

Short Notes for Thermodynamics

Symbol/Formula	Parameter
M	Molar mass (M/μ)
m	Mass (M)
$n = \frac{m}{M}$	Number of moles (μ)
E	Energy or general extensive property
$e = \frac{E}{m}$	Specific molar energy (energy per unit mass) or general extensive property per unit mass
$\bar{e} = \frac{E}{n} = eM$	Specific energy (energy per unit mole) or general extensive property per unit mole
P	Pressure ($ML^{-1}T^{-2}$)
V	Volume (L^3); Specific volume or volume per unit mass, v (L^3M^{-1}) and the volume per unit mole $\bar{v}$ ($L^3\mu^{-1}$)
T	Temperature (Θ)
ρ	Density (ML^{-3}); $\rho = 1/v$.
x	Quality
U	Thermodynamic internal energy (ML^2T^{-2}); Internal energy per unit mass, u (L^2T^{-2}), and the internal energy per unit mole, $\bar{u}$ ($ML^2T^{-2}\mu^{-1}$)
$H = U + PV$	Thermodynamic enthalpy (ML^2T^{-2}); Enthalpy per unit mass, $h = u + Pv$ (dimensions: L^2T^{-2}) and the internal energy per unit mole $\bar{h}$ ($ML^2T^{-2}\mu^{-1}$)
S	Entropy ($ML^2T^{-2}\Theta^{-1}$); Entropy per unit mass, s ($L^2T^{-2}\Theta^{-1}$) and the internal energy per unit mole $\bar{s}$ ($ML^2T^{-2}\Theta^{-1}\mu^{-1}$)
W	Work (ML^2T^{-2})
Q	Heat transfer (ML^2T^{-2})
$\dot{W}_u$:	The useful work rate or mechanical power (ML^2T^{-3})
$\dot{m}$:	The mass flow rate (MT^{-1})

$\frac{\bar{V}^2}{2}$:	The kinetic energy per unit mass (L^2T^{-2})
gz :	The potential energy per unit mass (L^2T^{-2})
E_{tot} :	The total energy = $m(u + \frac{\bar{V}^2}{2} + gz)$ (ML^2T^{-2})
$\dot{Q}$:	The heat transfer rate (ML^2T^{-3})
$\frac{dE_{cv}}{dt}$:	The rate of change of energy for the control volume. (ml^2t^{-3})
M	Molar mass (M/μ)
m	Mass (M)
$n = \frac{m}{M}$	Number of moles (μ)
E	Energy or general extensive property
$e = \frac{E}{m}$	Specific molar energy (energy per unit mass) or general extensive property per unit mass
$\bar{e} = \frac{E}{n} = eM$	Specific energy (energy per unit mole) or general extensive property per unit mole
P	Pressure ($ML^{-1}T^{-2}$)
V	Volume (L^3); Specific volume or volume per unit mass, v (L^3M^{-1}) and the volume per unit mole $\bar{v}$ ($L^3\mu^{-1}$)
T	Temperature (Θ)
ρ	Density (ML^{-3}); $\rho = 1/v$.
x	Quality
U	Thermodynamic internal energy (ML^2T^{-2}); Internal energy per unit mass, u (L^2T^{-2}), and the internal energy per unit mole, $\bar{u}$ ($ML^2T^{-2}\mu^{-1}$)
$H = U + PV$	Thermodynamic enthalpy (ML^2T^{-2}); we also have the enthalpy per unit mass, $h = u + Pv$ (dimensions: L^2T^{-2}) and the internal energy per unit mole $\bar{h}$ ($ML^2T^{-2}\mu^{-1}$)
S	Entropy ($ML^2T^{-2}\Theta^{-1}$); Entropy per unit mass, s ($L^2T^{-2}\Theta^{-1}$) and the internal energy per unit mole $\bar{s}$ ($ML^2T^{-2}\Theta^{-1}\mu^{-1}$)

