Chapter 4

Wave Guides

CHAPTER HIGHLIGHTS

- *Rectangular Wave Guide*
- *TM Wave*
- *TE Wave*
- *Cut Off Frequency*
- *Modes in Rectangular Waveguide*
- *Group Velocity*
- *Non – Existence of TEM Waves in Waveguides*
- *Circular Wave Guide*
- *Field Configuration for Lower Order Modes in a Rectangular waveguide*
- *S – parameters*
- *Properties of s – matrix*
- *Microwave Networks*
- *Non – Reciprocal Networks*

Introduction

A wave guide is a hollow conducing tube to transmit electromagnetic waves. Although system of conductors and insulators for conveying electromagnetic waves could be called a wave guide but is customary that specially designed hollow metallic pipes are usually referred as wave guides.

Transmission lines are used to transmit electrical energy at low frequencies, while wave guides are used at high frequencies

Practical wave guides usually take the form of rectangular or circular cylinders.

Solved Examples

Example 1

At microwave frequencies, we prefer the wave guides to transmit lines for transporting EM energy because of the following except that

(A) Losses in transmission lines are prohibitively large
(B) wave guides have larger bandwidths and lower signal attenuation
(C) Transmission lines are larger in size than wave guides
(D) Transmission line supports only TEM mode

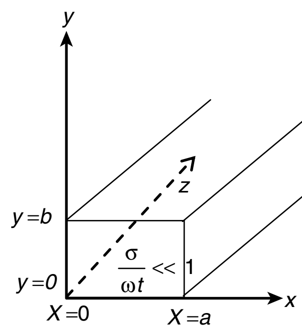
Solution

With respect to microwave frequencies, *a*, *b*, *d* are correct. Option (C) is wrong with respect to wave guides.

Rectangular Wave Guide

Consider the Rectangular wave guide of metal conductivity 'σ' with perfect dielectric such as air, of magnetic permeability μ and permittivity '$\in$' inside the guide for the purpose of study of characteristics.

Let the wave guide going to infinite extent in the z direction.



At dielectric conductor boundary:

E tangential = 0
D normal = ρ_S
H tangential = J
H normal = 0

$\therefore$ At $= x = 0$, $X = a$: $E_y = 0$, $E_z = 0$, $H_x = 0$

and at $y = 0$, $y = b$: $E_x = 0$, $E_z = 0$, $H_y = 0$

The medium inside the wave guide is dielectric

$\therefore\ \nabla \times H = J_D\ [J_C << J_D$ in a dielectric]

$$\nabla \times \overline{H} = j\omega \in \overline{E}$$

$$\nabla \times \overline{E} = -j\omega\mu \overline{H}$$

Considering E and H as

$$\overline{E} = E_x a_x + E_y a_y + E_z a_z$$

$$\overline{H} = H_x a_x + H_y a_y + H_z a_z$$

$$\nabla \times \overline{H} = j\omega \in \overline{E}$$

$$\nabla \times \overline{H} = \begin{vmatrix} a_x & a_y & a_z \\ \overline{\quad x} & \overline{\quad y} & \overline{\quad z} \\ H_x & H_y & H_z \end{vmatrix} = j\omega \in (E_x a_x + E_y a_y + E_z a_z)$$

Comparing the components

I. $$\frac{Hz}{y} - \frac{Hy}{z} = j\omega \in E_x \quad \text{(i)}$$

$$\frac{Hx}{z} - \frac{Hz}{x} = j\omega \in E_y \quad \text{(ii)}$$

$$\frac{Hz}{x} - \frac{Hx}{y} = j\omega \in E_z \quad \text{(iii)}$$

similarly

$$\nabla \times \overline{E} = -j\omega\mu \overline{H}$$

$$\nabla \times \overline{E} = \begin{vmatrix} a_x & a_y & a_z \\ \overline{\quad x} & \overline{\quad y} & \overline{\quad z} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu(H_x a_x + H_y a_y + H_z a_z)$$

II. $$\frac{Ez}{y} - \frac{Ey}{z} = -j\omega\mu H_x \quad \text{(i)}$$

$$\frac{Ex}{z} - \frac{Ez}{x} = -j\omega\mu H_y \quad \text{(ii)}$$

$$\frac{Ez}{x} - \frac{Ex}{y} = -j\omega\mu H_Z \quad \text{(iii)}$$

When a wave propagating in z direction, then

$$E(z) = E_0.e^{-\gamma z}$$

When a wave propagating in z direction then

$$E(z) = E_0.e^{-\gamma z}$$

$$\frac{}{z}(E(z)) = (-\gamma)E_0.e^{-\gamma z}$$

$$= -\gamma \overline{E}(z)$$

γ = propagation constant for the wave traveling in z direction.

