\left[\frac{t^3}{3}\right]_0^5 = 125$$

Therefore the distance travelled by the particle in the given time interval is 125 m.

Scalar and Vector Quantities

Do you know what a physical quantity is? A physical quantity is any physical property that can be expressed in numbers. For example, time is a physical quantity as it can be expressed in numbers, but beauty is not as it cannot be expressed in numbers.

Scalar Quantities

- If a physical quantity can be completely described only by its magnitude, then it is a **scalar quantity**. To measure the mass of an object, we only have to know how much matter is present in the object. Therefore, mass of an object is a physical quantity that only requires magnitude to be expressed. Therefore, we say that **mass is a scalar quantity**.
- Some more examples of scalar quantities are time, area, volume, and energy.
- We can add scalar quantities by simple arithmetic means.
- It is difficult to plot scalar quantities on a graph.

Vector Quantities

- There are some physical quantities that cannot be completely described only by their magnitudes. These physical quantities require direction along with magnitude. For example, if we consider force, then along with the magnitude of the force, we also have to know the direction along which the force is applied. Therefore, to describe a force, we require both its magnitude and direction. This type of physical quantity is called a vector quantity.

  Therefore, we can define **vector quantity** as **the physical quantity that requires both magnitude and direction to be described**.

- Some examples of vector quantities are velocity, force, weight, and displacement.
- Vector quantities cannot be added or subtracted by simple arithmetic means.
- Vector quantities can easily be plotted on a graph.

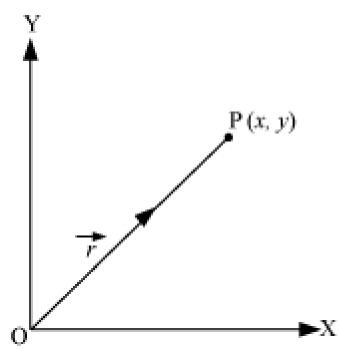
Scalars v/s Vectors

Scalars	Vectors

A scalar quantity has only magnitude.	A vector quantity has both magnitude and direction.
Scalars can be added, subtracted, multiplied, and divided just as ordinary numbers i.e., scalars are subjected to simple arithmetic operations.	Vectors cannot be added, subtracted, and multiplied following simple arithmetic laws. Arithmetic division of vectors is not possible at all.
Example: mass, volume, time, distance, speed, work, temperature	Example: displacement, velocity, acceleration, force

Position Vector

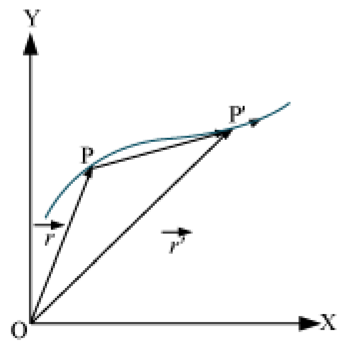
Position vector of a point in a coordinate system is the straight line that joins the origin and the point.



Magnitude of the vector is the length of the straight line and its direction is along the angle θ from the positive x-axis.

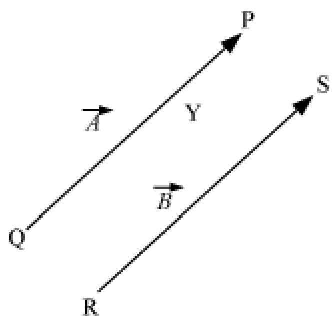
Displacement Vector

Displacement vector is the straight line joining the initial and final positions.

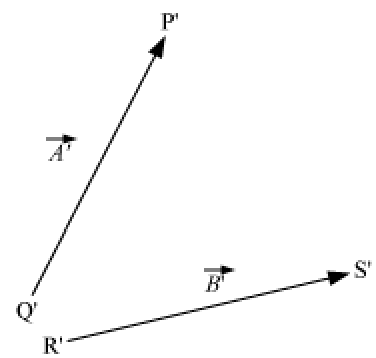


Equality of Vectors

Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal, if and only if they have the same magnitude and the same direction.



$\vec{A}$ and $\vec{B}$ are equal



$\vec{A'}$ and $\vec{B'}$ are unequal vectors; their magnitudes are same but directions are not same.

