JCECE
ENGINEERING ENTRANCE EXAM.
SOLVED PAPER 2003

Physics

1. The internal resistance of a cell is the resistance of :
(a) electrolyte used in cell
(b) electrodes of the cell
(c) material used in cell
(d) vessel of the cell

2. A device for generating an alternating current of a desired frequency is known as :
(a) an oscillator (b) an amplifier
(c) a rectifier (d) none of these

3. The angle of minimum deviation for a thin prism with respect to air and when dipped in water will be : $\left({}_a\mu_g = \frac{3}{2}, {}_a\mu_w = \frac{4}{3}\right)$
(a) $\frac{1}{4}$ (b) $\frac{1}{8}$
(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

4. Escape velocity of a body when projected from the earth's surface is 11.2 km/s. If it is projected at an angle of 50° from the horizontal, the escape velocity is :
(a) 11.8 km/s (b) 16.5 km/s
(c) 11.2 km/s (d) 14.5 km/s

5. A diamagnetic substance is brought near a strong magnet, then it is :
(a) attracted by a magnet
(b) repelled by a magnet
(c) repelled by north pole and attracted by south pole
(d) attracted by north pole and repelled by south pole

6. The ratio of intensities of two waves is 9 : 16. If they interfere, the ratio of maximum to minimum intensity will be :
(a) 4 : 1 (b) 1 : 25
(c) 1 : 3 (d) 49 : 1

7. The sun emits a light with maximum wavelength 510 nm while another star emits a light with maximum wavelength of 350 nm. The ratio of surface temperature of sun and the star will be :
(a) 0.69 (b) 0.46
(c) 1.45 (d) 2.1

8. If the equation of motion of standing wave is $y = 0.3\sin(314t - 1.57x)$, then the velocity of standing wave is :
(a) 400 unit (b) 350 unit
(c) 209 unit (d) 200 unit

9. For driving current of 2 A for 6 min in a circuit, 1000 J of work is to be done. The emf of the source in the circuit is :
(a) 2.03 V (b) 2.54 V
(c) 1.25 V (d) 1.39 V

10. The moment of momentum for an electron in second orbit of hydrogen atom as per Bohr's model is :
(a) $\frac{h}{\pi}$ (b) $2\pi h$ (c) $\frac{2h}{\pi}$ (d) $\frac{\pi}{h}$

11. The latent heat of vaporisation of water is 2250 J/kg. If the work done in the process of vaporisation of 1 kg is 168 J, then increase in internal energy will be :
(a) 1904 J (b) 1984 J
(c) 3202 J (d) 2082 J

12. Huygen's wave theory of light could not explain :
(a) photoelectric effect
(b) interference
(c) diffraction
(d) polarization

13. In a triode valve, when the plate potential is increased from 200 V to 220 V and grid potential is decreased from –0.5 V to – 1.3 V, there is no change in plate current. The amplification factor of the triode is :
(a) 25 (b) 14
(c) 11 (d) 73

14. A bar magnet of magnetic moment 220 Am^2 is suspended in a magnetic field of intensity 0.25 N/Am. The couple required to deflect it through 30° is :
(a) 27.5 Nm (b) 20.25 Nm
(c) 47.63 Nm (d) 12 Nm

15. A plano-convex lens has refractive index 1.5 and radius of curvature 50 cm. What is focal length of lens ?
(a) 100 cm (b) 200 cm
(c) 178 cm (d) 150 cm

16. A source and an observer move away from each other, with a velocity of 20 m/s. If the apparent frequency heard by the observer is 1840 Hz, the actual frequency of the source is : (Velocity of sound in air = 340 m/s) :
(a) 2486 Hz (b) 2070 Hz
(c) 2134 Hz (d) 1872 Hz

17. The speed of wave in a medium is 650 m/s. If 3500 waves are passing through a point in the medium in 1.67 min, then its wavelength will be :
(a) 16.25 m (b) 14.29 m
(c) 18.57 m (d) 20.50 m

18. The body is projected at such angle that the horizontal range is three times the greatest height. The angle of projection is :
(a) 43°8′ (b) 25°8′
(c) 33°7′ (d) 53°1′

19. On the horizontal surface of a truck, a block of mass 1 kg is placed ($\mu = 0.6$) and truck is moving with acceleration 5 m/s^2. The frictional force acting on the block will be : ($g = 10\ m/s^2$)
(a) 6 N (b) 5.88 N
(c) 7N (d) 9N

20. Calculate the work done when a force $\vec{F} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ units acts on a body producing a displacement $\vec{s} = 2\hat{i} + 4\hat{j} + 3\hat{k}$ units :
(a) 1 unit (b) 20 unit
(c) 5 unit (d) zero

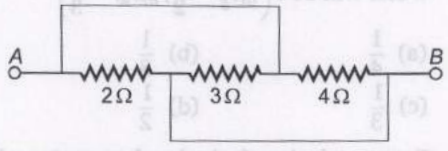
21. The efficiency of Carnot engine is 50% and temperature of sink is 500 K. If the temperature of source is kept constant and its efficiency is to be raised to 60%, then the required temperature of the sink will be :
(a) 600 K (b) 400 K
(c) 500 K (d) 100 K

22. What happens to the internal energy of a gas during isothermal expansion ?
(a) Internal energy will decrease
(b) Internal energy will increase
(c) Internal energy will become zero
(d) Internal energy will remain same

23. 100 g ice is mixed with 100 g of water at 100°C. What will be the final temperature of the mixture ?
(a) 10°C (b) 27°C
(c) 14°C (d) none of these

24. Black hole consists of :
(a) ozone layer
(b) super dense planetary material
(c) upper surface of atmosphere
(d) none of the above

25. Three resistors 2Ω, 3Ω and 4Ω are connected as shown in the given diagram. The equivalent resistance will be :



(a) $\frac{12}{13}$ (b) $\frac{11}{10}$ (c) 2 (d) $\frac{10}{11}$

26. If an electron jumps from first orbit to third orbit of hydrogen atom it will :
(a) not loose energy (b) release energy
(c) absorb energy (d) not gain energy

27. A body of mass 10 kg moving with velocity 10 m/s collides with a stationary body of mass 5 kg. After collision both bodies stick to each other, velocity of the joint body after collision is :
(a) $\frac{3}{10}$ m/s (b) 20 m/s
(c) $\frac{20}{3}$ m/s (d) 15 m/s

28. If a body starts from rest and travels 110 cm in the 9th second, then acceleration of the body is :
(a) 0.13 m/s^2 (b) 0.16 m/s^2
(c) 0.18 m/s^2 (d) 0.34 m/s^2

29. If the momentum of a particle is doubled, then its de-Broglie wavelength will :
(a) become four times
(b) become half
(c) become two times
(d) remain unchanged

30. Dimensions of torque are :
(a) $[M^2 L^2 T^{-2}]$ (b) $[ML^2 T^{-2}]$
(c) $[ML^2 T^{-1}]$ (d) $[ML^0 T^{-2}]$

31. Two vectors have magnitudes 3 and 5. If angle between them is 60°, then the dot product of two vectors will be :
(a) 7.5 (b) 6.5
(c) 8.4 (d) 7.9

32. The kinetic energy of 1 g molecule of a gas, at normal temperature and pressure is : ($R = 8.321$ J/mol-K)
(a) 1.2×10^2 J (b) 3.4×10^3 J
(c) 1.66×10^4 J (d) 2.97×10^4 J

33. If the coefficient of cubical expansion is x times of the coefficient of superficial expansion, then value of x is :
(a) 2.7 (b) 2
(c) 1.5 (d) 9.5

34. X-ray will not show the phenomenon of :
(a) interference
(b) deflection by electric field
(c) diffraction
(d) polarisation

35. Bernoulli's theorem is based on :
(a) conservation of mass, energy and momentum
(b) conservation of mass
(c) conservation of momentum
(d) conservation of energy

36. In bringing an electron towards another electron, the electrostatic potential energy of the system :
(a) decreases (b) increases
(c) remains same (d) becomes zero

37. The half-life of radium is 1600 yr. What is the mean life and disintegration constant of radium ?
(a) 2309 yr, $\frac{1}{2309}$/yr
(b) 3309 yr, $\frac{1}{3309}$/yr
(c) 1309 yr, $\frac{1}{1309}$/yr
(d) none of the above

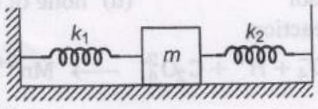
38. Energy obtained when 1mg mass is completely converted in to energy will be :
(a) 9×10^{13} J (b) 3×10^8 J
(c) 3×10^{15} J (d) 9×10^{15} J

39. A ray of light is incident on a surface of a plate of glass of refractive index 1.5 at polarising angle. The angle of refraction of the ray will be :
(a) 53.7° (b) 43.7°
(c) 33.7° (d) none of these

40. A closed organ pipe and an open organ pipe are tuned to the same fundamental frequency. The ratio of their lengths is :
(a) 1 : 2 (b) 4 : 1
(c) 1 : 4 (d) 2 : 1

41. A particle is moving along a circular path of radius 5m with uniform speed 5 m/s. What will be the average acceleration when the particle completes half revolution ?
(a) $\frac{10}{\pi}$ m/s^2 (b) 10 m/s^2
(c) 10π m/s^2 (d) Zero

42. A body of mass m is attached between two springs of force constants k_1 and k_2 as shown in figure. The other ends of the springs are fixed to firm supports. The frequency of the oscillation is :



(a) $n = \frac{1}{2\pi}\sqrt{\frac{k_1 - k_2}{m}}$
(b) $n = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{m}}$
(c) $n = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$
(d) none of the above

43. In a p-type semiconductor, germanium is doped with :
(a) boron (b) gallium
(c) aluminium (d) all of these

44. In a full wave rectifier circuit operating from 50 Hz mains frequency, what is the fundamental frequency in the ripple ?
(a) 50 Hz (b) 100 Hz
(c) 25 Hz (d) 70 Hz

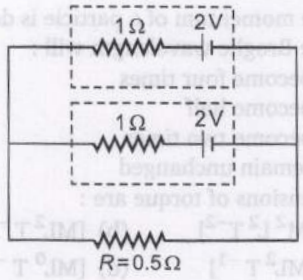
45. An ideal gas at 27° C is compressed adiabatically to $\frac{8}{27}$ of its original volume. The rise in temperature will be $\left(\gamma = \frac{5}{3}\right)$:

(a) 480°C (b) 275°C
(c) 450°C (d) 375°C

46. When you make ice cubes, entropy of water :
(a) does not change
(b) increases
(c) decreases
(d) may either increase or decrease depending on the process used

47. The focal length of the objective lens and eye-piece of an astronomical telescope are 2m and 0.05 m. Find the length of the telescope.
(a) 2.05 m (b) 1.16 m
(c) 1.05 m (d) 2.9 m

48. Two identical batteries, each of emf 2 V and internal resistance 1 Ω pass a current through external resistance $R = 0.5\Omega$. The maximum power that can be developed across R using these batteries is :



(a) 3.2W (b) 8.2W
(c) 4W (d) 2W

49. At what height from the earth's surface, the acceleration due to gravity will be half the value of g at surface ?
($R = 6400$ km)
(a) 6400 km (b) 8200 km
(c) 4800 km (d) 1600 km

50. In a semiconducting material the mobilities of electrons and holes are μ_e and μ_h respectively which of the following is true ?
(a) $\mu_e > \mu_h$ (b) $\mu_e < \mu_h$
(c) $\mu_e = \mu_h$ (d) $\mu_e < 0, \mu_h > 0$

Chemistry

1. Tyndall effect is shown by:
(a) solution (b) precipitate
(c) sol (d) none of these

2. In reaction,

$$MnO_4^- + H^+ + C_2O_4^{2-} \longrightarrow Mn^{2+} + H_2O + CO_2$$

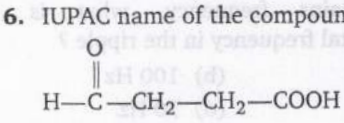
What is happening?
(a) Reduction of Mn (b) Reduction of $C_2O_4^{2-}$
(c) Oxidation of Mn (d) None of these

3. In which of the following ionic bond is present?
(a) BF_3 (b) CCl_4
(c) HCl (d) $BaCl_2$

4. Deficiency of vitamin E causes:
(a) scurvy (b) beri-beri
(c) sterility (d) xerophthalmia

5. The reaction of Lucas reagent is fast with:
(a) CH_3CH_2OH (b) $(CH_3)_3COH$
(c) $(CH_3)_2CHOH$ (d) $CH_3CH_2CH_2OH$

6. IUPAC name of the compound is:



(a) 3-oxo-butanoic acid
(b) 4-oxo-propanoic acid
(c) 3-formyl-propanoic acid
(d) none of the above

7. Tollen's reagent is:
(a) alkaline $CuSO_4$ solution
(b) ammoniacal cuproxide solution
(c) aqueous solution of sodium cupritartarate
(d) ammoniacal $AgNO_3$ solution

8. Epsom salt is:
(a) $MgSO_4 \cdot 2H_2O$ (b) $BaSO_4 \cdot 2H_2O$
(c) $MgSO_4 \cdot 7H_2O$ (d) $CaSO_4 \cdot 2H_2O$

9. Which of the following carbonate decompose most easily on heating?
(a) Rb_2CO_3 (b) K_2CO_3
(c) Na_2CO_3 (d) $MgCO_3$

10. In an isothermal expansion of an ideal gas:
(a) $W = 0$ (b) $\Delta E = 0$
(c) $q = 0$ (d) $\Delta V = 0$

11. The degree of hydrolysis of 0.01 M NH_4Cl is:
($K_h = 2.5 \times 10^{-9}$)
(a) 5×10^{-5} (b) 5×10^{-4}
(c) 5×10^{-3} (d) 5×10^{-7}