Combining I and II equation and an solving interms of E_z and H_z

III. $$E_x = -\frac{\overline{\gamma}}{h^2}\frac{E_z}{x} - \frac{j\omega\mu}{h^2}\frac{H_z}{y}$$

$$E_y = -\frac{-\overline{\gamma}}{h^2}\frac{E_z}{y} + \frac{j\omega\mu}{h^2}\frac{H_z}{x}$$

$$H_x = \frac{-\overline{\gamma}}{h^2}\frac{H_z}{x} - \frac{j\omega \in}{h^2}\frac{E_z}{y}$$

$$H_y = \frac{-\overline{\gamma}}{h^2}\frac{H_z}{x} - \frac{j\omega \in}{h^2}\frac{E_z}{x}$$

The wave guide does not support TEM wave.

Because when $E_z = 0$. and $H_z = 0$

All the Ex, Ey Hx, and Hy are zero. Therefore, no wave propagation, and wave guide support TE or TM waves.

1. TM wave

An electromagnetic wave launched into the wave guide is propagating such that the fields along with extra electric field describe wave motion. Then, the propagating wave is called as TM (or) E wave.

$\therefore$ For a TM wave, $H_z = 0$ and $E_z \neq 0$

IV. $$E_x = -\frac{\overline{\gamma}}{h^2}\frac{E_z}{x} \quad \text{(i)}$$

$$E_y = -\frac{-\gamma}{h^2}\frac{E_z}{y} \quad \text{(ii)}$$

$$H_x = -\frac{j\omega \in}{h^2}\frac{E_z}{y} \quad \text{(iii)}$$

$$H_y = \frac{-j\omega \in}{h^2}\frac{E_z}{x} \quad \text{(iv)}$$

Where $h^2 = \gamma^2 + \omega^2 \mu\in$

$$\eta_{TM} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\gamma}{j\omega \in}$$

TM wave propagating inside the guide is required to satisfy the wave equations.

$$\nabla^2 \overline{E} = -\omega^2 \mu \in \overline{E}$$

$$\nabla^2 \overline{H} = -\omega^2 \mu \in \overline{H}$$

It can be observed from IV. If E_z satisfies the wave equations, then all the remaining fields also bound to satisfy the wave equation

$$\therefore \nabla^2 E_z = -\omega^2 \mu \in E_z$$

$$\frac{^2E_z}{x^2} + \frac{^2E_z}{y^2} + \frac{^2E_z}{z^2} + \omega^2\mu \in E_z = 0$$

Where $(h^2 = \gamma^2 + \omega^2 \mu \in)$; and $\frac{}{z} = \gamma$

$$\frac{^2E_z}{x^2} + \frac{^2E_z}{y^2} + h^2 E_z = 0$$

Let $E_z\,(x, y) = x(x)\,y(y)$

$$\frac{^2xy}{x^2} + \frac{^2xy}{y^2} + h^2 xy = 0$$

$$Y\frac{^2x}{x^2} + X\frac{^2y}{y^2} + h^2 xy = 0$$

dividing through out by xy

$$\frac{1}{x}\frac{d^2x}{dx^2}+\frac{1}{y}\frac{d^2y}{dy^2}+h^2=0$$

$$\left(\frac{1}{x}\frac{d^2x}{dx^2}+h^2\right)+\left(\frac{1}{y}\frac{d^2y}{dy^2}\right)=0$$

Let $\frac{1}{x}\frac{d^2x}{dx^2}+h^2=A^2$

$$\frac{1}{y}\frac{d^2y}{dy^2}=-A^2$$

Rewriting the above

$$\frac{1}{x}\frac{d^2x}{dx^2}+h^2-A^2=0$$

Let $B^2=h^2-A^2$

$$\therefore \frac{1}{x}\frac{d^2x}{dx^2}+B^2=0$$

V. $$\frac{\partial^2 x}{\partial x^2}+B^2x=0 \quad \text{(i)}$$

$$\frac{\partial^2 y}{\partial y^2}+A^2y=0 \quad \text{(ii)}$$

Solving the (i) and (ii) of V

$X=C_1\cos Bx+c_2\sin Bx$

$y=C_3\cos Ay+c_4\sin Ay$

where c_1, c_2, c_3, and c_4 are constants.

