20 PRACTICE PAPER

Time allowed: 45 minutes **Maximum Marks: 200**

General Instructions: *As given in Practice Paper* – 1.

Section–A

Choose the correct option:

1. Let *I* be an identity matrix, then

(*a*) $I = [a_{ij}]_{n'}$ where $a_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$ (*b*) $I = [a_{ij}]_{n'}$ where $a_{ij} = \begin{cases} 0 \text{ if } i = j \\ 1 \text{ if } i \neq j \end{cases}$

(*c*) $I = [a_{ij}]_{n'}$ where $a_{ij} = k \; \forall \; i, j, k \in \mathbb{R}$ (*d*) None of these

2. If *a*, *b*, *c* are negative and different real numbers, then $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

(*a*) $\Delta < 0$ (*b*) $\Delta > 0$ (*c*) $\Delta = 0$ (*d*) $\Delta \leq 0$

3. If $|A| \neq 0$ then which of the following is not true?

(*a*) $(A^2)^{-1} = (A^{-1})^2$ (*b*) $(A')^{-1} = (A^{-1})'$ (*c*) $A^{-1} = |A|^{-1}$ (*d*) None of these

4. $\frac{d^2y}{dx^2}$ equals, if $x = at^2$, $y = 2at$

(*a*) $\frac{-1}{2at^3}$ (*b*) $\frac{1}{2at^2}$ (*c*) $\frac{-1}{2at^2}$ (*d*) 0

5. The values of *a* for which $y = x^2 + ax + 25$ touches the *x*-axis are

(*a*) 0 (*b*) ± 10 (*c*) 4, – 6 (*d*) ± 5

6. The value of $\int_0^{100} (x - [x])\, dx$ is

(*a*) 15 (*b*) 20 (*c*) 50 (*d*) none of these

7. $\int xe^{2x}(1 + x)\, dx$, equals

(*a*) $\frac{xe^x}{2} + C$ (*b*) $\frac{(e^x)^2}{2}$ (*c*) $\frac{(1+x)^2}{2}$ (*d*) $\frac{(xe^x)^2}{2} + C$

8. $\int x \log x \, (\log x - 1)\, dx$ is equal to

(*a*) $2(x \log x - x)^2 + C$ (*b*) $\frac{1}{2}(x \log x - x)^2 + C$ (*c*) $(x \log x)^2 + C$ (*d*) $\frac{1}{2}(x \log x)^3 + C$

9. $\int_0^{\pi/3} \frac{x\,dx}{1+\sec x}$ is equal to

(a) $\frac{x^2}{18}+\frac{\pi}{3\sqrt{3}}+2\log\frac{2}{\sqrt{3}}$ (b) $\frac{\pi^2}{18}-\frac{\pi}{3\sqrt{3}}+2\log\frac{2}{\sqrt{3}}$ (c) $\frac{\pi^2}{18}-\frac{\pi}{3\sqrt{3}}-2\log\frac{2}{\sqrt{3}}$ (d) none of these

10. Area bounded by the curve $x^2+y^2 \le 8x$ and $y^2 \ge 4x$ in the first quadrant is equal to

(a) $32\left(\frac{3\pi}{8}-1\right)$ sq. units. (b) $\frac{32}{3}\left(\frac{3\pi}{8}-1\right)$ sq. units. (c) $4\pi+\frac{32}{8}$ sq. units. (d) $\frac{1}{3}(12\pi-32)$ sq. units.

11. The solution of the differential equation $\frac{dy}{dx}=\frac{x+2y}{x}$ is

(a) $x+y=C$ (b) $x^2+y^2=C$ (c) $x+y=Cx$ (d) $x+y=Cx^2$

12. The solution of $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$ is

(a) $\sin^{-1}x-\sin^{-1}y=C$ (b) $\sin^{-1}x+\sin^{-1}y=C$ (c) $\sin^{-1}x=C\sin^{-1}y$ (d) $(\sin^{-1}x)(\sin^{-1}y)=C$

13. If X follows a Binomial Distribution with parameter $n=6$ and $4P(X=4)=P(X=2)$, then p equals

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

14. Let X be a discrete random variable. The probability distribution of X is given below.

X	30	10	–10
$P(X)$	1/5	3/10	1/2

Then $E(X)$ equals

(a) 6 (b) 4 (c) 3 (d) –5

15. By graphical method solution of LLP maximize

$Z = x + y$ subject to

$x + y \le 2$

$x\,;\, y \ge 0$

obtained at

(a) only one point (b) only two points (c) infinite number of points (d) none of these

Section–B (B1)

16. Consider the non–empty set consisting of children in a family and a relation R defined as aRb, if a is brother of b. Then, R is