12. The number of gram equivalent of H_2SO_4 in 1000 mL 3M solution, is:
(a) 3 (b) 6
(c) 4 (d) 1.5

13. The radioactive isotope used in cancer therapy, is:
(a) I^{128} (b) Co^{60}
(c) Co^{59} (d) P^{32}

14. The expression of angular momentum of an electron in a Bohr's orbit is:
(a) $\frac{nh}{3\pi}$ (b) $\frac{nh}{2\pi}$
(c) $\frac{h}{4\pi}$ (d) $\sqrt{l(l+1)}\cdot\frac{h}{2\pi}$

15. Which of the following is called Hofmann's bromamide reaction?
(a) $CH_3CN + 2H_2O \longrightarrow CH_3COO + NH_3$
(b) $CH_3CN + 4H \xrightarrow{Na/EtOH} CH_3CH_2NH_2$
(c) $CH_3CONH_2 \xrightarrow{Br_2/NaOH} CH_3NH_2$
(d) $CH_2COCl + CH_3OH \longrightarrow CH_3COOCH_3 + HCl$

16. Which one among the following contains a phenolic —OH group?
(a) Oxalic acid (b) Formic acid
(c) Picric acid (d) Citric acid

17. In the reaction, product 'X' is :
$CH_3—C\equiv CH + H_2O \xrightarrow{H^+/Hg^{2+}} X$
(a) CH_3CH_2CHO
(b) CH_3CH_2COOH
(c) $CH_3—C(OH)=CHOH$
(d) CH_3COCH_3

18. Which is an amphoteric oxide?
(a) MgO (b) K_2O
(c) Al_2O_3 (d) CuO

19. The oxidation number of S in $H_2S_2O_8$ is:
(a) +6 (b) −6
(c) +4 (d) +8

20. Bleaching powder is obtained by treating chlorine with :
(a) $CaCO_3$ (b) $Ca(OH)_2$
(c) CaO (d) none of these

21. For reaction,
$PCl_3(g) + Cl_2(g) \rightleftharpoons PCl_5(g)$
the value of K_c at 250°C is 26. At the same temperature, the value of K_p is:
(a) 0.46 (b) 0.61
(c) 0.95 (d) 0.73

22. The volume of a gas is reduced to 1.0 L at 25°C and 1 atm. pressure. Its pressure at 35°C would be :
(a) 0.96 atm (b) 1.03 atm
(c) 2.04 atm (d) 3.08 atm

23. The strongest acid among the following is:
(a) CH_3COOH (b) $CH_2ClCOOH$
(c) CH_2FCOOH (d) CH_3CH_2COOH

24. The compressibility factor for an ideal gas is:
(a) < 1 (b) $= 1$
(c) > 1 (d) always 2

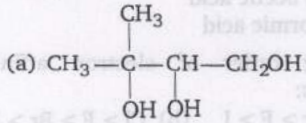
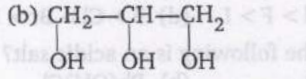
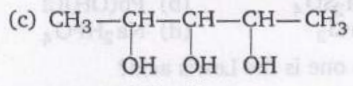
25. The standard reduction potential of Li^+, Ba^{2+}, Na^+ and Mg^{2+} are −3.05, −2.73, −2.71 and −2.37 volts respectively. Which one is the strongest reducing agent?
(a) Li (b) Na
(c) Mg (d) Ba

26. The unit of rate constant for Ist order reaction is :
(a) mol $L^{-1}s^{-1}$ (b) mol^{-1} L s^{-1}
(c) s^{-1} (d) mol^2 $L^{-2}s^{-1}$

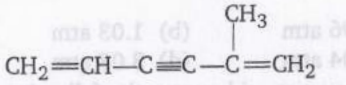
27. In which of the following, the dipole moment is zero?
(a) H_2O_2 (b) CO_2
(c) SO_2 (d) NH_3

28. The bond order of O_2^+ is:
(a) 2 (b) 2.5
(c) 1.5 (d) 0.5

29. Addition of $HgCl_2$ to $SnCl_2$ gives a black colour due to:
(a) oxidation of Sn
(b) reduction of $HgCl_2$
(c) formation of amalgum
(d) oxidation of Hg

30. Which one of the following is a trihydric alcohol containing only secondary hydroxyl group?
(a) 
(b) 
(c) 
(d) None of the above

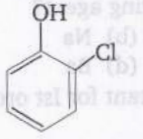
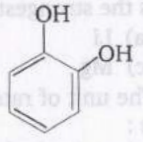
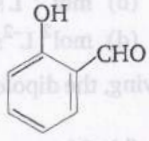
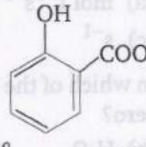
31. How many sigma and π-bonds are there in the following compound?



(a) 11 σ, 4π (b) 12σ, 3π
(c) 14σ, 4π (d) 15σ, 4π

32. In the nitration of benzene, the reactive species is:
(a) NO_2 (b) NO_2^+
(c) NO_2^- (d) NO^-

33. Reaction of phenol with CCl_4 and NaOH, followed by hydrolysis is likely to give:

(a) 
(b) 
(c) 
(d) 

34. In the reaction, $_{92}U^{238} \longrightarrow {}_{82}Pb^{206}$, the number of α and β-particles emitted are:
(a) 7α, 5β (b) 6α, 4β
(c) 4α, 3β (d) 8α, 6β

35. Which is the correct Gibb's Helmholtz equation?
(a) $\Delta H = \Delta G - T \cdot \Delta S$
(b) $\Delta S = \frac{1}{T}[\Delta G - \Delta H]$
(c) $\Delta S = \frac{1}{T}[\Delta H - \Delta G]$
(d) $-\Delta G = \Delta H - T \cdot \Delta S$

36. Vinegar is:
(a) 8-10% acetic acid
(b) 6-10% ethyl alcohol
(c) glacial acetic acid
(d) 10% formic acid

37. Decreasing order of electron affinity of halogens is:
(a) Cl > Br > F > I (b) Cl > F > Br > I
(c) Br > Cl > F > I (d) F > Cl > Br > I

38. Which of the following is an acidic salt?
(a) Na_2SO_4 (b) Pb(OH)Cl
(c) $BaCl_2$ (d) Na_2HPO_4

39. Which one is the Lewis acid?
(a) H_2O (b) $AlCl_3$
(c) RNH_2 (d) NH_3

40. 720 g water contain the number of moles:
(a) 2 (b) 190
(c) 40 (d) 55

41. $2HCHO \xrightarrow{50\%\ NaOH} CH_3OH + HCOONa$

This reaction is called:
(a) aldol condensation
(b) Tischenko reaction
(c) Cannizaro reaction
(d) Reimer Tiemann reaction

42. The geometry of the molecule having *s* and *p*-characters equal to 25% and 75% respectively, is:
(a) trigonal (b) linear
(c) tetrahedral (d) octahedral

43. Impossible configuration is:
(a) $1s^2, 2s^2 2p^2, 3s^2$ (b) $1s^2, 2s^2$
(c) $1s^2, 2s^2 2p^6$ (d) none of these

44. The solubility product of $PbBr_2$ is 10.8×10^{-5}. It is 70% dissociated in saturated solution. The solubility of salt is:
(a) 4.18×10^{-2} (b) 6.76×10^{-3}
(c) 3.4×10^{-4} (d) 5.44×10^{-2}

45. The latent heat of vaporisation of water is 540 cal g^{-1} at 100°C. What will be the entropy increase when one mole of water is evaporated at 100°C?
(a) 28 cal K^{-1} mol^{-1} (b) 26 cal $K^{-1}mol^{-1}$
(c) 540 cal $K^{-1}mol^{-1}$ (d) 1.82 cal $K^{-1}mol^{-1}$

46. The brown ring in the test of nitrate is formed due to:
(a) $[Fe(H_2O)_5NO_2]^{2+}$ (b) $Fe(NO_3)_3$
(c) $[Fe(H_2O)_5NO]^{2+}$ (d) $[Fe(H_2O)(NO)_5]^{2+}$

47. The corrosive sublimate is :
(a) Hg (b) Hg_2O
(c) Hg_2Cl_2 (d) $HgCl_2$

48. The half-life of radium is 1580 yr. Its average life is :
(a) 2.275×10^3 yr (b) 3.735×10^2 yr
(c) 1.62×10^3 yr (d) 7.28×10^2 yr

49. Cobalt is present in :
(a) vitamin B_{12} (b) vitamin B_6
(c) vitamin B_2 (d) vitamin B_1

50. Adipic acid and hexamethylene diamine, on polymerisation gives:
(a) nylon-6 (b) dacron
(c) nylon-66 (d) bakelite

Mathematics

1. If $z = x + iy$, then the area of a triangle whose vertices are points z, iz and $z + iz$ is :

(a) $2|z|^2$ (b) $\frac{1}{2}|z|^2$

(c) $|z|^2$ (d) $\frac{3}{2}|z|^2$

2. If the complex number z_1, z_2 and z_3 represent the vertices of an equilateral triangle such that $|z_1| = |z_2| = |z_3|$, then $z_1 + z_2 + z_3$ is equal to :

(a) 0 (b) 1

(c) -1 (d) none of these

3. If $i^2 = -1$, then the value of $\sum_{n=1}^{200} i^n$ is :

(a) 50 (b) -50

(c) 0 (d) 100

4. The sum to infinity of the series

$2 + \frac{1}{2} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^3} + \ldots$

will be :

(a) 3 (b) 4

(c) $\frac{7}{2}$ (d) $\frac{9}{2}$

5. If $x > 1, y > 1, z > 1$ are in GP, then $\frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z}$ are in :

(a) AP (b) HP

(c) GP (d) none of these

6. If $x^2 - 3x + 2$ be a factor of $x^4 - px^2 + q$, then (p, q) is equal to :

(a) (3, 4) (b) (4, 5)

(c) (4, 3) (d) (5, 4)

7. If $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$, have a common root and $a \neq 0$, then $\frac{a^3 + b^3 + c^3}{abc}$ is equal to :

(a) 1 (b) 2

(c) 3 (d) 4

8. If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, then the value of r is :

(a) 3 (b) 4

(c) 5 (d) 8

9. The number of divisors of 9600 including 1 and 9600 are :

(a) 60 (b) 58 (c) 48 (d) 46

10. If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ is equal to :

(a) $(n-1)a_n$ (b) na_n

(c) $\frac{1}{2}na_n$ (d) none of these

11. 6th term in expansion of $\left(2x^2 - \frac{1}{3x^2}\right)^{10}$ is :

(a) $\frac{4580}{17}$ (b) $-\frac{896}{27}$

(c) $\frac{5580}{17}$ (d) none of these

12. In the expansion of $(1+x)^m(1-x)^n$, the coefficient of x and x^2 are 3 and -6 respectively, then m is equal to :

(a) 6 (b) 9

(c) 12 (d) 24

13. $(1+3)\log_e 3 + \frac{1+3^2}{2!}(\log_e 3)^2 + \frac{1+3^3}{3!}(\log_e 3)^3 + \ldots \infty$ is equal to :

(a) 28 (b) 30 (c) 25 (d) 0

14. The sum of $\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2^3} + \frac{1}{5}\cdot\frac{1}{2^5} + \ldots \infty$ is :

(a) $\log_e \sqrt{\frac{3}{2}}$ (b) $\log_e \sqrt{3}$

(c) $\log_e \sqrt{\frac{7}{2}}$ (d) $\log_e 3$

15. $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ac \\ 1 & c & c^2 - ab \end{vmatrix}$ is equal to :

(a) 0 (b) $a^3 + b^3 + c^3 - 3abc$

(c) $3abc$ (d) $(a+b+c)^3$

16. $x + ky - z = 0$, $3x - ky - z = 0$ and $x - 3y + z = 0$ has non-zero solution for k is equal to :

(a) -1 (b) 0 (c) 1 (d) 2

17. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, then $(A')^{-1}$ is equal to :

(a) $\begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -1 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -9 & 8 & 5 \\ -8 & 7 & -4 \\ 2 & 2 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

18. Inverse matrix of $\begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix}$ is equal to :

(a) $\begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} -2 & 7 \\ 1 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -2 & 1 \\ 7 & -4 \end{bmatrix}$

19. If A lies in the second quadrant and $3 \tan A + 4 = 0$, then value of $2 \cot A - 5 \cos A + \sin A$ is equal to :

(a) $-53/10$ (b) $-7/10$
(c) $7/10$ (d) $23/10$

20. If $\sin\theta + \cos\theta = 1$, then the general value of θ is :

(a) $2n\pi$ (b) $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$

(c) $2n\pi + \frac{\pi}{2}$ (d) none of these

21. $\cos\left[2\cos^{-1}\frac{1}{5} + \sin^{-1}\frac{1}{5}\right]$ is equal to :

(a) $\frac{2\sqrt{6}}{5}$ (b) $\frac{-2\sqrt{6}}{5}$

(c) $\frac{1}{5}$ (d) $\frac{-1}{5}$

22. From a roof of a 15m high house the angle of elevation of a point located 15m distant to the base of the house is :

(a) 45° (b) 30°
(c) 60° (d) 90°

23. If A and B are two fixed point and P is a variable point such that $PA + PB = 4$, then the locus of P is a/an :

(a) parabola (b) ellipse
(c) hyperbola (d) none of these

24. A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$, its y-intercept is :

(a) 1/3 (b) 2/3
(c) 1 (d) 4/3

25. The angle between the pair of straight lines $x^2 - y^2 - 2xy - 1 = 0$ is :

(a) 90° (b) 60°
(c) 75° (d) 36°

26. The point of intersection of the line $4x - 3y - 10 = 0$ and the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are :

(a) $(-2, -6), (4, 2)$
(b) $(2, 6), (-4, -2)$
(c) $(-2, 6), (-4, 2)$
(d) none of the above

