5. Coordinate geometry

Exercise 5.1

1. Question

Find the midpoint of the line segment joining the points

(i) (1, – 1) and (–5, 3) (ii) (0,0) and (0,4)

Answer

(i). Midpoint of line–segment joining the points (x_1, y_1) and (x_2, y_2)

M (x, y) = $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Midpoint of line–segment joining the points (1, –1) and (–5, 3)

M (x, y) = $\left(\frac{1+(-5)}{2}, \frac{(-1)+3}{2}\right)$

= $\left(-\frac{4}{2}, \frac{2}{2}\right)$

= (–2, 1)

(ii). Midpoint of line–segment joining the points (x_1, y_1) and (x_2, y_2)

M (x, y) = $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Midpoint of line–segment joining the points (0, 0) and (0, 4)

M (x, y) = $\left(\frac{0+0}{2}, \frac{0+4}{2}\right)$

= $\left(\frac{0}{2}, \frac{4}{2}\right)$

= (0, 2)

2. Question

Find the centroid of the triangle whose vertices are

(i) (1,3), (2, 7) and (12, – 16)

(ii) (3, – 5), (–7, 4) and (10, – 2)

Answer

i). The centroid G (x, y) of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by:

G (x, y) = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

We have (x_1, y_1) = (1,3), (x_2, y_2) = (2, 7) and (x_3, y_3) = (12, – 16)

G (x, y) = $\left(\frac{1+2+12}{3}, \frac{3+7+(-16)}{3}\right)$

= $\left(\frac{15}{3}, -\frac{6}{3}\right)$

= (5, –2)

ii). The centroid G (x, y) of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by:

G (x, y) = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

We have (x_1, y_1) = (3, –5), (x_2, y_2) = (–7, 4) and (x_3, y_3) = (10, –2)

G (x, y) = $\left(\frac{3+(-7)+10}{3}, \frac{(-5)+4+(-2)}{3}\right)$

= $\left(\frac{6}{3}, -\frac{3}{3}\right)$

= (2, –1)

3. Question

The center of a circle is at (–6, 4). If one end of a diameter of the circle is at the origin, then find the other end.

Answer

Here, one end of the diameter is at origin that is A (0,0) and let B (a, b) is the required endpoint of the diameter.

Center {O (–6, 4)} of the circle will be exactly middle of the diameter. So, to find another we have to use the mid–point formula

Midpoint of line–segment joining the points (x_1, y_1) and (x_2, y_2)

M (x, y) = M$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

⇒ (-6, 4) = $\left(\frac{0+a}{2}, \frac{0+b}{2}\right)$

⇒ (-6, 4) = $\left(\frac{a}{2}, \frac{b}{2}\right)$

$\frac{a}{2} = -6$ and $\frac{b}{2} = 4$

a = –6 × 2 and b = 4 × 2

a = –12 and b = 8

Therefore, another end point of the diameter (–12, 8)

4. Question

If the centroid of a triangle is at (1, 3) and two of its vertices are (–7, 6) and (8, 5) then find the third vertex of the triangle.

Answer

Let A (–7, 8), B (8, 5) and C (a, b) are three vertices of the triangle and centroid of the triangle is (1, 3)

The centroid G (x, y) of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by:

G (x, y) = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

⇒ (1, 3) = $\left(\frac{(-7)+8+a}{3}, \frac{6+5+b}{3}\right)$

⇒ (1, 3) =$\left(\frac{1+a}{3}, \frac{11+b}{3}\right)$

$\frac{1+a}{3} = 1$ and $\frac{11+b}{3} = 3$

1 + a = 3× 1 and 11 + b = 3 × 3

1 + a = 3 and 11 + b = 9

a = 3 – 1 and b = 9 – 11

a = 2 and b = –2

Therefore, the missing vertex of the triangle is (2, –2).

5. Question

Using the section formula, show that the points A (1,0), B (5,3), C (2,7) and D (–2, 4) are the vertices of a parallelogram taken in order.

Answer

The mid–point of diagonals AC and diagonal BD coincide.

Thus, Section Formula internally = $\left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$

Where l = 1 and m = 1

Mid–point of diagonal AC

A (1, 0) and C (2, 7)

The mid–point of diagonal is in the ratio of 1:1

$= \left(\frac{(1\times2)+(1\times1)}{1+1}, \frac{(1\times7)+(1\times0)}{1+1}\right)$

$= \left(\frac{2+1}{2}, \frac{7+0}{2}\right)$

$= \frac{3}{2}, \frac{7}{2}$

Mid–point of diagonal AC

B (5, 3) and D (–2, 4)

The mid–point of diagonal is in the ratio of 1:1

$= \left(\frac{(1\times-2)+(1\times5)}{1+1}, \frac{(1\times4)+(1\times3)}{1+1}\right)$

$= \left(\frac{-2+5}{2}, \frac{4+3}{2}\right)$

$= \frac{3}{2}, \frac{7}{2}$

Two diagonals are meeting at the same point. So, the given vertex forms a parallelogram.

6. Question

Find the coordinates of the point which divides the line segment joining (3, 4) and (–6, 2) in the ratio 3: 2 externally.

Answer

Section formula externally = $\left(\frac{lx_2-mx_1}{l-m}, \frac{ly_2-my_1}{l-m}\right)$

Where l = 3 and m = 2

A (3, 4) and B (–6, 2)

$= \left(\frac{(3\times(-6))-(2\times3)}{3-2}, \frac{(3\times2)-(2\times4)}{3-2}\right)$

$= \left(\frac{-18-6}{1}, \frac{6-8}{1}\right)$

= (–24, –2)

Therefore, the coordinates of point which divides the line is (–24, –2)

7. Question

Find the coordinates of the point which divides the line segment joining (–3, 5) and (4, –9) in the ratio 1: 6 internally.

Answer

Section Formula internally = $\left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$

Where l = 1 and m = 6

A (–3, 5) and B (4, –9)

= $\left(\frac{(1\times4)+(6\times-3)}{1+6}, \frac{(1\times-9)+(6\times5)}{1+6}\right)$

= $\left(\frac{4-18}{7}, \frac{-9+30}{7}\right)$

= $\left(-\frac{14}{7}, \frac{21}{7}\right)$

= (–2, 3)

Therefore, the coordinates of point which divides the line is (–2, 3)

8. Question

Let A (–6, –5) and B (–6, 4) be two points such that a point P on the line AB satisfies AP = $\frac{2}{9}$ AB. Find the point P.

Answer

$AP = \frac{2}{9}AB$

⇒ 9 AP = 2 AB

⇒ 9 AP = 2(AP + PB)

⇒ 9AP = 2AP + 2PB

⇒ 9AP – 2AP = 2PB

⇒ 7AP = 2 PB

⇒ $\frac{AP}{PB} = \frac{7}{2}$

AP: PB = 7:2

So, P divides the line segment in the ratio is 2:7

Section Formula internally = $\left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$

Where l = 2 and m = 7

A (–6, –5) and B (–6, 4)

= $\left(\frac{(2\times(-6))+(7\times-6)}{2+7}, \frac{(2\times4)+(7\times-5)}{2+7}\right)$

= $\left(\frac{-12-42}{9}, \frac{8-35}{9}\right)$

= $\left(-\frac{54}{9}, -\frac{27}{9}\right)$

= (–6, –3)

Therefore, the point P is (–6, –3).

9. Question

Find the points of trisection of the line segment joining the points A (2, –2) and B (–7, 4).

Answer

Let P and Q are the points of the intersection of the line segment joining the points A and B.

Here, AP = PQ = QB

AP = 1 PQ = 1 QB = 1

Section Formula internally = $\left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$

P divides line segment AB in the ratio 1:2

Where l = 1 and m = 2

A (2, –2) and B (–7, 4)

$= \left(\frac{(1\times(-7))+(2\times2)}{1+2}, \frac{(1\times4)+(2\times-2)}{1+2}\right)$

$= \left(\frac{-7+4}{3}, \frac{4-4}{3}\right)$

$= \left(-\frac{3}{3}, \frac{0}{3}\right)$

= (–1,0)

Q divides line segment AB in the ratio 2: 1

Where l = 2 and m = 1

A (2, –2) and B (–7, 4)

$= \left(\frac{(2\times(-7))+(1\times2)}{2+1}, \frac{(2\times4)+(1\times-2)}{2+1}\right)$

$= \left(\frac{-14+2}{3}, \frac{8-2}{3}\right)$

$= \left(-\frac{12}{3}, \frac{6}{3}\right)$

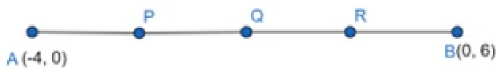
= (–4,2)

Therefore, the coordinates of point P (–1, 0) and Q (–4, 2)

10. Question

Find the points which divide the line segment joining A (–4 ,0) and B (0,6) into four equal parts.

Answer



Let P, Q and R are the points of the line segment joining the line segment A and B.

Here AP = PQ = QR= RB

AP = 1 PQ = 1 QR = 1 and PB = 1

Section Formula internally = $\left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$

P divides line segment AB in the ratio 1:3

Where l = 1 and m = 3

A (–4, 0) and B (0, 6)

$= \left(\frac{(1\times(0))+(3\times-4)}{1+3}, \frac{(1\times6)+(3\times 0)}{3+1}\right)$

$= \left(\frac{0-12}{4}, \frac{6-0}{4}\right)$

$= \left(-\frac{12}{4}, \frac{6}{4}\right)$

$= \left(-3, \frac{3}{2}\right)$

Section Formula internally $= \left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$

Q divides line segment AB in the ratio 2: 2

Where l = 2 and m = 2

A (–4, 0) and B (0, 6)

$= \left(\frac{(2\times(0))+(2\times-4)}{2+2}, \frac{(2\times6)+(2\times 0)}{2+2}\right)$

$= \left(\frac{0-8}{4}, \frac{12-0}{4}\right)$

$= \left(-\frac{8}{4}, \frac{12}{4}\right)$

= (–2, 3)

Section Formula internally $= \left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$

R divides line segment AB in the ratio 3:1

Where l = 3 and m = 1

A (–4, 0) and B (0, 6)

$= \left(\frac{(3\times(0))+(1\times-4)}{3+1}, \frac{(3\times6)+(1\times 0)}{3+1}\right)$

$= \left(\frac{0-4}{4}, \frac{18-0}{4}\right)$

$= \left(-\frac{4}{4}, \frac{18}{4}\right)$

$= \left(-1, \frac{9}{2}\right)$

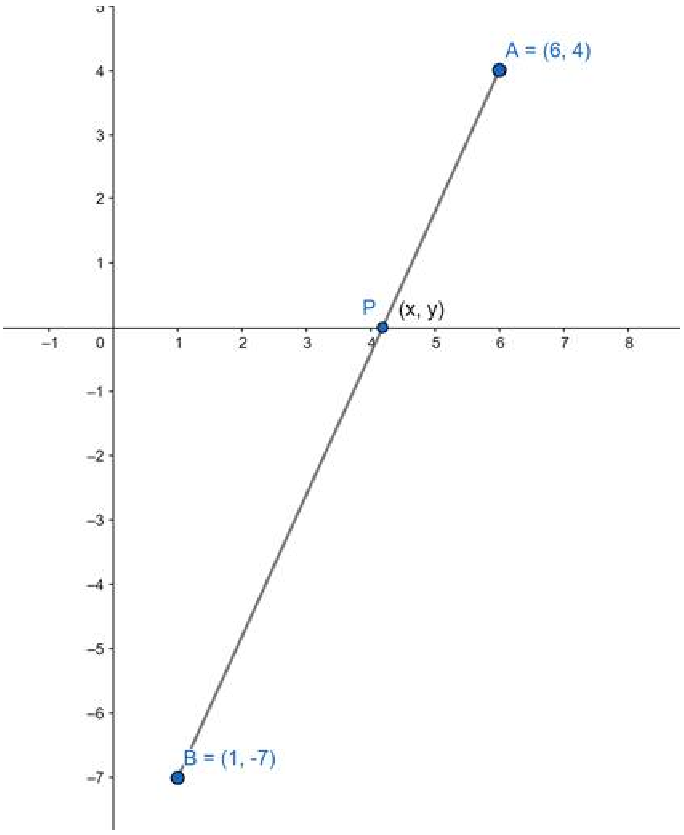
Therefore, the coordinates of point P$\left(-3, \frac{3}{2}\right)$, Q (–2, 3) and R$\left(-1, \frac{9}{2}\right)$.

11. Question

Find the ratio in which the x–axis divides the line segment joining the points (6, 4) and (1, –7).

Answer

Let l:m be the ratio of the line segment joining the points (6,4) and (1, –7) and let p (x, 0) be the point on x–axis.



Section formula internally: $\left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$

(x, 0) = $\left(\frac{l(1)+m(6)}{l+m}, \frac{l(-7)+m(4)}{l+m}\right)$

(x, 0) = $\left(\frac{l+6m}{l+m}, \frac{-7l+4m}{l+m}\right)$

Equating the y– coordinates

$$\frac{-7l+4m}{l+m} = 0$$

–7l +4 m = 0

–7l = –4m

$$\frac{l}{m} = \frac{4}{7}$$

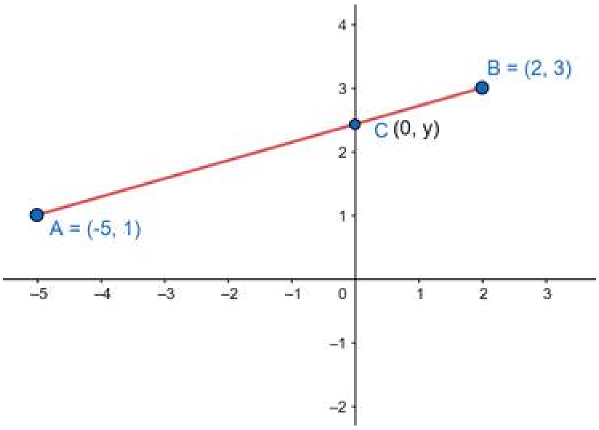
l:m = 4: 7

Therefore, x–axis divides the line segment in the ratio 4: 7 internally.

12. Question

In what ratio is the line joining the points (–5, 1) and (2, 3) divided by the y–axis? Also, find the point of intersection.

Answer



Let l:m be the ratio of the line segment joining the points (–5,1) and (2, 3) and let C (x, 0) be the point on x–axis.

Section formula internally: $\left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}\right)$

(x, 0) = $\left(\frac{l(2)+m(-5)}{l+m}, \frac{l(3)+m(1)}{l+m}\right)$

(x, 0) = $\left(\frac{2l-5m}{l+m}, \frac{3l+m}{l+m}\right)$

Equating the x– coordinates

$$\frac{2l-5m}{l+m} = 0$$

2l – 5m = 0

2l = 5m

$$\frac{l}{m} = \frac{5}{2}$$

l:m = 5: 2

Point of intersection = $\left(\frac{5(2)+2(-5)}{5+2}, \frac{5(3)+2(1)}{5+2}\right)$

= $\left(\frac{10-10}{7}, \frac{15+2}{7}\right)$

= $\left(0, \frac{17}{7}\right)$

Therefore, y–axis divides the line segment in the ratio 5: 2 internally and point of intersection is $\left(0, \frac{17}{7}\right)$.

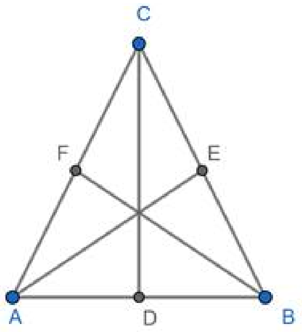
13. Question

Find the length of the medians of the triangle whose vertices are (1,–1), (0, 4) and (–5,3).

Answer

Let A (1, –1), B (0, 4) and C (–5, 3) are the points vertices of triangle.

Let D, E and F are the mid–points of the sides AB, BC and AC respectively.