(a) Symmetric but not Transitive (b) Transitive but not symmetric

(c) Neither symmetric nor transitive (d) Both symmetric and transitive

17. Let f be strictly decreasing function with range $[a, b]$ then domain of the function f is

(a) $[f^{-1}(b), f^{-1}(a)]$ (b) $[b, a]$ (c) $[f^{-1}(a), f^{-1}(b)]$ (d) None of these

18. The inverse of the function $f(x)=\log_2(x+\sqrt{x^2+1})$ is

(a) 2^x+2^{-x} (b) $\frac{2^x+2^{-x}}{2}$ (c) $\frac{2^{-x}-2^x}{2}$ (d) $\frac{2^x-2^{-x}}{2}$

19. If the binary operation $*$ on Z defined by $a*b=a^2-b^2+ab+4$, then the value of $(2*3)*4$ is

(a) 233 (b) 33 (c) 55 (d) – 55

20. Let * be a binary operation Q^+ (set of positive rationals) such that $a * b = \frac{ab}{8}$ then $2 * 3$ is equal to

(a) 3/8 (b) 1/4 (c) 3/4 (d) None of these

21. If $(\cot^{-1}x)^2 - 7(\cot^{-1}x) + 10 > 0$, then x lies in the interval

(a) (cot 5, cot 2) (b) $(-\infty, \cot 5) \cup (\cot 2, \infty)$ (c) $(-\infty, \cot 5)$ (d) $(\cot 2, \infty)$

22. The domain of $f(x) = \frac{\sin^{-1}x}{x}$ is

(a) $[-1, 1]$ (b) $\{0\}$ (c) $[-1, 0)$ (d) None of these

23. Let $f(x) = e^{\cos^{-1}\sin\left(x+\frac{\pi}{3}\right)}$, then $f\left(\frac{8\pi}{9}\right)$ equals

(a) $e^{\frac{7\pi}{12}}$ (b) $e^{\frac{13\pi}{18}}$ (c) $e^{\frac{5\pi}{18}}$ (d) $e^{\frac{\pi}{12}}$

24. Number of triplets (x, y, z) satisfying $\sin^{-1}x + \sin^{-1}y + \cos^{-1}z = 2\pi$ is

(a) 1 (b) 0 (c) 2 (d) ∞

25. Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then

(a) $A^2 = A$ (b) $A^2 = 0$ (c) $A^2 = I$ (d) None of these

26. There are two values of λ such that $\begin{vmatrix} 0 & 2 & 0 \\ \lambda & 3 & \lambda \\ \lambda & 5 & 6 \end{vmatrix} = -16$ then the sum of two values of λ is

(a) 5 (b) 0 (c) 3 (d) 6

27. If x, y, z are in AP, then the value of determinant $\begin{vmatrix} p+2 & p+3 & p+4 \\ p+3 & p+4 & p+5 \\ p+2x & p+2y & p+2z \end{vmatrix}$ equals

(a) $4a$ (b) 0 (c) $-4a$ (d) 1

28. If A is a singular matrix, then $A(\text{adj } A)$ is

(a) Null matrix (b) Scalar matrix (c) Identity matrix (d) None of these

29. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $adj\ A$ equal to

(a) $\begin{bmatrix} d & b \\ c & a \end{bmatrix}$ (b) $\begin{bmatrix} d & a \\ b & c \end{bmatrix}$ (c) $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (d) $\begin{bmatrix} a & b \\ d & c \end{bmatrix}$

30. Let $f(x) = \begin{cases} -1, & -2 \le x < 0 \\ x^2 - 1, & 0 < x \le 2 \end{cases}$ and $g(x) = |f(x)| + f(|x|)$ then the number of points at which $g(x)$ is non-differentiable is

(a) at most one point (b) two (c) exactly one point (d) infinite

31. Let $f(x)$ be differentiable function such that $f\left(\frac{x+y}{1-xy}\right) = f(x) + f(y)\ \forall\ x$ and y.

If $\lim_{x\to 0}\frac{f(x)}{x} = \frac{1}{3}$ then $f(1)$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{6}$ (d) None of these