W	Work (ML^2T^{-2})
Q	Heat transfer (ML^2T^{-2})
$\dot{W}_u$:	The useful work rate or mechanical power (ML^2T^{-3})
$\dot{m}$:	The mass flow rate (MT^{-1})
$\frac{\vec{V}^2}{2}$:	The kinetic energy per unit mass (L^2T^{-2})
gz:	The potential energy per unit mass (L^2T^{-2})
E_{tot} :	The total energy = $m(u + \frac{\vec{V}^2}{2} + gz)$ (ML^2T^{-2})
$\dot{Q}$:	The heat transfer rate (ML^2T^{-3})
$\frac{dE_{cv}}{dt}$:	The rate of change of energy for the control volume. (ml^2t^{-3})
M	Molar mass (M/μ)
m	Mass (M)
$n = \frac{m}{M}$	Number of moles (μ)
E	Energy or general extensive property
$e = \frac{E}{m}$	Specific molar energy (energy per unit mass) or general extensive property per unit mass
$\bar{e} = \frac{E}{n} = eM$	Specific energy (energy per unit mole) or general extensive property per unit mole
P	Pressure ($ML^{-1}T^{-2}$)
V	Volume (L^3); we also have the specific volume or volume per unit mass, v (L^3M^{-1}) and the volume per unit mole $\bar{v}$ ($L^3\mu^{-1}$)
T	Temperature (Θ)
ρ	Density (ML^{-3}); $\rho = 1/v$.
x	Quality
U	Thermodynamic internal energy (ML^2T^{-2}); we also have the internal energy per unit mass, u (L^2T^{-2}), and the internal energy per unit mole, $\bar{u}$ ($ML^2T^{-2}\mu^{-1}$)
$H = U + PV$	Thermodynamic enthalpy (ML^2T^{-2}); we also have the enthalpy per unit mass, $h = u + Pv$ (dimensions: L^2T^{-2}) and the internal energy per unit mole $\bar{h}$ ($ML^2T^{-2}\mu^{-1}$)

S	Entropy ($\text{ML}^2\text{T}^{-2}\Theta^{-1}$); we also have the entropy per unit mass, s ($\text{L}^2\text{T}^{-2}\Theta^{-1}$) and the internal energy per unit mole $\bar{s}$ ($\text{ML}^2\text{T}^{-2}\Theta^{-1}\mu^{-1}$)
W	Work (ML^2T^{-2})
Q	Heat transfer (ML^2T^{-2})
$\dot{W}_u$:	The useful work rate or mechanical power (ML^2T^{-3})
$\dot{m}$:	The mass flow rate (MT^{-1})
$\frac{\vec{V}^2}{2}$:	The kinetic energy per unit mass (L^2T^{-2})
gz:	The potential energy per unit mass (L^2T^{-2})
E_{tot} :	The total energy = $m(u + \frac{\vec{V}^2}{2} + gz)$ (ML^2T^{-2})
$\dot{Q}$:	The heat transfer rate (ML^2T^{-3})
$\frac{dE_{\text{cv}}}{dt}$:	The rate of change of energy for the control volume. (ml^2t^{-3})

Unit conversion factors

For metric units

- **Basic:**
 - $1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$;
 - $1 \text{ J} = 1 \text{ N} \cdot \text{m}$;
 - $1 \text{ W} = 1 \text{ J/s}$;
 - $1 \text{ Pa} = 1 \text{ N/m}^2$.
- **Others:**
 - $1 \text{ kPa} \cdot \text{m}^3 = 1 \text{ kJ}$;
 - $T(\text{K}) = T(^{\circ}\text{C}) + 273.15$;
 - $1 \text{ L (liter)} = 0.001 \text{ m}^3$;
 - $1 \text{ m}^2/\text{s}^2 = 1 \text{ J/kg}$.
- **Prefixes (and abbreviations):**
 - nano(n) – 10^{-9} ;
 - micro(μ) – 10^{-6} ;
 - milli(m) – 10^{-3} ;
 - kilo(k) – 10^3 ;
 - mega(M) – 10^6 ;
 - giga(G) – 10^9 .
 - A metric ton (European word: tonne) is 1000 kg.