Mid – point formula = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

<u>Mid – point of AB</u> = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

D = $\left(\frac{1+0}{2}, \frac{-1+4}{2}\right)$

D = $\frac{1}{2}, \frac{3}{2}$

<u>Mid – point of BC</u> = $\left(\frac{x_2+3}{2}, \frac{y_2+y_3}{2}\right)$

E = $\left(\frac{0+(-5)}{2}, \frac{4+3}{2}\right)$

E = $-\frac{5}{2}, \frac{7}{2}$

<u>Mid – point of AC</u> = $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

F = $\left(\frac{1+(-5)}{2}, \frac{-1+3}{2}\right)$

F = $\frac{-4}{2}, \frac{2}{2}$

F = (–2, 1)

Distance formula = √ $(x_1 - x_2)^2 + (y_1 - y_2)^2$

A (1, –1) and E $\left(-\frac{5}{2}, \frac{7}{2}\right)$

Length of AE = $\sqrt{\left(1+\frac{5}{2}\right)^2 + \left(-1-\frac{7}{2}\right)^2}$

$= \sqrt{\left(\frac{2+5}{2}\right)^2 + \left(\frac{-2-7}{2}\right)^2}$

$= \sqrt{\left(\frac{7}{2}\right)^2 + \left(\frac{-9}{2}\right)^2}$

$= \sqrt{\frac{49}{4} + \frac{81}{4}}$

$= \sqrt{\frac{49+81}{4}}$

$= \sqrt{\frac{130}{4}} = \frac{\sqrt{130}}{2}$

B (0, 4) and F (–2, 1)

Length of BF = $\sqrt{(0-(-2))^2 + (4-1)^2}$

$=\sqrt{(2)^2+(3)^2}$

$=\sqrt{4+9}$

$=\sqrt{13}$

A (–5,3) and D$\left(\frac{1}{2},\frac{3}{2}\right)$

Length of AE $=\sqrt{\left(-5-\frac{1}{2}\right)^2+\left(3-\frac{3}{2}\right)^2}$

$=\sqrt{\left(\frac{-10-1}{2}\right)^2+\left(\frac{6-3}{2}\right)^2}$

$=\sqrt{\left(\frac{-11}{2}\right)^2+\left(\frac{3}{2}\right)^2}$

$=\sqrt{\frac{121}{4}+\frac{9}{4}}$

$=\sqrt{\frac{121+9}{4}}$

$=\sqrt{\frac{130}{4}}=\frac{\sqrt{130}}{2}$

Exercise 5.2

1 A. Question

Find the area of the triangle formed by the points

(0, 0), (3, 0) and (0, 2)

Answer

(0, 0), (3, 0) and (0, 2)

Area of triangle $=\frac{1}{2}\{(x_1y_2+x_2y_3+x_3y_1)-(x_2y_1+x_3y_2+x_1y_3\}$

$x_1=0$, $x_2=3$ and $x_3=0$

$y_1=0$, $y_2=0$ and $y_3=2$

$=\frac{1}{2}\{\big((0\times 0)+(3\times 2)+(0\times 0)\big)-\big((3\times 0)+(0\times 0)+(0\times 2)\big)$

$=\frac{1}{2}\{(0+6+0)-(0+0+0)\}$

$=\frac{1}{2}\{6-0\}$

$=\frac{1}{2}\{6\}$

= 3sq. units

1 B. Question

Find the area of the triangle formed by the points

(5, 2), (3, –5) and (–5, –1)

Answer

(5, 2), (3, –5) and (–5, –1)

Let A (–5, –1) B (3, –5) and C (5, 2) be the vertices of triangle.

Area of triangle $= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3\}$

x_1 = –5, x_2 = 3 and x_3 = 5

y_1 = –1, y_2 = –5 and y_3 = 2

$= \frac{1}{2}\{((-5 \times -5) + (3 \times 2) + (5 \times -1)) - ((3 \times -1) + (5 \times -5) + (-5 \times 2))$

$= \frac{1}{2}\{(25 + 6 - 5) - (-3 - 25 - 10)\}$

$= \frac{1}{2}\{26 + 38\}$

$= \frac{1}{2}\{64\}$

= 32 sq. units

1 C. Question

Find the area of the triangle formed by the points

(–4, –5), (4, 5) and (–1, –6)

Answer

(–4, –5), (4, 5) and (–1, –6)

Let A (–4, –5) B (–1, –6) and C (4,5) be the vertices of triangle.

Area of triangle $= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3\}$

x_1 = –4, x_2 = –1 and x_3 = 4

y_1 = –5, y_2 = –6 and y_3 = 5

$= \frac{1}{2}\{((-4 \times -6) + (-1 \times 5) + (4 \times -5)) - ((-1 \times -5) + (4 \times -6) + (-4 \times 5))$

$= \frac{1}{2}\{(24 - 5 - 20) - (-5 - 24 - 20)\}$

$= \frac{1}{2}\{-1 - (-39)\}$

$= \frac{1}{2}\{-1 + 39\}$

$= \frac{1}{2}\{38\}$

= 19 sq. units

2 A. Question

Vertices of the triangles taken in order and their areas are given below. In each of the following find the value of a.

Vertices: (0, 0), (4, a), (6, 4)

Area (in sq. units): 17

Answer

Vertices of triangle A (0, 0), B (4, a) and C (6, 4)

Area of triangle = 17 sq. units

Area of triangle $= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3\}$

$x_1 = 0$, $x_2 = 4$ and $x_3 = 6$

$y_1 = 0$, $y_2 = a$ and $y_3 = 4$

$\Rightarrow 17 = \frac{1}{2}\{\big((0 \times a) + (4 \times 4) + (6 \times 0)\big) - \big((4 \times 0) + (6 \times a) + (0 \times 4)\big)$

$\Rightarrow 17 \ = \frac{1}{2}\{(0 + 16 + 0) - (0 + 6a + 0)\}$

$\Rightarrow 17 = \frac{1}{2}\{ 16 - 6a\}$

⇒ 17 × 2 = 16 – 6a

⇒ 34 = 16 – 6a

⇒ 34 + 6a = 16

⇒ 6a = 16 – 34

⇒ 6a = – 18

$\Rightarrow a = \frac{-18}{6}$

⇒ a = –3

Therefore, the required vertices are (4, –3)

2 B. Question

Vertices of the triangles taken in order and their areas are given below. In each of the following find the value of a.

Vertices: (a, a), (4, 5), (6, –1)

Area (in sq. units): 9

Answer

Vertices of triangle A (a, a), B (4, 5) and C (6, –1)

Area of triangle = 9 sq. units

Area of triangle $= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3\}$

$x_1 = a$, $x_2 = 4$ and $x_3 = 6$

$y_1 = a$, $y_2 = 5$ and $y_3 = -1$

$\Rightarrow 9 = \frac{1}{2}\{\big((a \times 5) + (4 \times -1) + (6 \times a)\big) - \big((4 \times a) + (6 \times 5) + (a \times -1)\big)$

$\Rightarrow 9 \ = \frac{1}{2}\{(5a - 4 + 6a) - (4a + 30 - a)\}$

$\Rightarrow 9 = \frac{1}{2}\{ 11a - 4 - 3a - 30\}$

$\Rightarrow 9 = \frac{1}{2}\{ 8a - 34\}$

⇒ 9 × 2 = 8a – 34

⇒ 18 = 8a – 34

⇒ 8a = 18 +34

$\Rightarrow 8a = 52$

$\Rightarrow a = \frac{52}{8}$

$\Rightarrow a = \frac{13}{2}$

Therefore, the required vertices are $\left(\frac{13}{2}, \frac{13}{2}\right)$

2 C. Question

Vertices of the triangles taken in order and their areas are given below. In each of the following find the value of a.

Vertices: (a, –3), (3, a), (–1,5)

Area (in sq. units): 12

Answer

Vertices of triangle A (a, –3), B (3, a) and C (–1, 5)

Area of triangle = 12 sq. units

Area of triangle $= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3\}$

$x_1 = a$, $x_2 = 3$ and $x_3 = -1$

$y_1 = -3$, $y_2 = a$ and $y_3 = 5$

$\Rightarrow 12 = \frac{1}{2}\{\big((a \times a) + (3 \times 5) + (-1 \times -3)\big) - \big((3 \times -3) + (-1 \times a) + (a \times 5)\big)$

$\Rightarrow 12 = \frac{1}{2}\{(a^2 + 15 + 3) - (-9 - a + 5a)\}$

$\Rightarrow 12 = \frac{1}{2}\{ a^2 + 18 + 9 - 4a\}$

$\Rightarrow 12 \times 2 = a^2 - 4a + 27$

$\Rightarrow 24 = a^2 - 4a + 27$

$\Rightarrow a^2 - 4a + 27 - 24 = 0$

$\Rightarrow a^2 - 4a + 3$

$\Rightarrow a^2 - 3a - a + 3 = 0$

$\Rightarrow a (a - 3) - (a - 3) = 0$

$\Rightarrow (a - 3) (a - 1) = 0$

$a - 3 = 0$ or $a - 1 = 0$

$a = 3$ or $a = 1$

Therefore, the required vertices are (3, –3) or (1, –3)

3 A. Question

Determine if the following set of points are collinear or not.

(4, 3), (1, 2) and (–2, 1)

Answer

(4, 3), (1, 2) and (–2, 1)

Let A (4, 3) B (1, 2) and C (–2, 1) be the vertices of triangle.

Area of triangle = $\frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3\}$

x_1 = 4, x_2 = 1 and x_3 = –2

y_1 = 3, y_2 = 2 and y_3 = 1

$\Rightarrow \frac{1}{2}\{\left((4\times 2)+(1\times 1)+(-2\times 3)\right)-\left((1\times 3)+(-2\times 2)+(4\times 1)\right)\}$

$\Rightarrow \frac{1}{2}\{(8+1-6)-(3-4+4)\}$

$\Rightarrow \frac{1}{2}\{3-3\}$

$\Rightarrow \frac{1}{2}\times 0 = 0$

Therefore, the given points are collinear.

3 B. Question

Determine if the following set of points are collinear or not.

(–2, –2), (–6, –2) and (–2, 2)

Answer

(–2, –2), (–6, –2) and (–2, 2)

Let A (–2, –2) B (–6, –2) and C (–2, 2) be the vertices of triangle.

Area of triangle = $\frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3\}$

x_1 = -2, x_2 = -6 and x_3 = -2

y_1 = -2, y_2 = -2 and y_3 = 2

$\Rightarrow \frac{1}{2}\{\left((-2\times -2)+(-6\times 2)+(-2\times -2)\right)-\left((-6\times -2)+(-2\times -2)+(-2\times 2)\right)\}$

$\Rightarrow \frac{1}{2}\{(4-12+4)-(12+4-4)\}$

$\Rightarrow \frac{1}{2}\{-4-12\}$

$\Rightarrow \frac{1}{2}\times(-16) = -8$

Therefore, the given points are non – collinear.

3 C. Question

Determine if the following set of points are collinear or not.

$\left(-\frac{3}{2},3\right)$, (6, –2) and (–3, 4)

Answer

$\left(-\frac{3}{2},3\right)(6,-2)$ and $(3,-4)$

Let A $\left(-\frac{3}{2},3\right)$ B (6, –2) and C (3, –4) be the vertices of triangle.

Area of triangle = $\frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3\}$

$x1 = -\frac{3}{2}$, x_2 = 6 and x_3 = 3

$y_1 = 3$, $y_2 = -2$ and $y_3 = 4$

$\Rightarrow \frac{1}{2}\left\{\left(\left(-\frac{3}{2} \times -2\right) + (6 \times 4) + (3 \times 3)\right) - \left((6 \times 3) + (4 \times -2) + \left(-\frac{3}{2} \times 4\right)\right)\right\}$

$\Rightarrow \frac{1}{2}\{(3 + 24 - 9) - (18 + 6 - 6)\}$

$\Rightarrow \frac{1}{2}\{18 - 18\}$

$\Rightarrow \frac{1}{2} \times 0 = 0$

Therefore, the given points are collinear

4 A. Question

In each of the following, find the value of k for which the given points are collinear.

(k, –1), (2, 1) and (4, 5)

Answer

(k, –1), (2, 1) and (4, 5)

Area of triangle $= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3\}$

$x_1 = k$, $x_2 = 2$ and $x_3 = 4$

$y_1 = -1$, $y_2 = 1$ and $y_3 = 5$

if points are collinear then, area of triangles are collinear.

$\Rightarrow 0 = \frac{1}{2}\left\{\left((k \times 1) + (2 \times 5) + (4 \times -1)\right) - \left((2 \times -1) + (4 \times 1) + (k \times 5)\right)\right\}$

$\Rightarrow 0 = \frac{1}{2}\{(k + 10 - 4) - (-2 + 4 + 5k)\}$

$\Rightarrow 0 = \frac{1}{2}\{(k + 6) - (2 + 5k)\}$

$\Rightarrow k + 6 - 2 - 5k = 0$

$\Rightarrow 4 - 4k = 0$

$\Rightarrow 4 = 4k$

$\Rightarrow k = \frac{4}{4}$

$\Rightarrow k = 1$

4 B. Question

In each of the following, find the value of k for which the given points are collinear.

(2, – 5), (3, – 4) and (9, k)

Answer

(2, –5), (3, 4) and (9, k)

Area of triangle $= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3\}$

$x_1 = 2$, $x_2 = 3$ and $x_3 = 9$

$y_1 = -5$, $y_2 = -4$ and $y_3 = k$

if points are collinear then, area of triangles are collinear.

$\Rightarrow 0 = \frac{1}{2}\left\{\left((2 \times -4) + (3 \times k) + (9 \times -5)\right) - \left((3 \times -5) + (9 \times 4) + (2 \times k)\right)\right\}$

$\Rightarrow 0 = \frac{1}{2}\{\,(-8 + 3k - 45) — (-15 - 36 + 2k)\}$

$\Rightarrow 0 = \frac{1}{2}\{\,(3k - 53) — (-51 + 2k)\}$

$\Rightarrow 0 = \frac{1}{2}\{\,3k - 53 + 51 - 2k\}$

$\Rightarrow k - 2 = 0$

$\Rightarrow k = 2$

4 C. Question

In each of the following, find the value of k for which the given points are collinear.

(k, k), (2, 3) and (4, – 1)

Answer

(k, k) (2, 3) and (4, –1)

Area of triangle $= \frac{1}{2}\{(x_1y_2 + \ x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3\}$

$x_1 = k$, $x_2 = 2$ and $x_3 = 4$

$y_1 = k$, $y_2 = 3$ and $y_3 = -1$

$\Rightarrow 0 = \frac{1}{2}\{\,\big((k \times 3) + (2 \times -1) + (4 \times k)\big) - \big((2 \times k) + (4 \times 3) + (k \times -1)\big)\}$

$\Rightarrow 0 = \frac{1}{2}\{(3k - 2 + 4k) - (2k + 12 - k)\}$

$\Rightarrow 0 = \frac{1}{2}\{(7k - 2) - (k + 12)\}$

$\Rightarrow 7k - 2 - k - 12 = 0$

$\Rightarrow 6k - 14 = 0$

$\Rightarrow 6k = 14$

$\Rightarrow k = \frac{14}{6} = \frac{7}{3}$

5 A. Question

Find the area of the quadrilateral whose vertices are

(6, 9), (7, 4), (4,2) and (3,7)

Answer

When the vertices of a quadrilateral is given then its area is given by $\frac{1}{2}\{(x_1-x_3)(y_2-y_4) - (x_2-x_4)(y_1-y_3)\}$

We must take all the vertices in counter clock wise direction otherwise it will give solution in negative.

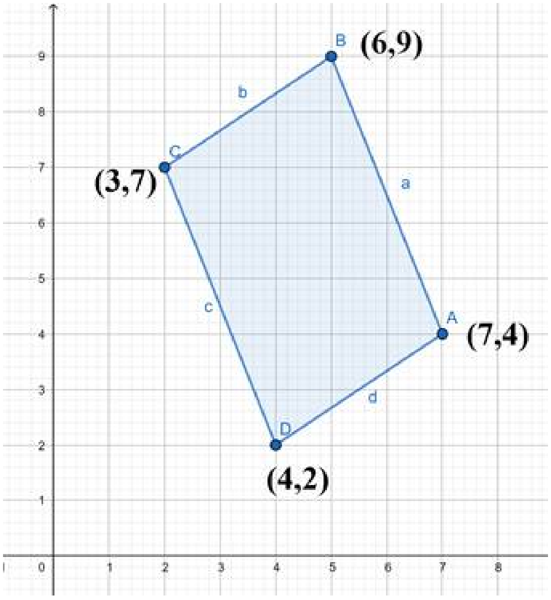
So, from the figure we assume that

A $(x_1, y_1) = (7,4)$

B $(x_2, y_2) = (6, 9)$

C $(x_3, y_3) = (3, 7)$

D $(x_4, y_4) = (4, 2)$



The area of the quadrilateral is $\frac{1}{2}\{(x_1-x_3)(y_2-y_4) - (x_2-x_4)(y_1-y_3)\}$

$= \frac{1}{2}\{(7-3)(9-2) - (6-4)(4-7)\}$

$= \frac{1}{2}\{28-(-6)\} = 17$ Sq. units

5 B. Question

Find the area of the quadrilateral whose vertices are

(–3, 4), (–5, – 6), (4, – 1) and (1, 2)

Answer

When the vertices of a quadrilateral is given then its area is given by $\frac{1}{2}\{(x_1-x_3)(y_2-y_4) - (x_2-x_4)(y_1-y_3)\}$

We must take all the vertices in counter clock wise direction otherwise it will give solution in negative.