Scalars and Vectors; Multiplication of Vectors by Real Numbers

Scalars vs. Vectors

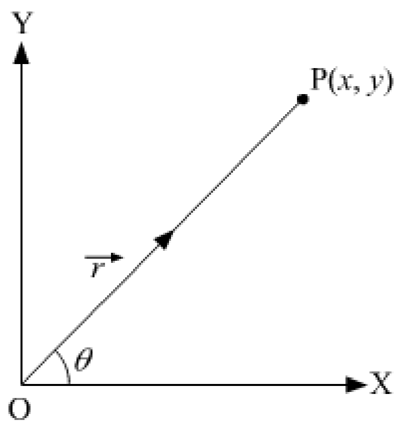
Scalars	Vectors
A scalar quantity has magnitude only.	A vector quantity has both magnitude and direction.

Scalar quantities can be added, subtracted, multiplied and divided just like ordinary numbers, i.e., scalars are subjected to simple arithmetic operations.	Vectors cannot be added, subtracted or multiplied following simple arithmetic rules. Arithmetic division of vectors is not possible at all.
Example: Mass, volume, time, distance, speed, work, temperature, etc.	Example: Displacement, velocity, acceleration, force, etc.

Distance & Displacement:

Position Vector

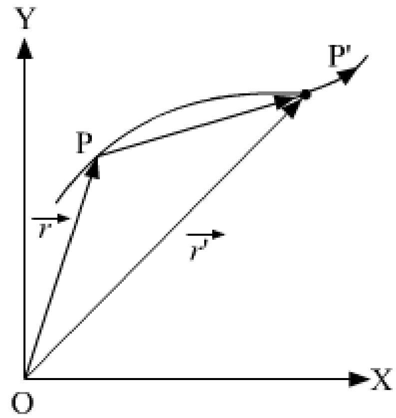
The position vector of a point in a coordinate system is the straight line that joins the origin and the point.



The magnitude of a vector is the length of the straight line. Its direction is along the angle θ from the positive x-axis.

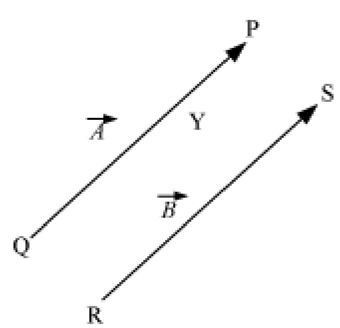
Displacement Vector

Displacement vector is the straight line joining the initial and the final position.

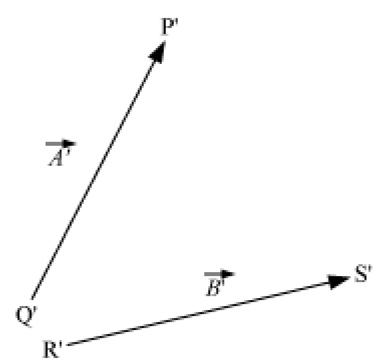


Equality of Vectors

Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal only if they have the same magnitude and the same direction.



$\vec{A}$ and $\vec{B}$ are equal



$\vec{A'}$ and $\vec{B'}$ are unequal vectors; their magnitudes are same but directions are not same.

Negative vector
Negative vector is a vector whose magnitude is equal to that of a given vector, but whose direction is opposite to that of the given vector.

Zero vector
Zero vector is a vector whose magnitude is zero and have an arbitrary direction.

Resultant vector
The resultant vector of two or more vectors is a vector which produces the same effect as produced by the individual vectors together.

Multiplication of Vectors by Real Numbers

- Multiplication of a vector $\vec{A}$ with a positive number *k* only changes the magnitude of the vector keeping its direction unchanged.

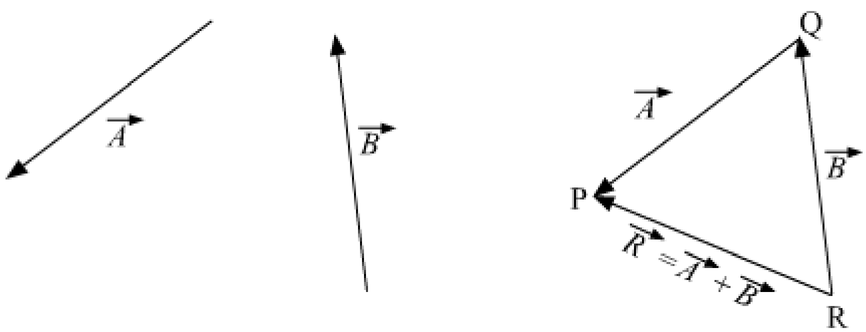
  $\left|k\vec{A}\right| = k\left|\vec{A}\right|$ if $k > 0$

- Multiplication of a vector $\vec{A}$ with a negative number $-k$ gives a vector $-k\vec{A}$ in the opposite direction.

