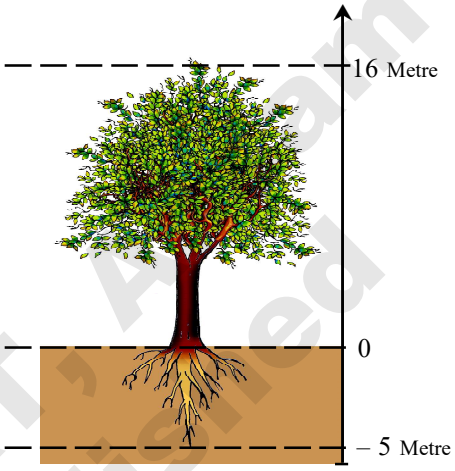
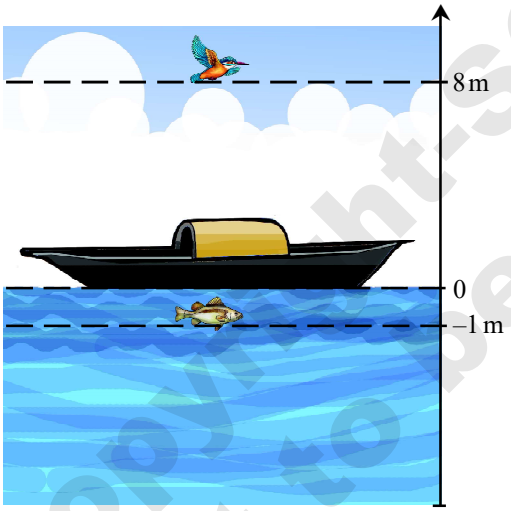
Lesson - 5

Integers

A tree is 16 metres tall. The roots of the tree go 5 metres beneath the gound. If we consider the surface of the earth to be 0, then the parts of the tree above the ground can be considered as 'positive' and the parts of the tree below the ground can be considered as 'negative' as they go in the opposite direction. Hence, as the tree grows 16 metres above the ground, it will be (+)16 metres and roots being 5 metres beneath the ground, can be cosidered as –5 metres.





Observe the adjacent picture. The bird is fllying 8 metres above the boat than the water level. The fish is 1 metre below the water level. If we consider the water level. to be 0, then the position of the fish which is 1 metre below the water level can be considered as –1 metre.

The value of the numbers with negative (– ve) sign is less than '0'. These are called Negative Numbers.

So, –1, –2, –3, – 4..... are negative numbers.

and 1, 2, 3, 4 ..... are positive numbers.

To represent a positive number, '+' sign is used in front of it. But if there is no sign in front of a number then also we consider the number as positive number. In front of a negative number, '–' sign is used.

Let us know

1 and –1, are opposite to each other

4 and – 4 are opposite to each other

We know that, temperature is measured with the help of a thermometer.

Temperature above 0^0C is denoted by '+' sign and temperature below 0^0C is denoted by '–' sign.

Let us denote the following as '+' or '–' sign

(a) Temperature 40^0C below 0^0C is → $+40^0$C

(b) Temperature 5^0C below 0^0C is → -5^0C

Do it yourself :

We can represent the conditions of profit and loss, increase and decrease, going up and down etc as shown below.

Loss of Rs. 10 → –Rs. 10

profit of Rs. 10 → Rs. 10

10 km above the sea level → 10 km.

10 km below the sea level → –10 km.

Do it yourself :

1. If the saving of Rs. 50 is written as + Rs. 50 than what will you write for expenditure of Rs. 40.
2. If the distance from a place towards North is represented by a positive number, then by what number shall we represent the distance from that place towards South.
3. Write the following in the blank boxes with appropriate signs.

(i) Profit of Rs. 70	+ Rs70
(ii) 50 cm above the ground.	
(iii) 30 cm below the ground	
(iv) Loss of Rs. 150	
(v) Saving of Rs. 30	
(vi) Temperature is 35^0C above 0^0C.	
(vii) Temperature is 8^0C below 0^0C	
(viii) 5 metre below the sea level.	
(ix) 5 metre above the sea level.	

Let us recall

1, 2, 3, 4, 5..... are natural numbers.