27. The focus of the parabola $y^2 = 4y - 4x$ is :

(a) (0, 2) (b) (1, 2)
(c) (2, 0) (d) (2, 1)

28. The eccentricity of the ellipse $4x^2 + 9y^2 + 8x + 36y + 4 = 0$ is :

(a) 5/6 (b) 3/5
(c) $\sqrt{2}/3$ (d) $\sqrt{5}/3$

29. The angle of intersection of the curves $y^2 = 2x/\pi$ and $y = \sin x$ is :

(a) $\cot^{-1}(-1/\pi)$ (b) $\cot^{-1}\pi$

(c) $\tan^{-1}(-\pi)$ (d) $\cot^{-1}\left(\frac{1}{\pi}\right)$

30. The projection of any line on coordinate axes be respectively 3, 4, 5, then its length is :

(a) 12 (b) 50
(c) $5\sqrt{2}$ (d) none of these

31. If the direction ratios of a line are 1, −3, 2, then the direction cosines of the line are :

(a) $\frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$ (b) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

(c) $\frac{-1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-2}{\sqrt{14}}$ (d) $\frac{-1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}$

32. The area of triangle whose vertices are $A(1, -1, 2)$, $B(2, 1, -1)$ and $C(3, -1, 2)$ is :

(a) 13 sq unit (b) $\sqrt{13}$ sq unit
(c) 56 sq unit (d) $\sqrt{6}$ sq unit

33. $|(\vec{a} \times \vec{b}) \cdot \vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$ if :

(a) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = 0$

(b) $\vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$

(c) $\vec{c} \cdot \vec{a} = \vec{a} \cdot \vec{b} = 0$

(d) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$

34. The angle between the vectors $(2\hat{i} + 6\hat{j} + 3\hat{k})$ and $(12\hat{i} - 4\hat{j} + 3\hat{k})$ is :

(a) $\cos^{-1}\left(\frac{1}{10}\right)$ (b) $\cos^{-1}\left(\frac{9}{11}\right)$

(c) $\cos^{-1}\left(\frac{9}{91}\right)$ (d) $\cos^{-1}\left(\frac{1}{9}\right)$

35. Domain of the function $\sin \log\left(\frac{\sqrt{4-x^2}}{1-x}\right)$ is :

(a) $[-2, 1]$ (b) $(-2, 1)$

(c) $[-2, 1)$ (d) $(-2, 1]$

36. If $f(9) = 9$, $f'(9) = 4$ and $\lim_{x \to 9} \frac{f(x) - 9}{x - 9} = 4$, then $\lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$ is equal to :

(a) 2 (b) 4

(c) -2 (d) -4

37. Let $f(x)$ be defined for all $x > 0$ and be continuous, let $f(x)$ satisfy $f\left(\frac{x}{y}\right) = f(x) - f(y)$ for all x, y, then :

(a) $f(x) = \log x$

(b) $f(x)$ is bounded

(c) $f\left(\frac{1}{2}\right) \to 0$ as $x \to 0$

(d) $xf(x) \to 1$ as $x \to 0$

38. The domain of the function $y = \frac{1}{\sqrt{|x| - x}}$ is :

(a) $(-\infty, 0)$

(b) $(+\infty, 0)$

(c) $(-\infty, -1)$

(d) $(-\infty, \infty)$

39. Differential coefficient of $\sin^{-1}\frac{1-x}{1+x}$ with respect to $\sqrt{x}$ is :

(a) $\frac{1}{2\sqrt{x}}$ (b) $\frac{\sqrt{x}}{\sqrt{1-x}}$

(c) 1 (d) none of these

40. The number that exceeds its square by the greatest number is :

(a) -1 (b) 0

(c) $\frac{1}{2}$ (d) 1

41. $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}\, dx$ is equal to :

(a) $\sin 2x + c$

(b) $-\frac{1}{2}\sin 2x + c$

(c) $\frac{1}{2}\sin 2x + c$

(d) $-\sin 2x + c$

42. If $f(x) = A \sin\left(\frac{\pi x}{2}\right) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)\, dx = \frac{2A}{\pi}$, then the constants A and B are respectively :

(a) $\frac{\pi}{2}$ and $\frac{\pi}{2}$

(b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$

(c) $\frac{4}{\pi}$ and 0

(d) 0 and $\frac{-4}{\pi}$

43. Let $f(x) = x - [x]$ for all real number, where $[x]$ is the integral part of x, then $\int_{-1}^{1} f(x)\, dx$ is equal to :

(a) 1 (b) 2

(c) 0 (d) 1/2

44. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is :

(a) $y = 2$ (b) $y = 2x$

(c) $y = 2x - 4$ (d) $y = 2x^2 - 4$

45. If A and B are two independent events, then $P(A + B)$ is equal to :

(a) $P(A) + P(B) - P(A)\,P(B)$

(b) $P(A) - P(B)$

(c) $P(A) + P(B)$

(d) $P(A) + P(B) + P(A)\,P(B)$

46. A bag contains 6 red, 5 white and 4 black balls, two balls are drawn the probability that none of them is red, is :

(a) 12/35

(b) 6/35

(c) 4/35

(d) none of these

47. If $P(A) = 0.65$, $P(B) = 0.15$, then $P(\bar{A}) + P(\bar{B})$ is equal to :
(a) 1.5 (b) 1.2
(c) 0.8 (d) none of these

48. If the regression equations of the variables x and y be $x = 19.13 - 0.83y$ and $y = 11.64 - 0.50x$, then the correlation coefficient between x and y is :
(a) 0.66 (b) –0.64
(c) 0.001 (d) –0.001

49. The roots of $x + \log_{10} x = 3.375$ is approximately :
(a) 2.09 (b) 2.909
(c) 2.990 (d) 3.003

50. After second iteration of Newton-Raphson method, the positive root of equation $x^2 = 3$ is (taking initial approximation $\frac{3}{2}$) :
(a) 3/2 (b) 7/4
(c) 97/56 (d) 347/200

ANSWERS

PHYSICS

1. (a)	2. (a)	3. (a)	4. (c)	5. (b)	6. (d)	7. (a)	8. (d)	9. (d)	10. (a)
11. (d)	12. (a)	13. (a)	14. (c)	15. (a)	16. (b)	17. (c)	18. (d)	19. (a)	20. (a)
21. (b)	22. (d)	23. (a)	24. (b)	25. (a)	26. (c)	27. (c)	28. (a)	29. (b)	30. (b)
31. (a)	32. (b)	33. (c)	34. (b)	35. (d)	36. (b)	37. (a)	38. (a)	39. (c)	40. (a)
41. (a)	42. (c)	43. (d)	44. (b)	45. (d)	46. (c)	47. (a)	48. (d)	49. (d)	50. (a)

CHEMISTRY

1. (c)	2. (a)	3. (d)	4. (c)	5. (b)	6. (c)	7. (d)	8. (c)	9. (d)	10. (b)
11. (b)	12. (b)	13. (b)	14. (b)	15. (c)	16. (c)	17. (d)	18. (c)	19. (a)	20. (b)
21. (b)	22. (b)	23. (c)	24. (b)	25. (a)	26. (c)	27. (b)	28. (b)	29. (b)	30. (c)
31. (c)	32. (b)	33. (d)	34. (d)	35. (c)	36. (a)	37. (b)	38. (d)	39. (b)	40. (c)
41. (c)	42. (c)	43. (a)	44. (a)	45. (c)	46. (c)	47. (c)	48. (a)	49. (b)	50. (b)

MATHEMATICS

1. (b)	2. (a)	3. (c)	4. (c)	5. (b)	6. (d)	7. (c)	8. (a)	9. (c)	10. (c)
11. (b)	12. (c)	13. (a)	14. (b)	15. (a)	16. (c)	17. (a)	18. (a)	19. (d)	20. (b)
21. (b)	22. (a)	23. (b)	24. (d)	25. (a)	26. (a)	27. (a)	28. (d)	29. (b)	30. (c)
31. (a)	32. (b)	33. (d)	34. (c)	35. (b)	36. (b)	37. (a)	38. (a)	39. (d)	40. (c)
41. (b)	42. (c)	43. (a)	44. (c)	45. (a)	46. (a)	47. (b)	48. (b)	49. (b)	50. (c)

HINTS & SOLUTIONS

Physics

1. If a load (a device that consumes electrical power) is connected externally to the electrodes of a cell, electrons will flow under the influence of a difference in potential from cathode to anode. The current that a cell delivers depends upon the resistance of the entire circuit, including that of the cell itself. The internal resistance of the cell depends upon the size of the electrodes, the distance between them in the electrolyte and the resistance of the electrolyte. The larger the electrodes and the closer together they are in the electrolyte (without touching) the lower is the internal resistance of the cell and more current the cell is capable of supplying to the load.

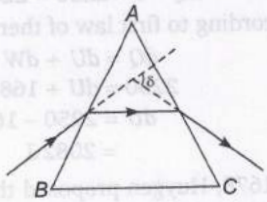
2. The oscillator may be defined as a circuit which generates an AC output signal without any externally input signal or a circuit which generates DC energy into AC energy at very high frequency.

Amplifier is a device which increases the amplitude of the input signal.

Rectifier is a circuit which converts AC to unidirectional pulsating output. In other words, it converts AC to DC.

3. For a prism of refractive index μ, angle of minimum deviation is given by

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$



When prism is thin, δ_m is small and

$$\frac{\sin A+\delta_m}{2} = \frac{A+\delta_m}{2} \text{ and } \sin\frac{A}{2} = \frac{A}{2}$$

$$\therefore \quad \mu = \frac{(A+\delta_m)/2}{A/2}$$

$$\Rightarrow \quad \delta_m = (\mu - 1)A$$

When in air,

$$\delta_m = ({}_a\mu_g - 1)A$$

$$\delta_m = \left(\frac{3}{2}-1\right)A = \frac{A}{2} \qquad ...(i)$$

When dipped in water

$$\delta'_m = ({}_w\mu_g - 1)A$$

$$= \left(\frac{{}_a\mu_g}{{}_a\mu_w} - 1\right)A$$

$$\delta'_m = \left(\frac{3/2}{4/3}-1\right)A = \frac{A}{8} \qquad ...(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{\delta'_m}{\delta_m} = \frac{1}{4}$$

4. **Key Idea :** *To escape from earth's field, body should be given as much energy which is equal to binding energy of body at earth's surface.*

The binding energy of particle at earth's surface kept at rest is $\frac{GMm}{R}$. If this much energy in the form of kinetic energy is supplied to the particle, it leaves the earth's gravitational field. So, if v_e is the escape velocity of the particle, then

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R}$$

or $$v_e = \sqrt{\frac{2GM}{R}}$$

or $$v_e = \sqrt{2gR} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

Since, escape velocity is independent of angle of projection , so it will be as before *i.e.*, 11.2 km/s.

5. When diamagnetic substances are placed in magnetic field of a strong magnet, then it is feebly magnetised in the opposite direction of field or it is repelled by strong magnet.

Note : The property 'Diamagnetism' is found in those materials whose atoms or molecules have even number of electrons. Thus, net magnetic moment of atom of diamagnetic substance is zero.

6. Key Idea : *Intensity of wave is proportional to square of amplitude of the wave.*

$$\text{Intensity} \propto (\text{Amplitude})^2$$

or $$I \propto a^2$$

or $$\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2$$

Given, $$\frac{I_1}{I_2} = \frac{9}{16}$$

$$\therefore \quad \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

Maximum intensity,

$$I_{max} = (a_1 + a_2)^2$$
$$= (3a + 4a)^2$$
$$= (7a)^2$$

$$I_{min} = (a_1 - a_2)^2$$
$$= (3a - 4a)^2$$
$$= (-a)^2$$

Hence, the ratio,

$$\frac{I_{max}}{I_{min}} = \frac{(7a)^2}{(-a)^2}$$
$$= \frac{49}{1}$$

7. According to Wien's law the product of wavelength corresponding to maximum intensity of radiation and temperature of body (in Kelvin) is constant, *i.e*, $\lambda_m T = b$ = constant.

$$\frac{(\lambda_m)_{sun}}{(\lambda_m)_{star}} = \frac{T_{star}}{T_{sun}}$$

Given, $T_{sun} = 510$ nm, $T_{star} = 350$ nm

$$\therefore \quad \frac{(\lambda_m)_{sun}}{(\lambda_m)_{star}} = \frac{350}{510} = 0.69$$

Note : This law is of great importance in 'Astrophysics' as through the analysis of radiations coming from a distant star, by finding λ_m the temperature of the star $T (= b/\lambda_m)$ is determined.

8. Key Idea : *Standard equation of standing wave is*

$$y = a\sin(\omega t - kx)$$

The given equation is

$$y = 0.3\sin(314t - 1.57x)$$

On comparing the two equations, we have

$$\omega t = 314t$$

and $$kx = 1.57x$$

$\Rightarrow$ $$\omega = 314$$

and $$k = 1.57$$

Hence, velocity of standing wave is

$$v = \frac{\omega}{k} = \frac{314}{1.57} = 200 \text{ unit.}$$

9. The emf of a cell is defined as work done by the cell in moving unit positive charge in the whole circuit including the cell once. Therefore, if W is the work done by a cell in moving charge q once around a circuit including the cell, then

$$\text{emf} \quad E = \frac{W}{q}$$

or $$E = \frac{W}{it}$$

Given, $i = 2$ A, $t = 6$ min $= 6 \times 60$ s, $W = 1000$ J

Hence, $$E = \frac{1000}{2 \times 6 \times 60} = 1.39 \text{ V}$$

Note : The term emf (electromotive force) is misleading introduced by volta who thought it to be force that causes the current to flow. Actually emf is not a force but work required to carry unit charge from lower potential to higher potential inside the cell.