Addition and Subtraction of Vectors

Addition of Vectors: Triangle Method

- The given vectors $\vec{A}$ and $\vec{B}$ have to be arranged head to tail, keeping their directions unchanged.

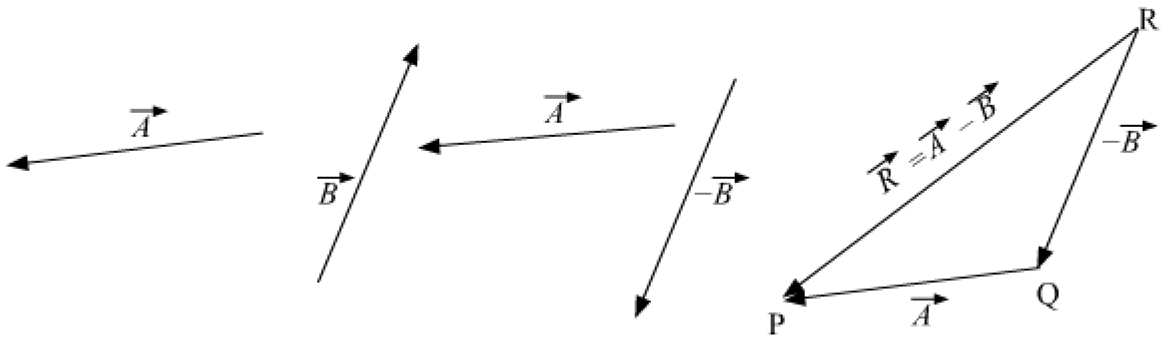


- The line PR, joining the ending point of $\vec{A}$ and the starting point of $\vec{B}$, represents a vector $\vec{R}$ (resultant vector) that is the sum of the vectors $\vec{A}$ and $\vec{B}$.
  i.e., $\vec{R} = \vec{A} + \vec{B}$

- Vector addition obeys commutative law and associative law.
  i.e., $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ and $\left(\vec{A} + \vec{B}\right) + \vec{C} = \vec{A} + \left(\vec{B} + \vec{C}\right)$

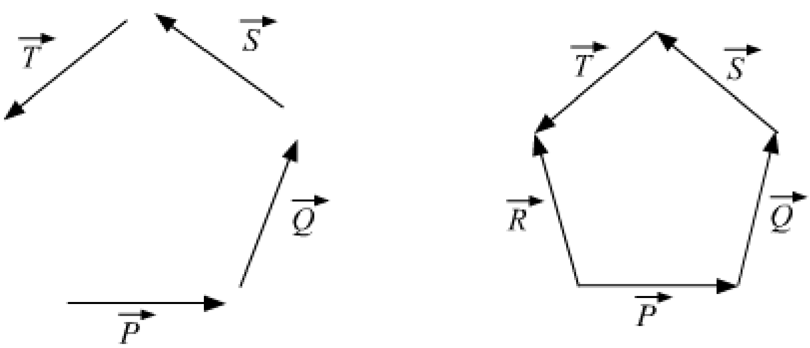
Subtraction of Vectors

- The difference between two vectors $\vec{A}$ and $\vec{B}$ is defined as the sum of two vectors $\vec{A}$ and $-\vec{B}$.
  i.e., $\vec{R} = \vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$

- 

Polygon law of vector

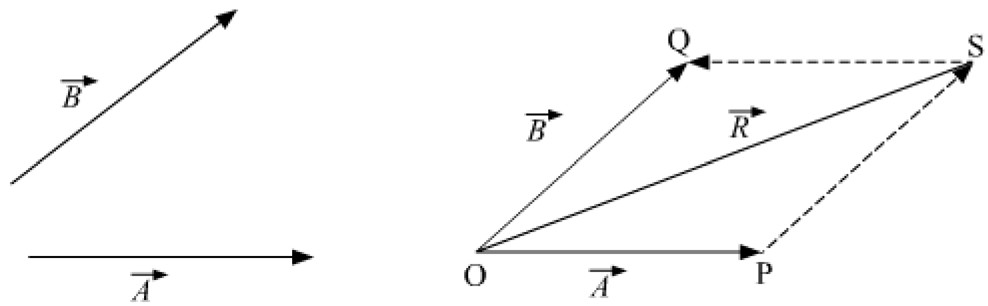
- According to this law, if a number of vectors acting in a plane are represented in magnitudes and directions by the sides of an open polygon taken in order, then the resultant vector is represented in magnitude and direction by the closing side of the polygon taken in the opposite order. The direction of the resultant vector is from the starting point of the first vector to the end point of the last vector.