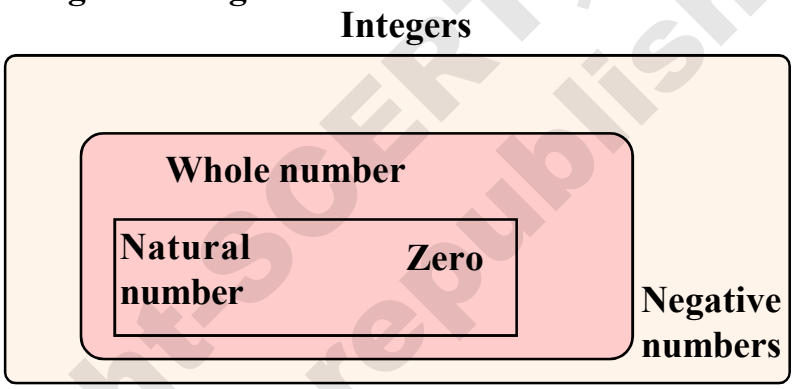
0, 1, 2, 3, 4, 5..... are whole numbers.

–1, –2, –3, –4, –5..... are negative numbers.

We have whole numbers, 0, 1, 2, 3, 4, 5..... and negative numbers, –1, –2, –3, –4, –5 ..... If we put them together, we get a new collection of numbers like, 0, 1, 2, 3, 4, 5.....,–1, –2, –3, –4, –5 ..... This collection of numbers is known as Integers.

Hence, from now onwords, the collection of positive numbers 1, 2, 3, ... and negative numbers, –1, –2, –3,–4, ... and 0 together will be called Integers.

Let us understand the relation between natural numbers, whole numbers and Integers through the diagram.



Write from the above diagram.

(a) Are all natural numbers also whole numbers?

(b) Are all natural numbers also integers?

(c) Are all whole numbers also integers?

(d) Are all integers also whole numbers?

(e) Are all integers also natural numbers?

Let us know :

1, 2, 3, 4, 5..... are positive integers.

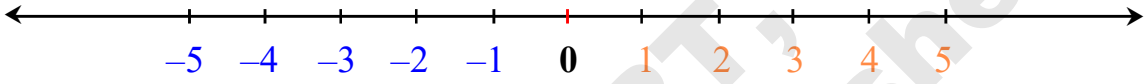
–1, –2, –3, –4, –5..... are negative integers.

Integers on the number line :

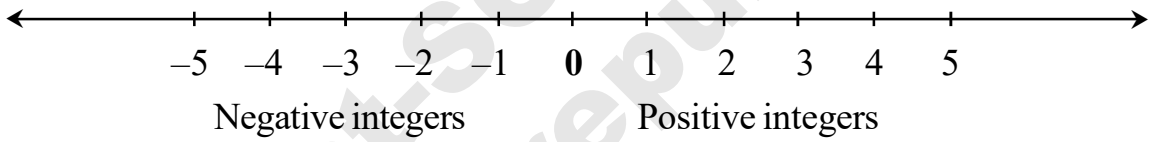
Like whole numbers, integers can also be represented on the number line.

Do it yourself :

- Draw a line.
- Put a mark on the middle of the line.
- Write '0' on that mark.
- On both the sides of the zero '0', go on putting marks after equal intervals.
- On the right of the zero (0), go on writing 1, 2, 3, 4, 5..... respectively on the marks put earlier and write, –1, –2, –3, –4, –5..... respectively on the left of zero.



The numbers on the right of '0' are positive integers and that of left are negative integers.



Observe carefully

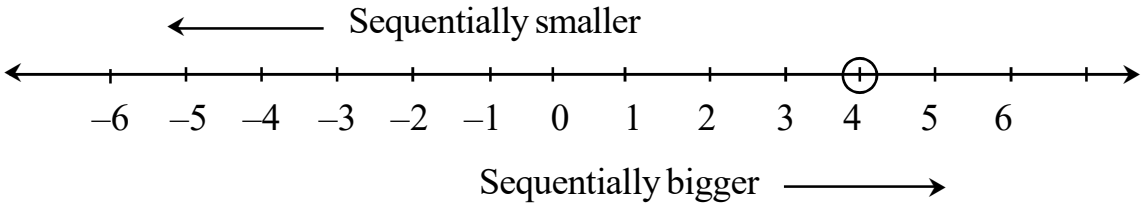
1 and –1, are equidistant from zero (0), but in the opposite direction.

2 and –2, are equidistant from zero (0), but in the opposite direction.