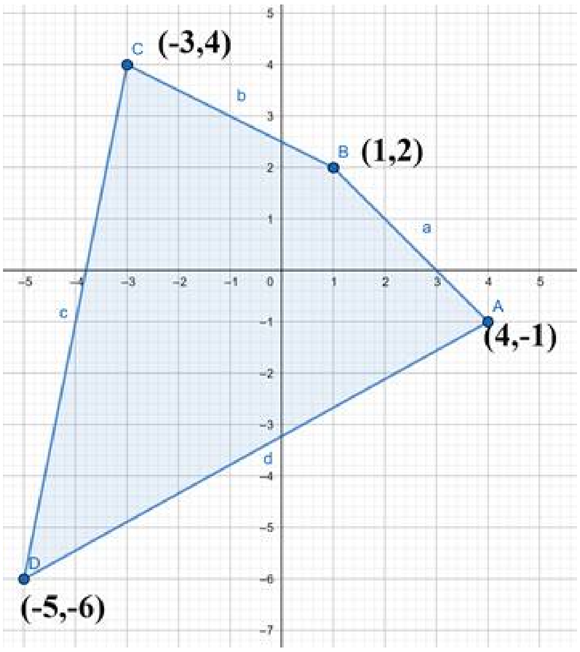
So, from the figure we assume that

A $(x_1, y_1) = (4, -1)$

B $(x_2, y_2) = (1, 2)$

C $(x_3, y_3) = (-3, 4)$

D $(x_4, y_4) = (-5, -6)$



The area of the quadrilateral is $\frac{1}{2}\{(x_1-x_3)(y_2-y_4) - (x_2-x_4)(y_1-y_3)\}$

$= \frac{1}{2}\{(4-(-3))(2-(-6)) - (-1-4)(1-(-5))\}$

$= \frac{1}{2}\{56+30\} = 43$ Sq. units

5 C. Question

Find the area of the quadrilateral whose vertices are

(–4, 5), (0, 7), (5, – 5) and (–4, – 2)

Answer

We must take all the vertices in counter clock wise direction otherwise it will give solution in negative.

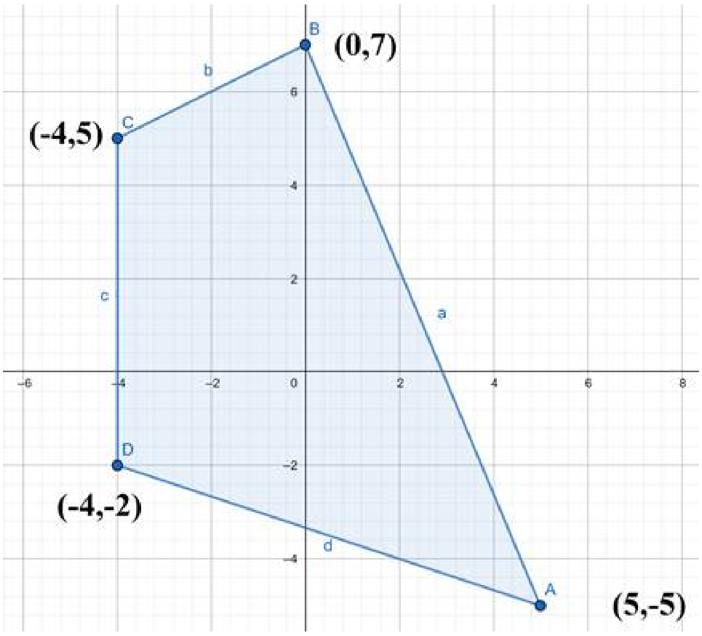
So, from the figure we assume that

A (x_1, y_1) = (5, –5)

B (x_2, y_2) = (0, 7)

C (x_3, y_3) = (–4, 5)

D (x_4, y_4) = (–4, –2)



The area of the quadrilateral is $\frac{1}{2}\{(x_1-x_3)(y_2-y_4) - (x_2-x_4)(y_1-y_3)\}$

$= \frac{1}{2}\{(5-(-4))(7-(-2)) -(-5-5)(0-(-4))\}$

$= \frac{1}{2}\{81+40\} = 60.5$ Sq. units

6. Question

If the three points (h, 0), (a, b) and (0, k) lie on a straight line, then using the area of the triangle formula, show that $\frac{a}{h}+\frac{b}{k}=1$ where h, k ≠ 0.

Answer

Let A(h,0) B(am) and C (0, k) are the three points

Since the three points A(h,0) B (a, b) and C (0, k) lie on a straight line we can say that the three points are collinear.

So, area of triangle ABC = 0

Area of triangle $= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3\}$

$x_1 = h$, $x_2 = a$ and $x_3 = 0$

$y_1 = 0$, $y_2 = b$ and $y_3 = k$

$\Rightarrow 0 = \frac{1}{2}\{((h \times b) + (a \times k) + (0 \times h)) - ((a \times 0) + (0 \times b) + (h \times k)\}$

$\Rightarrow 0 = \frac{1}{2}\{(hb + ak + 0) - (0 + 0 + kh)\}$

$\Rightarrow 0 = \frac{1}{2}\{hb + ak - kh\}$

$\Rightarrow 0 = hb + ak - kh$

$\Rightarrow hab + ak = kh$

Divided by (kh) on both sides we get

$\Rightarrow \frac{hb}{kh} + \frac{ak}{kh} = \frac{kh}{kh}$

$\Rightarrow \frac{b}{k} + \frac{a}{h} = 1$

Hence proved.

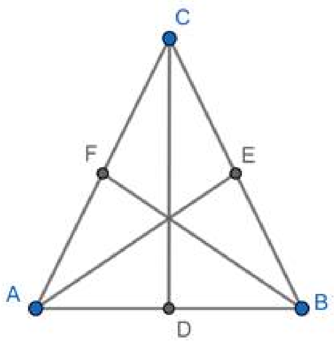
7. Question

Find the area of the triangle formed by joining the midpoints of the sides of a triangle whose vertices are (0, –1), (2,1) and (0,3). Find the ratio of this area to the area of the given triangle.

Answer

Let A (0, –1), B (2, 1) and C (0, 3) are the vertices of the triangle.

D, E and F are the mid–points of the sides AB, BC and AC respectively.



Mid – point formula = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

<u>Mid – point of AB</u> $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$D = \left(\frac{0+2}{2}, \frac{-1+1}{2}\right)$

$D = \frac{2}{2}, \frac{0}{2}$

D = (1,0)

<u>Mid – point of BC</u> = $\left(\frac{x_2+3}{2}, \frac{y_2+y_3}{2}\right)$

$E = \left(\frac{2+0}{2}, \frac{1+3}{2}\right)$

$E = \frac{2}{2}, \frac{4}{2}$

$E = (1, 2)$

<u>Mid – point of AC</u> = $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

$F = \left(\frac{0+0}{2}, \frac{3+(-1)}{2}\right)$

$F = \frac{0}{2}, \frac{2}{2}$

$F = (0, 1)$

<u>Area of Δ ABC:</u>

Area of triangle = $\frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3\}$

$x_1 = 0$, $x_2 = 2$ and $x_3 = 0$

$y_1 = -1$, $y_2 = 1$ and $y_3 = 3$

$\Rightarrow \frac{1}{2}\{((0 \times 1) + (2 \times 3) + (0 \times -1)) - ((2 \times -1) + (0 \times 1) + (0 \times 3))\}$

$\Rightarrow \frac{1}{2}\{(0 + 6 + 0) - (-2 + 0 + 0)\}$

$\Rightarrow \frac{1}{2}\{6 + 2\}$

$\Rightarrow \frac{1}{2} \times 8$

⇒ 4 sq. units

Area of Δ DEF:

Area of triangle = $\frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3\}$

$x_1 = 1$, $x_2 = 1$ and $x_3 = 0$

$y_1 = 0$, $y_2 = 2$ and $y_3 = 1$

$\Rightarrow \frac{1}{2}\{((1 \times 2) + (1 \times 1) + (0 \times 0)) - ((1 \times 0) + (0 \times 2) + (1 \times 1))\}$

$\Rightarrow \frac{1}{2}\{(2 + 1 + 0) - (0 + 0 + 1)\}$

$\Rightarrow \frac{1}{2}\{3 - 1\}$

$\Rightarrow \frac{1}{2} \times 2$

⇒ 1 sq. units

Area of triangle ABC: Area of triangle DEF

4: 1

Exercise 5.3

1. Question

Find the angle of inclination of the straight line whose slope is

(i) 1 (ii) √3 (iii) 0

Answer

If θ is the angle of inclination of the line, then the slope of the line is

m = tan θ where 0° ≤ θ ≤ 180°, θ ≠ 90°

i) tan θ = 1

⇒ θ = 45°

ii) tan θ = √3

⇒ θ = 60°

iii) tan θ = 0

⇒ θ = 0°

2. Question

Find the slope of the straight line whose angle of inclination is

(i) 30° (ii) 60° (iii) 90°

Answer

If θ is the angle of inclination of the line, then the slope of the line is m = tan θ

i) Given that m = tan 30°

$\Rightarrow \mathrm{m} = \frac{1}{\sqrt{3}}$

ii) Given that m = tan 60°

⇒ m = √3

iii) Given that m = tan 30°

$\Rightarrow \mathrm{m} = \frac{1}{0}$

∴ The slope is undefined.

3. Question

Find the slope of the straight line passing through the points

(i) (3, –2) and (7, 2) (ii) (2,–4) and origin

(iii) $\left(1+\sqrt{3}, 2\right)$ and $\left(3+\sqrt{3}, 4\right)$

Answer

Slope of straight line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$\mathrm{m} = \frac{y_2 - y_1}{x_2 - x_1}$$

i) Slope of straight line passing through the points (3, –2) and (7, 2) is

$$\mathrm{m} = \frac{2 + 2}{7 - 3} = \frac{4}{4} = 1$$

ii) Slope of straight line passing through the points (2, –4) and (0, 0) is

$$\mathrm{m} = \frac{0 + 4}{0 - 2} = \frac{4}{-2} = -2$$

iii) Slope of straight line passing through the points (1 + √3, 2) and (3 + √3, 4) is

$$\mathrm{m} = \frac{4 - 2}{3 + \sqrt{3} - 1 - \sqrt{3}} = \frac{2}{2} = 1$$

4. Question

Find the angle of inclination of the line passing through the points

(i) (1, 2) and (2, 3) (ii) $\left(3\sqrt{3}\right)$ and (0, 0)

(iii) (a, b) and (–a, –b)

Answer

Slope of straight line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$\mathrm{m} = \frac{y_2 - y_1}{x_2 - x_1}$$

i) Slope of straight line passing through the points (1, 2) and (2, 3) is

$$\mathrm{m} = \frac{3 - 2}{2 - 1} = \frac{1}{1} = 1$$

m = 1

If θ is the angle of inclination of the line, then the slope of the line is m = tan θ where 0° ≤ θ ≤ 180°, θ ≠ 90°

∴ tan θ = 1 ⇒ θ = 45°

ii) Slope of straight line passing through the points (3, √3) and (0, 0) is

$$m = \frac{0-\sqrt{3}}{0-3} = \frac{-\sqrt{3}}{-3} = \sqrt{3}$$

m = √3

If θ is the angle of inclination of the line, then the slope of the line is

m = tan θ where 0° ≤ θ ≤ 180°, θ ≠ 90°

∴ tan θ = √3 ⇒ θ = 30°

iii) Slope of straight line passing through the points (a, b) and (–a, –b) is

$$m = \frac{-b-b}{-a-a} = \frac{-2b}{-2a} = \frac{b}{a}$$

$$m = \frac{b}{a}$$

If θ is the angle of inclination of the line, then the slope of the line is

m = tan θ where 0° ≤ θ ≤ 180°, θ ≠ 90°

$\therefore \tan\theta = \frac{b}{a}$

5. Question

Find the slope of the line which passes through the origin and the midpoint of the line segment joining the points (0, – 4) and (8, 0).

Answer

We need to find slope of a line passing through origin (0, 0) & mid–point of P (0, –4) and Q (8, 0)

Let M mid–point of P (0, –4) and Q (8, 0)

Mid–point formula M (x, y) $= M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Mid–point of PQ $= \left(\frac{0+8}{2}, \frac{-4+0}{2}\right)$

$$= \left(\frac{8}{2}, -\frac{4}{2}\right)$$

$$= (4, -2)$$

We know that,

Slope of line passing through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2-y_1}{x_2-x_1}$$

Slope of line between points O (0, 0) and M (4, –2)

$$m = \frac{-2-0}{4-0}$$

$$m = -\frac{2}{4} = -\frac{1}{2}$$

Hence, slope of line is $-\frac{1}{2}$

6. Question

The side AB of a square ABCD is parallel to x–axis. Find the

(i) slope of AB (ii) slope of BC (iii) slope of the diagonal AC

Answer

i). Slope of AB

Since the side AB is parallel to x–axis,

Slope of side AB = 0

Therefore, the slope of AB is 0

ii). Slope of BC

The angle formed by the side BC is 90°

m = tan θ

θ = 90°

m = tan 90°

$= \frac{1}{0}$

Therefore, the slope of BC is not defined.

iii). Slope of the diagonal AC

The diagonal AC is the angle bisector of the angle ∠BAD

∴ θ = 45°

m = tan θ

= tan 45°

= 1

Therefore, the slope of the diagonal AC is 1

7. Question

The side BC of an equilateral 3ABC is parallel to x–axis. Find the slope of AB and the slope of BC.

Answer

Slope of line BC:

The side BC is parallel to x–axis.

Slope of the side BC = 0.

Slope of line AB:

Since, this is an equilateral triangle each angle is 60°

∴ θ = 60°

m = tan θ

= tan 60°

= √3

Hence, the slope of AB is √3 and the slope of BC is 0

8 A. Question

Using the concept of slope, show that each of the following set of points are collinear.

(2, 3), (3, –1) and (4, –5)

Answer

(2, 3), (3, –1) and (4, –5)

Let A (2, 3), B (3, –1) and C (4, –5) be the given points

If the given point is collinear then,

Slope of AB = slope of BC

Slope of line passing through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of AB:

$$m = \frac{-1-3}{3-2}$$

$$= -\frac{4}{1}$$

= –4

Slope of BC:

$$m = \frac{-5 + 1}{4-3}$$

$$= -\frac{4}{1}$$

= –4

Slope of AB = Slope of BC = –4

Therefore, the given points are collinear.

8 B. Question

Using the concept of slope, show that each of the following set of points are collinear.

(4, 1), (–2, –3) and (–5, –5)

Answer

(4, 1), (–2, –3) and (–5, –5)

Let A (4, 1), B (–2, –3) and C (–5, –5) be the given points

If the given point is collinear then,

Slope of AB = slope of BC

Slope of line passing through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of AB:

$$m = \frac{-3-1}{-2-4}$$

$$= \frac{-4}{-6}$$

$$= \frac{2}{3}$$

Slope of BC:

$$m = \frac{-5 + 3}{-5 + 2}$$

$$= \frac{-2}{-3}$$

$$= \frac{2}{3}$$

$$\text{Slope of AB} = \text{Slope of BC} = \frac{2}{3}$$

Therefore, the given points are collinear.

8 C. Question

Using the concept of slope, show that each of the following set of points are collinear.

(4, 4), (–2, 6) and (1, 5)

Answer

(4, 4), (–2, 6) and (1, 5)

Let A (4, 4), B (–2, 6) and C (1, 5) be the given points

If the given point is collinear then,

Slope of AB = slope of BC

Slope of line passing through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of AB:

$$m = \frac{6 - 4}{-2 - 4}$$

$$= \frac{2}{-6}$$

$$= -\frac{1}{3}$$

Slope of BC:

$$m = \frac{5 - 6}{1 + 2}$$

$$= \frac{1}{-3}$$

$$= -\frac{1}{3}$$

$$\text{Slope of AB} = \text{Slope of BC} = -\frac{1}{3}$$

Therefore, the given points are collinear.