$\therefore E_z=xy$

$=(C_1\cos Bx+c_2\sin Bx)(C_3\cos Ay+c_4\sin Ay)$

Applying boundary conditions

$E_z=0$, at $x=0$, a and At $y=0$, b

$$E_z|_{x=0}=(C_1C_3\cos Ay+C_1C_4\sin Ay)$$

$$E_{x=0}=0 \text{ only when } C_1=0$$

$$E_z|_{y=0}=(C_1C_3\cos Bx+C_2C_3\sin Bx)$$

$$E_z|_{y=0}=0 \text{ only when } C_3=0$$

$$\therefore E_z=C_2C_4\sin Bx\sin Ay$$

$$E_z|_{x=a}=C_2C_4\sin Ba\sin Ay$$

$$E_z|_{x=a}=0 \text{ only } Ba=m\pi$$

$$\therefore B=\frac{m\pi}{a}$$

$$E_z|_{y=b}=C_2C_4\sin Bx\sin Ab$$

$$E_z|_{y=b}=0 \text{ only when}$$

$$A=\frac{n\pi}{b}$$

$$\therefore E_z=C_2C_4\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y$$

Therefore, replacing E_z in III equations, we get

VI. $$E_x=\frac{-\gamma}{h^2}C_2C_4\cos Bx\sin Ay \quad \text{(i)}$$

$$E_y=\frac{-\gamma}{h^2}C_2C_4\text{A}\sin Bx\cos Ay \quad \text{(ii)}$$

$$H_x=\frac{j\omega\in}{h^2}C_2C_4\text{A}\sin Bx\cos Ay \quad \text{(iii)}$$

$$H_y=\frac{-j\omega\in}{h^2}C_2C_4B\cos Bx\sin Ay \quad \text{(iv)}$$

a: width-broad dimension

b: height-narrow dimension

$$(a>b)$$

The set VI governs the equations relating to TM wave (or) E – wave.

2. TE wave:

An EM wave launched into the wave guide is propagating such that the fields along with extra magnetic field describe the wave motion. Then, the propagating wave is called TE (or) H wave.

Therefore, for a T.E. wave

$$E_z=0,\ H_z\neq 0.$$

According to the equations of Set III, TE wave equations are

VII. $$E_x=\frac{-j\omega\mu}{h^2}\frac{H_z}{y} \quad \text{(i)}$$

$$E_y=\frac{j\omega\mu}{h^2}\frac{H_z}{x} \quad \text{(ii)}$$

$$H_x=\frac{-\gamma}{h^2}\frac{H_z}{x} \quad \text{(iii)}$$

$$H_y=\frac{-\gamma}{h^2}\frac{H_z}{y} \quad \text{(iv)}$$

Where

$$h^2=\gamma^2+\omega^2\mu\in$$

$$\eta_{TE}=\frac{Ex}{Hy}=\frac{-E_y}{Hx}$$

$$\eta_{TE}=\frac{j\omega\mu}{\gamma}$$

Considering the H_2 satisfies the wave equations

$$\nabla^2H_z=-\omega^2\mu\in H_z$$

$$\frac{{}^2H_z}{x^2}+\frac{{}^2H_z}{y^2}+\frac{{}^2H_z}{z^2}=-\omega^2\mu\in H_z$$

$$\frac{{}^2H_z}{x^2}+\frac{{}^2H_z}{y^2}+\gamma^2H_z+\omega^2\mu\in H_z=0$$

$$\frac{\partial^2H_z}{\partial x^2}+\frac{\partial^2H_z}{\partial y^2}+h^2H_z=0$$

