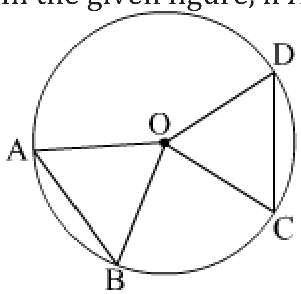
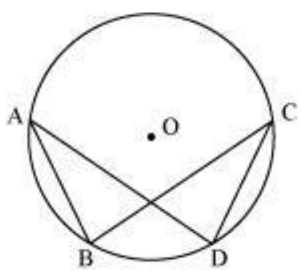
Circles

- Chords of a circle that are equal in length subtend equal angles at the centre of the circle.
  In the given figure, if AB and CD are two equal chords then ∠AOB = ∠COD

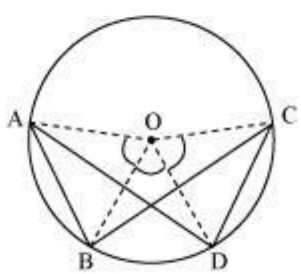


Converse of this property also holds true, which states that chords subtending equal angles at the centre of the circle are equal in length.

Example: If AB and CD are equal chords of a circle then show that AD and CB are also equal.

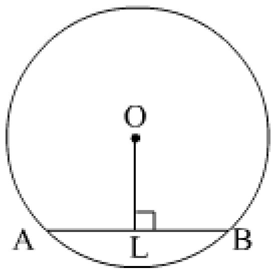


Solution: Construction: Let us join OA, OB, OC and OD.



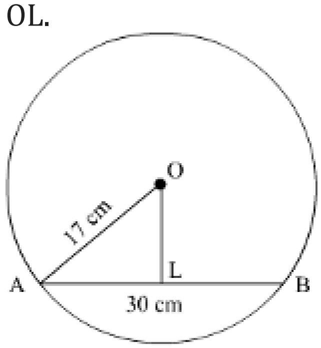
As AB = CD
∴ ∠AOB = ∠COD   [Equal chords subtend equal angles at the centre]
⇒ ∠AOB + ∠BOD = ∠COD + ∠BOD
⇒ ∠AOD = ∠BOC
⇒ AD = CB   [Chords subtending equal angles at the centre are equal in length]

- Perpendicular drawn from the centre of a circle to a chord bisects the chord.
  In the given figure, AL will be equal to LB if OL ⊥ AB, where O is the centre of the circle.



Converse of this property also holds true, which states that the line joining the centre of the circle to the mid-point of a chord is perpendicular to the chord.

Example: In the given figure, $OL \perp AB$. If OA = 17 cm and AB = 30 cm then find the length of OL.

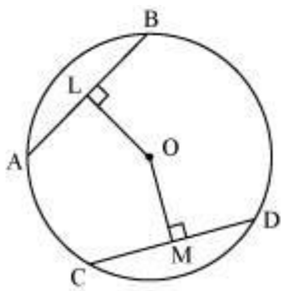


Solution: We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

$\therefore$ AL = BL = 15 cm
Now in right-angled triangle OLA, using Pythagoras theorem
$(OA)^2 = (OL)^2 + (AL)^2$
$\Rightarrow (17)^2 = (OL)^2 + (15)^2$
$\Rightarrow (OL)^2 = (17)^2 - (15)^2$
$\Rightarrow (OL)^2 = 289 - 225$
$\Rightarrow OL = \sqrt{64}$
$\therefore$ OL = 8 cm

- There is one and only one circle passing through three given non-collinear points. Therefore, at least three points are required to construct a unique circle.

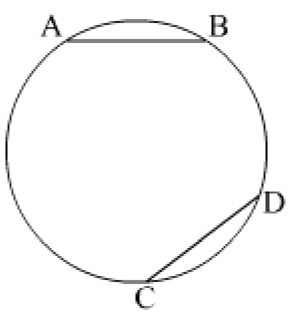
- Equal chords of a circle (or congruent circles) are equidistant from the centre of the circle. In the given figure, OL will be equal to OM if AB = CD, where O is the centre of the circle.



Converse of the property also holds true, which states that chords which are equidistant from the centre of a circle are equal in length.

- If two chords of a circle are equal then their corresponding arcs (minor or major) are congruent.