9. Question

If the points (a, 1), (1, 2) and (0, b + 1) are collinear, then show that $\frac{1}{a}+\frac{1}{b}=1$.

Answer

Let A (a, 1), B (1, 2) and C (0, b + 1) be the given points.

Slope of line passing through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of AB $= \frac{2-1}{1-a}$

$$= \frac{1}{1-a}$$

Slope of BC $= \frac{b+1-2}{0-1}$

$$= \frac{b-1}{-1} = -b + 1$$

If three points are collinear, then slope of AB is equal to slope of AC

∴ slope of AB = slope of BC

$$\Rightarrow \frac{1}{1-a} = -b + 1$$

⇒ 1 = (–b + 1)(1 – a)

⇒ 1 = –b(1 – a) + 1(1 – a)

⇒ 1 = –b + ab + 1 – a

⇒ –b + ab – a = 0

⇒ ab – b = a

Dividing both sides by ab, we get

$$\Rightarrow \frac{ab}{ab} - \frac{b}{ab} = \frac{a}{ab}$$

$$\Rightarrow 1 - \frac{1}{a} = \frac{1}{b}$$

$$\Rightarrow 1 = \frac{1}{b} + \frac{1}{a}$$

$$\frac{1}{a} + \frac{1}{a} = 1$$

Hence proved.

10. Question

The line joining the points A (–2, 3) and B (a, 5) is parallel to the line joining the points C (0, 5) and D (–2, 1). Find the value of a.

Answer

Line joining points A (–2, 3) and B (a, 5) IA parallel to the line joining the points C (0, 5) and D (–2, 1)

The two lines are parallel only when their slopes are equal.

∴ slope of AB = slope of CD

Slope of line passing through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of AB is:

A (–2, 3) and B (a, 5)

$$m = \frac{5 - 3}{a + 2}$$

$$= \frac{2}{a + 2}$$

Slope of CD is:

C (0, 5) and D (–2, 1)

$$m = \frac{1 - 5}{-2 - 0}$$

$$= \frac{-4}{-2}$$

$$= 2$$

As per the property, the two lines are parallel only when their slopes are equal.

i.e. Slope of AB = Slope of CD

$\Rightarrow \frac{2}{a + 2} = 2$

⇒ 2 = 2(a + 2)

⇒ 2 = 2a + 4

⇒ 2a = 2 – 4

⇒ 2a = –2

$\Rightarrow a = \frac{-2}{2}$

⇒ a = –1

11. Question

The line joining the points A (0, 5) and B (4, 2) is perpendicular to the line joining the points C (–1, –2) and D (5, b). Find the value of b.

Answer

Line joining points A (0, 5) and B (4, 2) IA parallel to the line joining the points C (–1, –2) and D (5, b)

The two lines are perpendicular only if the multiplication of their slope is equal to 1.

∴ (Slope of AB) × (Slope of CD) = 1

Slope of line passing through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of AB is:

A (0, 5) and B (4, 2)

$$m = \frac{2 - 5}{4 - 0}$$

$= \frac{-3}{4}$

Slope of CD is:

C (–1, –2) and D (5, b)

$m = \frac{b + 2}{5 + 1}$

$= \frac{b + 2}{6}$

The two lines are perpendicular only if the multiplication of their slope is equal to 1.

∴ (Slope of AB) × (Slope of CD) = –1

$\Rightarrow \left(\frac{-3}{4}\right) \times \left(\frac{b+2}{6}\right) = -1$

$\Rightarrow \left(\frac{-1}{4}\right) \times \left(\frac{b+2}{2}\right) = -1$

$\Rightarrow \frac{-b-2}{8} = -1$

⇒ –b – 2 = –1 × 8

⇒ –b – 2 = –8

⇒ –b = –8 + 2

⇒ – b = – 6

⇒ b = 6

12. Question

The vertices of ΔABC are A (1, 8), B (–2, 4), C (8, –5). If M and N are the midpoints of AB and AC respectively, find the slope of MN and hence verify that MN is parallel to BC.

Answer

Given: vertices of triangle ABC i.e. A (1, 8), B (–2, 4), C (8, –5)

M and N are mid – points of AB and AC.

Finding co–ordinates of M and N:

We know that,

M is the mid–point of AB

$x_1 = 1, x_2 = -2$

$y_1 = 8, y_2 = 4$

Mid–point formula M (x, y) $= M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Mid–point of AB $= \left(\frac{1 + (-2)}{2}, \frac{8 + 4}{2}\right)$

$= \left(\frac{-1}{2}, \frac{12}{2}\right)$

$= (\frac{-1}{2}, 6)$

N is the mid–point of AC

$x_1 = 1, x_2 = 8$

$y_1 = 8$, $y_2 = -5$

Mid–point of AC $= \left(\frac{1+8}{2}, \frac{8-5}{2}\right)$

$= \left(\frac{9}{2}, \frac{3}{2}\right)$

Slope of MN:

$M = \left(\frac{-1}{2}, 6\right)$ and $N = \left(\frac{9}{2}, \frac{3}{2}\right)$

Slope of line passing through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{\frac{3}{2} - 6}{\frac{9}{2} + \frac{1}{2}}$$

$$m_1 = \frac{\frac{3-12}{2}}{\frac{9+1}{2}}$$

$$m_1 = \frac{\frac{-9}{2}}{\frac{10}{2}}$$

$$m_1 = \frac{-9}{2} \times \frac{2}{10}$$

$$m_1 = -\frac{9}{10}$$

Verification of MN and BC are parallel:

If MN and BC are parallel, then their slopes must be equal.

Slope of BC:

B (–2, 4) and C (8, –5)

Slope of BC $(m_2) = \frac{-5-4}{8-(-2)}$

$$m_2 = \frac{-9}{8+2}$$

$$m_2 = \frac{-9}{10}$$

∴ Slope of MN = Slope of BC $= \frac{-9}{10}$

Hence, MN is parallel to BC.

13. Question

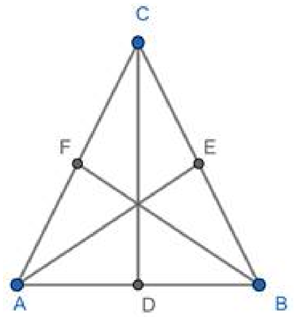
A triangle has vertices at (6, 7), (2, –9) and (–4, 1). Find the slopes of its medians.

Answer

Given: Vertices of triangle A (6, 7), B (2, –9) and C (–4, 1)

To find the slopes of medians. We need to know the mid–points of AB, BC and AC.

D, E and F are mid–points of AB, BC and AC respectively.



Mid – point formula = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Mid – point of AB = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$D = \left(\frac{6 + 2}{2}, \frac{7 - 9}{2}\right)$$

$$D = \frac{8}{2}, \frac{-2}{2}$$

$$D = (4, -1)$$

Mid – point of BC = $\left(\frac{x_2 + 3}{2}, \frac{y_2 + y_3}{2}\right)$

$$E = \left(\frac{2 - 4}{2}, \frac{-9 + 1}{2}\right)$$

$$E = -\frac{2}{2}, \frac{-8}{2}$$

$$E = (-1, -4)$$

Mid – point of AC = $\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$

$$F = \left(\frac{6 - 4}{2}, \frac{7 + 1}{2}\right)$$

$$F = \frac{2}{2}, \frac{8}{2}$$

F = (1, 4)

Slopes of median of triangles:

Slope of line passing through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

A (6, 7) and E (–1, –4)

Slope of AE = $\frac{-4-7}{-1-6}$

$$= \frac{-11}{-7}$$

$$= \frac{11}{7}$$

B (2, –9) and E (1, 4)

Slope of BF = $\frac{4+9}{1-2}$

$= \frac{13}{-1}$

$= -13$

C (–4, 1) and D (4, –1)

Slope of CD $= \frac{-1-1}{4+4}$

$= -\frac{2}{8}$

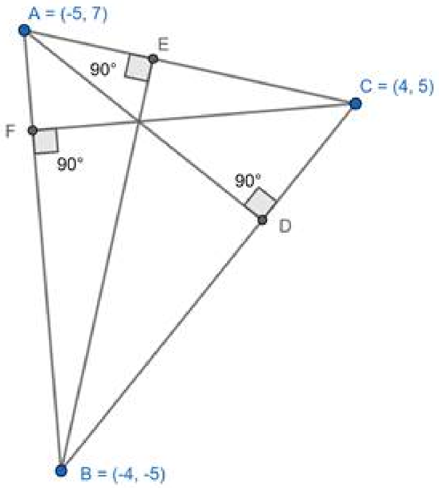
$= -\frac{1}{4}$

14. Question

The vertices of a ΔABC are A (–5, 7), B (–4, –5) and C (4, 5). Find the slopes of the altitudes of the triangle.

Answer

Let AD, BE and CF be the altitudes of a ΔABC.



Since, the altitude AD is perpendicular to BC,

Slope of BC $= \frac{5-(-5)}{4-(-4)}$

$= \frac{5+5}{4+4}$

$= \frac{10}{8} = \frac{5}{4}$

(slope of BC) × (Slope of AD) = –1 (∵ $m_1 m_2 = -1$)

Let slope of AD be m_1.

$\Rightarrow \frac{5}{4} \times m_1 = -1$

$\Rightarrow m_1 = -\frac{4}{5}$

Since, the altitude BE is perpendicular to AC,

Slope of AC $= \frac{5-7}{4-(-5)}$

$= \frac{-2}{4+5}$

$= \dfrac{-2}{9}$

(slope of AC) × (Slope of BE) = –1 (∵ m_1m_2 = –1)

Let slope of BE be m_2.

$\Rightarrow -\frac{2}{9} \times m_2 = -1$

$\Rightarrow m_2 = \frac{9}{2}$

Since, the altitude CF is perpendicular to AB,

Slope of AB $= \frac{-5-7}{-4-(-5)}$

$= \dfrac{-12}{-4 + 5}$

$= -\dfrac{12}{1} = -12$

(slope of AB) × (Slope of CF) = –1 (∵ m_1m_2 = –1)

Let slope of CF be m_3.

$\Rightarrow -12 \times m_3 = -1$

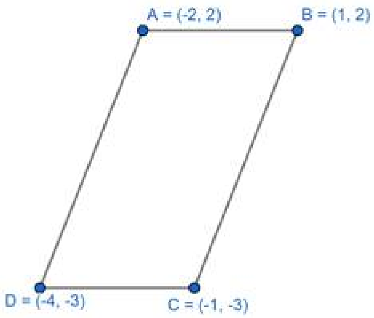
$\Rightarrow m_3 = \frac{1}{12}$

15. Question

Using the concept of slope, show that the vertices (1, 2), (–2, 2), (–4, –3) and (–1, –3) taken in order form a parallelogram.

Answer

Let A (–2, 2), B (1, 2), C (–1, –3) and D (–4, –3) be the given points taken in order.



Now,

Slope of line passing through (x_1, y_1) and (x_2, y_2) is

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Slope of AB $= \frac{2-2}{1-(-2)}$

$= \dfrac{0}{3}$

= 0

Slope of CD $= \frac{-3-(-3)}{-4-(-1)}$

$= \dfrac{-3\ +\ 3}{-4\ +\ 1}$

$= \dfrac{0}{-3}$

= 0

∴ Slope of AB = Slope of CD

Hence, AB is parallel to CD. … (1)

Now,

Slope of BC = $\frac{-3-2}{-1-1}$

$= \dfrac{-5}{-2}$

$= \dfrac{5}{2}$

Slope of AD = $\frac{-3-2}{-4-(-2)}$

$= \dfrac{-3-2}{-4\ +\ 2}$

$= \dfrac{-5}{-2}$

$= \dfrac{5}{2}$

∴ Slope of BC = Slope of AD

Hence, BC is parallel to AD. … (2)

From (1) and (2), we see that opposite sides of quadrilateral are parallel.

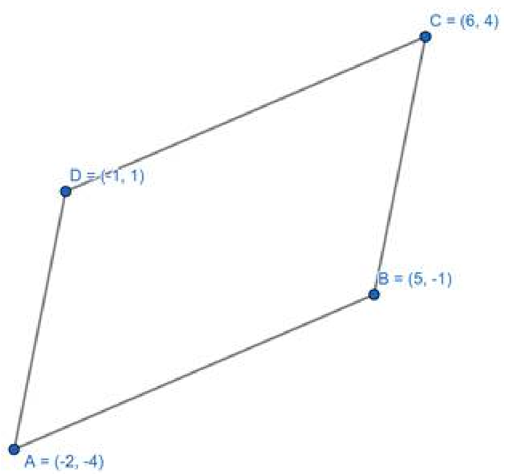
∴ ABCD is a parallelogram.

16. Question

Show that the opposite sides of a quadrilateral with vertices A (–2, –4), B (5, –1), C (6, 4) and D (–1, 1) taken in order are parallel.

Answer

Let A (–2, 2), B (1, 2), C (–1, –3) and D (–4, –3) be the given points taken in order.



Now,

Slope of line passing through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of AB $= \frac{-1-(-4)}{5-(-2)}$

$$= \frac{-1 + 4}{5 + 2}$$

$$= \frac{3}{7}$$

Slope of CD $= \frac{1-4}{-1-6}$

$$= \frac{-3}{-7}$$

$$= \frac{3}{7}$$

∴ Slope of AB = Slope of CD

Hence, AB is parallel to CD. … (1)

Now,

Slope of BC $= \frac{4-(-1)}{6-5}$

$$= \frac{4 + 1}{1}$$

$$= 5$$

Slope of AD $= \frac{1-(-4)}{-1-(-2)}$

$$= \frac{-1 + 4}{-1 + 2}$$

$$= \frac{5}{1}$$

$$= 5$$

∴ Slope of BC = Slope of AD

Hence, BC is parallel to AD. … (2)

From (1) and (2), we see that opposite sides of quadrilateral are parallel.

Exercise 5.4

1. Question

Write the equations of the straight lines parallel to x- axis which are at a distance of 5 units from the x–axis.

Answer

Given

The line is ∥ to x axis that means it is a horizontal line at the fixed distance from x axis.

The equation of this line would be of the form y = k where k is some constant.

Here it is given the distance from x axis is 5 units which could be either in positive or negative direction, so the required equation would be

$y = 5$ or $y = –5$

2. Question

Find the equations of the straight lines parallel to the coordinate axes and passing through the point (–5,–2).

Answer

A line parallel to x axis is horizontal line and when this line passes through the given point (–5,–2) the equation of the line would be $y = –2$

When the line is parallel to the y axis, it is a vertical straight line passing through the point (–5, –2), the required equation of the line would be $x = –5$

3. Question

Find the equation of a straight line whose

(i) slope is –3 and y–intercept is 4. (ii) angle of inclination is 60° and y–intercept is 3.

Answer

The equation of the straight line with the given slope (m) and y –intercept 'c' is given the slope–intercept form

i.e. $y = mx + c$

(i) Here given slope = –3 (m) and y intercept = 4 (c)

So the required equation is

$y = –3x + 4$

$\Rightarrow 3x + y–4 = 0$

(ii) Given angle of inclination = 60° and y intercept = 3 (c)

Here slope of the line (m) = $\tan \theta = \tan 60° = \sqrt{3}$

The required equation is

$y = mx + c$

$\Rightarrow y = \sqrt{3}x + 3$

$\Rightarrow \sqrt{3}x –y + 3 = 0$

4. Question

Find the equation of the line intersecting the y- axis at a distance of 3 units above the origin and $\tan \theta = \frac{1}{2}$, where θ is the angle of inclination.