Let $H_z = x.y$

$$\frac{\partial^2 xy}{\partial x^2} + \frac{\partial^2 xy}{\partial y^2} + h^2 xy = 0$$

$$y\frac{d^2 x}{dx^2} + x\frac{d^2 y}{dy^2} + h^2 xy = 0$$

$$\frac{1}{x}\frac{d^2 x}{dx^2} + \frac{1}{y}\frac{d^2 y}{dy^2} + h^2 = 0$$

Let $\frac{1}{x}\frac{d^2 x}{dx^2} + h^2 = A^2$

$$\frac{1}{y}\frac{d^2 y}{dy^2} = -A^2$$

$$\therefore \frac{1}{x}\frac{d^2 x}{dx^2} + h^2 - A^2 = 0$$

Let $h^2 - A^2 = B^2$

$$\therefore \frac{1}{x}\frac{d^2 x}{dx^2} + B^2 = 0$$

VIII. $\frac{\partial^2 x}{\partial x^2} + B^2 x = 0$ (i)

And $\frac{\partial^2 y}{\partial y^2} + A^2 y = 0$ (ii)

Solving i and ii of VIII
We get
$H_z = xy = (C_5 \cos Bx + C_6 \sin By)$
$(C_7 \cos Ay + C_6 \sin Ay)$
According to boundary conditions,
At $x = 0, a$: $E_y = 0, H_x = 0$
$y = 0, b$: $E_x = 0, H_y = 0$

$Ey = 0$, $Hx = 0$, only when $\therefore \frac{\partial Hz}{\partial x} = 0$

$Ex = 0$, and $Hy = 0$, when $\therefore \frac{\partial Hz}{\partial y} = 0$

Using the set VII equation Hz and boundary conditions, on solving we get

$$\therefore B = \frac{m\pi}{a} \qquad A = \frac{n\pi}{b}$$

and

$$H_z = C_5 C_7 \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

IX. $E_x = \frac{j\omega\mu}{h^2} C_5 C_7 (A) \cos Bx \sin Ay$ (i)

$E_y = \frac{-j\omega\mu}{h^2} C_5 C_7 (B) \sin Bx \cos Ay$ (ii)

$H_x = \frac{\gamma}{h^2} C_5 C_7 (B) \sin Bx \cos Ay$ (iii)

$H_y = \frac{\gamma}{h^2} C_5 C_7 (A) \cos Bx \sin Ay$ (iv)

The set IX equations governs the TE wave
Consider

$$h^2 = \gamma^2 \in \omega^2 \mu\in$$

$$B^2 + A^2 = \gamma^2 + \omega^2 \mu\in$$

$$\gamma^2 = B^2 + A^2 - \omega^2\mu\in$$

$$\gamma = \sqrt{B^2 + A^2 - \omega^2\mu \in}$$

For wave propagation.

$$\gamma = j\bar{\beta}$$

i.e. $j\bar{\beta} = \sqrt{B^2 + A^2 - \omega^2\mu \in}$

$-\bar{\beta} = B^2 + A^2 - \omega^2\mu \in$

$\bar{\beta} = \sqrt{\omega^2\mu \in -\left(B^2 + A^2\right)}$

$\therefore$ For wave propagation, $\bar{\beta}$ should be real.

$\therefore\ \omega^2\mu \in > \left(B^2 + A^2\right)$

When $\bar{\beta} = 0$ wave guide is at cut-off frequency.

$\therefore\ \omega^2\mu \in = B^2 + A^2$

$\omega_c^2\mu \in = B^2 + A^2$ at $f = fc$

$$\omega^2\mu \in = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\omega_c = \frac{\pi}{\sqrt{\mu \in}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{1}{2\sqrt{\mu \in}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

f_c depend on m and n

Cut-off frequency

The minimum frequency above which the wave propagation occurs through the wave guide is called the cut-off frequency.

$$f_c = \frac{1}{2\sqrt{\mu \in}}\left[\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\right]$$

f_c depends on m and n.

Example 2

A wave guide operated below the cut-off frequency can be used as

(A) Phase shifter (B) attenuator
(C) isolator (D) None of the above

Solution

There is no wave through wave guide in the cut-off frequency, and hence, the wave guide can be used as an attenuator.

3. **Modes in rectangular wave guide**
The set VI and IX equations are TM and TE wave equation. Substituting different values for m and n, we will get different mode equations and the brief summary is as follows.