10. According to Bohr's model of atom, electrons in an atom can revolve in those orbits (as suggested by classical theory) in which its angular momentum about the nucleus is an integer multiple of $\frac{h}{2\pi}$, where h is Planck's constant $(= 6.6 \times 10^{-34}$ J-s).

Thus, $$I\omega = mvr = \frac{nh}{2\pi}$$

For 2nd orbit, $n = 2$

$$\therefore \quad I\omega = \frac{2h}{2\pi} = \frac{h}{\pi}$$

11. Heat supplied to vaporise the water

$$dQ = mL$$

where L is the latent heat of vaporisation of water.

$$\therefore \quad dQ = 1 \times 2250 = 2250 \text{ J}$$

According to first law of thermodynamics,

$$dQ = dU + dW$$

or $$2250 = dU + 168$$

or $$dU = 2250 - 168$$
$$= 2082 \text{ J}$$

12. In 1678, Huygen proposed the wave theory of light. According to which light travels in the form of waves. These waves after emerging from the light source travel in all directions with velocity of light. This wave character of light successfully explained the phenomenon of polarisation, diffraction and interference.

While it is failed to explain photoelectric effect. This was later explained by Einstein on the basis of Planck's quantum theory which states that light travels in the form of small bundles or packets of energy called photons.

13. Amplification factor is defined as the ratio of change in plate potential (ΔV_p) to produce certain change in plate current (Δi_p) to the change in grid potential (ΔV_g) for the same change in plate current *i.e,*.

$$\mu = -\left(\frac{\Delta V_p}{\Delta V_g}\right)_{\Delta i_p = \text{constant}}$$

Here, $\Delta V_p = 220 - 200 = 20$ V,

$\Delta V_g = -1.3 - (-0.5)$

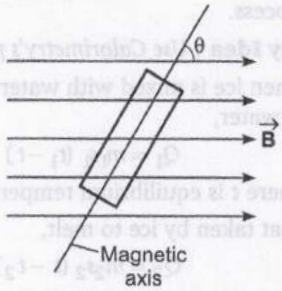
$= -0.8$ V

$$\therefore \quad \mu = -\left(\frac{20}{-0.8}\right) = 25$$

Note : Amplification factor is unitless and dimensionless.

14. **Key Idea** : *A bar magnet suspended in a uniform magnetic field sets itself with its axis parallel to the field.*

A magnet placed in the magnetic field experiences a torque which rotates the magnet to a position in which the axis of the magnet is parallel to the field.



The magnitude of torque acting on a current loop placed in a magnetic field $\vec{B}$ with its axis at an angle θ with the direction of $\vec{B}$ is given by

$\tau = iAB \sin\theta$

Here, magnitude of dipole moment, $M = i\,A$

$$\therefore \quad \tau = MB \sin\theta$$

Putting the numerical values, we have

$M = 220$ Am^2, $B = 0.25$ N/Am, $\theta = 30°$

$$\therefore \quad \tau = 220 \times 0.25 \times \cos 30°$$

$$= 220 \times 0.25 \times \frac{\sqrt{3}}{2}$$

$= 47.63$ Nm

15. If R_1 and R_2 are the radii of curvature of first and second refracting surfaces of a thin lens with optical centre C of focal length f and refractive index μ then according to lens Maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where μ is refractive index of material of lens with respect to surrounding medium.

For plano-convex lens,

$R_1 = 50$ cm, $R_2 = \infty$ (for plane surface)

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{50} - \frac{1}{\infty}\right)$$

or $$\frac{1}{f} = 0.5 \times \frac{1}{50}$$

or $$f = \frac{50}{0.5} = 100 \text{ cm}$$

16. **Key Idea** : *Movement of the source and observer alters the wavelength and the perceived frequency of sound.*

As both source and observer are moving away, so observed frequency decreases.

$$\therefore \quad f' = \left(\frac{v - v_o}{v + v_s}\right) f$$

where v is the velocity of sound = 340 m/s

Here, $v_o = 20$ m/s, $v_s = 20$ m/s, $f' = 1840$ Hz

$$\therefore \quad 1840 = \left(\frac{340 - 20}{340 + 30}\right) f$$

or $$f = \frac{1840 \times 360}{320} = 2070 \text{ Hz}$$

17. As we have given that 3500 waves are passing through a point in the medium in 1.67 min, so frequency of the wave,

$$n = \frac{3500}{1.67 \times 60} \approx 35/\text{s}$$

Hence, wavelength of the wave,

$$\lambda = \frac{v}{n}$$

$$= \frac{650}{35} = 18.57 \text{ m}$$

18. Horizontal range of a body is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

where θ is the angle at which body is projected.

The greatest height of body is given by

$$H = \frac{u^2 \sin^2\theta}{2g}$$

Given, $R = 3H$

$$\frac{u^2 \sin 2\theta}{g} = 3 \times \frac{u^2 \sin^2 \theta}{2g}$$

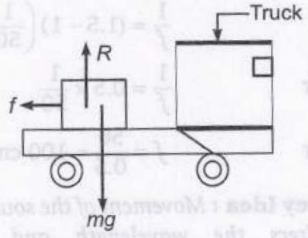
or $$2 \sin\theta \cos\theta = \frac{3}{2} \sin^2\theta$$

or $$\tan\theta = \frac{4}{3}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{4}{3}\right) = 53°1'$$

19. The frictional force acting on the block

$$f = \mu R$$



but from figure, $R = mg$

$$\therefore \quad f = \mu\, m\, g$$

Here, $\mu = 0.6$, $m = 1$ kg, $g = 10$ m/s^2

$$\therefore \quad f = 0.6 \times 1 \times 10 = 6 \text{ N}$$

Note : Due to acceleration of truck, the block experiences a forward force

$$F = ma = 1 \times 5 = 5 \text{ N}$$

Since, $f > F$, so block is not displaced from its previous position.

20. Work (W) is measured by the vector product of the applied force ($\vec{F}$) and the displacement ($\vec{s}$) of the body in the direction of the force ($\vec{F}$), that is,

$$W = \vec{F} \cdot \vec{s}$$

Given, $\vec{F} = 2\hat{i} + 3\hat{j} - 5\hat{k}$, $\hat{s} = 2\hat{i} + 4\hat{j} + 3\hat{k}$

$$\therefore W = (2\hat{i} + 3\hat{j} - 5\hat{k}) \cdot (2\hat{i} + 4\hat{j} + 3\hat{k})$$

$$= (2)(2) + (3)(4) + (-5)(3)$$

$$= 4 + 12 - 15$$

$$= 1 \text{ unit}$$

Note : Both force and displacement are vector quantities but work is a scalar quantity.

21. Efficiency of the Carnot engine is given by

$$\eta = 1 - \frac{T_2}{T_1} \quad \ldots(i)$$

where T_1 = temperature of source

T_2 = temperature of sink

Given, $\eta = 50\% = 0.5$, $T_2 = 500$ K

Substituting in relation (i), we have

or $$0.5 = 1 - \frac{500}{T_1}$$

or $$\frac{500}{T_1} = 0.5$$

$$\therefore \quad T_1 = \frac{500}{0.5} = 1000 \text{ K}$$

Now, the temperature of sink is changed to T'_2 and the efficiency becomes 60% *i.e,* 0.6.

Using relation (i), we get

$$0.6 = 1 - \frac{T'_2}{1000}$$

or $$\frac{T'_2}{1000} = 1 - 0.6 = 0.4$$

or $$T'_2 = 0.4 \times 1000 = 400 \text{ K}$$

Note : Carnot engine is not a practical engine because many ideal situations have been assumed while designing this engine which can practically not be obtained.

22. For a process to be isothermal, any heat flow into or out of the system must occur slowly enough, so that thermal equilibrium is maintained. Hence, in a isothermal process, temperature during the whole process does not change *i.e.,* internal energy which is a function of temperature remains same throughout the process.

23. **Key Idea** : *Use Calorimetry's principle.*

When ice is mixed with water, then heat given by water,

$$Q_1 = m_1 s_1 (t_1 - t)$$

where t is equilibrium temperature.

Heat taken by ice to melt,

$$Q_2 = m_2 s_2 (t - t_2) + m_2 L$$

From principle of Calorimetry.

Heat given by water = Heat taken by ice

i.e., $m_1 s_1 (t_1 - t) = m_2 s_2 (t - t_2) + m_2 L$

Given, $m_1 = m_2 = 100$ g, $t_1 = 100°$ C,

$t_2 = 0°$ C, $s_1 = 1$ cal/g° C, $s_2 = 1$ cal/g° C,

$L = 80$ cal/g

Substituting the values in Eq. (i), we get

$$100 \times (100 - t) = 100 \times 1 \times (t - 0) + 100 \times 80$$

or $$10000 - 100t = 100t + 8000$$

or $$200t = 2000$$

or $$t = 10° \text{ C}$$

24. When a star considerably larger than our sun burns out, the gravitational force between all its particles can cause the star to collapse on itself and thereby to form a black hole. The gravitational force at the surface of such a collapsed star is so strong that neither particles nor light can escape from its surface (thus the term "black hole").

Any star coming to near a black hole can be ripped apart by strong gravitational force and pulled into the hole. So, a black hole consists of super dense planetary material.

25. The three resistances shown in figure are connected in parallel. So, their equivalent resistance is given by

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$$

$$= \frac{6+4+3}{12} = \frac{13}{12}$$

$$\therefore \quad R = \frac{12}{13}\Omega$$

Note : The equivalent resistance in parallel combination is less than either of the resistances.

26. The energy of electron in higher orbit is greater than in lower orbit. If in a hydrogen atom, electron jumps from lower orbit to higher orbit, it gains, (absorb) difference of energy of both orbits. While if it jumps from higher orbit to lower orbit, it releases difference of energy of both orbits. Since, here electron jumps from first (lower) orbit to third (higher) orbit, it absorbs some energy.

27. **Key Idea** : *In an inelastic collision, linear momentum is conserved.*

Since, in given collision the two bodies stick to each other, so collision is inelastic. Applying conservation of linear momentum.

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v \quad \ldots(i)$$

where v is the velocity of joint body.

Given, $m_1 = 10$ kg, $v_1 = 10$ m/s, $m_2 = 5$ kg, $v_2 = 0$.

Substituting the values in Eq. (i), we get

$$10 \times 10 + 5 \times 0 = (10 + 5)v$$

or $$100 = 15v$$

or $$v = \frac{100}{15} = \frac{20}{3} \text{ m/s}$$

28. The distance travelled by the body in nth second is given by

$$s_n = u + \frac{1}{2}a(2n-1) \quad \ldots(i)$$

Since, body starts from rest, so $u = 0$.

Given, $s_n = 110$ cm, $n = 9$

Substituting the values in Eq. (i), we get

$$110 = 0 + \frac{1}{2}a(2 \times 9 - 1)$$

or $$110 = \frac{a}{2} \times 17$$

$$\therefore \quad a = \frac{2 \times 110}{17} \approx 13 \text{ cm/s}^2$$

or $$a = 0.13 \text{ m/s}^2$$

29. Momentum of a particle is related to de-Broglie wavelength through the relation given by

$$p = \frac{h}{\lambda}$$

where h is Planck's constant.

or $$\lambda = \frac{h}{p}$$

or $$\lambda \propto \frac{1}{p}$$

i.e., de-Broglie wavelength is inversely proportional to the momentum of the particle, so, if momentum is doubled, then de-Broglie wavelength will become half.

30. **Key Idea** : *Torque is measured in units of newton-metres.*

The force applied to a lever, multiplied by its distance from the lever's fulcrum is torque.

i.e., $$\tau = r \cdot F$$

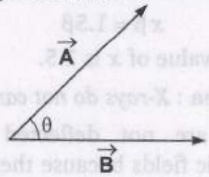
or $$\tau = \text{N-m}$$

$$\therefore \quad [\tau] = [ML^2T^{-2}]$$

Note : Torque is the time derivative of angular momentum. Also dimensions of work, kinetic energy and torque are same.

31. The scalar product of two vectors can be constructed by taking the component of one vector in the direction of the other and multiplying it times the magnitude of the other vector.

This can be expressed in the form.



$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

$\vec{A}$ denotes vector and A denotes the magnitude of this vector.

Given, $A = 3, B = 5$ and $\theta = 60°$

Putting these values in above equation, we have

$$\vec{A}\cdot\vec{B} = 3\times 5\cos 60°$$

$$\vec{A}\cdot\vec{B} = 15\times\frac{1}{2} = 7.5$$

32. **Key Idea :** *Average kinetic energy per molecule is equal to product of mass of 1g molecule and square of mean square velocity.*

The kinetic energy of 1g-mol is

$$E = \frac{1}{2}M\bar{v}^2 = \frac{1}{2}M\left(\frac{3RT}{M}\right) \qquad \left[\because \bar{v} = \sqrt{\frac{3RT}{M}}\right]$$

$$E = \frac{3}{2}RT$$

where R is gas constant.

Putting the numerical values, we have

$$E = \frac{3}{2}\times 8.31\times 273 = 3.4\times 10^3\ \text{J}$$

33. The linear expansivity of a solid is also referred to as the thermal coefficient of linear expansion, and is frequently given the symbol (α). As solids expand, so does their surface areas and volumes. The terms superficial expansivity (β) or thermal coefficient of superficial expansion is used when describing area changes due to temperature changes. For volume changes, the term cubic expansivity (γ) or thermal coefficient of cubic expansion is used.

For most solids, the relationships

$$\gamma = 3\alpha \text{ and } \beta = 2\alpha \text{ hold.}$$

Hence, we have

$$\gamma = \frac{3\beta}{2} = 1.5\beta$$

$$\Rightarrow \qquad x\beta = 1.5\beta \qquad (\because \gamma = x\beta)$$

Hence, value of x is 1.5.

34. **Key Idea** : *X-rays do not carry any charge.*

X-rays are not deflected by electric and magnetic fields because they do not carry any charge. *X*-rays are electromagnetic waves, they show all properties of light rays hence, they undergo reflection, refraction, interference, diffraction and polarisation.

