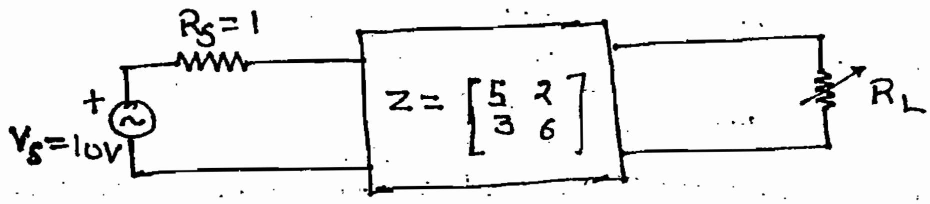
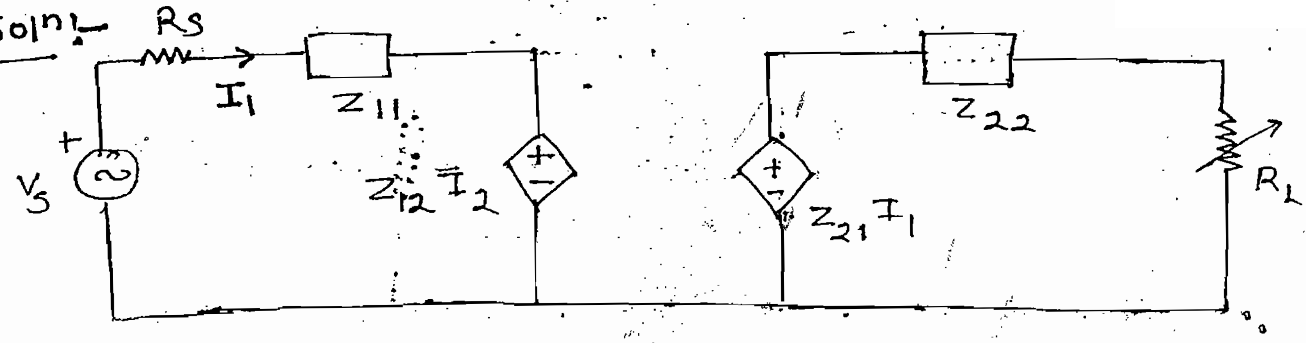
Lecture-12