Answer

Given

Angle of inclination = $\tan \theta = 1/2$ = slope of the line (m)

Also, the y intercept = 3 units (c) as the line is intersecting y axis at a distance of 3 units above origin

The equation of the straight line with the given slope (m) and y –intercept 'c' is given the slope–intercept form

i.e. $y = mx + c$

$$y = \frac{1}{2}x + 3$$

$\Rightarrow 2y = x + 6$

$\Rightarrow x – 2y + 6 = 0$

x – 2y + 6 = 0

5. Question

Find the slope and y–intercept of the line whose equation is

(i) y = x + 1 (ii) 5x = 3y (iii) 4x – 2y + 1 = 0 (iv) 10x + 15y + 6 = 0

Answer

The equation of the straight line with the slope (m) and y –intercept ‘c’ in the slope–intercept form is

y = mx + c

(i) here y = x + 1

⇒ slope of the line (m) = 1 and y–intercept (c) = 1

(ii) here 5x = 3y

3y = 5x

⇒ $y = \frac{5}{3}x$

Thus the slope of the line = $\frac{5}{3}$ and y intercept (c) = 0

(iii) 4x– 2y + 1 = 0

⇒ –2y = –4x –1

⇒ $y = 2x + \frac{1}{2}$

Thus the slope of the line (m) = 2

and y –intercept (c) = 1/2

(iv) 10x + 15y + 6 = 0

⇒ 15y = = 10x – 6

⇒ $y = \frac{10}{15}x - \frac{6}{15} = \frac{2}{3}x - \frac{2}{5}$

Thus the slope of the line (m) = $\frac{2}{3}$ and y –intercept (c) = $\frac{2}{5}$

6. Question

Find the equation of the straight line whose

(i) slope is –4 and passing through (1, 2) (ii) slope is $\frac{2}{3}$ and passing through (5, –4)

Answer

The equation of the line with the given slope m and passing through the point (x_1, y_1) is given by the slope–point form

$y - y_1 = m(x - x_1)$

(i) Here given the slope (m) = –4 and the point = (1,2)

Thus the equation of the line is

y–2 = –4(x – 1)

⇒ y –2 = –4x + 4

⇒ y = –4x + 6

⇒ 4x + y – 6 = 0

(ii) Here given slope (m) = 2/3 and point = (5,–4)

Thus the required equation is

$$y - (-4) = \frac{2}{3}(x - 5)$$

⇒ 3y + 12 = 2x –10

⇒ –2x + 3y + 22 = 0

⇒ 2x – 3y – 22 = 0

7. Question

Find the equation of the straight line which passes through the midpoint of the line segment joining (4, 2) and (3, 1) whose angle of inclination is 30°.

Answer

The equation of the line with the given slope m and passing through the point (x_1, y_1) is given by the slope–point form

$y - y_1 = m(x - x_1)$

here given angle of inclination = 30°

⇒ slope (m) = tan 30 ° $= \frac{1}{\sqrt{3}}$

Also the line passes through the midpoint of the line segment joining points (4, 2) and (3, 1)

⇒ the line passes through the point $\left(\frac{(x_1+x_2)}{2}, \frac{y_1+y_2}{2}\right) \Rightarrow \frac{4+3}{2}, \frac{2+1}{2} \Rightarrow \left(\frac{7}{2}, \frac{3}{2}\right)$

The equation of the required line is

$$y - \frac{3}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{7}{2}\right)$$

$$\Rightarrow \sqrt{3}(2y - 3) = \frac{2(2x-7)}{2}$$

$$\Rightarrow 2\sqrt{3}y - 3\sqrt{3} = 2x - 7$$

⇒ 2x –2 √3 y + (3√3–7) = 0

8. Question

Find the equation of the straight line passing through the points

(i) (–2, 5) and (3, 6) (ii) (0, –6) and (–8, 2)

Answer

The equation of the line passing through the two given (x_1, y_1) and (x_2,y_2)points is given by form

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

(i) Here, (–2,5) and (3,6) are tow given points

The required equation is

$$\frac{y - 5}{6 - 5} = \frac{x - (-2)}{3 - (-2)}$$

$$\Rightarrow \frac{y-5}{1} = \frac{x+2}{5}$$

⇒ 5y –25 = x + 2

⇒ x–5y + 27 = 0

(ii) Here two points are (0, –6) and (–8, 2)

The equation is given by

$$\frac{y-(-6)}{2-(-6)}=\frac{x-0}{-8-0}$$

$$\Rightarrow \frac{y+6}{8}=\frac{x}{-8}$$

⇒ –8y – 48 = 8x

⇒ x + y + 6 = 0 (dividing the whole equation by 8)

9. Question

Find the equation of the median from the vertex R in a ΔPQR with vertices at

P(1, –3), Q(–2, 5) and R(–3, 4).

Answer

The given points are

P (1,–3), Q(–2,5) and R (–3,4)

The median from R will pass through the midpoint of line PQ

(∵ Median passes through the mid–point of the side opposite to the vertex)

Let the mid–point of PQ be A = $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\frac{1-2}{2},\frac{-3+5}{2}=-\frac{1}{2},1$

The equation of the line passing through the two given (x_1, y_1) and (x_2,y_2)points is given by form

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

Thus here the required equation is

$$\frac{y-4}{1-4}=\frac{x-(-3)}{-\frac{1}{2}-(-3)}$$

$$\Rightarrow \frac{y-4}{-3}=\frac{x+3}{-\frac{1}{2}+3}$$

$$\Rightarrow \frac{y-4}{-3}=\frac{x+3}{\frac{5}{2}}$$

$$\Rightarrow \frac{y-4}{-3}=\frac{2x+6}{5}$$

⇒ 5y –20 = –6x –18

⇒ 6x + 5y –2 = 0

10. Question

By using the concept of the equation of the straight line, prove that the given three points are collinear.

(i) (4, 2), (7, 5) and (9, 7)

(ii) (1, 4), (3, –2) and (–3, 16)

Answer

The equation of the line passing through the two given (x_1, y_1) and (x_2,y_2)points is given by form

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

(i) The equation of the line passing through (4, 2), (7, 5)is

$$\frac{y - 2}{5 - 2} = \frac{x - 4}{7 - 4}$$

⇒ y–2 = x–4

⇒ x – y –2 = 0

Substituting the 3rd point in the equation of the line obtained

x–2 = y

⇒ 7 = 9–2

⇒ 7 = 7

Hence LHS = RHS

Since the third point satisfies the equation of the line obtained

⇒ All the three points lies in the same straight line or are collinear.

(ii) (1, 4), (3, –2) and (–3, 16)

The equation of the line passing through the points (1, 4), (3, –2) is

$$\frac{y - 4}{-2 - 4} = \frac{x - 1}{3 - 1}$$

⇒ $\frac{y-4}{-6} = \frac{x-1}{2}$

⇒ y–4 = –3x + 3

⇒ 3x + y –7 = 0

Substituting the 3rd point in the equation of line obtained

y = –3x + 7

⇒ 16 = –3 (–3) + 7

⇒ 16 = 16

LHS = RHS

Thus the three points are in the same straight line or are collinear.

11. Question

Find the equation of the straight line whose x and y–intercepts on the axes are given by

(i) 2 and 3 (ii) $-\frac{1}{3}$ and $\frac{3}{2}$

(iii) $\frac{2}{5}$ and $-\frac{3}{4}$

Answer

The equation of the straight line whose x and y intercepts are given as ‘a’ and ‘b’ respectively, is of the form

$$\frac{x}{a} + \frac{y}{b} = 1$$

(i) Here given x intercept as 2 and y intercept as 3

The equation of the line

$= \frac{x}{2} + \frac{y}{3} = 1$

⇒ 3x + 2y = 6

⇒ 3x + 2y –6 = 0

(ii) Given x intercept as $-\frac{1}{3}$ and y intercept as $\frac{3}{2}$

The equation of the line

$\Rightarrow \frac{x}{-\frac{1}{3}} + \frac{y}{\frac{3}{2}} = 1$

$\Rightarrow -3x + \frac{2y}{3} = 1$

⇒ –9x + 2y = 3

⇒ –9x + 2y –3 = 0

(iii) Given x intercept as $\frac{2}{5}$ and y intercept as $-\frac{3}{4}$

The equation of the line

$\Rightarrow \frac{x}{\frac{2}{5}} + \frac{y}{-\frac{3}{4}} = 1$

$\Rightarrow \frac{5x}{2} - \frac{4y}{3} = 1$

$\Rightarrow 15x - 8y = 6$

⇒ 15x –8y –6 = 0

12. Question

Find the x and y intercepts of the straight line

(i) 5x + 3y – 15 = 0 (ii) 2x – y + 16 = 0

(iii) 3x + 10y + 4 = 0

Answer

The equation of the straight line whose x and y intercepts are given as ‘a’ and ‘b’ respectively, is of the form

$\frac{x}{a} + \frac{y}{b} = 1$

(i) The given equation is

5x + 3y – 15 = 0

⇒ 5x + 3y = 15

$\Rightarrow \frac{5x}{15} + \frac{3y}{15} = 1$

$\Rightarrow \frac{1x}{3} + \frac{1y}{5} = 1$

Hence the x intercept is 3 and y intercept is 5

(ii) The given equation is

2x – y + 16 = 0

⇒ 2x– y = –16

$\Rightarrow \frac{2x}{-16} - \frac{y}{-16} = -\frac{16}{-16}$

$\Rightarrow -\frac{1x}{8} + \frac{y}{16} = 1$

Hence the x intercept is –8 and y intercept is 16.

(iii) Given equation is

3x + 10y + 4 = 0

⇒ 3x + 10y = –4

$\Rightarrow \frac{3x}{-4} + \frac{10y}{-4} = -\frac{4}{-4}$

$\Rightarrow \frac{3}{-4}x - \frac{5}{2}y = 1$

$\Rightarrow \frac{x}{\frac{4}{-3}} + \frac{y}{\frac{-2}{5}} = 1$

Hence the x intercept is $-\frac{4}{3}$ and y intercept is $-\frac{2}{5}$

13. Question

Find the equation of the straight line passing through the point (3, 4) and has intercepts which are in the ratio 3 : 2.

Answer

Given that the intercepts are in the ratio 3:2

Let the x–intercept be 3k and 2k, where k is some constant

Using the intercept –form, the equation of the line is

$\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow \frac{x}{3k} + \frac{y}{2k} = 1$

$\Rightarrow \frac{x}{3} + \frac{y}{2} = k$

The point (3, 4) lies on this line, thus the point will satisfy the equation of the line

$\Rightarrow \frac{x}{3} + \frac{y}{2} = k$

$\Rightarrow \frac{3}{3} + \frac{4}{2} = k$

⇒ 1 + 2 = k

⇒ K = 3

Thus the equation of the line is

$\Rightarrow \frac{x}{3} + \frac{y}{2} = 3$

⇒ 2x + 3y = 18

⇒ 2x + 3y –18 = 0

14. Question

Find the equation of the straight lines passing through the point (2, 2) and the sum of the intercepts is 9.

Answer

Let the x intercept be ‘a’ and the y intercept be ‘b’ . it is given that

a + b = 9

The equation of the line using the intercept form is

$\frac{x}{a} + \frac{y}{b} = 1$

Substituting b = 9– a in this equation

$\frac{x}{a} + \frac{y}{9-a} = 1$

$\Rightarrow \frac{x(9-a)+ya}{a(9-a)} = 1$

⇒ x (9–a) + ya = a(9 –a)

The point (2, 2) lies on this line and thus it satisfies the equation

x (9–a) + ya = a(9 –a)

⇒ 2(9 –a) + 2a = a (9–a)

⇒ 18 –2a + 2a = 9a –a^2

⇒ a^2 – 9a + 18 = 0

⇒ a^2 – 3a –6a + 18 = 0

⇒ a (a– 3) –6 (a –3) = 0

⇒ (a–6) (a–3) = 0

⇒ a = 6 or 3

The equation of the line is

x(9–3) + y(3) = 3(9–3)

⇒ 6x + 3y = 18

⇒ 2x + y –6 = 0

Or

x(9–6) + y(6) = 6(9–6)

3x + 6y = 18

⇒ x + 2y –6 = 0

15. Question

Find the equation of the straight line passing through the point (5, –3) and whose intercepts on the axes are equal in magnitude but opposite in sign.

Answer

Let the x intercept be ‘a’. it is given the y intercept is equal in magnitude but opposite in sign

So y intercept = ‘–a’

The equation of the line using the intercept form is

$\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow \frac{x}{a} + \frac{y}{-a} = 1$

$\Rightarrow \frac{x-y}{a} = 1$

⇒ x–y = a

The point (5, –3) lie son this line, thus it satisfies the given equation of the line

5 –(–3) = a

$\Rightarrow$ a = 8

The equation of the line is

x– y – 8 = 0

16. Question

Find the equation of the line passing through the point (9, –1) and having its x–intercept thrice as its y–intercept.

Answer

Let the x intercept be ‘a’. it is given that x intercept is thrice the y intercept

$\Rightarrow$ y intercept = $\frac{a}{3}$

The equation of the line using the intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

Substituting b = a/3 in the equation

$$\frac{x}{a} + \frac{y}{\frac{a}{3}} = 1$$

$\Rightarrow \frac{x + 3y}{a} = 1$

$\Rightarrow$ x + 3y = a

the point (9,–1) lies on the equation of the line thus it must satisfies it

$\Rightarrow$ 9 + 3(–1) = a

$\Rightarrow$ a = 6

Hence the equation of the line is

x + 3y –6 = 0

17. Question

A straight line cuts the coordinate axes at A and B. If the midpoint of AB is (3, 2), then

find the equation of AB.

Answer

The mid–point of AB is in the 1st quadrant as both the intercepts are positive.

Thus A will lie on the y –axis and B will lie on the x axis

Let A be (0,a) and B be (b,0)

By the mid–point theorem, the coordinates of mid–point are

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ = (3,2)

$\Rightarrow \frac{0 + b}{2}, \frac{a + 0}{2} = (3, 2)$

$\Rightarrow \frac{b}{2} = 3$ and $\frac{a}{2} = 2$

$\Rightarrow$ B = 6 and a = 4

Thus y intercept is 4 and x intercept is 6

The equation of the line using the intercept form is

$\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow \frac{x}{6} + \frac{y}{4} = 1$

$\Rightarrow 4x + 6y = 24$

$\Rightarrow 2x + 3y - 12 = 0$

18. Question

Find the equation of the line passing through (22, –6) and having intercept on x–axis exceeds the intercept on y–axis by 5.

Answer

Let the x intercept be 'a'. It is given that x intercept exceeds the y intercept by 5

$\Rightarrow$ y intercept = a– 5

The equation of the line using the intercept form is

$\frac{x}{a} + \frac{y}{b} = 1$

Substituting Value of b

$\frac{x}{a} + \frac{y}{a-5} = 1$

$\Rightarrow \frac{x(a-5) + ya}{a(a-5)} = 1$

$\Rightarrow x(a-5) + ya = a(a-5)$

Given that the point (22, –6) lies on this equation of the line, hence it should satisfy it

$22(a-5) + (-6)a = a(a-5)$

$\Rightarrow 22a - 110 - 6a = a^2 - 5a$

$\Rightarrow a^2 - 21a + 110 = 0$

$\Rightarrow a(a-10) - 11(a-10) = 0$

$\Rightarrow (a-11)(a-10)$

$\Rightarrow a = 11$ or 10

The equation of the line is

$x(11-5) + 11y = 11(11-5)$

$\Rightarrow 6x + 11y - 66 = 0$

Or

$x(10-5) + 10y = 10(10-5)$

$\Rightarrow 5x + 10y - 50 = 0$

$\Rightarrow x + 2y - 10 = 0$

19. Question

If A(3, 6) and C(–1, 2) are two vertices of a rhombus ABCD, then find the equation of straight line that lies along the diagonal BD.

Answer

In a rhombus, the diagonals are perpendicular to each other and also bisect each other

Thus, the product of the slope of AC and BD will be –1 and BD will pass through the mid–point of AC

The points are given as A (3,6) and C (–1, 2)

Slope of AC is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-6}{-1-3} = -\frac{4}{-4} = 1$$

Thus, the slope of BD (m) = –1

The mid–point of AC is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \frac{3-1}{2}, \frac{6+2}{2} = (1,4)$$

Thus BD passes through the point (1,4)

Now using the slope point form, the equation of the line BD is:

$(y - y_1) = m\ (x - x_1)$

Here m = 1 and point is (1, 4)

⇒ y – 4 = (–1) (x –1)

⇒ y – 4 = –x + 1

⇒ x + y –5 = 0

20. Question

Find the equation of the line whose gradient is $\frac{3}{2}$ and which passes through P, where P divides the line segment joining A(–2, 6) and B (3, –4) in the ratio 2 : 3 internally.

Answer

According to the section formula, the coordinates of the point P (x, y) with the given ratio $m_1 : m_2$ is

$$P = \left(\frac{m_2x_2 + m_1x_1}{2+3}, \frac{m_2y_2 + m_1y_1}{2+3}\right)$$

$$= \left(\frac{2(3) + 3(-2)}{5}, \frac{2(-4) + 3(6)}{5}\right)$$

$$= \left(\frac{0}{5}, \frac{10}{5}\right)$$

$$= (0, 2)$$

The gradient is the slope, thus the slope of the line is given as $\frac{3}{2}$

Using the slope –point form, the equation of the line is

$y - y_1 = m\ (x - x_1)$

$$\Rightarrow y - 2 = \frac{3}{2}(x - 0)$$

⇒ 2y – 4 = 3x

⇒ 3x – 2y + 4 = 0

Exercise 5.5

1 A. Question

Find the slope of the straight line

3x + 4y – 6 = 0

Answer

Here we have the straight line : 3x + 4y –6 = 0

The slope intercept form is y = mx + b, where m is the slope and b is the y intercept.