35. Bernoulli's theorem states that when an incompressible and non-viscous liquid (or gas) flows in stream-lined motion from one place to another, then at every point of its path the total energy per unit volume (pressure energy + kinetic energy + potential energy) is constant. That is

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

where P is pressure, ρ is density, v is velocity, g is gravity and h is height. Thus, Bernoulli's theorem is one way the principle of conservation of energy for a flowing liquid (or gas).

36. The electron has negative charge. When an electron is bringing towards another electron, then due to same negative charge repulsive force is produced between them. So, to bring them closer a work is done against this repulsive force. This work is stored in the form of electrostatic potential energy. Thus, electrostatic potential energy of system increases.

Alternative : Electrostatic potential energy of system of two electrons

$$U = \frac{1}{4\pi\varepsilon_0}\frac{(-e)(-e)}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

Thus, as r increases, potential energy U increases.

37. The sum of half-lives of all atoms divided by the number of all atoms is called the mean-life (τ) of radioactive substance.

$$\tau = T_{1/2}/0.693$$

where $T_{1/2}$ is the half-life of radium.

$$\therefore \qquad \tau = \frac{1600}{0.693}\ \text{yr} \approx 2309\ \text{yr}$$

Also mean-life of a radioactive substance is given by

$$\tau = \frac{1}{\lambda}$$

or $$\lambda = \frac{1}{\tau} = \frac{1}{2309}/\text{yr}$$

38. According to Einstein's theory of relativity, matter is completely converted into energy or *vice-versa*. The relation is

$$E = \Delta m c^2$$

Given, $\Delta m = 1\text{mg} = 1\times 10^{-3}\ \text{g}$, $c = 3\times 10^8$ m/s

$$\therefore \qquad E = 1\times 10^{-3}\times(3\times 10^8)^2$$

$$= 9\times 10^{13}\ \text{J}$$

39. Key Idea : *Apply Brewster's law.*

According to Brewster's law, when unpolarised light is incident at an angle called polarising angle, i_p on an interface separating air from a medium of refractive index n, then the reflected light is fully polarised (perpendicular to the plane of incidence), provided

$$n = \tan i_p$$

Given, $n = 1.5$

$\therefore$ $1.5 = \tan i_p$

$\Rightarrow$ $i_p = \tan^{-1}(1.5) = 56.3°$

If r is the angle of refraction them

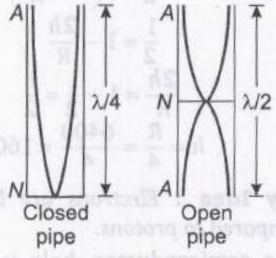
$$i_p + r = 90°$$

$\Rightarrow$ $r = 90° - i_p$

$= 90° - 56.3° = 33.7°.$

40. Frequency of fundamental note in closed pipe is

$$n' = \frac{v}{4l} \quad ...(i)$$



Frequency of fundamental note in open pipe is

$$n'' = \frac{v}{2l_2} \quad ...(ii)$$

Given, $n' = n''$

$\therefore$ $\frac{v}{4l_1} = \frac{v}{2l_2}$

$\Rightarrow$ $l_2 : l_2 = 1 : 2$

Note : A closed pipe produces only odd harmonics while an open pipe produces both even and odd harmonics.

41. Change in velocity when the particle completes half revolution is

$$\Delta v = 5 - (-5)$$

$$= 10 \text{ m/s}$$

Time taken in completing half revolution is

$$T = \frac{\pi r}{v} = \frac{\pi \times 5}{5} = \pi \text{ second}$$

Hence, average acceleration when particle completes half revolution

$$a_{av} = \frac{\Delta v}{T} = \frac{10}{\pi} \text{ m/s}^2$$

42. The two springs in the arrangement form parallel combination. So, spring constant in parallel combination is givne by

$$k = k_1 + k_2$$

where k_1 and k_2 are force constants of springs.

Frequency of oscillation of the system is given by

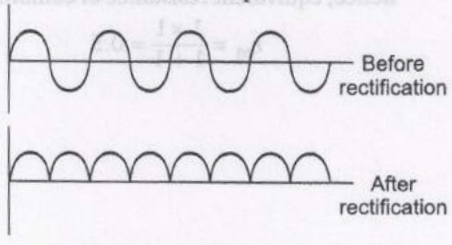
$$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

or $$n = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$$

43. If a trivalent impurity is mixed in a pure (intrinsic) semiconductor, then it becomes a p-type semiconductor. As given in problem, all boron, gallium and aluminium are trivalent impurities so, they must be doped to Ge (or Si) to make it p-type semiconductor.

Note : The added impurity is very small of the order of 1 atom million atoms of the pure semiconductor. When impurity atoms are incorporated in a pure semiconductor crystal, the electric properties are altered and most of the charge carriers originate from impurity atoms.

44. When diode acts as full wave rectifier two diode valves are connected to obtain a continuous full wave output.



Hence, fundamental frequency in the ripple is twice of operating frequency, that is 100 Hz.

45. When a system undergoes a change under the condition that no exchange of heat takes place between system and surrounding them such a process is called an adiabatic one. The relation between temperature (T), volume (V) and ratio of specific heats (γ) is

$$T V^{\gamma-1} = \text{constant}$$

$$\therefore \frac{T'}{T} = \left(\frac{V}{V'}\right)^{\gamma-1} = \left(\frac{27}{8}\right)^{\frac{5}{3}-1} = \left(\frac{27}{8}\right)^{\frac{2}{3}}$$

$$\Rightarrow \frac{T'}{T} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$\Rightarrow T' = \frac{9}{4}T = \frac{9}{4} \times (273 + 27)$$

$$\Rightarrow T' = \frac{9}{4} \times 300 = 675 \text{ K}$$

$$\therefore \Delta T = T' - T = 675 - 300 = 375 \text{ K} = 375^\circ \text{C}$$

46. The entropy function gives us a numerical measure of the irreversibility of a given process *i.e.*, it is a measure of disorder of a system. During formation of ice cubes orderedness increases *i.e.*, disorderness decreases, hence entropy decreases.

47. An astronomical telescope consists of two converging lenses one is objective lens and other is eyepiece. In relaxed eye state of telescope, length of telescope tube is given by

$$L_\infty = f_o + f_e$$

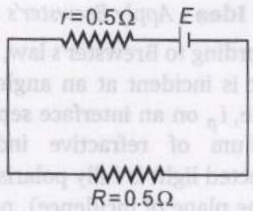
Given, $f_o = 2\text{m}, f_e = 0.05 \text{ m}$

$$\therefore L_\infty = 2 + 0.05 = 2.05 \text{ m}$$

48. **Key Idea :** *In parallel combination positive poles of all cells are connected to one point and negative poles to another point.*

The resistances of 1Ω are connected in parallel hence, equivalent resistance of combination is

$$r_{eq} = \frac{1 \times 1}{1 + 1} = 0.5$$



From Ohm's law,

$$E = i(R + r)$$

$$\therefore i = \frac{2}{0.5 + 0.5} = 2\text{A}$$

Power is given by

$$P = i^2R = (2)^2 \times 0.5 = 2\text{W}$$

49. Acceleration due to gravity at a height h is given by

$$g' = g\left(1 - \frac{2h}{R}\right) \quad \ldots(\text{i})$$

At height h, $g' = \frac{g}{2}$ (given)

Thus, Eq. (i) becomes

$$\therefore \frac{g}{2} = g\left(1 - \frac{2h}{R}\right)$$

or $$\frac{1}{2} = 1 - \frac{2h}{R}$$

or $$\frac{2h}{R} = 1 - \frac{1}{2} = \frac{1}{2}$$

or $$h = \frac{R}{4} = \frac{6400}{4} = 1600 \text{ km}$$

50. **Key Idea :** *Electrons are lighter in weight compared to protons.*

In a semiconductor, hole is equivalent to a positively charged particle which moves in a direction opposite to that of an electron. Since, electrons are lighter in mass compared to positively charged particles, they move easily in the semiconducting material. Hence, the mobility of holes in p-type semiconductor is less than mobility of electrons in n-type semiconductor *i.e.*, $\mu_e > \mu_h$.

Chemistry

1. Key Idea : *Tyndall effect is property of colloidal sol.*

Tyndall effect is scattering of light by the colloidal particles in a colloidal sol.

2. Key Idea : (i) *Loss of electron and increase in oxidation number is oxidation.*

(ii) *Gain of electron and decrease in oxidation number is reduction.*

$MnO_4^- + H^+ + C_2O_4^{2-} \longrightarrow Mn^{2+} + H_2O + CO_2$

Oxidation number of Mn in $MnO_4^- = +7$

Oxidation number of Mn in $Mn^{2+} = +2$

Oxidation number of C in $C_2O_4^{2-} = +3$

Oxidation number of C in $CO_2 = +4$

For Mn, oxidation number is decreasing from +7 to +2

∴ Mn is getting reduced during reaction.

For C, oxidation number is increasing from +3 to +4

∴ C is oxidised during reaction.

3. Key Idea : *Ionic bonds are formed when difference in electronegativity of elements is more than 1.7.* BF_3, CCl_4 *and* HCl *have covalent bonds and* $BaCl_2$ *has ionic bond.*

4. Key Idea : *Find deficiency diseases caused by vitamins.*

(a) Scurvy—vitamin C

(b) Beri-Beri— vitamin B

(c) Sterility—vitamin E

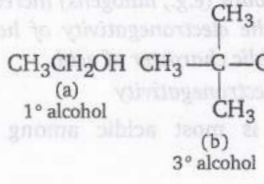
(d) Xerophthalmia—vitamin A

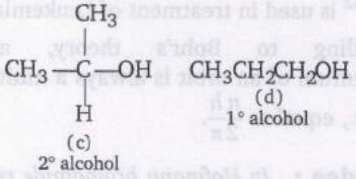
5. Key Idea : *Lucas reagent is used to distinguish between 1°, 2° and 3° alcohols. Lucas reagent is* $HCl + ZnCl_2$.

3° alcohol + Lucas reagent ⟶ Immediate terbidity

2° alcohol + Lucas reagent ⟶ Terbidity after 5-10 min

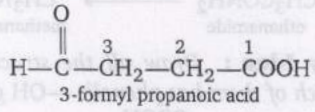
1° alcohol + Lucas reagent ⟶ No terbidity





∴ (b) a tertiary alcohol will give fastest reaction.

6. Key Idea : *If the organic compound has two groups having C atom placed at ends of carbon chain, only the carbon of group of higher priority is included in chain.*



7. Tollen's reagent is ammoniacal $AgNO_3$. It is used to distinguish between aldehydes and ketones.

8. $MgSO_4 \cdot 7H_2O$ is Epsom salt.

9. Key Idea : *Alkali metals carbonates do not decompose on heating but alkaline earth metal carbonates decompose easily.*

$$MgCO_3 \xrightarrow{\Delta} MgO + CO_2$$

∴ $MgCO_3$ will decompose most easily among given choices.

10. Key Idea : *Find basic definition of thermodynamics.*

(a) In isothermal process, temperature and internal energy remains constant. ($\Delta E = 0$)

(b) $\Delta q = 0$ – Adiabatic process

(c) $\Delta V = 0$ – Isochoric process

11. Key Idea : *Degree of hydrolysis* $(h) = \sqrt{\frac{K_h}{C}}$

where K_h = hydrolysis constant = 2.5×10^{-9}

C = concentration = 0.01 M

$$\therefore \quad h = \sqrt{\frac{2.5 \times 10^{-9}}{0.01}}$$

$$= 5 \times 10^{-4}$$

12. $H_2SO_4 \longrightarrow 2H^+ + SO_4^{2-}$

∵ 1 mole H_2SO_4 = 2 g-equivalent of H_2SO_4

∴ 3 mole H_2SO_4 = 2 × 3 g-equivalent of H_2SO_4

= 6 g-equivalent of H_2SO_4

13. (i) $_{27}Co^{60}$ emits γ-rays and is used to destroy cancer cells.

(ii) P^{32} is used in treatment of Leukemia.

14. According to Bohr's theory, angular momentum of an orbit is always a multiple of $\frac{h}{2\pi}$, *i.e.*, equal to $\frac{nh}{2\pi}$.

15. Key Idea : *In Hofmann bromamide reaction acid amide reacts with* Br_2/Cl_2 *in* NaOH/KOH *to produce amine having one carbon atom less than amide. So the following is Hofmann bromamide reaction.*

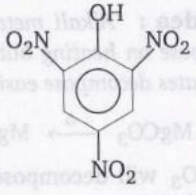
$$\underset{\text{ethanamide}}{CH_3CONH_2} \xrightarrow{Br_2/NaOH} \underset{\text{methanamine}}{CH_3NH_2}$$

16. Key Idea : *Draw all the structures to find which of them has phenolic —OH group.*

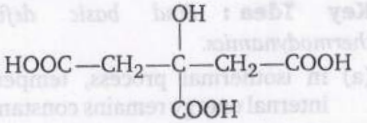
(a) Oxalic acid $\begin{vmatrix} COOH \\ COOH \end{vmatrix}$

(b) Formic acid HCOOH

(c) Picric acid



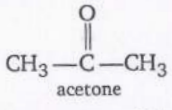
(d) Citric acid



∴ Picric acid has phenolic —OH group.