TE_{mn} and TM_{mn}		TM_{mn} (H_z=0)	TE_{mn} (E_z = 0)
$m = 0$	$n = 0$	no field	Only H_z is present
$m > 0$	$n = 0$	no field	E_y, H_x, and H_z present
$m = 0$	$n > 0$	no field	E_x, H_y, and H_z present
$m > 0$	$n > 0$	E_x, E_y, H_x, H_y and E_z present	E_x, E_y, H_x, $H_y H_z$ present

Example 3

If the wave guide extended infinitely in the z direction, and broad dimensions along x-axis and narrow dimension along y-axis, respectively. Then which of the following field components exist for TE_{20}

(A) H_z (B) H_y (C) E_x (D) E_y

Solution

According to the given information, TE_{20} contains E_y, H_x, and E_z components

Integers m and n governs field term associated with TM wave (or) TE wave. The propagating waves are designated as TM_{mn} waves and TE_{mn} waves, respectively

n \ m	0	1	2	3
0		TE_{10}	TE_{20}	TE_{30}…..
1	TE_{01}	TE_{11} TM_{11}	TE_{21} TM_{21}	……….
2	TE_{02}	TE_{12} TM_{12}	TE_{22} TM_{22}	………..
3	TE_{03}	TE_{13} TM_{13}	TE_{23} TM_{23}	………..

The lowest TM_{mn} wave is
$TM_{m=1\ n=1}$ i.e., TM_{11}

$$f_c = \frac{1}{2\sqrt{\mu \in}}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{1/2}$$

$$TE_{10}: f_c = \frac{1}{2\sqrt{\mu \in}}\frac{1}{a}$$

$$TE_{01}: f_c = \frac{1}{2\sqrt{\mu \in}}\frac{1}{b}$$

Since $a > b$

$$f_c|_{TE10} < f_c|_{TE01}$$

∴ First propagating TE wave is TE_{10}

TE_{11} (or) TM_{11}:

$$f_c = \frac{1}{2\sqrt{\mu \in}}\frac{\sqrt{a^2+b^2}}{ab}$$

Example 4

A rectangular wave guide of internal dimensions (a = 4 cm and b = 3 cm) is to be operated in TE_{11} mode the minimum operating frequencies

(A) 6.25 GHz (B) 5.0 GHz
(C) 6.0 GHz (D) 3.75 GHz

Solution

$$f_c|_{TE11} = \frac{1}{2\sqrt{\mu \in}}\frac{\sqrt{a^2+b^2}}{ab}$$

$$= \frac{3x10^8}{2} x \frac{5}{12} \times 10^2 = \frac{5}{8} x10^{10} = \frac{50}{8}\text{GHz}$$

$$f_c|_{TE11} = 6.25\text{GHz}$$

∴ Cut-off frequency of TE_{10} wave is the lowest among the first few propagating waves. Therefore, TE_{10} wave is termed as 'dominant wave'.

The wave that prescribes lowest frequency is generally referred to as 'Dominant wave'. For rectangular wave guide, the dominant wave is TE_{10} wave.

Wave guide when allows wave propagation, it acts like a high pass filter. In other words, when signal frequency f is greater than cut-off frequency of the guide, wave propagates inside the guide.

Example 5

A rectangular wave guide having a cut-off frequency of 18 GHZ for TE30 mode and dominant mode as TE10 mode. Then, the inner broad wall dimension of wave guide is.

(A) 1.67 cm (B) 2.5 cm (C) 3.75 cm (D) 5 cm

Solution

$$f_c|_{TEmn} = \frac{1}{2\sqrt{\mu \in}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c|_{TE30} = \frac{1}{2\sqrt{\mu \in}}\frac{3}{a}$$

$$18\text{ GHz} = \frac{3\times10^8\times3}{2a}$$

$$a = 2.5\text{ cm}$$

Example 6

A rectangular metal wave guide filled with a material of relative primitively $\varepsilon_r = 9$ has the inside dimensions of 3.0 cm × 1.2 cm. The cut-off frequency for the dominant mode is.