Therefore, 3x + 4y –6 = 0

⇒ 4y + 3x = 6

⇒ 4y = 6–3x

⇒ $\frac{4y}{4} = \frac{6}{4} - \frac{3x}{4}$ (Divide both sides of the equation by 4)

⇒ $y = \frac{3}{2} - \frac{3x}{4}$

Now we will rewrite it in the slope intercept form

$$y = \frac{-3}{4}x + \frac{3}{2}$$

Hence according to the slope intercept form, y = mx + b

m i.e. slope is $\frac{-3}{4}$.

1 B. Question

Find the slope of the straight line

y = 7x + 6

Answer

Here we have the straight line : y = 7x + 6

The slope intercept form is y = mx + b, where m is the slope and b is the y intercept.

Therefore when, y = 7x + 6

Hence according to the slope intercept form,

y = mx + b

m i.e. slope is 7.

1 C. Question

Find the slope of the straight line

4x = 5y + 3.

Answer

Here we have the straight line : 4x = 5y + 3

The slope intercept form is y = mx + b, where m is the slope and b is the y intercept.

Therefore, 4x = 5y + 3

⇒ –5y – 3 = –4x (Commutative law)

⇒ 5y + 3 = 4x (multiply by – on both sides of the equation)

⇒ $\frac{5y}{5} + \frac{3}{5} = \frac{4x}{5}$ (Divide both sides of the equation by 5)

⇒ $y + \frac{3}{5} = \frac{4x}{5}$

$\Rightarrow y = \frac{4x}{5} - \frac{3}{5}$

Now we will rewrite it in the slope intercept form

$$y = \frac{4x}{5} - \frac{3}{5}$$

Hence according to the slope intercept form, y = mx + b

m i.e. slope is $\frac{4}{5}$.

2. Question

Show that the straight lines x + 2y + 1 = 0 and 3x + 6y + 2 = 0 are parallel.

Answer

Given: Here the straight lines are x + 2y + 1 = 0 and

3x + 6y + 2 = 0.

To Prove: x + 2y + 1 = 0 and 3x + 6y + 2 = 0 are parallel.

Proof: If two lines are parallel then their slopes are equal.

Here slope of the first line x + 2y + 1 = 0 will be

$$m = -\frac{\text{coefficient of } x}{\text{coefficient of } y}$$

(When the line is in the form ax + by + c = 0 then the slope of the line is $m = -\frac{\text{coefficient of } x}{\text{coefficient of } y}$)

$\Rightarrow m = -\frac{1}{2}$

Here slope of the second line 3x + 6y + 2 = 0 will be

$$m = -\frac{\text{coefficient of } x}{\text{coefficient of } y}$$

$\Rightarrow m = -\frac{3}{6}$

$\Rightarrow m = -\frac{1}{2}$

Now both the slopes are equal.

Hence both the lines are parallel.

3. Question

Show that the straight lines 3x – 5y + 7 = 0 and 15x + 9y + 4 = 0 are perpendicular.

Answer

Given: The straight lines are 3x – 5y + 7 = 0 and

15x + 9y + 4 = 0 .

To Prove: The straight lines are 3x – 5y + 7 = 0 and

15x + 9y + 4 = 0 are perpendicular.

Proof: If two lines are perpendicular, then the product of their slopes is equal to –1.

The slope of the first line,3x –5y + 7 = 0 is

$m1 = -\frac{3}{(-5)} = \frac{3}{5}$ $(m = -\frac{\text{coefficient of } x}{\text{coefficient of } y})$

The slope of the second line, 15x + 9y + 4 = 0

is $m2 = -\frac{15}{9}$ $(m = -\frac{\text{coefficient of x}}{\text{coefficient of y}})$

⇒ $m2 = -\frac{5}{3}$

Now the product of these slopes is m1×m2.

⇒ $m1 \times m2 = \frac{3}{5} \times \frac{-5}{3} = -1$

As the product of the slopes is –1, the lines are perpendicular to

each other.

4. Question

If the straight lines $\frac{y}{2} = x - p$ and ax + 5 = 3y are parallel, then find a.

Answer

Given: The straight lines are $\frac{y}{2} = x - p$ and ax + 5 = 3y are parallel.

Since the lines are parallel their slopes should be equal.

Now, $\frac{y}{2} = x - p$

⇒ y = 2(x–p)

⇒ y = 2x –2p

⇒ m1 = $\frac{2}{1}$ = 2

ax + 5 = 3y

⇒ 3y = ax + 5

⇒ $\frac{3y}{3} = \frac{ax}{3} + \frac{5}{3}$

⇒ $y = \frac{a}{3}x + \frac{5}{3}$

⇒ $m2 = \frac{a}{3}$

Now as the lines are parallel ,m1 = m2

⇒ $2 = \frac{a}{3}$

⇒ 2×3 = a

⇒ a = 6

5. Question

Find the value of a if the straight lines 5x – 2y – 9 = 0 and ay + 2x – 11 = 0 are perpendicular to each other.

Answer

Given: The straight lines 5x – 2y – 9 = 0 and ay + 2x – 11 = 0 are perpendicular to each other.

As the lines are perpendicular to each other, the product of their slopes is equal to –1.

Slope of the first line 5x – 2y – 9 = 0 is m1.

$m1 = -\frac{5}{(-2)} = \frac{5}{2}$ $(m = -\frac{\text{coefficient of x}}{\text{coefficient of y}})$

Slope of the second line ay + 2x –11 = 0 is m2.

$$m2 = -\frac{2}{a}$$

Therefore, $m1 \times m2 = \frac{5}{2} \times \frac{-2}{a}$

$\Rightarrow m1 \times m2 = \frac{-5}{a}$

As the lines are perpendicular m1× m2 = –1

$\Rightarrow -1 = \frac{-5}{a}$

⇒ a = 5

6. Question

Find the values of p for which the straight lines 8px + (2 – 3p)y + 1 = 0 and px + 8y – 7 = 0 are perpendicular to each other.

Answer

Given: The straight lines 8px + (2 – 3p)y + 1 = 0 and

px + 8y – 7 = 0 are perpendicular to each other.

Since the lines are perpendicular to each other, product of their slopes is equal to –1.

Slope of the first line 8px + (2 – 3p)y + 1 = 0 is m1.

i.e. $m1 = -\frac{8p}{(2-3p)}$

Slope of the second line px + 8y – 7 = 0 is m2.

i.e. $m2 = -\frac{p}{8}$

As the lines are perpendicular to each other m1× m2 = –1.

$\Rightarrow -\frac{8p}{2-3p} \times \frac{-p}{8} = -1$

$\Rightarrow \frac{8p^2}{8(2-3p)} = -1$

⇒ p^2 = -1×(2–3p)

⇒ p^2 = 3p–2

⇒ p^2–3p + 2 = 0

⇒ p.p–2p–p + 2 = 0

⇒ p(p–2)–1(p–2) = 0

⇒ (p–2)(p–1) = 0

⇒ p–2 = 0 ,p–1 = 0

⇒ p = 2,p = 1

Hence p = 1,2.

7. Question

If the straight line passing through the points (h, 3)and (4, 1) intersects the line 7x – 9y – 19 = 0 at right angle, then find the value of h.

Answer

Given: The straight line passing through the points (h, 3)

and (4, 1) intersects the line 7x – 9y – 19 = 0 at right angle.

As these lines intersect at right angle, they are perpendicular to each other.

When the lines are perpendicular to each other, the product of their slopes is equal to –1.

Slope of the first line passing through the points (h,3) and (4,1)

$m1 = \frac{y2-y1}{x2-x1}$ (here the two points are(x1,y1) and (x2,y2))

Now, $m1 = \frac{1-3}{4-h}$

$\Rightarrow m1 = \frac{-2}{4-h}$

Slope of the second line 7x – 9y –19 = 0 is

$$m2 = -\frac{7}{(-9)} = \frac{7}{9}$$

Therefore, product of the slopes is $m1 \times m2 = \frac{-2}{4-h} \times \frac{7}{9}$

$\Rightarrow \frac{-2}{4-h} \times \frac{7}{9} = -1$

$\Rightarrow \frac{14}{9(4-h)} = 1$

$\Rightarrow \frac{14}{36-9h} = 1$

⇒ 14 = 36–9h

⇒ 9h = 36–14 = 22

$\Rightarrow h = \frac{22}{9}$

8. Question

Find the equation of the straight line parallel to the line 3x – y + 7 = 0 and passing through the point (1, –2).

Answer

Here it’s given that the straight line is parallel to the line

3x – y + 7 = 0 and passing through the point (1, –2).

As the lines are parallel to each other their slopes are equal.

Slope of the given line 3x – y + 7 = 0 is

$$m = -\frac{3}{(-1)} = 3$$

Equation of the line passing through the point(1,–2) is

(y–y1) = m(x–x1),where (x1,y1) is (1,–2)

⇒ (y–(–2)) = 3(x–1)

⇒ y + 2 = 3x–3

⇒ y + 2–3x + 3 = 0

⇒ –3x + y + 5 = 0

⇒ 3x–y–5 = 0 (multiplied by –1 on both sides of the equation)

Hence the equation of the line is 3x–y–5 = 0 .

9. Question

Find the equation of the straight line perpendicular to the straight line x – 2y + 3 = 0 and passing through the point (1, –2).

Answer

Here it’s given that the straight line is perpendicular to the

straight line x – 2y + 3 = 0 and passing through the point (1, –2).

As the lines are perpendicular to each other , the product of their

slopes is equal to –1.

Slope of the given line x–2y + 3 = 0 is

$$m = -\frac{1}{(-2)} = \frac{1}{2}$$

Equation of the line passing through the point(1,–2) is

(y–y1) = m(x–x1),where (x1,y1) is (1,–2)

$$(y - (-2)) = \frac{1}{2}(x - 1)$$

⇒ 2(y + 2) = (x–1)

⇒ 2y + 4 = x–1

10. Question

Find the equation of the perpendicular bisector of the straight line segment joining the points (3, 4) and (–1, 2).

Answer

Given: There is a perpendicular bisector of the straight line segment joining the points (3, 4) and (–1, 2).

We have to find the equation of the perpendicular bisector.

As it is perpendicular to the given line segment,the product of their slopes is equal to –1 and as it bisects the line segment,it implies it divides the line segment into 2 equal parts.

Thus,mid–point of the line segment joining the points (3, 4) and (–1, 2) is:

$\frac{x1 + x2}{2}, \frac{y1 + y2}{2}$; where (x1,y1) and (x2,y2) are the end points of the line segment

$= \frac{3 + (-1)}{2}, \frac{4 + 2}{2}$

$= \frac{3-1}{2}, \frac{6}{2}$

$= \frac{2}{2}, 3$

= 1,3

Therefore the mid–point is (1,3).

The slope of the line segment joining the points (3, 4) and

(–1, 2) is:

$$m = \frac{y2 - y1}{x2 - x1}$$

$\Rightarrow m = \frac{2-4}{-1-3}$

$\Rightarrow m = \frac{-2}{-4} = \frac{1}{2}$

Therefore the equation of the perpendicular bisector is

(y–y1) = m(x–x1)

Now substitute the value of the mid–point(1,3) and slope $\frac{1}{2}$ in the above equation.

$$(y-3) = \frac{1}{2}(x-1)$$

⇒ 2(y–3) = (x–1)

⇒ 2y –6 = x–1

⇒ 2y–6–x + 1 = 0

⇒ –x + 2y–5 = 0

⇒ x–2y + 5 = 0 (multiply by –1 on both the sides of the equation)

11. Question

Find the equation of the straight line passing through the point of intersection of the lines 2x + y – 3 = 0 and 5x + y – 6 = 0 and parallel to the line joining the points (1, 2) and (2, 1).

Answer

Here it is given that the straight line passing through the point of intersection of the lines 2x + y – 3 = 0 and 5x + y – 6 = 0 and parallel to the line joining the points (1, 2) and (2, 1).

As the straight line passes through the point of intersection of the lines 2x + y – 3 = 0 and 5x + y – 6 = 0 , we should find the intersection point by solving these equations:

$$\begin{array}{l} 2x + y - 3 = 0 \\ 5x + y - 6 = 0 \\ \underline{-\quad - \quad +\qquad} \\ \underline{-3x \quad + \quad 3 = 0} \end{array}$$

i.e –3x = –3

⇒ x = 1

Substitute x = 1 in 2x + y–3 = 0

We get,2(1) + y–3 = 0

⇒ y–1 = 0

⇒ y = 1

Therefore , the intersection point is (1,1).

As the line passing through(1,1) is parallel to the line segments

joining the points (1, 2) and(2, 1),their slopes are equal.

Slope of the line joining the points (1, 2) and(2, 1) is

$$m = \frac{y2 - y1}{x2 - x1}$$

$$\Rightarrow m = \frac{1-2}{2-1} = \frac{-1}{1} = -1$$

Hence the equation of the line passing through the point (1,1) with slope m equal to –1 is

(y–y1) = m(x–x1)

⇒ (y–1) = –1(x–1)

⇒ y–1 = –x + 1

⇒ y–1 + x–1 = 0

$\Rightarrow x + y - 2 = 0$

12. Question

Find the equation of the straight line which passes through the point of intersection of the straight lines 5x – 6y = 1 and 3x + 2y + 5 = 0 and is perpendicular to the straight line 3x – 5y + 11 = 0.

Answer

Here we have the straight line which passes through the point of intersection of the straight lines 5x – 6y = 1 and 3x + 2y + 5 = 0 and is perpendicular to the straight line 3x – 5y + 11 = 0.

As the straight line passes through the point of intersection of the lines 5x – 6y = 1 and 3x + 2y + 5 = 0, we should find the intersection point by solving these equations:

5x –6y = 1

3x + 2y + 5 = 0

$\Rightarrow$ 5x – 6y– 1 = 0 –––(1)

and 3x + 2y + 5 = 0–––(2)

Now multiply equation (2) by 3 on both the sides.

Thus we have 9x + 6y + 15 = 0–––(3)

Now, we have

$$\begin{array}{l} 5x - 6y - 1 = 0 \text{ from (1)} \\ \underline{9x + 6y + 15 = 0} \text{ from (3)} \\ \underline{14x \quad + 14 = 0} \end{array}$$

$\Rightarrow$ 14x = –14

$\Rightarrow$ x = –1

Now, substituting x = –1 in the equation 5x–6y = 1, we have,

5(–1)–6y = 1

$\Rightarrow$ –5–6y = 1

$\Rightarrow$ –6y = 1 + 5 = 6

$\Rightarrow y = \frac{6}{-6} = -1$

$\Rightarrow$ y = –1

Therefore the point of intersection is (–1,–1).

Slope of the line 3x –5y + 11 = 0 is :

$$m1 = -\frac{3}{(-5)} = \frac{3}{5}$$

As the line which passes through (–1,–1) is perpendicular to 3x –5y + 11 = 0 ,the product of their slopes will be –1.

Therefore, $\frac{3}{5} \times m2 = -1$

$\Rightarrow m2 = -1 \times \frac{5}{3}$ (Here we have multiplied $\frac{5}{3}$ on both the sides)

$\Rightarrow m2 = \frac{-5}{3}$

Hence the equation of the line passing through (–1,–1) and slope as $\frac{-5}{3}$ is:

(y–y1) = m(x–x1)

$\Rightarrow (y-(-1)) = \frac{-5}{3}(x-(-1))$

$\Rightarrow 3(y + 1) = -5(x + 1)$

⇒ 3y + 3 = –5x–5

⇒ 3y + 3 + 5x + 5 = 0

⇒ 5x + 3y + 8 = 0

13. Question

Find the equation of the straight line joining the point of intersection of the lines 3x – y + 9 = 0 and x + 2y = 4 and the point of intersection of the lines 2x + y – 4 = 0 and x – 2y + 3 = 0.

Answer

Here we have the straight line which joins the point of intersection of the lines 3x – y + 9 = 0 and x + 2y = 4 and the point of intersection of the lines 2x + y – 4 = 0 and x – 2y + 3 = 0.