Ques:- Find max. power dissipation in load resistor

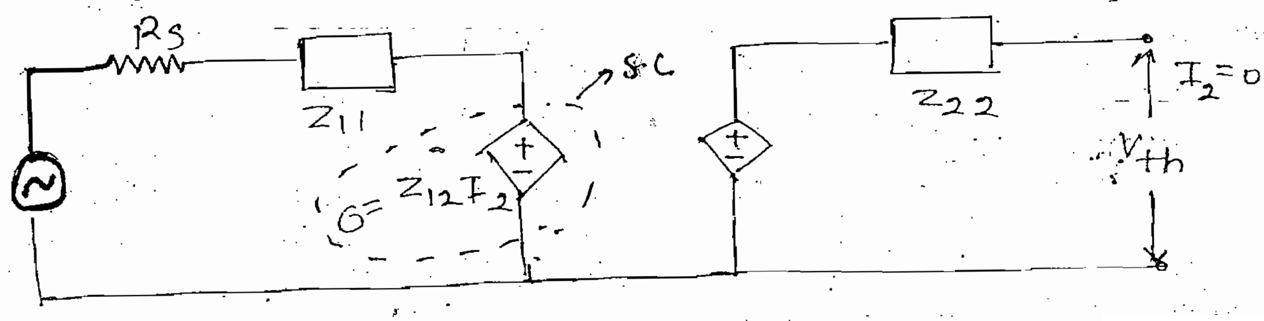


Soln:-



Case-(1):- V_{th}

Disconnect load resistor



$$I_1 = \frac{V_S}{R_S + Z_{11}} \quad \text{--- (I)}$$

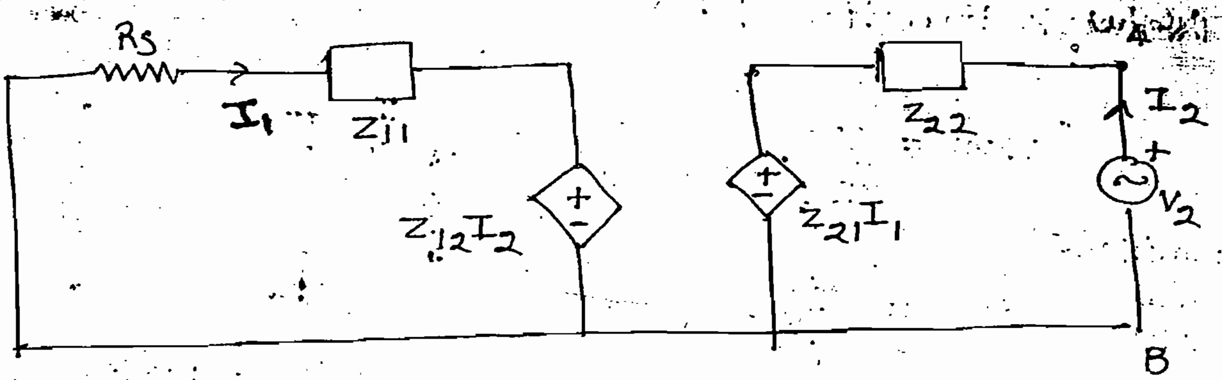
$$V_{th} = Z_{21} I_1 \quad \text{--- (II)}$$

Substitute eq-(I) in eq- (II)

$$V_{th} = Z_{21} \frac{V_S}{R_S + Z_{11}}$$

$$V_{th} = \frac{3 \times 10}{1+5} = 5$$

Case-(ii) (R_{th}) :—



$$Z_{th} = \frac{V_2}{I_2} \qquad I_2 = \frac{V_2 - Z_{21} I_1}{Z_{22}}$$

$$I_1 = -\frac{Z_{12} I_2}{R_S + Z_{11}} \quad \text{—(i)} \qquad I_2 Z_{22} = V_2 - Z_{21} I_1$$

$$V_2 = I_2 Z_{22} + Z_{21} I_1 \quad \text{—(ii)}$$

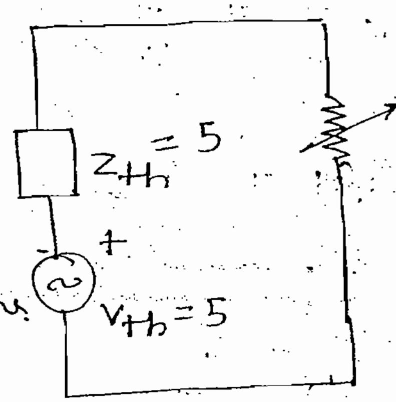
Substitute eq-(i) in eq-(ii).

$$V_2 = I_2 Z_{22} - \frac{Z_{12} Z_{21} I_2}{R_S + Z_{11}}$$

$$Z_{th} = \frac{V_2}{I_2} = Z_{22} - \frac{Z_{12} Z_{21}}{R_S + Z_{11}}$$

$$= 6 - \frac{2 \times 3}{1+5} \Rightarrow Z_{th} = 5$$

$$R_L = R_{th} = 5\Omega$$



$$P_{max} = \frac{V_{th}^2}{4 R_L}$$

$$= \frac{\cancel{25}\ 5}{4 \times \cancel{5}} = 1.25\,W, \text{ Ans}$$

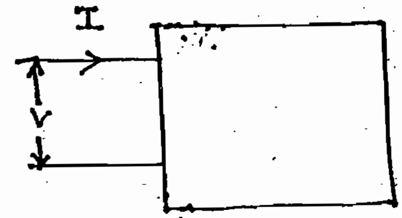
IW 10 P

Network Functions :-

Immittance (common Name)

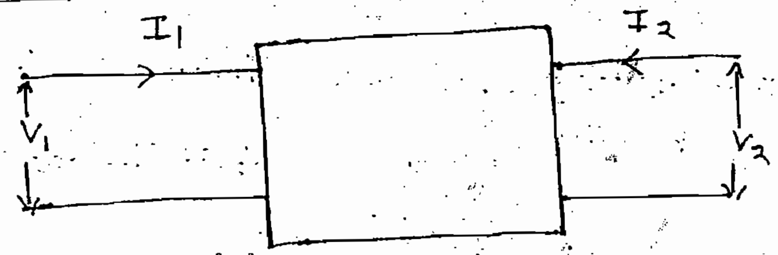
$$Z(s) = \frac{V(s)}{I(s)}$$

$$Y(s) = \frac{I(s)}{V(s)}$$



Single Port

Two Port :-



Driving Point I/p impedance

(I) $Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$ $\quad Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$