For engineering units

- **Energy:**
 - $1 \text{ Btu} = 5.40395 \text{ psia} \cdot \text{ft}^3 = 778.169 \text{ ft} \cdot \text{lb}_f = (1 \text{ kWh})/3412.14 = (1 \text{ hp} \cdot \text{h})/2544.5 = 25,037 \text{ lb}_m \cdot \text{ft}^2/\text{s}^2$.
- **Pressure:**
 - $1 \text{ psia} = 1 \text{ lb}_f/\text{in}^2 = 144 \text{ psfa} = 144 \text{ lb}_f/\text{ft}^2$.
- **Others:**
 - $T(\text{R}) = T(^{\circ}\text{F}) + 459.67$;
 - $1 \text{ lb}_f = 32.174 \text{ lb}_m \cdot \text{ft}/\text{s}^2$;
 - $1 \text{ ton of refrigeration} = 200 \text{ Btu}/\text{min}$.

Concepts & Definitions

	Formula	Units
Pressure	$P = \frac{F}{A}$	Pa
• Units	$1 \text{ Pa} = 1 \text{ N} / \text{m}^2$ $1 \text{ bar} = 10^5 \text{ Pa} = 0.1 \text{ Mpa}$ $1 \text{ atm} = 101325 \text{ Pa}$	
Specific Volume	$v = \frac{V}{m}$	m^3 / kg
Density	$\rho = \frac{m}{V} \rightarrow \rho = \frac{1}{v}$	kg / m^3
Static Pressure Variation	$\Delta P = \rho gh \quad \uparrow = - , \downarrow = +$	Pa
Absolute Temperature	$T(\text{K}) = T(^{\circ}\text{C}) + 273.15$	

Properties of a Pure Substance

	Formula	Units
Quality	$x = \frac{m_{\text{vapor}}}{m_{\text{tot}}} \text{ (vapour mass fraction)}$ $1 - x = \frac{m_{\text{liquid}}}{m_{\text{tot}}} \text{ (Liquid mass fraction)}$	
Specific Volume	$v = v_f + xv_{fg}$	m^3 / kg
Average Specific Volume	$v = (1 - x)v_f + xv_g \text{ (only two phase mixture)}$	m^3 / kg
Ideal -gas law	$P \ll P_c \quad T \ll T_c \quad Z = 1$	
• Equations	$Pv = RT \quad PV = mRT = n\bar{R}T$	

• Universal Gas Constant	$\bar{R} = 8.3145$	$\text{kJ} / \text{kmol K}$
• Gas Constant	$R = \frac{\bar{R}}{M}$ $M = \text{molecular mass}$	$\text{kJ} / \text{kg K}$
Compressibility Factor Z	$Pv = ZRT$	
Reduced Properties	$P_r = \frac{P}{P_c}$, $T_r = \frac{T}{T_c}$	

Work & Heat

	Formula	Units
Displacement Work	$W = \int_1^2 F dx = \int_1^2 P dV$	J
Integration	$W = \int_1^2 P dV = P(V_2 - V_1)$	J
Specific Work	$w = \frac{W}{m}$ (work per unit mass)	J / kg
Power (rate of work)	$\dot{W} = F\bar{V} = P\dot{V} = T\omega$	W
• Velocity	$\bar{V} = r\omega$	rad / s
• Torque	$T = Fr$	Nm
Polytropic Process ($n \neq 1$)	$PV^n = \text{Const} = P_1V_1^n = P_2V_2^n$ $Pv^n = C$	
• Polytropic Exponent	$n = \frac{\ln\left(\frac{P_2}{P_1}\right)}{\ln\left(\frac{V_1}{V_2}\right)}$	
• $n=1$	$PV = \text{Const} = P_1V_1 = P_2V_2$	
Polytropic Process Work	$W_2 = \frac{1}{1-n} (P_2V_2 - P_1V_1)$ $n \neq 1$	J
• $n=1$	$W_2 = P_2V_2 \ln\left(\frac{V_2}{V_1}\right)$	J
Adiabatic Process	$Q = 0$	
Conduction Heat Transfer	$\dot{Q} = -kA \frac{dT}{dx}$, $k = \text{conductivity}$	W
Convection Heat Transfer	$\dot{Q} = hA\Delta T$, $h = \text{convection coefficient}$	W
Radiation Heat Transfer	$\dot{Q} = \varepsilon\sigma A(T_s^4 - T_{amb}^4)$	W

Terminology:

Q_1 = heat

Q_2 = heat transferred during the process between state 1 and state 2

Q = rate of heat transfer

W = work

${}_1W_2$ = work done during the change from state 1 to state 2

$\dot{W}$ = rate of work = Power. $1 \text{ W} = 1 \text{ J/s}$

The First Law of Thermodynamics

	Formula	Units
Total Energy	$E = U + KE + PE \rightarrow dE = dU + d(KE) + d(PE)$	J
Energy	$dE = \delta Q - \delta W \rightarrow E_2 - E_1 = {}_1Q_2 - {}_1W_2$	J
Kinetic Energy	$KE = 0.5m\bar{V}^2$	J
Potential Energy	$PE = mgZ \rightarrow PE_2 - PE_1 = mg(Z_2 - Z_1)$	J
Internal Energy	$U = U_{liq} + U_{vap} \rightarrow mu = m_{liq}u_f + m_{vap}u_g$	
Specific Internal Energy of Saturated Steam (two-phase mass average)	$u = (1-x)u_f + xu_g$ $u = u_f + xu_{fg}$	kJ / kg
Total Energy	$U_2 - U_1 + \frac{m(\bar{V}_2^2 - \bar{V}_1^2)}{2} + mg(Z_2 - Z_1) = {}_1Q_2 - {}_1W_2$	J
Specific Energy	$e = u + 0.5\bar{V}^2 + gZ$	
Enthalpy	$H = U + PV$	
Specific Enthalpy	$h = u + Pv$	kJ / kg
For Ideal Gasses	$Pv = RT$ and $u = f(T)$	
• Enthalpy	$h = u + Pv = u + RT$	
• R Constant	$u = f(T) \rightarrow h = f(T)$	
Specific Enthalpy for Saturation State (two-phase mass average)	$h = (1-x)h_f + xh_g$ $h = h_f + xh_{fg}$	kJ / kg
Specific Heat at Constant Volume	$C_v = \frac{1}{m} \left(\frac{\delta Q}{\delta T} \right)_v = \frac{1}{m} \left(\frac{\delta U}{\delta T} \right)_v = \left(\frac{\delta u}{\delta T} \right)_v$ $\rightarrow (u_e - u_i) = C_v(T_e - T_i)$	
Specific Heat at Constant Pressure	$C_p = \frac{1}{m} \left(\frac{\delta Q}{\delta T} \right)_p = \frac{1}{m} \left(\frac{\delta H}{\delta T} \right)_p = \left(\frac{\delta h}{\delta T} \right)_p$ $\rightarrow (h_e - h_i) = C_p(T_e - T_i)$	
Solids & Liquids	Incompressible, so $v = \text{constant}$ $C = C_c = C_p$ (Tables A.3 & A.4) $u_2 - u_1 = C(T_2 - T_1)$ $h_2 - h_1 = u_2 - u_1 + v(P_2 - P_1)$	

Ideal Gas	$h = u + Pv = u + RT$ $u_2 - u_1 \cong C_v(T_2 - T_1)$ $h_2 - h_1 \cong C_p(T_2 - T_1)$
Energy Rate	$\dot{E} = \dot{Q} - \dot{W} \quad (\text{rate} = +\text{in} - \text{out})$ $\rightarrow E_2 - E_1 = {}_1Q_2 - {}_1W_2 \quad (\text{change} = +\text{in} - \text{out})$