(A) 2.5 GHz (B) 5 GHz
(C) 5/3 GHz (D) 7.5 GHz

Solution

$$f_c = \frac{1}{2\sqrt{\mu \in}}\frac{1}{a}$$

$$f_c = \frac{3\times 10^8}{2\times 3}\times\frac{1}{3\times 10^{-2}}$$

$$f_c = \frac{10}{6}\text{GHz}$$

$$f_c = \frac{5}{3}\text{GHz}$$

$b < a/2$ $\qquad$ $b > a/2$

$$TE_{10} : f_c = \frac{1}{2\sqrt{\mu \in}}.\frac{1}{a}$$

$f_c|TE_{01} > f_c|TE_{20}$ $\qquad$ $f_c|TE_{01} < f_c|TE_{20}$

$b = a/2 \xleftrightarrow[f_c|TE_{10}]{f_c|TE_{01}} = f_c|TE_{20}$ Bandwidth is fixed

$b < a/2 \xleftrightarrow[f_c|TE_{10}\, f_c|TE_{20}\, f_c|TE_{01}]{x}$ Bandwidth is fixed

$b > a/2 \xleftrightarrow[f_c|TE_{10}\, f_c|TE_{01}\, f_c|TE_{20}]{x}$ Bandwidth is decreased

For maximum band width in rectangular wave guide.

Wave guide in general acts like a high-pass filter. In order to allow the wave guide behave like a band pass filter with maximum bandwidth, the dimensions are related as follows.

$$b \le a/2 \text{ or } \frac{a}{b} \ge 2$$

Example 7

A rectangular wave guide has dimensions of $a = 4$ cm and $b = 3$ cm. The modes that propagate through wave guide at a frequency of 4 GHz

(A) TE_{10} (B) TE_{10}, TE_{01}
(C) $TE_{10}, TE_{01}, TE_{11},$ (D) $TE_{10}, TE_{01}, TE_{11}, TM_{01},$

Solution

TE_{10} is dominant mode.

$$\therefore f_c = \frac{3\times 10^8}{2\times 4\times 10^{-2}} = \frac{3}{8}\times 10^{10}$$

$$f_c|_{TE10} = \frac{30}{8}\text{GHz} = 3.75\text{GHz}$$

$$f_c|_{TE11} = \frac{3\times 10^8}{2}\times\frac{5}{12\times 10^{-2}} = \frac{50}{8}\text{ GHz} = 6.25\text{ GHz}$$

$$f_c|_{TE01} = \frac{1}{2\sqrt{\mu \in}}.\frac{1}{b}$$

$$= \frac{3\times 10^8}{2\times 3\times 10^{-2}}$$

$$f_c|_{TE01} = 5\text{ GHz}$$

∴ Given $f = 4$ GHz only

$$a = 2.5\text{cm } f_c|_{TE10} < \text{f}$$

∴ Only TE_{10} mode propagate through the wave guide

Fields of dominant wave

$$H_z = C_5\, C_7 \cos\left(\frac{\pi}{a}.x\right)$$

$$E_y = \frac{j\omega\mu}{h^2}\, C_5\, C_7\left(-\pi/a\right)\sin\left(\frac{\pi}{a}.x\right)$$

$$E_y = \frac{j\omega\mu}{\left(\pi/a\right)^2}\, C_5\, C_7\left(-\pi/a\right)\sin\left(\pi/a.x\right)$$

$$_x = \frac{-\gamma}{\left(\pi/a\right)^2}\, C_5\, C_7\left(-\pi/a\right)\sin\left(\left(\pi/a\right).x\right)$$

Rewriting.

X. $H_z = C_5\, C_7 \cos\left[\left(.\frac{\pi}{a}\right)x\right]$

$$E_y = E_o \sin\left[\left(.\frac{\pi}{a}\right)x\right]$$

$$H_x = \frac{-E_o}{\left(\frac{j\omega\mu}{\gamma}\right)}\sin\left(\frac{\pi}{a}.x\right)$$

$$H_x = \frac{-E_o}{\eta_{TE}}\sin\left(\frac{\pi}{a}.x\right)$$

$$E_o = \frac{j\omega\mu C_5 C_7}{\pi}$$

$$\eta_{TE} = \frac{\eta}{\sqrt{1-\left(f_c/f\right)^2}}$$

$$f_c = \frac{1}{2\sqrt{\mu \in}}\frac{1}{a}$$

All the X field equations are depends on frequency and a. wave guide having $b \le a/2$ supports only dominant wave when $f > f_c|_{TE10}$ and $f < f_c|_{TE20}$

Power flow

Average power flow $= \dfrac{E_o{}^2.ab}{4\eta_{TE}}$

The average power flow through rectangular wave guide supporting dominant wave is a function of narrow dimension 'b'.