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{6}$ (d) None of these

32. Let $f(x)=\begin{cases}1-3x, & x<0\\ 3, & x=0\\ x^2+3, & x>0\end{cases}$ then at $x=0$

(a) $f(x)$ is continuous from left (b) $f(x)$ is continuous

(c) $f(x)$ is right continuous (d) $f(x)$ has removable discontinuity

33. If $a>b>0$, the minimum value of $a\sec\theta - b\tan\theta$ is

(a) $b-a$ (b) $\sqrt{a^2+b^2}$ (c) $\sqrt{a^2-b^2}$ (d) $2\sqrt{a^2-b^2}$

34. Given that $\frac{dy}{dx}=ye^x$ such that at $x=0$, $y=e$. The value of $y(y>0)$ when $x=1$ will be

(a) e (b) $\frac{1}{e}$ (c) e^e (d) None of these

35. The general solution of $\frac{dy}{dx}+ay=e^{mx}$ is

(a) $(m+a)\,y=e^{mx}+ke^{-ax}$ (b) $y=e^{mx}+k$ (c) $y=e^{-ax}+k$ (d) None of these

36. $\int_{-\pi}^{\pi}\frac{x\sin x\,dx}{e^x+1}$ is equal to

(a) $\frac{3\pi}{2}$ (b) $\frac{\pi}{2}$ (c) π (d) 0

37. The value of $\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ is

(a) 2 (b) $\frac{3}{4}$ (c) 0 (d) -2

38. $\int\frac{dx}{\sqrt{9x-4x^2}}$ is equal to

(a) $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$ (c) $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$ (d) $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{9}\right)+C$

39. Area bounded by the curves satisfying the conditions $\frac{x^2}{25}+\frac{y^2}{36}\le 1\le\frac{x}{5}+\frac{y}{6}$ is given by

(a) $15\left(\frac{\pi}{2}-2\right)$ sq. units. (b) $\frac{15}{4}\left(\frac{\pi}{2}-1\right)$ sq. units.

(c) $30(\pi-1)$ sq. units. (d) $\frac{15}{2}(\pi-2)$ sq. units.

40. The vectors from origin to the points A and B are $\vec{a}=2\hat{i}-3\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}+\hat{k}$, respectively, then the area of triangle OAB is

(a) 340 sq. units (b) $\sqrt{25}$ sq. units (c) $\sqrt{229}$ sq. units (d) $\frac{1}{2}\sqrt{229}$ sq. units

41. For any vector $\vec{a}$, the value of $(\vec{a}\cdot\hat{i})^2+(\vec{a}\cdot\hat{j})^2+(\vec{a}\cdot\hat{k})^2$ is equal to

(a) $\vec{a}^{\,2}$ (b) $3\vec{a}^{\,2}$ (c) $4\vec{a}^{\,2}$ (d) $2\vec{a}^{\,2}$

42. If the sum of two unit vectors is also a unit vector then the angle between the two vectors is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{2\pi}{3}$

43. The value of $\vec{A}\cdot\{(\vec{B}+\vec{C})\times(\vec{A}+\vec{B}+\vec{C})\}$ is

(a) 0 (b) $[\vec{A}\,\vec{B}\,\vec{C}][\vec{B}\,\vec{C}\,\vec{A}]$ (c) $[\vec{A}\,\vec{B}\,\vec{C}]$ (d) $\vec{A}\times(\vec{B}\times\vec{C})$

44. The distance of the point (1, 6, 3) to the line $\vec{r}=(\hat{j}+2\hat{k})+\lambda(\hat{i}+2\hat{j}+3\hat{k})$ is

(a) $\sqrt{13}$ (b) 13 (c) $2\sqrt{13}$ (d) None of these

45. If $AB \perp BC$ where $A(2k, 2, 3)$, $B(k, 1, 5)$, $C(3 + k, 2, 1)$ then the value of k equals

(*a*) 3 (*b*) $\frac{1}{3}$ (*c*) -3 (*d*) $-\frac{1}{3}$

46. The ratio of distances from $(-1, 1, 3)$ and $(3, 2, 1)$ to the plane $2x + 5y - 7z + 9 = 0$ is

(*a*) 2 : 1 (*b*) 1 : 3 (*c*) 1 : 1 (*d*) 1 : 2

47. If the lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are parallel, then

(*a*) $a = a', c = c'$ (*b*) $a = c', c = a'$ (*c*) $a = c = a' = c'$ (*d*) None of these

48. If $P(A) = \frac{3}{10}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, then $P(B/A) + P(A/B)$ is equal to

(*a*) $\frac{11}{4}$ (*b*) $\frac{11}{3}$ (*c*) $\frac{5}{12}$ (*d*) $\frac{7}{12}$

49. If two events are independent, then

(*a*) they must be mutually exclusive (*b*) the sum of their probabilities must be equal to 1

(*c*) Both (*a*) and (*b*) are correct (*d*) None of the above is correct

50. If a die is thrown and a card is selected at random from a deck of 52 playing cards, then the probability of getting an even number on the die and a spade card is

(*a*) $\frac{11}{2}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{1}{8}$ (*d*) $\frac{3}{4}$