As the straight line which joins the point of intersection of the lines 3x – y + 9 = 0 and x + 2y = 4,let us solve these 2 equations:

3x –y + 9 = 0–––(1)

X + 2y–4 = 0–––(2)

⇒ 3x –y + 9 = 0–––(1)

3x + 6y–12 = 0–––(2)(multiply (2) equation by 3 on both the sides)

Now,

(Subtract equation (2) from (1))

$$\begin{array}{l} 3x - y + 9 = 0 \text{ from } (1) \\ 3x + 6y - 12 = 0 \text{ from } (2) \\ - \quad - \quad\quad + \\ \hline \quad -7y + 21 = 0 \\ \hline \end{array}$$

⇒ –7y = –21

$\Rightarrow y = \frac{21}{7} = 3$

Substitute y = 3 in the first equation

3x–y + 9 = 0

⇒ 3x–3 + 9 = 0

⇒ 3x + 6 = 0

⇒ 3x = –6

$\Rightarrow x = \frac{-6}{3} = -2$

Thus, the point of intersection is(–2,3).

Now,

2x + y – 4 = 0–––(3)

x – 2y + 3 = 0–––(4)

(multiply (4) equation by 2 on both the sides) we get

2x – 4y + 6 = 0 --------(5)

Now,

$$\begin{array}{l} 2x + y - 4 = 0 \text{ from } (3) \\ 2x - 4y + 6 = 0 \text{ from } (5) \\ \underline{-\quad + \qquad -} \\ \hline \quad 5y - 10 = 0 \end{array}$$

⇒ 5y = 10

$\Rightarrow y = \frac{10}{5} = 2$

Substitute y = 2 in the (3) equation:

2x + y–4 = 0

⇒ 2x + 2–4 = 0

⇒ 2x–2 = 0

⇒ 2x = 2

$\Rightarrow x = \frac{2}{2} = 1$

The point of intersection is (1,2).

Hence equation of the line is

$$\frac{y - y1}{y2 - y1} = \frac{x - x1}{x2 - x1}$$

Here we have the points (–2,3) and (1,2)

Therefore,

$$\frac{y - 3}{2 - 3} = \frac{x - (-2)}{1 - (-2)}$$

$$\Rightarrow \frac{y - 3}{-1} = \frac{x + 2}{1 + 2}$$

$$\Rightarrow \frac{y - 3}{-1} = \frac{x + 2}{3}$$

⇒ 3(y–3) = –1(x + 2)

⇒ 3y–9 = –x–2

⇒ 3y–9 + x + 2 = 0

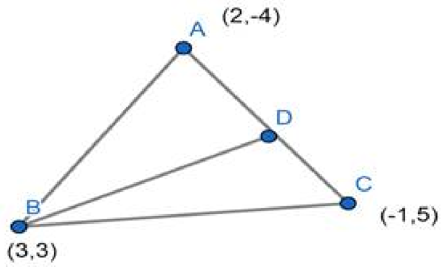
⇒ 3y + x–7 = 0

⇒ x + 3y–7 = 0

14. Question

If the vertices of a Δ ABC are A(2, –4), B(3, 3) and C(–1, 5). Find the equation of the straight line along the altitude from the vertex B.

Answer



Given: A ΔABC has vertices A(2,–4),B(3,3) and C(–1,5).

The straight line BD drawn from vertex B is perpendicular to the line AC. So the product of the slopes of BD and AC is equal to –1.

Slope of AC is m1.

$$m1 = \frac{y2 - y1}{x2 - x1}$$

Where (x1,y1) and (x2,y2) are (2,–4) and (–1,5)

Therefore, $m = \frac{5-(-4)}{-1-2} = \frac{9}{(-3)} = -3$

Slope of the BD is $\frac{-1}{m}$

$= \frac{1}{3}$

Hence the equation of the line BD drawn from the vertex B is

(y–y1) = m(x–x1),here B is (3,3) and m = $\frac{1}{3}$

$\Rightarrow (y-3) = \frac{1}{3}(x-3)$

⇒ 3(y–3) = (x–3)

⇒ 3y –9 = x–3

⇒ 3y–9–x + 3 = 0

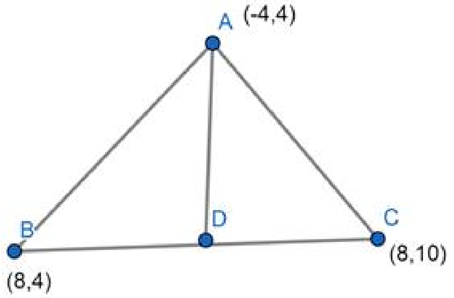
⇒ –x + 3y–6 = 0

⇒ x–3y + 6 = 0

15. Question

If the vertices of a Δ ABC are A(–4,4), B(8 ,4) and C(8,10). Find the equation of the straight line along the median from the vertex A.

Answer



Given: The ΔABC with vertices A(–4,4),B(8,4) and C(8,10) and

AD is the median,i.e. it passes through the mid point of BC.

Therefore D is midpoint of BC where B(8,4) and C(8,10).

$$\left[\frac{(x1 + x2)}{2}, \frac{(y1 + y2)}{2}\right]$$

$= \left[\frac{8+8}{2}, \frac{4+10}{2}\right]$

$= \left(\frac{16}{2}, \frac{14}{2}\right)$

$= (8,7)$

Equation of AD is

$\frac{y-y1}{y2-y1} = \frac{x-x1}{x2-x1}$,where (x1,y1) and (x2,y2) are (–4,4) and (8,7)

⇒ $\frac{y-(4)}{7-4} = \frac{x-(-4)}{8-(-4)}$

⇒ $\frac{y-4}{3} = \frac{x+4}{8+4}$

⇒ $\frac{y-4}{3} = \frac{x+4}{12}$

⇒ 12(y–4) = 3(x + 4)

⇒ 12y–48 = 3x + 12

⇒ 12y–48–3x–12 = 0

⇒ –3x + 12y–60 = 0

⇒ 3x–12y + 60 = 0

⇒ x–4y + 15 = 0(Divide both sides of the equation by 3)

Hence Equation of AD is x–4y + 15 = 0.

16. Question

Find the coordinates of the foot of the perpendicular from the origin on the straight line 3x + 2y = 13.

Answer

Here we have a perpendicular from the origin i.e.(0,0) to the

straight line 3x + 2y = 13.

We have to find the foot of the perpendicular i.e the intersection point at the line 3x + 2y = 13.

As these lines are perpendicular the product of their slopes is equal to –1.

Slope of the 3x + 2y–13 = 0 is m.

$$m = -\frac{3}{2}$$

Therefore the slope of perpendicular is $-\frac{1}{m}$

i.e.$-\frac{1}{\frac{-3}{2}} = \frac{2}{3}$

Hence the equation of the perpendicular from (0,0) and slope as $\frac{2}{3}$

is (y–y1) = m(x–x1)

$$y - 0 = \frac{2}{3}(x - 0)$$

⇒ $y = \frac{2}{3}(x)$

⇒ 3y = 2x

⇒ 3y–2x = 0

⇒ 2x–3y = 0

Now solve the two equations 3x + 2y–13 = 0 and 2x–3y = 0.

3x + 2y–13 = 0–––(1)

2x–3y = 0–––––(2)

Multiply (1) by 3 and (2) by 2 and add

$$\begin{array}{l} 9x + 6y - 39 = 0 \\ \underline{4x - 6y = 0} \\ 13x = 39 \end{array}$$

$$\Rightarrow x = \frac{39}{13} = 3$$

Substitute x = 3 in the equation 2x–3y = 0.

2(3)–3y = 0

⇒ –3y = –6

⇒ y = 2(Divide both the sides of the equation by –3)

Hence the coordinates of the foot of the perpendicular is(3,2)

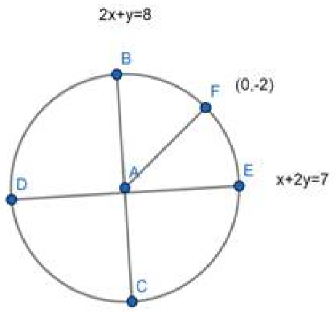
17. Question

If x + 2y = 7 and 2x + y = 8 are the equations of the lines of two diameters of a circle, find the radius of the circle if the point (0, –2) lies on the circle.

Answer

Given: The equations of the lines of two diameters of a circle

are x + 2y = 7 and 2x + y = 8 and F(0,–2)



Now to find the center of the circle,we have to find the

intersection of the lines x + 2y = 7 and 2x + y = 8

x + 2y = 7–––(1)

2x + y = 8–––(2)

Multiplying (1) by 2,we get

$$\begin{array}{l} 2x + 4y = 14 \text{ from } (1) \\ 2x + y = 8 \\ \underline{- \quad - \quad\quad -} \\ \quad 3y = 6 \end{array}$$

$$\Rightarrow y = \frac{6}{3} = 2$$

Substitute y = 2 in x + 2y = 7 we get,

x + 2(2) = 7

⇒ x = 7 – 4 = 3

The point of intersection is (3,2) i.e.the center.

Therefore distance between the points (3,2) and (0,–2) is

$$\sqrt{(x2 - x1)^2 + (y2 - y1)^2}$$

$$= \sqrt{(0 - 3)^2 + (-2 - 2)^2}$$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

= 5 units

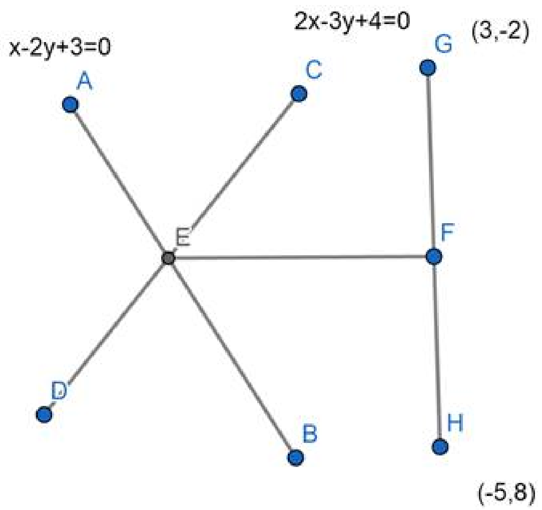
Hence the radius of the circle is 5 units.

18. Question

Find the equation of the straight line segment whose end points are the point of intersection of the straight lines

2x – 3y + 4 = 0, x – 2y + 3 = 0 and the midpoint of the line joining the points (3, –2) and (–5, 8).

Answer



Here its given that the straight line segment has end points as the point of intersection of the straight lines

2x – 3y + 4 = 0, x – 2y + 3 = 0 and the midpoint of the line joining the points (3, –2) and (–5, 8).

As one end point is point of intersection of the straight lines

2x – 3y + 4 = 0, x – 2y + 3 = 0,we will solve these equations.

2x–3y + 4 = 0––––(1)

x–2y + 3 = 0–––––(2)

Multiply (2) by 2,then we have

$$\begin{array}{l} 2x - 3y + 4 = 0 \text{ from } (1) \\ 2x - 4y + 6 = 0 \text{ from } (2) \\ \underline{- \quad + \quad -\qquad\qquad} \\ \underline{\qquad y - 2 = 0\ } \end{array}$$

i.e. y = 2

Substitute y = 2 in 2x–3y + 4 = 0

2x–3(2) + 4 = 0

⇒ 2x–6 + 4 = 0

⇒ 2x–2 = 0

⇒ 2x = 2

⇒ x = 1

Therefore the lines intersect at (1,2).

Now the line has one end point as (1,2) and other end point as mid– point of the line joining (3, –2) and (–5, 8).

The mid– point of (3, –2) and (–5, 8) is

$$\left(\frac{x1 + x2}{2}, \frac{y1 + y2}{2}\right)$$

$$= \frac{3 + (-5)}{2}, \frac{-2 + 8}{2}$$

$$= \frac{-2}{2}, \frac{6}{2}$$

= (–1,3)

Hence to find the equation of the line with end points as (x1,y1) and (x2,y2) ,we use:

$\frac{y-y1}{y2-y1} = \frac{x-x1}{x2-x1}$,where (x1,y1) and (x2,y2) are (1,2) and (–1,3)

$\Rightarrow \frac{y-2}{3-2} = \frac{x-1}{-1-1}$

$\Rightarrow \frac{y-2}{1} = \frac{x-1}{-2}$

⇒ –2(y–2) = (x–1)

⇒ –2y + 4 = x–1

⇒ –2y + 4–x + 1 = 0

⇒ –2y–x + 5 = 0

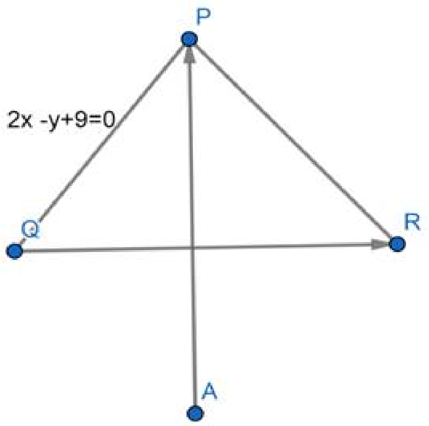
⇒ x + 2y–5 = 0(multiply by –1 on both the sides of the equation)

19. Question

In an isosceles Δ PQR, PQ = PR. The base QR lies on the x–axis, P lies on the y– axis and 2x – 3y + 9 = 0 is the equation of PQ. Find the equation of the straight line along PR.

Answer

Here it’s given that in an isosceles Δ PQR, PQ = PR. The base QR lies on the x–axis, P lies on the y– axis and 2x – 3y + 9 = 0 is the equation of PQ.



The point P lies on the y–axis, so we have to put x = 0 in the equation for PQ

i.e.2x –3y + 9 = 0

⇒ 2(0)–3y + 9 = 0

⇒ –3y + 9 = 0

⇒ –3y = –9

$\Rightarrow y = \frac{-9}{-3} = 3$

Therefore, the point P is (0,3).

Now, to find the point Q which is on the x axis ,we have to substitute 0 in the place of y in the given equation QR.

2x –3y + 9 = 0

⇒ 2x–3(0) + 9 = 0

⇒ 2x + 9 = 0

⇒ 2x = –9

⇒ $x = \frac{-9}{2}$

Therefore,the point Q becomes $(\frac{-9}{2}, 0)$

Now to find the equation of PR, where P is (0,3) and Q is $(\frac{-9}{2}, 0)$,we

have to use ,

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

⇒ $\frac{y-3}{0-3} = \frac{x-0}{\frac{-9}{2}-0}$

⇒ $\frac{y-3}{-3} = \frac{x}{\frac{-9}{2}}$

⇒ $\frac{-9}{2}(y - 3) = -3(x)$

⇒ –9y + 27 = 2(–3x)

⇒ –9y + 27 = -6x

⇒ –9y + 27 + 6x = 0

⇒ 2x–3y + 9 = 0(Divide by 3 on both the sides)

Exercise 5.6

1. Question

The midpoint of the line joining (a,– b) and (3a, 5b) is

A. (–a, 2b)

B. (2a, 4b)

C. (2a, 2b)

D. (–a,– 3b)

Answer

The midpoint of the line joining (a,– b) and (3a, 5b) is

$\frac{a+3a}{2}, \frac{-b+5b}{2}$,(mid point of line segment is $\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$; where (x1,y1) and (x2,y2) are end points of the line segment)

i.e.$(\frac{4a}{2}, \frac{4b}{2})$

= (2a,2b)

2. Question

The point P which divides the line segment joining the points A(1,– 3)and B(–3, 9) internally in the ratio 1:3 is

A. (2,1)

B. (0, 0)

C. $\left(\frac{5}{3}, 2\right)$

D. (1, –2)

Answer

Here P divides the line segment joining the points A(1,– 3)and B(–3, 9) internally in the ratio 1:3 .

Therefore coordinates of P are

$x_p = \frac{k1x_B + k2x_A}{k1 + k2}, y_p = \frac{k1x_B + k2x_A}{k1 + k2}$;where k1 and k2 are the ratio in which the line is divided.

Now we substitute the values :

$\therefore x_p = \frac{1(-3) + 3(1)}{1 + 3}, y_p = \frac{1(-3) + 3(1)}{1 + 3}$

$\Rightarrow x_p = \frac{-3 + 3}{4}, y_p = \frac{-3 + 3}{4}$

$\Rightarrow x_p = \frac{0}{4}, y_p = \frac{0}{4}$

$\Rightarrow x_p = 0, y_p = 0$

3. Question

If the line segment joining the points A(3, 4) and B (14,– 3)meets the x–axis at P, then the ratio in which P divides the segment AB is

A. 4 : 3

B. 3 : 4

C. 2 : 3

D. 4 : 1

Answer

Here the line segment joining the points A(3, 4) and

B (14,– 3)meets the x–axis at P.