For the remaining numbers, the fact is same.

Comparison of integers and its order :

Every integer on the number line is greater than the integer on its left. This is true for all the integers.



Let us remember

- Smallest positive integer is 1.
- Largest negative integer is –1
- As the number line extends to infinity in both the directions, we donot have largest positive integer and smallest negative integer.
- The integers on the right side of zero on the number line are greater than the integers on the left side of 0.

  For every positive integer, we have a negative integer in the opposite direction of zero and at an equal distance as the positive integer on the right of zero. Both of these integers are called **additive inverse** of one another.

Example :

Plot 5 and 2 on the number line. Then answer the following questions.

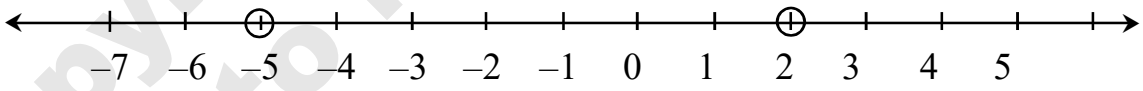
(i) Which one is greater between –5 and 2?

(ii) Write 3 integers smaller than –4.

(iii) Write the integers between – 4 and 2. Write the largest and the smallest integer from the list you found.

(iv) Write two integers greater than 2.

Let us find the answers of the above questions with the help of a number line.



(i) 2 is bigger, because 2 is on the right of –5.

(ii) Three integer smaller than –4 are –5, –6, –7 [numbers on the left of –4 are smaller than –4].

(iii) The integers between –4 and 2 are –3, –2, –1, 0, 1. 1 is the largest and –3 is the smallest integer among them.

(iv) Two integers greater than 2 are 3 and 4.

[All the numbers on the right of 2 are greater than 2.]

Put > or < between the following pair of numbers.

0	<	7		–3		0
4		– 4		6		–2
3		8		–7		–1

Do it yourself :

1. The integer between –2 and 1 is -

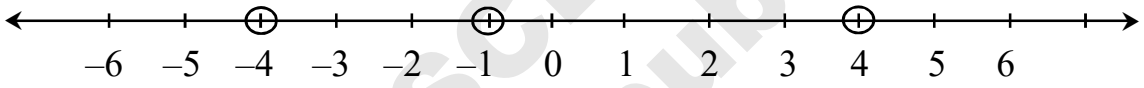
   (a) –4 (b) 0 (c) 2 (d) 3

2. The integer greater than –6 is.

   (a) –7 (b) –9 (c) 0 (d) –10

   What do the encircled '0' numbers on the number line represent?

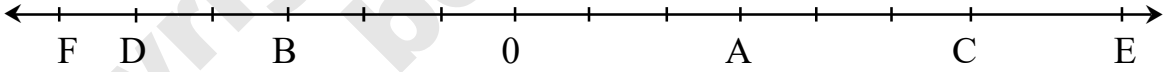
3. Which numbers on the number line is represented by the symbol ⊖.



4. Represent the following numbers on a number line.

   (a) 7 (b) –5 (c) –3 and 3 (d) –5, 0, 5, –4

5.



Observe the number line shown above and answer the following questions.

(a) If point A is +3 then which point is –3.

(b) Write an integer for point C.

(c) Is point D a negative integer or a positive integer?

(d) Which point marked on this number line has the least value?

(e) Arrange the numbers indicated by points in ascending order.

6. Compare the pairs of numbers by using > or < symbols in the blank boxes.

(i) 0 ☐ –4 (ii) 0 ☐ 4

(iii) –3 ☐ 3 (iv) 7 ☐ –6

(v) –1 ☐ –3 (vi) –8 ☐ –1

(vii) 5 ☐ 8 (viii) –6 ☐ 2

7. Arrange the following integers in ascending order.

(i) –1, 6, 0, 3, –3, –2, –7, 5

(ii) –20, –30, –10, 0, 10, 30, 20

(iii) 4, -4, 3, -3, 7, –7, 9, –9

8. Write all the integers between the given pairs.

(i) 0 and 6 (ii) –6 and 6

(iii) –7 and 0 (iv) –4 and 5

(v) –11 and –1 (vi) –1 and 7

9. Write the integers between –7 and 2 in an ascending order. Write the largest and the smallest number among them.

10. Write 5 negative integers greater than –12.

11. Write 5 positive integers greater than–12.

12. Answer the following by using number line.

(i) Write two integers which lie just to the left of and just to the right of –3.