First-Law Analysis for Control Volume

	Formula	Units
Volume Flow Rate	$\dot{V} = \int \vec{V} dA = A\bar{V}$ (using average velocity)	
Mass Flow Rate	$\dot{m} = \int \rho \vec{V} dA = \rho A\bar{V} = A \frac{\bar{V}}{v}$ (using average values)	kg / s
Power	$\dot{W} = \dot{m} C_p \Delta T$ $\dot{W} = \dot{m} C_v \Delta T$ $\dot{m} = \frac{\dot{V}}{v}$	W
Flow Work Rate	$\dot{W}_{flow} = P\dot{V} = \dot{m}Pv$	
Flow Direction	From higher P to lower P unless significant KE or PE	
• Total Enthalpy	$h_{tot} = h + \frac{1}{2}\bar{V}^2 + gZ$	
Instantaneous Process		
• Continuity Equation	$\dot{m}_{C.V.} = \sum \dot{m}_i - \sum \dot{m}_e$	
• Energy Equation	$\dot{E}_{C.V.} = \dot{Q}_{C.V.} - \dot{W}_{C.V.} + \sum \dot{m}_i h_{toti} - \sum \dot{m}_e h_{tote} \rightarrow \text{First Law}$ $\rightarrow \dot{Q} + \sum \dot{m}_i (h_i + \frac{1}{2}\bar{V}^2 + gZ_i) = \frac{dE}{dt} + \sum \dot{m}_e (h_e + \frac{1}{2}\bar{V}^2 + gZ_e) - \dot{W}$	
Steady State Process A steady-state has no storage effects, with all properties constant with time		
• No Storage	$\dot{m}_{C.V.} = 0, \quad \dot{E}_{C.V.} = 0$	
• Continuity Equation	$\sum \dot{m}_i = \sum \dot{m}_e$ (in = out)	
• Energy Equation	$\dot{Q}_{C.V.} + \sum \dot{m}_i h_{toti} = \dot{W}_{C.V.} + \sum \dot{m}_e h_{tote}$ (in = out) $\rightarrow$ First Law $\rightarrow \dot{Q} + \sum \dot{m}_i (h_i + \frac{1}{2}\bar{V}^2 + gZ_i) = \dot{W} + \sum \dot{m}_e (h_e + \frac{1}{2}\bar{V}^2 + gZ_e)$	
• Specific Heat Transfer	$q = \frac{\dot{Q}_{C.V.}}{\dot{m}}$	kJ / kg
• Specific Work	$w = \frac{\dot{W}_{C.V.}}{\dot{m}}$	kJ / kg
• SS Single Flow Eq.	$q + h_{toti} = w + h_{tote}$ (in = out)	
Transient Process Change in mass (storage) such as filling or emptying of a container.		

• Continuity Equation	$m_2 - m_1 = \sum m_i - \sum m_e$
• Energy Equation	$E_2 - E_1 = Q_{C.V.} - W_{C.V.} + \sum m_i h_{toti} - \sum m_e h_{tote}$ $E_2 - E_1 = m_2 \left(u_2 + \frac{1}{2} \bar{V}_2^2 + gZ_2 \right) - m_1 \left(u_1 + \frac{1}{2} \bar{V}_1^2 + gZ_1 \right)$
→	$Q_{C.V.} + \sum m_i h_{toti} = \sum m_e h_{tote} + \left[m_2 \left(u_2 + \frac{1}{2} \bar{V}_2^2 + gZ_2 \right) - m_1 \left(u_1 + \frac{1}{2} \bar{V}_1^2 + gZ_1 \right) \right]_{C.V.} - W_{C.V.}$

The Second Law of Thermodynamics

	Formula	Units
	All W , Q can also be rates $\dot{W}$, $\dot{Q}$	
Heat Engine	$W_{HE} = Q_H - Q_L$	
• Thermal efficiency	$\eta_{HE} = \frac{W_{HE}}{Q_H} = 1 - \frac{Q_L}{Q_H}$	
• Carnot Cycle	$\eta_{Thermal} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$	
• Real Heat Engine	$\eta_{HE} = \frac{W_{HE}}{Q_H} \leq \eta_{Carnot HE} = 1 - \frac{T_L}{T_H}$	
Heat Pump	$W_{HP} = Q_H - Q_L$	
• Coefficient of Performance	$\beta'_{HP} = \frac{Q_H}{W_{HP}} = \frac{Q_H}{Q_H - Q_L}$	
• Carnot Cycle	$\beta'_{HP} = \frac{Q_H}{Q_H - Q_L} = \frac{T_H}{T_H - T_L}$	
• Real Heat Pump	$\beta_{HP} = \frac{Q_H}{W_{HP}} \leq \beta_{Carnot HP} = \frac{T_H}{T_H - T_L}$	
Refrigerator	$W_{REF} = Q_H - Q_L$	
• Coefficient of Performance	$\beta_{REF} = \frac{Q_L}{W_{REF}} = \frac{Q_L}{Q_H - Q_L}$	
• Carnot Cycle	$\beta = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L}$	
• Real Refrigerator	$\beta_{REF} = \frac{Q_L}{W_{REF}} \leq \beta_{Carnot REF} = \frac{T_L}{T_H - T_L}$	
Absolute Temperature	$\frac{T_L}{T_H} = \frac{Q_L}{Q_H}$	