(II) $Y_{11}(s) = \frac{I_1(s)}{V_1(s)}$ $\quad Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$ → Driving pt admittance function

(III) $Z_{12}(s) = \frac{V_1(s)}{\cancel{V_2}(s)}$ $I_2 \leftarrow$ $\quad Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$ → Transfer impedance ratio

(IV) $Y_{12}(s) = \frac{I_1(s)}{V_2(s)}$ $\quad Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$ → Transfer admittance ratio

(V) $G_{12} = \frac{V_1(s)}{V_2(s)}$ $\quad G_{21} = \frac{V_2(s)}{V_1(s)}$ → Transfer voltage ratio

(VI) $\alpha_{12} = \frac{I_1(s)}{I_2(s)}$ $\quad \alpha_{21} = \frac{I_2(s)}{I_1(s)}$ → Transfer current ratio

→ N/w parameter is calculated at pre-defined condition (either o.c or s.c)

eg:- $Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \rightarrow$ o.c $\quad Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \rightarrow$ s.c.

→ To find the N/w function no pre-defined condition is required

→ To represent the N/w simultaneously four parameter are required

eg:- To develop T- N/w Z_{11}, Z_{12}, Z_{21} & Z_{22} are required.

→ By using only one N/w function it is possible to represent the N/w.

Network Synthesis:—

→ In the N/w analysis for a given N/w it is possible to find either voltage response or current response or impedance function or admittance function

→ In a N/w synthesis, for a given function N/w is designed (It is reverse procedure of N/w analysis)

→ In a N/w synthesis it is possible to design the N/w for the following functions

(I) Single port → 1. $Z(s)$, 2. $Y(s)$ → Immittance — Driving Point Immittance func

(II) Two port → 1. $Z_{11}(s)$, $Z_{22}(s)$, $Y_{11}(s)$, $Y_{22}(s)$ → func

2. $Z_{12}(s)$, $Z_{21}(s)$, $Y_{12}(s)$, $Y_{21}(s)$, G_{12}, G_{21}, α_{12}, α_{21} → Transfer function

→ In the N/w synthesis for given function it is possible to design the following N/w

(I) Series (II) Parallel (III) Ladder

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(I): Series $\longrightarrow$ Foster-I form

(II) Parallel $\longrightarrow$ Foster-II form

} Partial fraction of expansion

(III) Ladder $\longrightarrow$ Cauer-I form / Cauer-II form } Continued fraction of expansion

Necessary condition to design a N/w:-

(I) F(S) should be ~~primary~~ positive real function (PRF)

$R \geq 0$, $L \geq 0$, $C \geq 0$ $\longrightarrow$ Positive Real

(II) If F(S) is PRF then $\frac{1}{F(S)}$ also PRF

(III) If $F_1(S)$ and $F_2(S)$ are PRF then

$$F(S) = F_1(S) + F_2(S)$$ are always PRF.

(iv) All the poles of the function should be present in the left half of the plane

(V) Imaginary poles and zeroes should be conjugate pairs

(VI) In the partial fraction of expansion residue should be positive real

(VII) Numerator and denominator polynomials should satisfy Hurwitz.

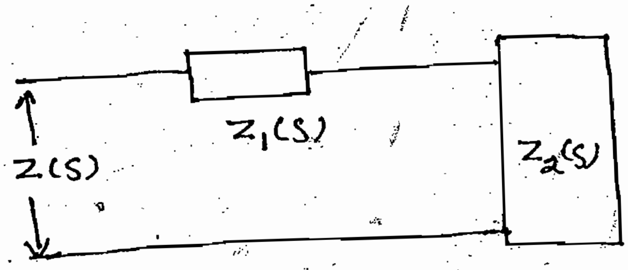
(VIII) The highest power of Numerator and denominator of polynomial should be differ at ~~by~~ atmost unity. This condition prohibits multiple pole and zero at infinity.

9 → The lowest power of Numerator and Denominator should be differ by atmost unity. This conditions prohibits multiple poles and zeroes at origin

10 → Total No. of poles of a function should be equal to total no. of zeroes.