- Vectors $\vec{P}$, $\vec{Q}$, $\vec{S}$ and $\vec{T}$ are placed in order to represent an incomplete polygon. Their resultant vector $\vec{R}$ represents the closing side of the polygon.

Parallelogram Method of Vector Addition

- The given vectors $\vec{A}$ and $\vec{B}$ have to be arranged, keeping their directions unchanged such that their starting point is a common point O.



- If a parallelogram OQSP is drawn with these two vectors as its sides, then the diagonal OS is the sum $\overrightarrow{R}$ (resultant vector) of the two vectors.
- The length of the diagonal is the magnitude of the resultant vector and its direction is along the diagonal OS.
- The magnitude of the resultant vector *R* is given by

(1) If $\overrightarrow{A}$ and $\overrightarrow{B}$ are perpendicular to each other, then θ = 90°

$$R = \sqrt{A^2 + B^2 + 2ABcos90^\circ}$$
$$\Rightarrow R = \sqrt{A^2 + B^2} \quad (\because\ cos90^\circ = 0)$$

$$\text{Now, } \tan\theta = \frac{B\ \sin 90^\circ}{A + B\cos 90^\circ}$$
$$\Rightarrow \tan\theta = \frac{B}{A}$$
$$\Rightarrow \theta = \tan^{-1}\left(\frac{B}{A}\right)$$

(2) If $\overrightarrow{A}$ and $\overrightarrow{B}$ are parallel to each other, then θ = 0°

$$R = \sqrt{A^2 + B^2 + 2AB\cos 0^\circ}$$
$$\Rightarrow R = \sqrt{A^2 + B^2 + 2AB}$$
$$\Rightarrow R = A + B$$
$$\text{Now, } \tan\theta = \frac{B\sin 0^\circ}{A + B\cos 0^\circ}$$
$$\Rightarrow \theta = 0 \qquad (\because\ \sin 0^\circ = 0)$$

(3) If $\overrightarrow{A}$ and $\overrightarrow{B}$ are antiparallel to each other, then θ =180°

$$R = \sqrt{A^2 + B^2 + 2AB\cos 180^\circ}$$
$$\Rightarrow R = \sqrt{A^2 + B^2 - 2AB} \qquad (\because\ \cos 180^\circ = -1)$$
$$\Rightarrow R = A - B$$
$$\text{Now, } \tan\theta = \frac{B\sin 180^\circ}{A + B\cos 180^\circ}$$
$$\Rightarrow \theta = 0^\circ \qquad (\because \sin 180^\circ = 0)$$

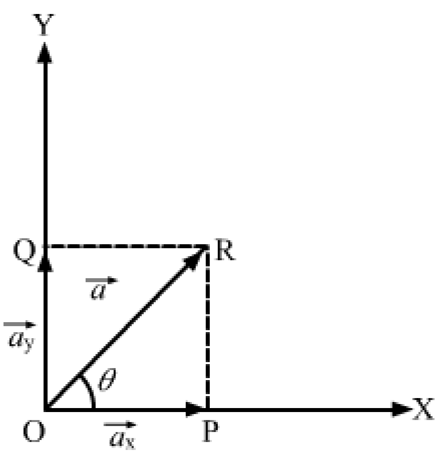
Resolution of Vectors

Unit Vector

- A unit vector is a vector of unit magnitude and points towards a particular direction.
    - Unit vector can be expressed as $\widehat{a} = \frac{\vec{a}}{\left|\vec{a}\right|}$
    - $\hat{i},\ \hat{j}$ and $\hat{k}$ are three special unit vectors along X, Y, and Z axes respectively.

Resolution of Vector in Rectangular Components (in Two Dimensions)

- The process of splitting a vector into rectangular components is called resolution of vector.
- The components of a vector are found by projecting the vector on the axes of a rectangular coordinate system. The coordinate system can be considered according to our convenience.