Entropy

	Formula	Units
Inequality of Clausius	$\oint \frac{\delta Q}{T} \leq 0$	
Entropy	$dS \equiv \left(\frac{\delta Q}{T} \right)_{rev}$	kJ / kgK
Change of Entropy	$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{rev}$	kJ / kgK
Specific Entropy	$s = (1-x)s_f + xs_g$ $s = s_f + xs_{fg}$	kJ / kgK
Entropy Change		
• Carnot Cycle	Isothermal Heat Transfer: $S_2 - S_1 = \frac{1}{T_H} \int_1^2 \delta Q = \frac{Q_2}{T_H}$ Reversible Adiabatic (Isentropic Process): $dS = \left(\frac{\delta Q}{T} \right)_{rev}$ Reversible Isothermal Process: $S_4 - S_3 = \int_3^4 \left(\frac{\delta Q}{T} \right)_{rev} = \frac{Q_4}{T_L}$ Reversible Adiabatic (Isentropic Process): Entropy decrease in process 3-4 = the entropy increase in process 1-2.	
• Reversible Heat-Transfer Process	$s_2 - s_1 = s_{fg} = \frac{1}{m} \int_1^2 \left(\frac{\delta Q}{T} \right)_{rev} = \frac{1}{mT} \int_1^2 \delta Q = \frac{Q_2}{T} = \frac{h_{fg}}{T}$	
Gibbs Equations	$Tds = du + Pdv$ $Tds = dh - vdP$	
Entropy Generation	$dS = \frac{\delta Q}{T} + \delta S_{gen}$ $\delta W_{irr} = PdV - T\delta S_{gen}$ $S_2 - S_1 = \int_1^2 dS = \int_1^2 \frac{\delta Q}{T} + S_{2,gen}$	
Entropy Balance Equation	$\Delta \text{Entropy} = +in - out + gen$	
Principle of the Increase of Entropy	$dS_{net} = dS_{c.m.} + dS_{surr} = \sum \delta S_{gen} \geq 0$	
Entropy Change		
• Solids & Liquids	$s_2 - s_1 = c \ln \frac{T_2}{T_1}$ Reversible Process: $ds_{gen} = 0$ Adiabatic Process: $dq = 0$	

- **Ideal Gas**

Constant Volume: $s_2 - s_1 = \int_1^2 C_{v0} \frac{dT}{T} + R \ln \frac{v_2}{v_1}$

Constant Pressure: $s_2 - s_1 = \int_1^2 C_{p0} \frac{dT}{T} - R \ln \frac{P_2}{P_1}$

Constant Specific Heat: $s_2 - s_1 = C_{v0} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$
 $s_2 - s_1 = C_{p0} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$

Standard Entropy

$$s_T^0 = \int_{T_0}^T \frac{C_{p0}}{T} dT \quad kJ / kgK$$

Change in Standard Entropy

$$s_2 - s_1 = (s_{T2}^0 - s_{T1}^0) - R \ln \frac{P_2}{P_1} \quad kJ / kgK$$

Ideal Gas Undergoing an Isentropic Process

$$s_2 - s_1 = 0 = C_{p0} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$\rightarrow \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{R/C_{p0}}$$

$$\text{but } \frac{R}{C_{p0}} = \frac{C_{p0} - C_{v0}}{C_{p0}} = \frac{k-1}{k},$$

$$k = \frac{C_{p0}}{C_{v0}} = \text{ratio of specific heats}$$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{k-1}, \quad \frac{P_2}{P_1} = \left(\frac{v_1}{v_2} \right)^k$$

Special case of polytropic process where $k = n$: $Pv^k = \text{const}$

Reversible Polytropic Process for Ideal Gas

$$PV^n = \text{const} = P_1 V_1^n = P_2 V_2^n$$

$$\rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^n, \quad \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{n-1/n} = \left(\frac{V_1}{V_2} \right)^{n-1}$$