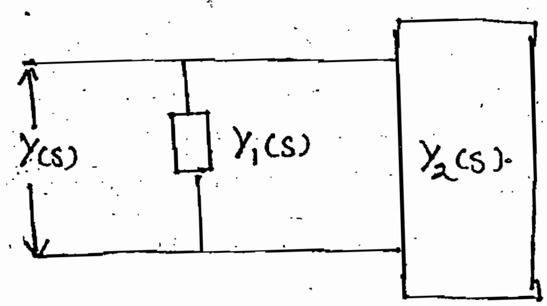
$$Z(s) = Z_1(s) + Z_2(s)$$

$$\Rightarrow Z_2(s) = Z(s) - Z_1(s)$$



$$Y(s) = Y_1(s) + Y_2(s)$$

$$\Rightarrow Y_2(s) = Y(s) - Y_1(s)$$



Removal of Pole at ∞ :—

$$Z(s) = \frac{b_{n+1}s^{n+1} + b_n s^n + \cdots + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_0}$$

Numerator Power > Denominator power
→ pole exist at ∞.

By long division

$$Z(s) = \frac{b_{n+1}s^{n+1}}{a_n s^n} + Z_2(s)$$

$$\left(H = \frac{b_{n+1}}{a_n}\right)$$

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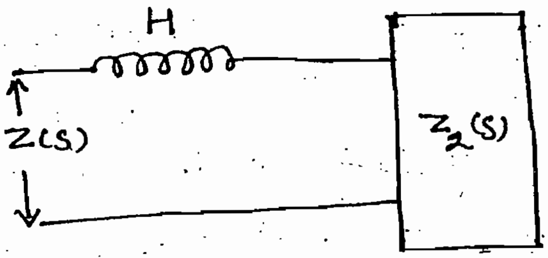
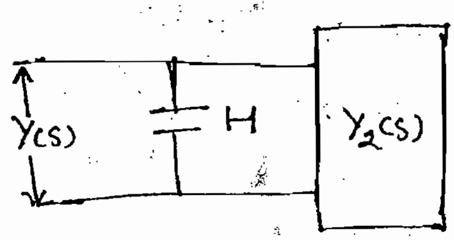
$$Z(s) = Hs + Z_2(s) \qquad X_L = Ls$$

$$Z_2(s) = Z(s) - Hs$$



$$Y(s) = \underset{\substack{\downarrow \\ B_C = sC}}{Hs} + Y_2(s)$$

$$Y_2(s) = Y(s) - Hs$$



Removal of pole at origin:-

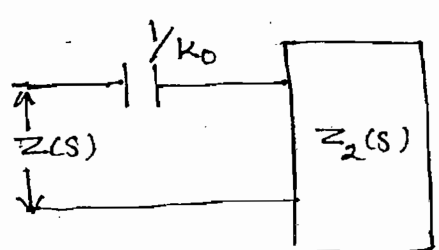
$$Z(s) = \frac{b_0 + \cdots + b_{n-1}s^{n-1} + b_n s^n}{s\,(a_0 + \cdots + a_{n-1}s^{n-1} + a_n s^n)}$$

By long division

$$Z(s) = \frac{b_0}{s a_0} + Z_2(s) \qquad \left(k_0 = \frac{b_0}{a_0}\right)$$

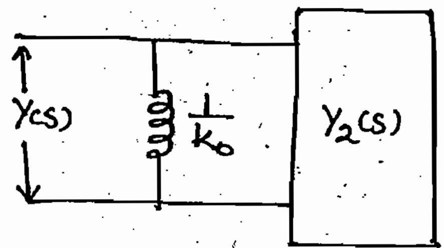
$$Z(s) = \frac{k_0}{s} + Z_2(s)$$

$$\Rightarrow Z_2(s) = Z(s) - \frac{k_0}{s} \qquad X_C = \frac{1}{Cs}$$



$$Y(s) = \frac{k_0}{s} + Y_2(s)$$

$$Y_2(s) = Y(s) - \frac{k_0}{s}$$



Removal of conjugate Pair of Poles: —

$$Z(s) = Z_1(s) + Z_2(s)$$

$$Z_2(s) = Z(s) - Z_1(s)$$

$$Z_1(s) \longrightarrow \text{Poles} \Rightarrow \pm j\omega$$

$$Z_1(s) = \frac{k_1}{s+j\omega} + \frac{k_2}{s-j\omega} \qquad (k_1 = k_2 = k)$$