- $\overrightarrow{a}_x$ and $\overrightarrow{a}_y$ are the components of vector $\overrightarrow{a}$ along X-axis and Y-axis respectively.

From triangle law of vector addition, we have

$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{OQ}$$

$$\text{or } \overrightarrow{a} = \overrightarrow{a}_x + \overrightarrow{a}_y \qquad \left[\because \overrightarrow{OP} = \overrightarrow{a}_x \text{ and } \overrightarrow{OQ} = \overrightarrow{a}_y\right]$$

Let $\hat{i}$ and $\hat{j}$ be the unit vectors along X-axis and Y-axis respectively.

$$\therefore \overrightarrow{a}_x = a_x \hat{i} \text{ and } \overrightarrow{a}_y = a_y \hat{j}$$

Hence, $\overrightarrow{a} = a_x \hat{i} + a_y \hat{j}$

From right-angled triangle ORP, $a_x = a \cos\theta$ and $a_y = a \sin\theta$

Thus, the magnitudes of the components are

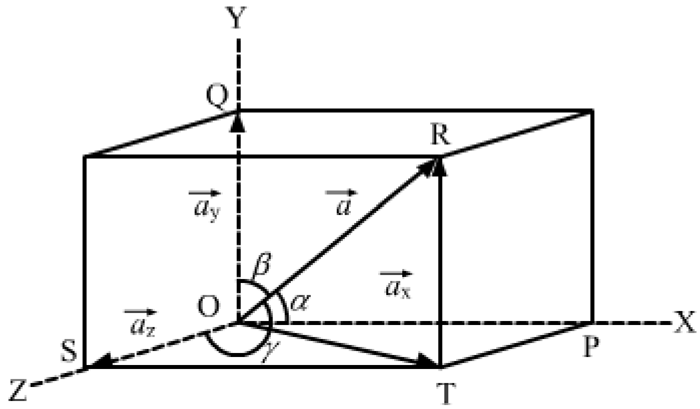
$$\boxed{a_x = a\cos\theta \text{ and } a_y = a\sin\theta}$$

Therefore, if the components of a vector are known, then its magnitude and direction can be determined by using the following equations.

$$\boxed{a = \sqrt{a_x^2 + a_y^2} \text{ and } \tan\theta = \frac{\text{PR}}{\text{OP}} = \frac{a_y}{a_x}}$$

Resolution of vectors explained by teacher:

Rectangular Components of a Vector in Three Dimensions



Using triangle law of vectors,

$$\overrightarrow{OR} = \overrightarrow{OT} + \overrightarrow{TR}$$

Using parallelogram law of vectors,

$$\overrightarrow{OT} = \overrightarrow{OS} + \overrightarrow{OP}$$

$$\therefore \overrightarrow{OR} = \overrightarrow{OS} + \overrightarrow{OP} + \overrightarrow{TR}$$

$$= \overrightarrow{OS} + \overrightarrow{OP} + \overrightarrow{OQ} \qquad \left[\because \overrightarrow{TR} = \overrightarrow{OQ}\right]$$

$$\text{or } \vec{a} = \vec{a}_z + \vec{a}_x + \vec{a}_y$$

$$\text{Let } \vec{a}_z = a_z\hat{k},\ \vec{a}_x = a_x\hat{i},\ \vec{a}_y = a_y\hat{j}$$

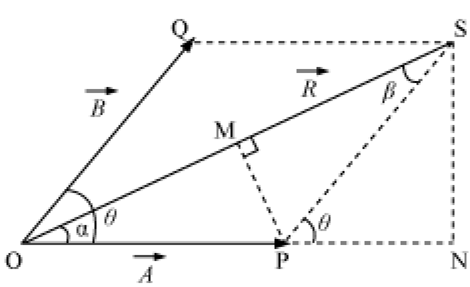
$$\therefore \vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

If, α, β, and γ are the angles which $\vec{a}$ makes with X, Y and Z axes respectively, then

$a_x = a\cos\alpha$, $a_y = a\cos\beta$, $a_z = a\cos\gamma$

And, $a = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Addition of Vectors by Analytical Method



Let $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ represent the two vectors $\vec{A}$ and $\vec{B}$, making an angle θ.