- **Work**

$${}_1W_2 = \int_1^2 P dV = \text{const} \int_1^2 \frac{dV}{V^n} = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{mR(T_2 - T_1)}{1-n}$$

- **Values for n**

Isobaric process: $n = 0, \quad P = \text{const}$

Isothermal Process: $n = 1, \quad T = \text{const}$

Isentropic Process: $n = k, \quad s = \text{const}$

Isochronic Process: $n = \infty, \quad v = \text{const}$

Second-Law Analysis for Control Volume

	Formula	Units
2nd Law Expressed as a Change of Entropy	$\frac{dS_{c.m.}}{dt} = \sum \frac{\dot{Q}}{T} + \dot{S}_{gen}$	
Entropy Balance Equation	rate of change = +in - out + generation $\rightarrow \frac{dS_{C.V.}}{dt} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{C.V.}}{T} + \dot{S}_{gen}$ where $S_{C.V.} = \int \rho s dV = m_{c.v.} s = m_A s_A + m_B s_B + \dots$ and $\dot{S}_{gen} = \int \rho \dot{s}_{gen} dV = \dot{S}_{gen,A} + \dot{S}_{gen,B} + \dots$	
Steady State Process	$\frac{dS_{C.V.}}{dt} = 0$ $\rightarrow \sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \sum \frac{\dot{Q}_{C.V.}}{T} + \dot{S}_{gen}$	
• Continuity equation	$\dot{m}_i = \dot{m}_e = \dot{m} \quad \Rightarrow \dot{m}(s_e - s_i) = \sum \frac{\dot{Q}_{C.V.}}{T} + \dot{S}_{gen}$	
• Adiabatic process	$s_e = s_i + s_{gen} \geq s_i$	
Transient Process	$\frac{d}{dt}(ms)_{C.V.} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{C.V.}}{T} + \dot{S}_{gen}$ $\rightarrow (m_2 s_2 - m_1 s_1)_{C.V.} = \sum m_i s_i - \sum m_e s_e + \int_0^t \frac{\dot{Q}_{C.V.}}{T} dt + {}_1\dot{S}_{2gen}$	
Reversible Steady State Process		
If Process Reversible & Adiabatic	$s_e = s_i$ $h_e - h_i = \int_i^e v dP$ $w = (h_i - h_e) + \frac{\bar{V}_i^2 - \bar{V}_e^2}{2} + g(Z_i - Z_e)$ $= -\int_i^e v dP + \frac{\bar{V}_i^2 - \bar{V}_e^2}{2} + g(Z_i - Z_e)$	
If Process is Reversible and Isothermal	$\dot{m}(s_e - s_i) = \frac{1}{T} \sum \dot{Q}_{C.V.} = \frac{\dot{Q}_{C.V.}}{T}$ $\text{or } T(s_e - s_i) = \frac{\dot{Q}_{C.V.}}{\dot{m}} = q$ $\rightarrow T(s_e - s_i) = (h_e - h_i) - \int_i^e v dP$	

Incompressible Fluid	$v(P_e - P_i) + \frac{\bar{V}_e^2 - \bar{V}_i^2}{2} + g(Z_e - Z_i) = 0 \rightarrow \text{Bernoulli Eq.}$
Reversible Polytropic Process for Ideal Gas	$w = -\int_i^e v dP \quad \text{and} \quad Pv^n = \text{const} = C^n$ $w = -\int_i^e v dP = -C \int_i^e \frac{dP}{P^{1/n}}$ $= -\frac{n}{n-1} (P_e v_e - P_i v_i) = -\frac{nR}{n-1} (T_e - T_i)$
Isothermal Process (n=1)	$w = -\int_i^e v dP = -C \int_i^e \frac{dP}{P} = -P_i v_i \ln \frac{P_e}{P_i}$
Principle of the Increase of Entropy	$\frac{dS_{net}}{dt} = \frac{dS_{C.V.}}{dt} + \frac{dS_{sur}}{dt} = \sum \dot{S}_{gen} \geq 0$
Efficiency	
Turbine	$\eta = \frac{w_a}{w_s} = \frac{h_i - h_e}{h_i - h_{es}} \quad \text{Turbine work is out}$
Compressor (Pump)	$\eta = \frac{w_s}{w_a} = \frac{h_i - h_{es}}{h_i - h_e} \quad \text{Compressor work is in}$
Cooled Compressor	$\eta = \frac{w_T}{w}$
Nozzle	$\eta = \frac{\frac{1}{2} \bar{V}_e^2}{\frac{1}{2} \bar{V}_{es}^2} \quad \text{Kinetic energy is out}$