17. Key Idea : H_2O *adds across triple bond in presence of* Hg^{2+} *salt.*

$$\underset{\text{propyne-1}}{CH_3—C\equiv CH} + H_2O \xrightarrow{H^+/Hg^{2+}}$$



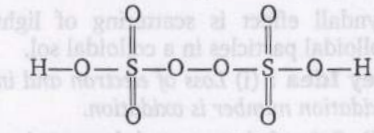
18. Key Idea : *Basicity of oxides decreases while moving in a period. Amphoteric oxides react both with acid and base.*

(i) MgO, K_2O and CaO are basic oxides

(ii) Al_2O_3 is amphoteric oxide

$$Al_2O_3 + 6HCl \longrightarrow Al_2Cl_6 + 3H_2O'$$
$$Al_2O_3 + 2NaOH \longrightarrow 2NaAlO_2 + H_2O$$

19. Key Idea : *Oxidation state of oxygen in peroxide linkage is* –1.



∴ In $H_2S_2O_8$ there is one peroxide linkage

∴ Oxidation state of two oxygens is –1

and oxidation state of 6 oxygens is –2

Let oxidation state of S in $H_2S_2O_8 = x$

$\therefore (+1\times 2) + 2x + (6\times -2) + (2\times -1) = 0$

or $2x + 2 - 12 - 2 = 0$

or $2x = 12$

$\therefore x = +6$

20. $Ca(OH)_2 + Cl_2 \longrightarrow \underset{\text{bleaching powder}}{CaOCl_2} + H_2O$

21. Key Idea : $K_p = K_c\,(RT)^{\Delta n}$

$$PCl_3(g) + Cl_2(g) \rightleftharpoons PCl_5(g)$$

$\therefore \Delta n_g = 1 - 2 = -1$

Given $K_c = 26$

$T = 250°C = 250 + 273 = 523\ K$

$R = 0.0821$

$\therefore K_p = K_c(RT)^{\Delta n}$

$= 26(0.0821\times 523)^{-1}$

$= 26 \times \frac{1}{0.0821\times 523}$

$= 0.61$

22. Key Idea : $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

Given $P_1 = 1$ atm, $P_2 = ?$

$T_1 = 25°C = 25 + 273 = 298\ K$

$T_2 = 35°C = 35 + 273 = 308\ K$

$\frac{1}{298} = \frac{P_2}{308}$

or $P_2 = \frac{308}{298} = 1.03$ atm

23. Key Idea : *Proton donors are acids. Electron withdrawing groups (e.g., halogens) increase the acidity. More the electronegativity of halogen, more will be acidic character of acid.*

F *has highest electronegativity*

∴ CH_2FCOOH is most acidic among given choices.

24. Key Idea : *Compressibility factor* $Z = \frac{PV}{nRT}$

For ideal gas $PV = nRT$

$\therefore$ $Z = 1$ for ideal gas.

25. Key Idea : *Smaller the value of reduction potential; stronger will be reducing agent. Reduction potential value indicate the ease of oxidation. Reducing agent itself undergoes oxidation.*

$\because Li^+$ has least reduction potential.

$\therefore$ It is strongest reducing agent among given choices.

26. Key Idea : *For reaction of nth order*

$$k = \frac{1}{time} \times \frac{1}{(conc.)^{n-1}}$$

For first order reaction $n = 1$

$$\therefore k = \frac{1}{\text{time}} \times \frac{1}{(\text{conc.})^{1-1}} = \frac{1}{\text{time}} = \text{time}^{-1}$$

$$= s^{-1}$$

27. Key Idea : *The dipole moment of molecules having regular geometry or linear structure is zero.*

Dipole moment is vector quantity.

(a) H_2O_2—open book structure

(b) CO_2—linear structure

$$O = C = O$$

$\therefore$ Dipole moment is zero

(c) SO_2—bent structure

(d) NH_3—pyramidal structure

28. Key Idea : *Write electronic configuration of* O_2^+ *according to molecular orbital theory and then calculate bond order.*

$O_2^+ = 8 + 8 - 1 = 15$

$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2, \pi 2p_y^2, \pi^* 2p_x^1$

Bond order $= \frac{1}{2}$ [number of bonding electrons – number of antibonding electrons]

$= \frac{1}{2}(10 - 5) = \frac{5}{2} = 2.5$

29. Key Idea : *Write the equation to find composition of black colour.*

$$2HgCl_2 + SnCl_2 \longrightarrow Hg_2Cl_2 + SnCl_4$$

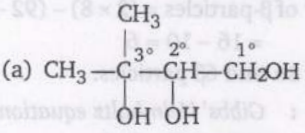
$$Hg_2Cl_2 + SnCl_2 \longrightarrow \underset{\text{black}}{2Hg} + SnCl_4$$

$\therefore$ Reduction of $HgCl_2$ to Hg results in formation of black coloured mercury.

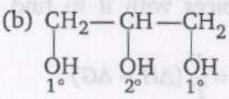
30. Key Idea : *1° carbon atom has only alkyl group attached to it.*

2° carbon atom has 2 alkyl group attached to it.

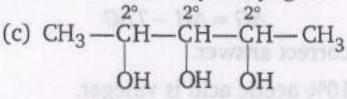
3° carbon atom has 3 alkyl group attached to it.

(a) 

$\therefore$ It has 1, 2, 3° hydroxyl groups.

(b) 

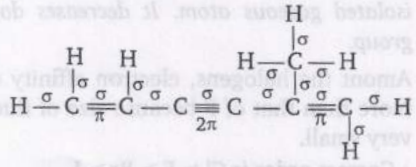
$\therefore$ It has 1° and 2° hydroxyl gas.

(c) 

$\therefore$ It has all 2° hydroxyl groups.

31. Key Idea : *First bond between any two atoms is sigma and rest are pi bonds.*

Structure

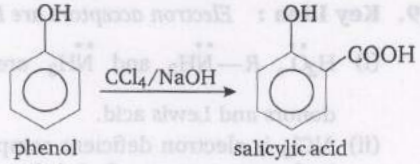


14σ and 4π bonds are there.

32. Key Idea : *Nitration of benzene is electrophilic substitution reaction.*

NO_2^+ is electrophile in this reaction.

33. Key Idea : *This reaction is called Reimer-Tiemann reaction.*



phenol salicylic acid

34. Key Idea :

(i) *Number of α particles lost*

$$= \frac{\text{atomic mass of reactant} - \text{atomic mass of product}}{4}$$

(ii) *Number of β-particles*

= (2 × *number of α-particles*) − (*difference in atomic number of product and reactant*)

Given $_{92}U^{238} \longrightarrow {}_{82}Pb^{206}$

$\therefore$ Number of α-particles $= \frac{238-206}{4} = \frac{32}{4} = 8$

$\therefore$ Number of β-particles $= (2 \times 8) - (92 - 82)$
$= 16 - 10 = 6$

$\therefore$ Total of 8α and 6β particles.

35. Key Idea : *Gibbs' Helmholtz equation is* $\Delta G = \Delta H - T\Delta S$

Compare the choices with it to find correct answer

(c) $\Delta S = \frac{1}{T}(\Delta H - \Delta G)$

or $T\Delta S = \Delta H - \Delta G$

or $\Delta G = \Delta H - T\Delta G$

$\therefore$ It is correct answer.

36. (a) 8-10% acetic acid is vinegar.
(c) Glacial acetic acid is 100% acetic acid.
(d) 10% formic acid is formaline.

37. Key Idea : *Electron affinity is the amount of energy released when an electron is added to isolated gaseous atom. It decreases down the group.*

Amont the halogens, electron affinity of Cl is more than that of F because size of fluorine is very small.

$\therefore$ Correct order is Cl > F > Br > I

38. Key Idea : *Acidic salts are those salts which are formed by incomplete neutralisation of acid*

(i) Pb(OH)Cl is basic salt

(ii) Na_2HPO_4 is acidic salt which is formed by incomplete neutralisation of phosphoric acid.

39. Key Idea : *Electron acceptors are Lewis acid*

(i) $H_2\ddot{O}$, $R-\ddot{N}H_2$ and $\ddot{N}H_3$ are electron donors and Lewis acid.

(ii) $AlCl_3$ is electron deficient compound and is electron acceptor. So it is Lewis acid.

40. Key Idea : *Number of moles* $= \frac{mass}{molecular\ mass}$

Given mass of $H_2O = 720$ g

$\therefore$ Moles of water $= \frac{720}{18} = 40$

41. Key Idea : *Define all reactions to find correct answer.*

(i) Aldol condensation : carbonyl compound which have α–H atom + NaOH $\longrightarrow$ β-hydroxy carbonyl compound

(ii) Reimer Tiemann reaction :

Phenol + CCl_4 + NaOH $\longrightarrow$ Salicylic acid

(iii) Cannizaro reaction :

Aldehydes (which do not have α-H atom) + NaOH $\longrightarrow$ alcohol + acid

e.g., $\underset{\text{formaldehyde}}{2HCHO} \xrightarrow{50\%\ NaOH} \underset{\text{methanal}}{CH_3OH} + \underset{\text{sodium formate}}{HCOONa}$

42. Key Idea : *25% s and 75% p character means it is* sp^3 *hybridisation.*

$\therefore$ Shape of sp^3 hybrid orbital is tetrahedral.

43. Key Idea : *Check all configurations on aufbau configuration.*

(a) $1s^2, 2s^2, 2p^2, 3s^2$ is not possible because orbitals of lower energy level must be completely filled, before the orbital of higher energy level is filled.

(b) $1s^2, 2s^2$ is possible as it follows aufbau principle.

(c) $1s^2, 2s^2, 2p^6$ is possible as it follows aufbau principle

44. Key Idea : *First find relationship between solubility and solubility product.*

$PbBr_2 \rightleftharpoons Pb^{2+} + 2Br^-$

$K_{sp} = [Pb^{2+}][Br^-]^2$

given K_{sp} $PbBr_2 = 10.8 \times 10^{-5}$

$\alpha = 70\%$

Solubility $= x$ mol/L

$\therefore$ $[Pb^{2+}] = 0.7x$,

$[Br^-] = 2 \times 0.7 \times x = 1.4x$

$\therefore$ $K_{sp} = [Pb^{2+}][Br^-]^2$

$= (0.7x)(1.4x)^2$

or $10.8 \times 10^{-5} = 1.372x^3$

or $x = \sqrt[3]{\frac{10.8 \times 10^{-5}}{1.372}}$

$= 4.18 \times 10^{-2}$

48. Key Idea : *Average life* $= 1.44 \times$ *half-life*

Given, half-life $= 1580$ yr

$\therefore$ Average life $= 1.44 \times 1580$

$= 2275$ yr

$= 2.275 \times 10^3$ yr

49. Cobalt is present in vitamin B_{12}, it is also called cyanocobalamine. Its deficiency is called pernicious aneamia.

50. Key Idea : *Adipic acid + Hexamethylene diamine*

↓ Polymerisation

nylon-66.

$$n H_2N(CH_2)_6—NH_2 + n HO—\overset{\overset{O}{\|}}{C}(CH_2)_4—COOH$$

$\downarrow -nH_2O$

$$\left[HN—(CH_2)_6—NH—\overset{\overset{O}{\|}}{C}—(CH_2)_4—\overset{\overset{O}{\|}}{C} \right]_n$$

nylon-66.

Mathematics

1. Since, $z = x + iy$

$\therefore$ $iz = -y + xi$

and $z + iz = x + iy + i(x + iy)$

$= (x - y) + i(x + y)$

Since, z, iz and $z + iz$ are the vertices of a triangle.

$$\therefore \text{Area of triangle} = \frac{1}{2}\begin{Vmatrix} x & y & 1 \\ -y & x & 1 \\ x-y & x+y & 1 \end{Vmatrix}$$

$$= \frac{1}{2}|[x(x - x - y) - y(-y - x + y) + 1(-xy - y^2 - x^2 + xy)]|$$

$$= \frac{1}{2}|[-xy + xy - y^2 - x^2]|$$

$$= \frac{1}{2}[x^2 + y^2]$$

$$= \frac{|z|^2}{2}$$

2. Let the complex numbers z_1, z_2, z_3 denotes the vertices A, B, C of an equilateral triangle ABC, then if O be the origin we have

$\overline{OA} = z_1, \overline{OB} = z_2, \overline{OC} = z_3$

$\because$ $|z_1| = |z_2| = |z_3|$

$\Rightarrow$ $OA = OB = OC$

$\therefore$ O is the circumcentre of ΔABC, hence $z_1 + z_2 + z_3 = 0$.

3. $\therefore \sum_{n=1}^{200} i^n = i + i^2 + i^3 + \ldots + i^{200}$

($\because$ it is a GP series)

$$= \frac{i(1 - i^{200})}{1 - i} = \frac{i(1 - 1)}{1 - i}$$

$= 0$

4. Let $S = 2 + \frac{1}{2} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^3} + \ldots$

$$= \left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\right) + \left(1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots\right)$$

$$= \frac{1}{1 - \frac{1}{2}} + \frac{1}{1 - \frac{1}{3}} \quad (\because \text{GP series})$$

$$= \frac{2}{1} + \frac{3}{2}$$

$$= \frac{7}{2}$$

5. Since, x, y and z are in GP

$\therefore$ $y^2 = xz$

On taking log on both sides, we get

$2 \log y = \log x + \log z$

On adding 2 on both sides, we get

$2 \log y + 2 = \log x + 1 + \log z + 1$

$\Rightarrow$ $1 + \log x, 1 + \log y, 1 + \log z$ are in AP

$\Rightarrow$ $\frac{1}{1 + \log x}, \frac{1}{1 + \log y}, \frac{1}{1 + \log z}$ are in HP.

6. Key Idea : *If $ax^2 + bx + c$ is a factor of $x^4 + dx^2 + e$, then the roots of the quadratic equation will satisfy the equation.*

Let $f(x) = x^2 - 3x + 2 = 0$

$\Rightarrow$ $(x - 2)(x - 1) = 0$

$\Rightarrow$ $x = 1, 2$

Since, $x^2 - 3x + 2$ be a factor of $x^4 - px^2 + q = 0$.

$\therefore$ The value of x will satisfy it.