$\therefore \vec{R} = \vec{A} + \vec{B}$

For right-angled triangle ONS,

$OS^2 = ON^2 + SN^2$

However, $ON = OP + PN = A + B\cos\theta$

$SN = B\sin\theta$

$OS^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$

$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$ ... (i)

In ΔOSN, $SN = OS\sin\alpha = R\sin\alpha$

In ΔPSN, $SN = PS\sin\theta = B\sin\theta$

$$\Rightarrow \frac{R}{\sin\theta} = \frac{B}{\sin\alpha} \quad \text{...(ii)}$$

Similarly,

$PM = A\sin\alpha = B\sin\beta$

$$\frac{A}{\sin\beta} = \frac{B}{\sin\alpha} \quad \text{...(iii)}$$

Combining equations (ii) and (iii), we obtain

$$\Rightarrow \frac{R}{\sin\theta} = \frac{A}{\sin\beta} = \frac{B}{\sin\alpha} \quad \text{...(iv)}$$

Using equation (iv), we obtain

$$\sin\alpha = \frac{B}{R}\sin\theta \quad \text{...(v)}$$

Where 'R' is given by equation (i)

$\Rightarrow \tan\alpha = \dfrac{SN}{OP+PN} = \dfrac{B\sin\theta}{A+B\cos\theta}$ ...(vi)

Equation (i) gives the magnitude of the resultant and equation (v) and (vi) its directions.

Equation (i) is known as the law of cosines and equation (iv) as the law of sines.

Problems Based on Addition of Vectors by Analytical Method

Example – Two forces 10 N and 15 N are acting at an angle of 120°between them. Find the resultant force in magnitude and direction.

Solution

Here, A = 10 N, B = 15 N

θ = 120°; R =?; α= ?

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\Rightarrow R = \sqrt{(10)^2 + (15)^2 + 2\times10\times15\cos120°}$

$\Rightarrow R = \sqrt{100 + 225 + 300\left(-\dfrac{1}{2}\right)}$

$R = \sqrt{100 + 225 - 150}$

$\Rightarrow R = \sqrt{175}$

⇒ R= 13.2 N

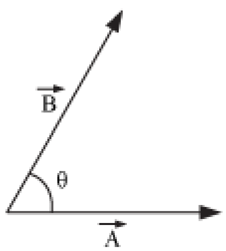
$\tan\alpha = \dfrac{B\sin\theta}{A+B\cos\theta}$

$\Rightarrow \tan\alpha = \dfrac{15\sin120°}{10+15\cos120°}$

$$\Rightarrow \tan\ \alpha = \frac{15\times\frac{\sqrt{3}}{2}}{10+15\times\left(-\frac{1}{2}\right)} = \frac{15\times0.87}{10-7.5}$$
$$= \frac{13.05}{2.5} = 5.22$$
$$\Rightarrow \tan\ \alpha = 5.22$$
$$\Rightarrow \alpha = \tan^{-1}(5.22)$$
$$= \alpha = 79.15^\circ$$

- **Scalar Product**Scalar product of two vectors $\vec{A}$ and $\vec{B}$ is given by

  $\vec{A}\cdot\vec{B} = AB\cos\theta$

- It is also known as dot product.
- The result of the scalar product of two vectors is a scalar quantity.
- When two vectors are parallel, θ = 0°, cos 0° = 1

$$\therefore\ \vec{A}\,.\,\vec{B} = AB\cos\ 0^\circ = AB$$

- ∴ A→.B→=ABcos 0°=AB
- For unit vectors, $\hat{i}\cdot\hat{i} = |\hat{i}||\hat{i}|\cos 0°$

  $= 1\times1\times1 = 1$

  Similarly, $\hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

- When two vectors are perpendicular, θ = 90°, cos 90° = 0

$\therefore \vec{A} \cdot \vec{B} = AB\cos 90° = 0$

It means the dot product of two perpendicular vectors is zero.