Note:

$$^{\circ}\text{F} = (^{\circ}\text{C} \times 9/5) + 32$$

$$^{\circ}\text{C} = (^{\circ}\text{F} - 32) \times (5/9)$$

$$^{\circ}\text{K} = ^{\circ}\text{C} + 273$$

$$Q = mC\Delta T \text{ thermal energy} = \text{mass} \times \text{specific heat} \times \text{change in } T$$

$$Q = mH_f \text{ thermal energy} = \text{mass} \times \text{heat of fusion}$$

$$Q = mH_v \text{ thermal energy} = \text{mass} \times \text{heat of vaporization}$$

$$\Delta L = \alpha L_i \Delta T \text{ change in length} = \text{coefficient of expansion} \times \text{initial length} \times \text{change in } T$$

$$\Delta V = \beta V_i \Delta T \text{ change in volume} = \text{coefficient of expansion} \times \text{initial volume} \times \text{change in } T$$

$$\Delta U = Q - W \text{ internal energy} = \text{heat energy} - \text{work}$$

Plausible Physical Situations		Name	State Variables	P	V	T	ΔE_{th}	Q	W_s	W
Insulated sleeve or rapid process	Add weight to or push down on piston	Adiabatic compression	$PV^\gamma = \text{Const}; TV^{\gamma-1} = \text{Const}$	Up	Down	Up	$nC_v\Delta T > 0$	0	$-nC_v\Delta T < 0$	$nC_v\Delta T > 0$
Insulated sleeve or rapid process	Remove weight from or pull up on piston	Adiabatic expansion	$PV^\gamma = \text{Const}; TV^{\gamma-1} = \text{Const}$	Down	Up	Down	$nC_v\Delta T < 0$	0	$-nC_v\Delta T > 0$	$nC_v\Delta T < 0$
Heat gas	Locked piston or rigid container	Isochoric	V fixed; $P \propto T$	Up	Fixed	Up	$nC_v\Delta T > 0$	$nC_v\Delta T > 0$	0	0
Cool gas	Locked piston or rigid container	Isochoric	V fixed; $P \propto T$	Down	Fixed	Down	$nC_v\Delta T < 0$	$nC_v\Delta T < 0$	0	0
Heat gas	Piston free to move, load unchanged	Isobaric expansion	P fixed; $V \propto T$	Fixed	Up	Up	$nC_v\Delta T > 0$	$nC_p\Delta T > 0$	$P\Delta V > 0$	$-P\Delta V < 0$
Cool gas	Piston free to move, load unchanged	Isobaric compression	P fixed; $V \propto T$	Fixed	Down	Down	$nC_v\Delta T < 0$	$nC_p\Delta T < 0$	$P\Delta V < 0$	$-P\Delta V > 0$
Immerse gas in large bath	Add weight to piston	Isothermal compression	T fixed at temperature of bath, $PV = \text{Const}$	Up	Down	Fixed	$nC_v\Delta T = 0$	$nRT \cdot \ln(V_f/V_i) < 0$	$nRT \cdot \ln(V_f/V_i) < 0$	$-nRT \cdot \ln(V_f/V_i) > 0$
Immerse gas in large bath	Remove weight from piston	Isothermal expansion	T fixed at temperature of bath, $PV = \text{Const}$	Down	Up	Fixed	$nC_v\Delta T = 0$	$nRT \cdot \ln(V_f/V_i) > 0$	$nRT \cdot \ln(V_f/V_i) > 0$	$-nRT \cdot \ln(V_f/V_i) < 0$
Unknown	Unknown	No Name	$PV/T = \text{Const}$	?	?	?	$nC_v\Delta T$	$\Delta E_{th} + W_s$	$\int PdV = \pm$ area under curve in PV diagram	$-\int PdV = \pm$ area under curve in PV diagram