(ii) Write the integer which is 5 units distance on the right of –2.

(iii) Write two integers which are 3 units away from –3 on both the sides equally.

(iv) Which number is to the left of the 5 numbers from –1?

14. We know, –1>–5 and –5>–9; Now, compare –1 and –9 by inserting appropriate symbols between them.

From the number line given in question number 3, it can be said that

3>2, as 3 is to the right of 2.

2>1, as 2 is to the right of 1.

1>0, as 1 is to the right of 0.

0>–1, as 0 is to the right of –1.

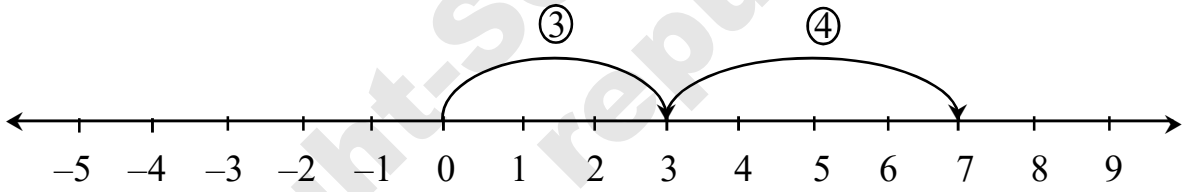
–1>–2, as –1 is to the right of –2.

So, we can say that,

- Every positive integer is larger than 0.
- Every negative integer is smaller than 0.
- Every positive integer is larger than every negative integer.

Addition of integers on a number line :

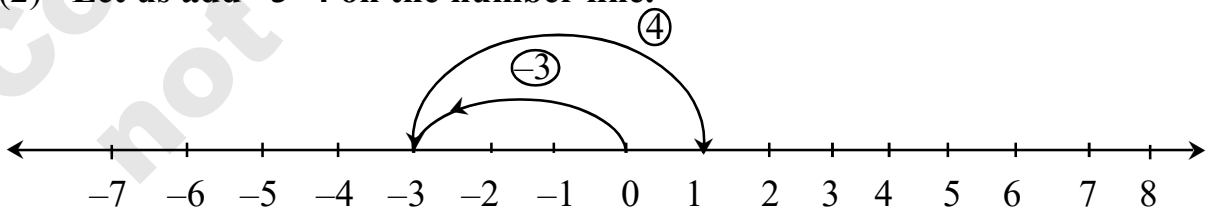
(1) **Let us add 3 and 4 on the number line.**



On the number line, we first move 3 steps to the right from 0 reaching at 3, then we move 4 steps to the right of 3 and reach 7.

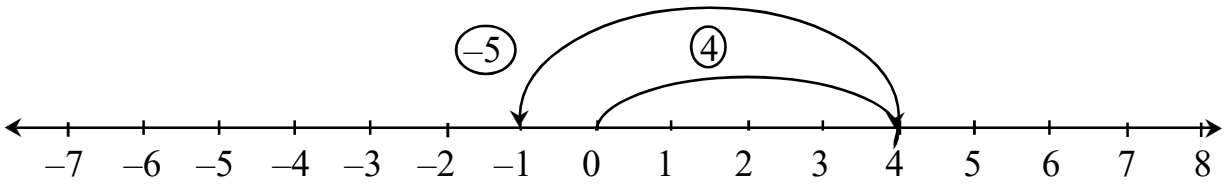
Thus, we get, 3 + 4 = 7

(2) **Let us add –3+4 on the number line.**



First we move 3 steps to the left of 0 reaching –3 and then from –3 we move 4 steps to the right and reach the point 1. Thus, we get, –3+ 4 =1; That means sum of –3 and 4 is 1.