- For unit vectors, $\hat{i} \cdot \hat{j} = |\hat{i}||\hat{j}|\cos 90°$

  OR

  $\hat{i} \cdot \hat{j} = 1 \times 1 \times 0 = 0$

  Similarly, $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

Properties of Scalar Product of two vectors

- Scalar product of two vectors is commutative, i.e.,

  $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

- Scalar product is distributive, i.e.,

  $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

- Scalar product of a vector with itself gives the square of its magnitude, i.e.,

  $\vec{A} \cdot \vec{A} = A^2$

Dot Product in Cartesian Coordinates

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

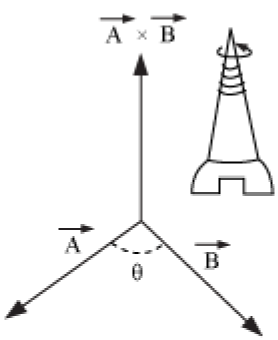
$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\begin{aligned}\vec{A}\cdot\vec{B} = {} & A_x\hat{i}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) \\ & + A_y\hat{j}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) \\ & + A_z\hat{k}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) \\ = {} & A_xB_x(\hat{i}\cdot\hat{i}) + A_xB_y(\hat{i}\cdot\hat{j}) + A_xB_z(\hat{i}\cdot\hat{k}) + \\ & A_yB_x(\hat{j}\cdot\hat{i}) + A_yB_y(\hat{j}\cdot\hat{j}) + A_yB_z(\hat{j}\cdot\hat{k}) + A_zB_x \\ & (\hat{k}\cdot\hat{i}) + A_zB_y(\hat{k}\cdot\hat{j}) + A_zB_z(\hat{k}\cdot\hat{k}) \\ = {} & A_xB_x(1) + A_xB_y(0) + A_xB_z(0) + A_yB_x(0) \\ & + A_yB_y(1) + A_yB_z(0) + A_zB_x(0) + A_zB_y(0) + A_zB_z(1)\end{aligned}$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Vector Product

- The magnitude of the vector product of two vectors $\vec{A}$ and $\vec{B}$ is defined as the product of the magnitude of the vectors $\vec{A}$ and $\vec{B}$ and sine of the smaller angle between them.



$\vec{A} \times \vec{B} = AB\sin\theta\hat{n}$ ... (i)

Here, $\hat{n}$ is the unit vector perpendicular to both $\vec{A}$ and $\vec{B}$.

- The cross product of two vectors $\vec{A}$ and $\vec{B}$ is a vector, which is at right angles to both $\vec{A}$ and $\vec{B}$ and points in the direction in which a right-handed screw will advance.

- The vector product of two like vectors is zero.

  Example:

$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = (1)(1)\sin 0°(\hat{n}) = 0$

- $\hat{i} \times \hat{j} = (1)(1)\sin 90°(\hat{k}) = \hat{k}$

  Here, $\hat{k}$ is the unit vector perpendicular to the plane of $\hat{i}$ and $\hat{j}$ and is in the direction in which a right-handed screw will move, when rotated from $\hat{i}$ to $\hat{j}$.

  Also, $\hat{j} \times \hat{i} = (1)(1)\sin 90°(-\hat{k}) = -\hat{k}$

  Similarly,

  $\hat{j} \times \hat{k} = -\hat{k} \times \hat{j} = \hat{i}$ and $\hat{k} \times \hat{i} = -\hat{i} \times \hat{k} = \hat{j}$

- Let $\overrightarrow{A}$ and $\overrightarrow{B}$ be two vectors.

  $\vec{A} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

  $\vec{B} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

  Then,

  $$\vec{A} \times \vec{B} = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) \times (x_2\hat{i} + y_2\hat{j} + z_2\hat{k})$$
  $$\vec{A} \times \vec{B} = (y_1z_2 - z_1y_2)\hat{i} + (z_1x_2 - x_1z_2)\hat{j} + (x_1y_2 - y_1x_2)\hat{k}$$
  $$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

Properties of Vector Product

- The cross product of a vector with itself is a null vector.

  $\vec{A} \times \vec{A} = (A)(A)\sin 0°\hat{n} = \vec{0}$

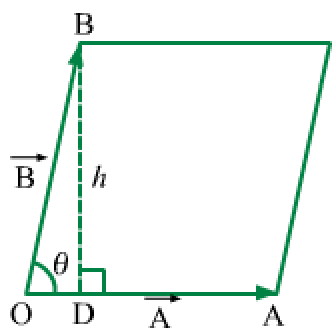
- The cross product of two vectors does not obey commutative law.

$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

- The cross product of vectors obeys the distributive law.

$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

If the vectors $\vec{A}$ and $\vec{B}$ represent the two adjacent sides of a parallelogram, the magnitude of cross product of $\vec{A}$ and $\vec{B}$ will represent the area of the parallelogram.



- Area of parallelogram ABCD = $\left|\vec{A} \times \vec{B}\right|$