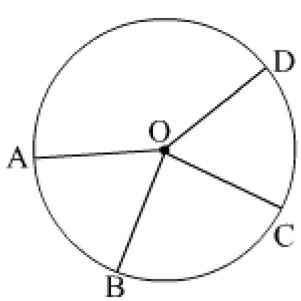
  In the given figure, arc AB will be congruent to arc CD if chord AB = chord CD.



  Converse of the property also holds true, which states that if two arcs of a circle are congruent then their corresponding chords are equal.

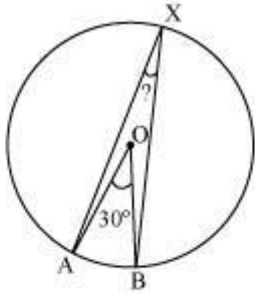
- Congruent arcs subtend equal angles at the centre of the circle.

  In the given figure, ∠AOB will be equal to ∠COD if arcs AB and CD are congruent.



  Converse of the property is also true, which states that two arcs subtending equal angles at the centre of the circle are congruent.

- The angle subtended by an arc at the centre of the circle is double the angle subtended by the arc at the remaining part of the circle.
  In the given figure, ∠AOB and ∠AXB are the angles subtended by arc AB at the centre and at remaining part of the circle.

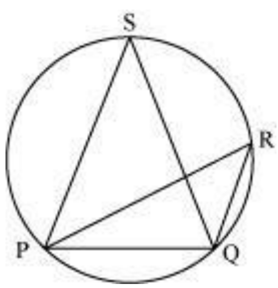


$\therefore 2 \times \angle AXB = \angle AOB$

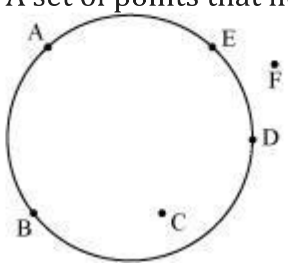
$\Rightarrow \angle AXB = \frac{1}{2} \times 30° = 15°$

- The angle lying in the major segment is an acute angle and the angle lying in the minor segment is an obtuse angle. This statement is true for all major and minor segments in a circle.

- Angles in the same segment of a circle are equal.
  In the given figure, ∠PRQ and ∠PSQ lie in the same segment of the circle.
  $\therefore \angle PRQ = \angle PSQ$



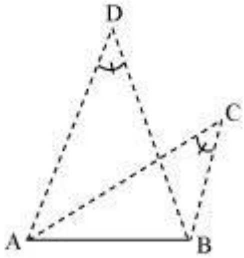
- A set of points that lie on a common circle are known as **concyclic points**.



  Here, points A, B, D and E are concyclic points.

  - If a line segment joining two points subtends equal angles at two other points lying on the same side of the line segment then the four points are concyclic.

In the given figure, if ∠ACB = ∠ADB then the points A, B, C and D are concyclic as C and D lie on the same side of the line segment.



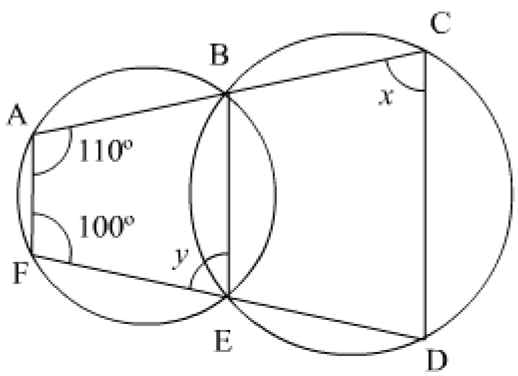
- A quadrilateral whose vertices lie on a circle is known as a **cyclic quadrilateral**.
  Properties of cyclic quadrilateral:

  - The sum of each pair of opposite angles of a cyclic quadrilateral is 180°.

- Converse of the property also holds true, which states that if the sum of a pair of opposite angles of a quadrilateral is 180° then the quadrilateral is cyclic.

  - Exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

Example: In the given figure, find the value of *x* and *y*.



Solution: We know that in a cyclic quadrilateral opposite angles are supplementary.
In cyclic quadrilateral ABEF, ∠A + ∠BEF = 180°
⇒∠BEF = y = 70°
Also,∠BEF + BED = 180° (Linear pair)
⇒∠BED = 110°
In cyclic quadrilateral BCDE, ∠BED + ∠C = 180°
⇒∠C = x =70°