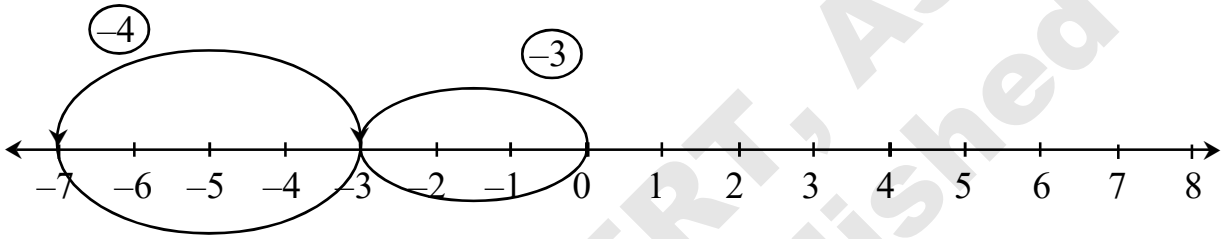
(3) **Let us add 4 and –5 on the number line.**



First we move to the right of 0 by 4 steps reaching at 4. Then we move 5 steps to the left of 4 reaching –1.

Thus, 4 + (–5) = –1. That means that sum of 4 and –5 is –1.

(4) **Let us add (–3) and (–4) on the number line.**



On the number line, we first move 3 steps to the left of 0 reaching (–3) then we move 4 steps to the left of (–3) and reach (–7).

Thus –3 +(–4) = –7. That means, sum of (–3) and (–4) is –7.

Addition of 0 to any integer

$0 + 9 = 9$

$0 + (-6) = -6$

$-9 + 0 = -9$

What did we get from the examples?

Sum of 0 and any other integer is the integer itself, isn't it? So, the addition of 0 with other integer results in the integer itself.

Add quickly,

$0 + 2 = ?$ $\qquad$ $0 + (-7) = ?$

$-6 + 0 = ?$ $\qquad$ $-9 + 0 = ?$

Do it yourself :

1. (a) Sum of (–5) and 5 is.

A. 10 $\qquad$ B. –10 $\qquad$ C. 0 $\qquad$ D. 5

(b) Sum of (–10) and (–3) is.

A. –13 $\qquad$ B. –7 $\qquad$ C. 4 $\qquad$ D. 7

2. Use the number line and find the sum of the following

(i) 8+(–5) (ii) –8+(–5)

(iii) –8 +5 (iv) 7 + (–3)

(v) –12+8 (vi) –2+5+(–3)

(vii) –3+(–4)+(–2) (viii) –6 +(–2) + 7

3. Add the following.

(i) –3+ 0 (ii) 3 + 0

(iii) 0 +(7) (iv) 0 + (–9)

4. Find the sum of

(i) 308 and –470 (ii) 614 and – 312

(iii) 518 and – 609 (iv) 708 and –807

(v) 108, –57 and – 51 (vi) –306, –501 and –409

(vii) 319, –54 and – 78

5. Using the number line find the following.

(i) The integer, which is 5 more than –5.

(ii) The integer, which is 2 more than –7.

(iii) The integer, which is 8 more than –4.

(iv) The integer, which is 4 more than –1.

(v) The integer, which is 7 more than 2.

6. Put (✓) for correct answers and (×) for incorrect answers in the following blank boxes.

(i) 3+ (–5) = –5+ 3 ☐

(ii) –8+(–6) = 6 + 8 ☐

(iii) –15 +15 = 0 ☐

(iv) –9 + 6 = – 6 + 9 ☐

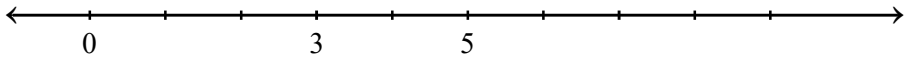
Sum of two negative integers is a negative integer.

Subtraction of integers on a number line :

For the subtraction of natural numbers we have seen that,

5–3 = 2, 8–3 = 5 etc.

Let us look at these on the number line.



5 is 5 steps right to 0 on the number line. Now, we move 3 steps left to zero (0) and reach 2. That means, to subtract 3 from 5, we have to move 3 steps left to 5 and reach 2.

Similarly, to subtract 3 from 8, we have to move 3 steps left from 8.

So, we get 8 – 3 = 5.

Now, what to do, to subtract negative number from a positive number. Let us consider to subtract (–3) from 5, for that we have to observe the following pattern.

5 – 3 = 2 ..... (i)

5 – 2 = 3 ..... (ii)

5 – 1 = 4 ..... (iii)

5 – 0 = 5 ..... (iv)

5 – (–1) = ? ..... (v)

From the above, it is clear that when subtracting from 5, in each case, subtrand is decreasing by 1 and the difference is increasing by 1.