Therefore coordinates of P are

$x_p = \frac{k1x_B + k2x_A}{k1 + k2}, y_p = \frac{k1x_B + k2x_A}{k1 + k2}$;where k1 and k2 are the ratio in which the line is divided.

Now,

$x_p = \frac{k1(14) + k2(3)}{k1 + k2}, y_p = \frac{k1(-3) + k2(4)}{k1 + k2}$

$\Rightarrow x_p = \frac{14k1 + 3k2}{k1 + k2}, 0 = \frac{-3k1 + 4k2}{k1 + k2}$

(here $y_{p=0} \because$ the line meets the x–axis at P)

$\Rightarrow \frac{-3k1 + 4k2}{k1 + k2} = 0$

$\Rightarrow -3k1 + 4k2 = 0$

$\Rightarrow -3k1 = -4k2$

$\Rightarrow \frac{k1}{k2} = \frac{4}{3}$

4. Question

The centroid of the triangle with vertices at (–2,– 5), (–2,12) and (10, – 1)is

A. (6, 6)

B. (4, 4)

C. (3, 3)

D. (2, 2)

Answer

Given the triangle with vertices at (–2,– 5),

(–2,12) and (10, – 1).

The centroid of the triangle ABC is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

∴ centroid of the given ΔABC is$\left(\frac{-2 + (-2) + 10}{3}, \frac{-5 + 12 + (-1)}{3}\right)$

i.e. $(\frac{6}{3}, \frac{6}{3})$ = (2,2)

5. Question

If (1, 2), (4, 6), (x, 6)and (3, 2)are the vertices of a parallelogram taken in order, then the value of x is

A. 6

B. 2

C. 1

D. 3

Answer

Here we have the parallelogram with vertices

(1, 2), (4, 6), (x, 6)and (3, 2).

Let the vertices be A(1,2),B(4,6),C(x,6) and D(3,2)

Since the vertices are taken in order AC and BD are the diagonals of the parallelogram.

In a parallelogram diagonals bisect each other.

∴Mid–point of AC = Mid–point of BD

Mid–point of two points (x_1,y_1) and (x_2,y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Here Mid–point of AC = $\left(\frac{1 + x}{2}, \frac{2 + 6}{2}\right) = \left(\frac{1 + x}{2}, \frac{8}{2}\right) = \left(\frac{1 + x}{2}, 4\right)$---(1)

Mid–point of BD = $\left(\frac{4 + 3}{2}, \frac{6 + 2}{2}\right) = (\frac{7}{2}, \frac{8}{2}) = (\frac{7}{2}, 4)$---(2)

(1) = (2)

⇒ $\left(\frac{1 + x}{2}, 4\right) = \left(\frac{7}{2}, 4\right)$

Now we will equate the corresponding coordinates.

∴ $\frac{1 + x}{2} = \frac{7}{2}$

⇒ 2(1 + x) = 7×2

⇒ 2 + 2x = 14

⇒ 2x = 12

⇒ x = 6

6. Question

Area of the triangle formed by the points (0,0), (2, 0)and (0, 2)is

A. 1 sq. units

B. 2 sq. units

C. 4 sq. units

D. 8 sq. units

Answer

Here we have the triangle with vertices (0,0), (2, 0)and (0, 2).

Area of the ΔABC with vertices as A(x1,y1),B(x2,y2) and C(x3,y3) is

$$\text{Area} = |\frac{1}{2}(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))|$$

Let us say the vertices are A(0,0),B(2,0) and C(0,2).

$$\text{Area} = |\frac{1}{2}\left(0(0-2) + 2(2-0) + 0(0-0)\right)|$$

$$\Rightarrow \text{Area} = |\frac{1}{2}(0(-2) + 4 + 0)$$

$$\Rightarrow \text{Area} = |\frac{1}{2}(0 + 4)|$$

$$\Rightarrow \text{Area} = |\frac{1}{2}(4)|$$

⇒ Area = 2 sq.units

Hence Area is 2 sq.units.

7. Question

Area of the quadrilateral formed by the points (1,1), (0,1), (0, 0)and (1, 0)is

A. 3 sq. units

B. 2 sq. units

C. 4 sq. units

D. 1 sq. units

Answer

Here we have the quadrilateral formed by the points (1,1), (0,1), (0, 0)and (1, 0).

We have to calculate the Area of the quadrilateral with vertices as A(1,1), B(0,1), C(0, 0)and D(1, 0)

Let us divide the quadrilateral into 2 triangles,so Area of the quadrilateral will be sum of Areas of two triangles.

Let us say one Δ is ABC and other Δ is ADC

Now Area of ΔABC is

$$\text{Area} = |\frac{1}{2}(x1(y2 - y3) + x2(y3 - y1) + x3(y1 - y2))|$$

When we substitute the values of the coordinates of the vertices

as A(1,1), B(0,1), C(0, 0),we get

$\text{Area} = |\frac{1}{2}(1(1-0) + 0(0-1) + 0(1-1))|$

$\Rightarrow \text{Area} = \left|\frac{1}{2}(1 + 0 + 0)\right| = \frac{1}{2}$

Now Area of ΔADC with vertices A(1,1), D(1, 0)

C(0, 0) is

$\text{Area} = |\frac{1}{2}(1(0-0) + 1(0-1) + 0(1-0))|$

$\Rightarrow \text{Area} = |\frac{1}{2}(0-1 + 0)| = \frac{1}{2}$

Hence area is 1 sq.units.

8. Question

The angle of inclination of a straight line parallel to x–axis is equal to

A. 0°

B. 60°

C. 45°

D. 90°

Answer

Since the line is parallel to x–axis makes an angle of

0 degree with x–axis,the angle of inclination becomes 0°.

9. Question

Slope of the line joining the points (3,– 2)and (–1, a) is $-\frac{3}{2}$, then the value of a is equal to

A. 1

B. 2

C. 3

D. 4

Answer

Slope of the line joining the points (3,– 2)and (–1, a) is $-\frac{3}{2}$.

Slope of the line with end points as (x1,y1) and (x2,y2)

is $m = \frac{y2-y1}{x2-x1}$.

$\therefore m = \frac{a-(-2)}{-1-3} = \frac{a+2}{-4}$

Given $m = \frac{-3}{2}$.

$\therefore \frac{a+2}{-4} = \frac{-3}{2}$

⇒ 2(a + 2) = -3(-4)

⇒ 2a + 4 = 12

⇒ a + 2 = 6 (Divide by 2 on both the sides)

⇒ a = 6–2 = 4

∴ a = 4

10. Question

Slope of the straight line which is perpendicular to the straight line joining the points (–2, 6)and (4, 8)is equal to

A. $\frac{1}{3}$

B. 3

C. –3

D. $-\frac{1}{3}$

Answer

The straight line is perpendicular to the straight line joining the points(–2, 6)and (4, 8).

The slope of line joining the points (–2, 6)and (4, 8) is m1.

i.e. $m1 = \frac{y2-y1}{x2-x1} = \frac{8-6}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$

Now the perpendicular line will have the slope m2 and as

the product of the slopes of 2 perpendicular lines is –1.

m1×m2 = –1

⇒ $\frac{1}{3} \times m2 = -1$

⇒ m2 = –3

11. Question

The point of intersection of the straight lines 9x – y – 2 = 0 and 2x + y – 9 = 0 is

A. (–1, 7)

B. (7,1)

C. (1, 7)

D. (–1,– 7)

Answer

We can get the point of intersection of the straight lines 9x – y – 2 = 0 and 2x + y – 9 = 0 by solving these equations.

9x – y – 2 = 0 from (1)
2x + y – 9 = 0 from (2)
11x – 11 = 0

⇒ 11x = 11

⇒ x = 1

Substitute x = 1 in 9x–y–2 = 0.

⇒ 9(1)–y–2 = 0

⇒ 9–2–y = 0

⇒ 7 = y or y = 7

∴ The point of intersection is (1,7)

12. Question

The straight line 4x + 3y – 12 = 0 intersects the y– axis at

A. (3, 0)

B. (0, 4)

C. (3, 4)

D. (0, – 4)

Answer

When the straight line 4x + 3y – 12 = 0 intersects the y– axis ,then the x coordinate of that point is 0.Thus the point is(0,y).

Now we will substitute (0,y) in the equation for the straight line

4x + 3y – 12 = 0.

⇒ 4(0) + 3y–12 = 0

⇒ 3y–12 = 0

⇒ 3y = 12

⇒ $y = \frac{12}{3} = 4$

Hence the intersection point is (0,4).

13. Question

The slope of the straight line 7y – 2x = 11 is equal to

A. $-\frac{7}{2}$

B. $\frac{7}{2}$

C. $\frac{2}{7}$

D. $-\frac{2}{7}$

Answer

The slope of the line ax + by + c = 0 is

$$m = -\frac{\text{coefficient of } x}{\text{coefficient of } y}$$

∴ slope of the straight line 7y – 2x = 11 is equal to

$$m = -\frac{(-2)}{7} = \frac{2}{7}$$

14. Question

The equation of a straight line passing through the point (2 , –7) and parallel to x–axis is

A. x = 2

B. x = – 7

C. y = – 7

D. y = 2

Answer

The equation of the straight line passing through the point (x1 , y1) with slope as m is

(y–y1) = m(x–x1)

So here the straight line passing through the point (2,–7) and parallel to x axis is

(y–(–7)) = 0(x–2) (Slope of the line parallel to x axis is 0)

⇒ y + 7 = 0

⇒ y = –7

15. Question

The x and y–intercepts of the line 2x – 3y + 6 = 0, respectively are

A. 2, 3

B. 3, 2

C. –3, 2

D. 3, –2

Answer

The x –intercept can be found by substituting y = 0 in the equation 2x–3y + 6 = 0.

⇒ 2x –3(0) + 6 = 0

⇒ 2x + 6 = 0

⇒ 2x = –6

⇒ $x = \frac{-6}{2} = -3$

Now to calculate the y–intercept,substitute x = 0 in the equation 2x–3y + 6 = 0.

i.e.2(0)–3y + 6 = 0

⇒ –3y + 6 = 0

⇒ –3y = –6

⇒ $y = \frac{-6}{-3} = 2$

∴ x–intercept is –3 and y– intercept is 2 i.e.(–3,2).

16. Question

The centre of a circle is (–6, 4). If one end of the diameter of the circle is at (–12, 8), then the other end is at

A. (–18, 12)

B. (–9, 6)

C. (–3, 2)

D. (0, 0)

Answer

Here the centre of a circle is (–6, 4) and one end of the diameter of the circle is at (–12, 8).

As (–6,4) is the center of the circle it becomes the mid –point

of the diameter.

$\therefore \left(\frac{x1+x2}{2}, \frac{y1+y2}{2}\right)$ is the mid–point or the center of the circle.

$\therefore \frac{x1+(-12)}{2}, \frac{y1+8}{2} = (-6,4)$

$\Rightarrow \frac{x1-12}{2} = -6$ and $\frac{y1+8}{2} = 4$

⇒ x1–12 = –6×2 = –12

⇒ x1 = –12 + 12 = 0

Now, $\frac{y1+8}{2} = 4$

⇒ y1 + 8 = 4×2

⇒ y1 = 8–8 = 0

Hence the other end of the diameter is (0,0).

17. Question

The equation of the straight line passing through the origin and perpendicular to the straight line 2x + 3y – 7 = 0 is

A. 2x + 3y = 0

B. 3x – 2y = 0

C. y + 5 = 0

D. y – 5 = 0

Answer

The straight line is passing through the origin and perpendicular to the straight line 2x + 3y – 7 = 0.

Since the straight line is perpendicular to the straight line

2x + 3y – 7 = 0,the product of their slopes will be equal to –1.

Slope of 2x + 3y–7 = 0 is m1.

$$m1 = -\frac{2}{3}$$

∴ slope of the perpendicular line will be m2.

Now m1×m2 = –1

$\Rightarrow \frac{-2}{3} \times m2 = -1$

$\Rightarrow m2 = \frac{3}{2}$

The equation of the line passing through (x1,y1) and slope as m is:

(y–y1) = m(x–x1)

∴ The equation of the line passing through the origin(0,0) and slope as $\frac{3}{2}$ is:

$$(y-0) = \frac{3}{2}(x-0)$$

⇒ 2(y–0) = 3(x–0)

⇒ 2y = 3x–0

⇒ 3x–2y = 0

18. Question

The equation of a straight line parallel to y–axis and passing through the point (–2, 5) is

A. x – 2 = 0

B. x + 2 = 0

C. y + 5 = 0

D. y – 5 = 0

Answer

The straight line parallel to y–axis and passing through the point (–2, 5) will have slope as tan 90°.

(∵ Slope of a line is tan θ, where θ is the angle formed by the line with the x–axis)

Equation of a line passing through (x1,y1) is

(y–y1) = m(x–x1),where m is the slope.

Here m = tan 90°.

i.e.

(y–5) = tan 90°(x–(–2))

⇒ $(y - 5) = \frac{1}{0}(x + 2)$

⇒ 0(y–5) = x + 2

⇒ 0 = x + 2

or x + 2 = 0

19. Question

If the points (2, 5), (4, 6) and (a, a) are collinear, then the value of a is equal to

A. –8

B. 4

C. –4

D. 8

Answer

Here we have the points (2, 5), (4, 6) and (a, a) collinear.

∴ Area of triangle formed by these vertices is 0.Let us say that the vertices are A(2,5),B(4,6) and C(a,a).

$\text{Area} = |\frac{1}{2}(x1(y2 - y3) + x2(y3 - y1) + x3(y1 - y2))|$

⇒ $\text{Area} = \left|\frac{1}{2}\big(2(6 - a) + 4(a - 5) + a(5 - 6)\big)\right| = 0$

⇒ $\left|\frac{1}{2}(12 - 2a + 4a - 20 + 5a - 6a)\right| = 0$

⇒ $\left|\frac{1}{2}(-8 + a)\right| = 0$

⇒ $-8 + a = 2 \times 0$

⇒ –8 + a = 0

⇒ a = 8

20. Question

If a straight line y = 2x + k passes through the point (1, 2), then the value of k is equal to

A. 0

B. 4

C. 5

D. –3

Answer

When a straight line y = 2x + k passes through the point (1, 2), then the value of k can be obtained by substituting x = 1 and y = 2

in the equation of the straight line y = 2x + k.

i.e. 2 = 2×(1) + k

⇒ k + 2 = 2 (commutative property)

⇒ k = 2–2 = 0

21. Question

The equation of a straight line having slope 3 and y–intercept –4 is

A. 3x – y – 4 = 0

B. 3x + y – 4 = 0

C. 3x – y + 4 = 0

D. 3x + y + 4 = 0

Answer

The equation of a straight line having slope m and y–intercept as c is

y = mx + c

Now ,when slope is 3 and y–intercept –4 then equation of the straight line will be

y = 3x + (–4)

⇒ y = 3x–4

22. Question

The point of intersection of the straight lines y = 0 and x = – 4 is

A. (0,– 4)

B. (–4, 0)

C. (0, 4)

D. (4, 0)

Answer

The straight line x = –4 is a line parallel to y–axis and

perpendicular to x–axis and it intersects the x–axis or the straight

line y = 0 at (–4,0).

23. Question

The value of k if the straight lines 3x + 6y + 7 = 0 and 2x + ky = 5 are perpendicular is

A. 1

B. –1

C. 2

D. $\frac{1}{2}$

Answer

When the straight lines 3x + 6y + 7 = 0 and 2x + ky = 5 are perpendicular the product of their slopes should be equal to –1 .

Slope of 3x + 6y + 7 = 0 is:

$\mathrm{m1} = -\frac{3}{6} = \frac{-1}{2}$; ($\mathrm{m} = -\frac{\text{coefficient of x}}{\text{coefficient of y}}$)

Slope of 2x + ky = 5 is:

$$\mathrm{m2} = -\frac{2}{\mathrm{k}}$$

Now we have $\mathrm{m1} \times \mathrm{m2} = -1$.

∴ when we substitute the values of m1 and m2 we get:

$$\frac{-1}{2} \times \frac{-2}{\mathrm{k}} = -1$$

⇒ $\frac{1}{\mathrm{k}} = -1$

⇒ 1 = (–1)×k

⇒ 1 = –k or k = –1