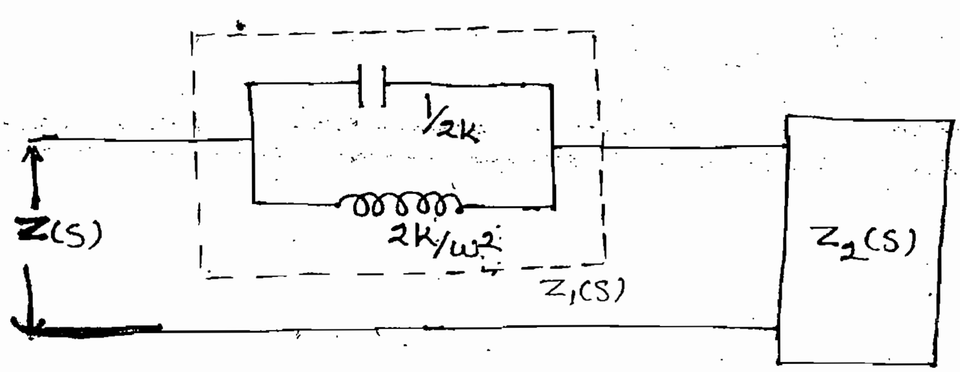
$$Z_1(s) = \frac{k}{s+j\omega} + \frac{k}{s-j\omega}$$

$$Z_1(s) = \frac{2ks}{s^2+\omega^2}$$

$$Z_1(s) = \frac{1}{\frac{s^2}{2ks} + \frac{\omega^2}{2ks}} = \frac{1}{Y_a + Y_b}$$

$B_C = sC$

$B_L = \frac{1}{Ls}$



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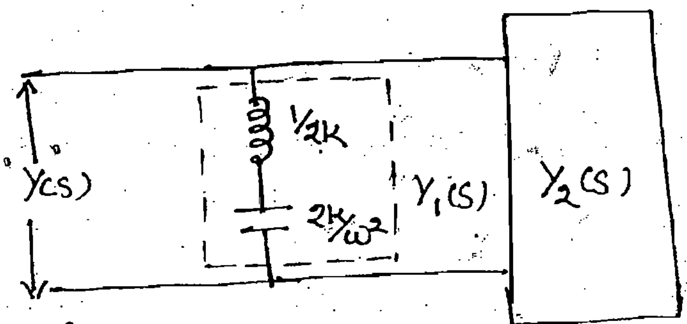
$$Y(s) = Y_1(s) + Y_2(s)$$

$$Y_2(s) = Y(s) - Y_1(s)$$

$$Y_1(s) = \frac{1}{\frac{s}{2K} + \frac{\omega^2}{2Ks}} = \frac{1}{Z_a + Z_b}$$

$X_L = Ls$

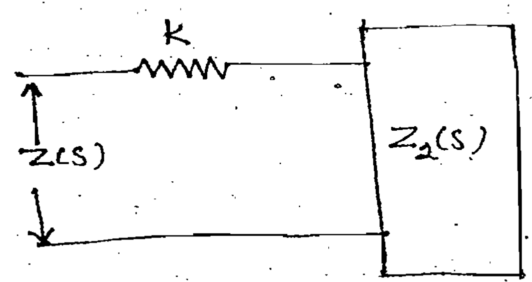
$X_C = \frac{1}{cs}$



Removal of constant:—

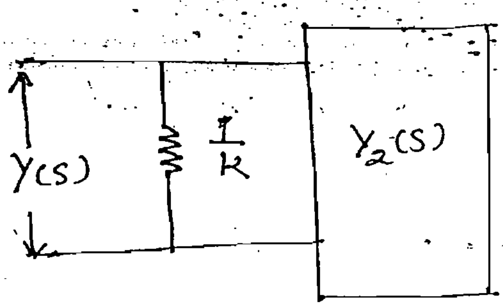
$$Z(s) = K + Z_2(s)$$

$$Z_2(s) = Z(s) - K$$



$$Y(s) = K + Y_2(s)$$

$$Y_2(s) = Y(s) - K$$



LC N/w Foster-I form :- (Series)

$$Z(S) = \frac{k_0}{S} + \sum_{i=1}^{n} \frac{2k_i S}{S^2+\omega_i^2} + HS$$

$$X_C = \frac{1}{CS}$$

$(X_L = LS)$