So, in 5 – (–1), the difference is 1 more than the difference in step (iv) ie 5 – 0 = 5, as compared earlier in step (iii) and step (iv).

5 – (–1) = 6

Similarly 5 – (–2) = 7

5 – (–3) = 8

But from the sum of the positive integers, we get,

5 + 3 = 8

Hence 5 – (–3) = 5 + 3 = 8

Also, 7 – (–4) = 7 + 4 = 11

0 – (–2) = 0 + 2 = 2

–4 – (–3) = –4 + 3 = –1

So, from the above, it is clear that we can extend the addition and subtraction process for more than two integers.

Example :

Find the value of 14 – 18–12 + 15 – 30.

Solution : We can rearrange the numbers so that the positive integers and the negative integers are grouped together.

That means, here, we have to subtract the sum of 12, 18 and 30 from the sum of 14 and 15.

Thus, $14 + 15 = 29$

and $12 + 18 + 30 = 60$

That is to subtract 60 from 29.

So, $29 - 60 = -31$

Hence, $14 - 18 - 12 + 15 - 30 = -31$

Alternatively, $14 - 18 - 12 + 15 - 30$

$= (14 + 15) - (18 + 12 + 30)$

$= 29 - 60$

$= -31$

1. **Let us simplify :**

(a) $15 - 12 + 20 - 30$

Here, we have to subtract 12 and 30 from the sum of 15 and 20. That means we have to subtract the sum of 12 and 30 (12 + 30) from the sum of 15 and 20 (15 + 20).

Hence, $15 - 12 + 20 - 30$

$= (15 + 20) - (12 + 30)$

$= 35 - 42$

$= -7$

(b) $20 - 12 + 11 - 13 - 30$

$= (20 + 11) - (12 + 13 + 30)$

$= 31 - 55$

$= -24$

[Forming a group of positive integers and adding them up. Also forming a group of negative integers and adding them up. Subtract the second one from the first.]

(c) $-42 - 17 + 80 - 12$

$= 80 - (42+17+12)$

$= 80 - 71$

$= 9$

2. Sum of two integers is –550. One of them is –270, find the other.

Sum of two integers $= -550$

One of them $= -270$

Other integer $= -550 - (-270)$

$= -550 + 270$

$= -280$

Exercise

1. Fill in the blanks :

(i) $(-8) + \square = 0$ (ii) $-15 - \square = 0$

(iii) $15 - (-15) = \square$ (iv) $\square - (-8) = -12$

(v) $15 + (-15) = \square$ (vi) $13 + \square = -20$

2. Fill in the blanks with (>, < or = sign)

(i) $-9 + (-7) \square (-7) + (-9)$

(ii) $-11 - 9 \square -9 + 11$

(iii) $-35 + 20 \square -10 - 5$

(iv) $40 + (-30) \square -20 + (-10)$

(v) $-15 - (-8) \square 15 - 8$

3. Subtract the following using number line.

(i) Subtract –2 from –5 (ii) Subtract 3 from –3 (iii) Subtract –4 from –4

(iv) Subtract –2 from –6 (v) Subtract –3 from –7 (vi) Subtract –1 from –3

4. Subtract the following.

(i) Subtract 142 from –120 (ii) Subtract –150 from –250

(iii) Subtract –60 from 80 (iv) Subtract –50 from 70

(v) Subtract 4000 from –5080 (vi) Subtract 3000 from –10000

5. Simplify

(i) –97 + (–18) –47+ 57

(ii) –47–48 + 80–62+ 20

(iii) –26 – (–48) – 50–(–20)–10

(iv) 50–18– (–12)+ (–12)

6. Which integer is to be added to –32 to get the sum 0 ?

7. Sum of two integers is –410. One of them is 180. Find the other.

8. What is to be subtracted from –80 to get the difference 0 ?

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Answer

1. (i) + 8 (ii) – 15 (iii) 30 (iv) – 20 (v) 0 (vi) – 33

2. (i) = (ii) < (iii) = (iv) > (v) <

3. (i) – 3 (ii) – 6 (iii) 0 (iv) 8 (v) 10 (vi) – 2

4. (i) – 260 (ii) – 100 (iii) 140 (iv) 120 (v) – 9080 (vi) – 7000

5. (i) – 105 (ii) – 57 (iii) – 18 (iv) 32

6. 32 7. –590 8. – 80