$\Rightarrow \quad 1 - p + q = 0$ and $16 - 4p + q = 0$

$\Rightarrow \quad p = 5, q = 4$

7. The condition for common roots gives

$$(bc - a^2)^2 = (ca - b^2)(ab - c^2)$$

$$\Rightarrow b^2c^2 + a^4 - 2a^2bc = a^2bc - c^3a - b^3a + b^2c^2$$

$$\Rightarrow a^4 + c^3a + b^2a = 3a^2bc$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

8. We have, ${}^{15}C_{3r} = {}^{15}C_{r+3}$

$$\Rightarrow {}^{15}C_{15-3r} = {}^{15}C_{r+3}$$

$$\Rightarrow 15 - 3r = r + 3$$

$$\Rightarrow 4r = 12$$

$$\Rightarrow r = 3$$

Note : ${}^nC_r$ is defined only when $n, r > 0$ and $n \geq r$.

9. $9600 = 2^7 \times 3 \times 5^2$

$\therefore$ The number of divisors

$$= (7+1)(1+1)(2+1)$$
$$= 8 \times 2 \times 3$$
$$= 48$$

10. Given, $a_n = \sum_{r=0}^{1} \frac{1}{{}^nC_r}$ ... (i)

Let $b_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$

$$\Rightarrow b_n = \frac{0}{{}^nC_0} + \frac{1}{{}^nC_1} + \frac{2}{{}^nC_2} + \ldots + \frac{n-1}{{}^nC_{n-1}} + \frac{n}{{}^nC_n} \quad \ldots(ii)$$

Or it can be rewritten as

$$b_n = \frac{n}{{}^nC_n} + \frac{n-1}{{}^nC_{n-1}} + \ldots + \frac{1}{{}^nC_1} + \frac{0}{{}^nC_0} \quad \ldots(iii)$$

On adding Eqs. (ii) and (iii), we get

$$2b_n = \frac{n}{{}^nC_0} + \frac{n}{{}^nC_1} + \ldots + \frac{n}{{}^nC_n}$$
$$= n\left[\frac{1}{{}^nC_0} + \frac{1}{{}^nC_1} + \ldots + \frac{1}{{}^nC_n}\right]$$
$$= na_n \quad \text{[from (i)]}$$

$$\Rightarrow b_n = \frac{1}{2}na_n$$

11. **Key Idea :** *The general term in the expansion of* $(x + a)^n$ *is*

$$T_{r+1} = {}^nC_r\, x^{n-r} a^r.$$

Given expansion is $\left(2x^2 - \frac{1}{3x^2}\right)^{10}$

$$\therefore \quad T_{r+1} = {}^{10}C_r\,(2x^2)^{10-r}\left(-\frac{1}{3x^2}\right)^r$$

$$\Rightarrow \quad T_6 = {}^{10}C_5\,(2x^2)^5\left(-\frac{1}{3x^2}\right)^5$$
$$= -\frac{10! \times 32}{5!\,5!} \times \frac{1}{243}$$
$$= -\frac{896}{27}$$

12. $\therefore (1+x)^m (1-x)^n$

$$= \left[1 + mx + \frac{m(m-1)x^2}{2!} + \ldots\right] \times \left[1 - nx + \frac{n(n-1)x^2}{2!} - \ldots\right]$$
$$= 1 + (m-n)x + \left(\frac{n^2 - n}{2} - mn + \frac{m^2 - m}{2}\right)x^2 + \ldots$$

Since, the coefficient of x and x^2 are 3 and -6 respectively.

$\therefore \quad m - n = 3$

and $\quad \frac{n^2 - n}{2} - mn + \frac{m^2 - m}{2} = -6$

$$\Rightarrow \frac{(m-3)(m-4)}{2} - m(m-3) + \frac{m^2 - m}{2} = -6$$

$$\Rightarrow m^2 - 7m + 12 - 2m^2 + 6m + m^2 - m + 12 = 0$$

$$\Rightarrow -2m + 24 = 0$$

$$\Rightarrow m = 12$$

13. Let $S = (1+3)\log_e 3 + \frac{1+3^2}{2!}(\log_e 3)^2 + \ldots$

$$S = \left(\log_e 3 + \frac{1}{2!}(\log_e 3)^2 + \ldots\right) + \left(3\log_e 3 + \frac{1}{2!}(3\log_e 3)^2 + \ldots\right)$$
$$= (e^{\log_e 3} - 1) + (e^{3\log_e 3} - 1)$$
$$= (3-1) + (27-1)$$
$$= 28$$

14. Let $S = \frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2^3} + \frac{1}{5}\cdot\frac{1}{2^5} + \ldots$

$$= \frac{1}{2}\log\left(\frac{1 + \frac{1}{2}}{1 - \frac{1}{2}}\right)$$

$$\left[\because \log\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \ldots\right)\right]$$

$$= \frac{1}{2}\log\left(\frac{3}{1}\right) = \log 3^{1/2}$$

$$= \log\sqrt{3}$$

15. Let $\Delta = \begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ac \\ 1 & c & c^2 - ab \end{vmatrix}$

Applying $(R_2 \to R_2 - R_1, R_3 \to R_3 - R_1)$

$$= \begin{vmatrix} 1 & a & a^2 - bc \\ 0 & b-a & (b-a)(a+b+c) \\ 0 & c-a & (c-a)(a+b+c) \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a & a^2 - bc \\ 0 & 1 & a+b+c \\ 0 & 1 & a+b+c \end{vmatrix}$$

$= 0$ ($\because$ two rows are identical)

16. **Key Idea :** *If the system of equations has non-zero solution, then value of determinant should be zero.*

Given, system of equations are $x + ky - z = 0$, $3x - ky - z = 0$ and $x - 3y + z = 0$ has non-zero solution.

$$\therefore \quad \begin{vmatrix} 1 & k & -1 \\ 3 & -k & -1 \\ 1 & -3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 1(-k-3) - k(3+1) - 1(-9+k) = 0$$

$$\Rightarrow \quad -6k + 6 = 0$$

$$\Rightarrow \quad k = 1$$

Note : If the system of equations has zero solution, then the value of determinant should not be zero.

17. Given that, $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$

$$\therefore \quad A' = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$\Rightarrow$

$$|A'| = 1(-1-8) + 0 - 2(-8+3)$$

$$= -9 + 10 = 1$$

Cofactors of A' are

$C_{11} = -9, C_{12} = 8, C_{13} = -5$

$C_{21} = -8, C_{22} = 7, C_{23} = -4$

$C_{31} = -2, C_{32} = 2, C_{33} = -1$

$$\therefore \quad \text{adj}(A') = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

$$\therefore \quad (A')^{-1} = \frac{\text{adj}(A')}{|A'|}$$

$$= \frac{1}{1}\begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

Note : For the existence of an inverse, the value of that determinant is not equal to zero.

18. Let $A = \begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix}$

$|A| = 8 - 7 = 1$

Cofactors of A are

$C_{11} = 2, C_{12} = -1$

$C_{21} = -7, C_{22} = 4$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{1}\begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}$$

Note : If a matrix, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

$$adj\ (A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

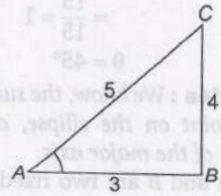
19. **Key Idea :** *In second quadrant tan, cot, cos are negative and sin are positive.*

Given that,

$$3\tan A + 4 = 0 \Rightarrow \tan A = -\frac{4}{3}$$

Since, A lies in IInd quadrant

$$\therefore \quad \cot A = -\frac{3}{4}, \cos A = -\frac{3}{5}, \sin A = \frac{4}{5}$$



$$\therefore \quad 2\cot A - 5\cos A + \sin A$$

$$= 2\left(-\frac{3}{4}\right) - 5\left(-\frac{3}{5}\right) + \frac{4}{5}$$

$$= -\frac{3}{2} + 3 + \frac{4}{5}$$

$$= \frac{-15 + 30 + 8}{10} = \frac{23}{10}$$

20. We have,

$$\sin\theta + \cos\theta = 1$$

$$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\frac{\pi}{4} + \theta\right) = \sin\frac{\pi}{4}$$

$$\Rightarrow \frac{\pi}{4} + \theta = n\pi + (-1)^n \frac{\pi}{4}$$

$$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$$

21. **Key Idea :** $\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}, \forall\, x \in R$

$$\cos\left[2\cos^{-1}\frac{1}{5} + \sin^{-1}\frac{1}{5}\right]$$

$$= \cos\left[\cos^{-1}\frac{1}{5} + \sin^{-1}\frac{1}{5} + \cos^{-1}\frac{1}{5}\right]$$

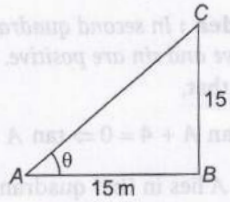
$$= \cos\left(\frac{\pi}{2} + \cos^{-1}\frac{1}{5}\right)$$

$$= -\sin\left(\cos^{-1}\frac{1}{5}\right)$$

$$= -\sin\left(\sin^{-1}\sqrt{\frac{24}{25}}\right)$$

$$= -\sqrt{\frac{24}{25}} = \frac{-2\sqrt{6}}{5}$$

22. Let the height of the roof be 15 m.
In ΔABC,



$$\tan\theta = \frac{BC}{AB}$$

$$= \frac{15}{15} = 1$$

$$\Rightarrow \theta = 45°$$

23. **Key Idea :** *We know, the sum of focal distance of any point on the ellipse, always equal to the length of the major axis.*

$\therefore$ If A and B are two fixed point and P is any variable point, such that $PA + PB = 4$, then the locus of P is an ellipse.

24. The equation of a line passing through a point (2, 2) and is perpendicular to the line $3x + y = 3$ is

$$y - 2 = \frac{1}{3}(x - 2)$$

$$\Rightarrow 3y - 6 = x - 2$$

$$\Rightarrow x - 3y + 4 = 0$$

Since, this line intercept the y-axis, *i. e.*, $x = 0$

$$\Rightarrow -3y + 4 = 0 \Rightarrow y = \frac{4}{3}$$

25. **Key Idea :** *If the equation of pair of lines is* $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, *then*

$$\theta = \frac{2\sqrt{h^2 - ab}}{a + b}.$$

Given equation of pair of lines is

$$x^2 - y^2 - 2xy - 1 = 0$$

Here, $a = 1, b = -1, h = -1$

$$\therefore \tan\theta = \frac{2\sqrt{(-1)^2 + 1}}{1 - 1} = \infty$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

Note : If $h^2 = ab$, then the pair of lines are parallel.

26. Given equation of line is $4x - 3y - 10 = 0$

$$\text{or } y = \frac{4x - 10}{3} \quad ...(i)$$

and equation of circle is

$$x^2 + y^2 - 2x + 4y - 20 = 0$$

$$\Rightarrow x^2 + \left(\frac{4x - 10}{3}\right)^2 - 2x + 4\left(\frac{4x - 10}{3}\right) - 20 = 0$$

$$\Rightarrow x^2 + \frac{(16x^2 + 100 - 80x)}{9} - 2x + \frac{16x - 40}{3} - 20 = 0$$

$$\Rightarrow 25x^2 - 50x - 200 = 0$$

$$\Rightarrow x^2 - 2x - 8 = 0$$

$$\Rightarrow (x - 4)(x + 2) = 0$$

$$\Rightarrow x = 4, -2$$

On putting the values of x in Eq. (i), we get

$$y = 2, -6$$

$\therefore$ Required point of intersections are $(-2, -6), (4, 2)$.

27. Given equation of parabola is

$$y^2 = 4y - 4x$$

$$\Rightarrow y^2 - 4y + 4 = -4x + 4$$

$$\Rightarrow (y - 2)^2 = -4(x - 1)$$

$$\Rightarrow Y^2 = -4X$$

where $Y = y - 2, X = x - 1$

The coordinates of focus are $(-1, 0)$.

i. e., $x - 1 = -1, y - 2 = 0$

$$\Rightarrow x = 0, y = 2$$

$\therefore$ Required coordinate is (0, 2).

28. **Key Idea :** *Let the equation of ellipse be*

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

When $a>b$, $e=\sqrt{1-\frac{b^2}{a^2}}$ *and*

when $b>a$, $e=\sqrt{1-\frac{a^2}{b^2}}$

Given equation is

$$4x^2+9y^2+8x+36y+4=0$$

$$\Rightarrow 4(x^2+2x+1)+9(y^2+4y+4)+4 = 4+36$$

$$\Rightarrow 4(x+1)^2+9(y+2)^2=36$$

$$\Rightarrow \frac{(x+1)^2}{9}+\frac{(y+2)^2}{4}=1$$

$$\therefore e=\sqrt{1-\frac{4}{9}} \quad (\because a>b)$$

$$=\frac{\sqrt{5}}{3}$$

Note : Eccentricity cannot be negative or imaginary.

29. Given equation of curves are

$$y^2=2x/\pi \text{ and } y=\sin x$$

$\therefore$ The point of intersections are (0, 0) and $(\pi/2, 1)$.

$\therefore$ The slope of curve $y^2=2x/\pi$ at point $(\pi/2, 1)$ is

$$m_1=\frac{dy}{dx}=\frac{1}{y\pi}=\frac{1}{\pi}$$

And slope of curve $y=\sin x$ at point $(\pi/2, 1)$ is

$$m_2=\frac{dy}{dx}=\cos x=0$$

$$\therefore \tan\theta=\frac{m_1-m_2}{1+m_1m_2}$$

$$=\frac{\frac{1}{\pi}-0}{1+\frac{1}{\pi}(0)}=\frac{1}{\pi}$$

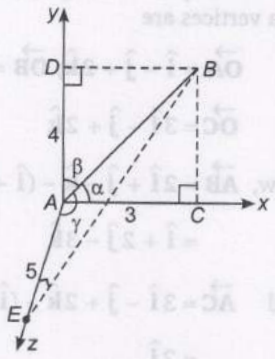
$$\Rightarrow \cot\theta=\pi \Rightarrow \theta=\cot^{-1}\pi$$

Note : Whenever we find the angle of intersection of two curves, we have to find the angle between the tangents at that point.

30. **Key Idea :** *If* $\cos\alpha$, $\cos\beta$ *and* $\cos\gamma$ *are the direction cosines of a line, then*

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$

In ΔABC,



$$\cos\alpha=\frac{3}{AB}$$

In ΔABD,

$$\cos\beta=\frac{4}{AB}$$

And in ΔABE,

$$\cos\gamma=\frac{5}{AB}$$

We know,

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$

$$\Rightarrow \left(\frac{3}{AB}\right)^2+\left(\frac{4}{AB}\right)^2+\left(\frac{5}{AB}\right)^2=1$$

$$\Rightarrow AB^2=9+16+25$$

$$\Rightarrow AB^2=50$$

$$\Rightarrow AB=5\sqrt{2}$$

31. **Key Idea :** *If* a, b *and* c *are the direction ratios of a line, then the direction cosines of a line are*

$$l=\frac{a}{\sqrt{a^2+b^2+c^2}}, m=\frac{b}{\sqrt{a^2+b^2+c^2}},$$

$$n=\frac{c}{\sqrt{a^2+b^2+c^2}}$$

Given direction ratios are 1, – 3 and 2.

$\therefore$ Direction cosines are

$$l=\frac{1}{\sqrt{1^2+(-3)^2+2^2}}, m=-\frac{3}{\sqrt{1^2+(-3)^2+2^2}},$$

$$n=\frac{2}{\sqrt{1^2+(-3)^2+(2)^2}}$$

i.e., $l=\frac{1}{\sqrt{14}}, m=-\frac{3}{\sqrt{14}}, n=\frac{2}{\sqrt{14}}$

Note : In any line has only one direction cosine but the direction ratios may be more than one.

32. If O be the point of origin, the position vectors of a vertices are

$$\vec{OA} = \hat{i} - \hat{j} + 2\hat{k}, \vec{OB} = 2\hat{i} + \hat{j} - \hat{k},$$

$$\vec{OC} = 3\hat{i} - \hat{j} + 2\hat{k}$$

Now, $\vec{AB} = 2\hat{i} + \hat{j} - \hat{k} - (\hat{i} - \hat{j} + 2\hat{k})$

$$= \hat{i} + 2\hat{j} - 3\hat{k}$$

and $\vec{AC} = 3\hat{i} - \hat{j} + 2\hat{k} - (\hat{i} - \hat{j} + 2\hat{k})$

$$= 2\hat{i}$$

$\therefore$ Area of triangle $= \frac{1}{2}|| \vec{AB} \times \vec{AC}||$

$$= \frac{1}{2}\begin{Vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 0 & 0 \end{Vmatrix}$$

$$= \frac{1}{2}|[\hat{i}(0) - \hat{j}(6) + \hat{k}(-4)]|$$

$$= \frac{1}{2}|[-6\hat{j} - 4\hat{k}]|$$

$$= \frac{1}{2}\sqrt{36 + 16} = \frac{1}{2} \times 2\sqrt{13}$$

$$= \sqrt{13} \text{ sq unit.}$$

Note : If area of triangle is zero, then points are collinear.

33. We have,

$$|(\vec{a} \times \vec{b}) \cdot \vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$$

$$\Rightarrow \quad ||\vec{a}||\vec{b}| \sin\theta \hat{n} \cdot \vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$$

Where $\hat{n}$ is a unit vector $\perp$ to $\vec{a} \times \vec{b}$

$$\Rightarrow |\vec{a}||\vec{b}||\vec{c}||\sin\theta \cos\alpha| = |\vec{a}||\vec{b}||\vec{c}|$$

$$\Rightarrow \quad |\sin\theta||\cos\alpha| = 1$$

$$\Rightarrow \quad \theta = \frac{\pi}{2} \text{ and } \alpha = 0$$

$$\Rightarrow \quad \vec{a} \perp \vec{b} \text{ and } \vec{c} \,||\, \hat{n}$$

$$\Rightarrow \vec{a} \perp \vec{b} \text{ and } \vec{c} \perp \text{to both } \vec{a} \text{ and } \vec{b}$$

$$\Rightarrow \vec{a}, \vec{b} \text{ and } \vec{c} \text{ are mutually perpendicular.}$$

$$\Rightarrow \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$$

34. Let $\vec{a} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ and $\vec{b} = 12\hat{i} - 4\hat{j} + 3\hat{k}$

$$\therefore \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{(2\hat{i} + 6\hat{j} + 3\hat{k}) \cdot (12\hat{i} - 4\hat{j} + 3\hat{k})}{\sqrt{4 + 36 + 9}\sqrt{144 + 16 + 9}}$$

$$= \frac{24 - 24 + 9}{7 \cdot 13} = \frac{9}{91}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{9}{91}\right)$$

35. **Key Idea :** *log x is defined for* $x > 0$.

Let $f(x) = \sin \log\left(\frac{\sqrt{4 - x^2}}{1 - x}\right)$

For domain of $f(x)$,

$$\frac{\sqrt{4 - x^2}}{1 - x} > 0$$

$$\Rightarrow \quad 4 - x^2 > 0 \text{ and } 1 - x > 0$$

$$\Rightarrow \quad -2 < x < 2 \text{ and } x < 1$$

$$\Rightarrow \quad x \in (-2, 1)$$

36. Given that

$$\lim_{x \to 9} \frac{f(x) - 9}{x - 9} = 4$$

$$\Rightarrow \lim_{x \to 9} \frac{(\sqrt{f(x)})^2 - (3)^2}{(\sqrt{x})^2 - 3^2} = 4$$

$$\Rightarrow \lim_{x \to 9} \frac{(\sqrt{f(x)} - 3)}{(\sqrt{x} - 3)} \times \lim_{x \to 9} \frac{\sqrt{f(x)} + 3}{\sqrt{x} + 3} = 4$$

$$\Rightarrow \lim_{x \to 9} \frac{(\sqrt{f(x)} - 3)}{(\sqrt{x} - 3)} \times \frac{\sqrt{f(9)} + 3}{3 + 3} = 4$$

$$\Rightarrow \lim_{x \to 9} \frac{(\sqrt{f(x)} - 3)}{(\sqrt{x} - 3)} \times \frac{6}{6} = 4$$

$$[\because f(9) = 9 \text{ given}]$$

$$\Rightarrow \lim_{x \to 9} \frac{(\sqrt{f(x)} - 3)}{(\sqrt{x} - 3)} = 4$$

37. Let $f(x) = \log(x), x > 0$

$\therefore$ It is continuous for every positive value of x.

$$\therefore \quad f\left(\frac{x}{y}\right) = \log\left(\frac{x}{y}\right) = \log(x) - \log(y)$$

$$= f(x) - f(y)$$

$\therefore$ Option (a) is correct.

38. Given, $y = \frac{1}{\sqrt{|x| - x}}$

When $x \geq 0$

$$y = \frac{1}{\sqrt{x - x}} = \infty \quad \text{(not defined)}$$

When $x < 0$

$$y = \frac{1}{\sqrt{-x - x}} = \frac{1}{\sqrt{-2x}}$$

$\therefore$ Given function is defined for every negative values of x.

$\therefore$ Required domain is $(-\infty, 0)$.

39. Let $u = \sin^{-1}\frac{1-x}{1+x}$ and $v = \sqrt{x}$

On differentiating w.r.t. x, respectively

$$\therefore \quad \frac{du}{dx} = \frac{1}{\sqrt{1 - \left(\frac{1-x}{1+x}\right)^2}} \times \frac{(1+x)(-1) - (1-x)(1)}{(1+x)^2}$$

$$= \frac{1}{\sqrt{4x}} \times \frac{(-2)}{1+x}$$

and $$\frac{dv}{dx} = \frac{1}{2\sqrt{x}}$$

$$\therefore \quad \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{-2}{\sqrt{4x}\,(1+x)} \times 2\sqrt{x}$$

$$= \frac{-2}{1+x}$$

40. Let the number be x, then $y = x - x^2$

On differentiating w.r.t. x, on both sides, we get

$$\frac{dy}{dx} = 1 - 2x$$

For greatest or least number, put $\frac{dy}{dx} = 0$

$$\therefore \quad 1 - 2x = 0 \Rightarrow x = \frac{1}{2}$$

Again, differentiating, we get

$$\frac{d^2y}{dx^2} = -2 \text{ which is greatest}$$

$\therefore$ Greatest number is $\frac{1}{2}$.

41. Let $I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}\,dx$

$$= \int \frac{(\sin^4 x - \cos^4 x)(\sin^4 x + \cos^4 x)}{(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x}\,dx$$

$$= \int \frac{(\sin^4 x - \cos^4 x)(\sin^4 x + \cos^4 x)}{(\sin^4 x + \cos^4 x)}\,dx$$

$$= \int (\sin^4 x - \cos^4 x)\,dx$$

$$= \int (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)\,dx$$

$$= -\int (\cos^2 x - \sin^2 x)\,dx$$

$$= -\int \cos 2x\,dx$$

$$= -\frac{\sin 2x}{2} + c$$

42. We have, $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x) = \frac{2A}{\pi}$

$$\Rightarrow \quad \int_0^1 A\sin\left(\frac{\pi x}{2}\right) + B = \frac{2A}{\pi}$$

$$\Rightarrow \quad \left[\frac{-2A\cos\left(\frac{\pi x}{2}\right)}{\pi} + Bx\right]_0^1 = \frac{2A}{\pi}$$

$$\Rightarrow -\frac{2A}{\pi}\cos\left(\frac{\pi}{2}\right) + B + \frac{2A}{\pi}\cos 0° = \frac{2A}{\pi}$$

$$\Rightarrow \quad B + \frac{2A}{\pi} = \frac{2A}{\pi}$$

$$\Rightarrow \quad B = 0$$

Now, $f(x) = A\sin\left(\frac{\pi x}{2}\right)$

$$\Rightarrow \quad f'(x) = \frac{\pi A}{2}\cos\left(\frac{\pi x}{2}\right)$$

$$\Rightarrow \quad f'\left(\frac{1}{2}\right) = \frac{\pi A}{2}\cos\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \quad \sqrt{2} = \frac{\pi A}{2\sqrt{2}}$$

$$\Rightarrow \quad A = \frac{4}{\pi}$$

$$\therefore \quad A = \frac{4}{\pi},\ B = 0$$

43. Given, $f(x) = x - [x]$

$$\therefore \quad \int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} (x - [x])\,dx$$

$$= \int_{-1}^{0} (x - [x])\,dx + \int_{0}^{1} (x - [x])\,dx$$

$$= \int_{-1}^{0} (x + 1)\,dx + \int_{0}^{1} x\,dx$$

$$= \left[\frac{x^2}{2} + x\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1}$$

$$= \left[0 - \frac{1}{2} + 1\right] + \left[\frac{1}{2} - 0\right]$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

Note : For every real value of x, the value of $[x]$ is always an integers.

44. Given differential equation is

$$\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$$

Let $\frac{dy}{dx} = p$

$\therefore \quad p^2 - px + y = 0$

On differentiating w.r.t. x, we get

$$2p\frac{dp}{dx} - p - x\frac{dp}{dx} + \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dp}{dx}(2p - x) = 0 \qquad \left[\because \frac{dy}{dx} = p\right]$$

$$\Rightarrow \quad \frac{dp}{dx} = 0$$

On integrating, we get

$$p = c$$

$$\Rightarrow \quad \frac{dy}{dx} = c$$

On integrating, we get

$$y = cx - c^2$$

If $c = 2$, then $y = 2x - 4$

45. **Key Idea :** *If A and B are independent, then*

$$P(A \cap B) = P(A)\,P(B)$$

$\therefore \quad P(A \cup B) = P(A + B)$

$$= P(A) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - P(A)\,P(B)$$

Note : If A and B are two mutually exclusive events, then $P(A \cap B) = 0$.

46. Total number of balls = 15

$\therefore$ Required probability $= \frac{{}^9C_2}{{}^{15}C_2} = \frac{9 \times 8}{15 \times 14}$

$$= \frac{12}{35}$$

47. Given that, $P(A) = 0.65$, $P(B) = 0.15$

$\therefore \quad P(\bar{A}) + P(\bar{B}) = 1 - P(A) + 1 - P(B)$

$$= 2 - 0.65 - 0.15$$

$$= 1.20$$

48. **Key Idea :** *If both regression coefficients are positive or negative, then the correlation coefficient is positive or negative.*

Given regression equations are

$x = 19.13 - 0.83y$ and $y = 11.64 - 0.50x$

Their corresponding regression coefficients are

$b_{xy} = -0.83$ and $b_{yx} = -0.50$

$\therefore$ Correlation coefficient $= -\sqrt{b_{xy} \times b_{yx}}$

$$= -\sqrt{0.83 \times 0.50}$$

$$= -0.64$$

Note : If regression coefficients are of opposite sign, then correlation coefficient cannot be determined.

49. Roots of $x + \log_{10} x = 3.375$ is approximately 2.909

50. **Key Idea :** *The Newton-Raphson formula for the function $f(x)$ is*

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Let $f(x) = x^2 - 3$

On differentiating w.r.t. x, we get

$$f'(x) = 2x$$

$\therefore$ By using Newton-Raphson of first iteration.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= \frac{3}{2} - \frac{\frac{9}{4} - 3}{2 \times \frac{3}{2}} = \frac{3}{2} + \frac{1}{4} = \frac{7}{4}$$

Second iteration is

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= \frac{7}{4} - \frac{\frac{1}{16}}{\frac{7}{2}} = \frac{7}{4} - \frac{1}{56}$$

$$= \frac{97}{56}.$$