Phase constant

Consider $\bar{\beta} = \sqrt{\omega^2 \mu \in -\left(B^2 + A^2\right)}$

$$\therefore\; B^2 + A^2 = \omega_c^2 \mu \in$$

$$\bar{\beta} = \omega\sqrt{\mu \in}\sqrt{1-\left({}^{\omega_c}\!/_{\omega}\right)^2}$$

$$\bar{\beta} = \beta\sqrt{1-\left({}^{\omega_c}\!/_{\omega}\right)^2}$$

$$\bar{\beta} < \beta$$

$\bar{\beta}$ – guide phase constant (or)

Phase constant inside the wave guide

$$\bar{\beta} = \beta\sqrt{1-\left({}^{\omega_c}\!/_{\omega}\right)^2}$$

Example 8

In a wave guide, the evanescent modes are said to occur if
(A) phase constant is zero
(B) phase constant is real
(C) Phase constant is imaginary
(D) The signal has constant frequency

Solution

An evanescent mode occur in wave guide when $f < f_c$

$\therefore$ if $f < f_c$

$$\bar{\gamma} = h\sqrt{1-\left({}^{f}\!/_{f_c}\right)^2}$$

$\bar{\gamma}$ is real when $f < f_c$.

$$\therefore\; \bar{\beta} = 0$$

Is phase constant is zero

Phase velocity:

$$\overline{V_p} = \frac{\omega}{\beta} = \frac{\omega}{\beta\sqrt{1-\left({}^{\omega c}\!/_{\omega}\right)^2}}$$

$$V_p = \frac{V}{\sqrt{1-\left({}^{\omega c}\!/_{\omega}\right)^2}} \quad \text{Where } V = \frac{1}{\sqrt{\mu \in}}$$

$$\bar{v}_p > v$$

Phase velocity inside the wave guide is greater than the phase velocity outside the wave guide.

$$\bar{\beta} = \beta\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

$$\frac{2\pi}{\bar{\lambda}} = \frac{2\pi}{\lambda}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

$$\bar{\lambda} = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\frac{1}{\lambda^2} = \frac{1}{\bar{\lambda}^2} + \frac{1}{\lambda_c^2}$$

λ – wave length outside the wave guide
$\bar{\lambda}$ – wave length inside the wave guide
λ_c – cut of wave length
$\lambda_c > \lambda$

Example 9

A rectangular hollow metal wave guide has dimensions $a = 2.29$ cm and $b = 1.02$ cm microwave power at 10 Hz is transmitted through the wave guide in the TE_{10} mode then the cut-off wave length for this mode is

(A) 1.145 cm (B) 1.5cm
(C) 4.58cm (D) 6.87 cm

Solution

$$\lambda_c = 2a = 2 \times 2.29 \text{ cm}$$

$$\lambda_c = 4.58 \text{ cm}$$

Example 10

In Example 11, wave guide length (or) wave length inside the wave guide is

(A) 3 cm (B) 4 cm
(C) 4.58 cm (D) None of these

Solution

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left({}^{fc}\!/_{f}\right)^2}}$$

$$\lambda = {}^{c}\!/_{f} = 3\text{cm}$$

$$f_c = \frac{c}{2a} = 6.55 \text{ GHZ}$$

$$\lambda_g = 4 \text{ cm}$$

Wave impedance

$$\eta_{TM} = \frac{\bar{\gamma}}{j\omega \in} = \frac{j\bar{\beta}}{j\omega \in} = \frac{\bar{\beta}}{\omega \in}$$

$$\eta_{TM} = \sqrt{\frac{\mu}{\in}}\sqrt{1-\left({}^{f_c}\!/_{f}\right)^2}$$

$$\eta_{TM} = \eta\sqrt{1-\left({}^{f_c}\!/_{f}\right)^2}$$

$$\eta_{TE} = \frac{j\omega\mu}{\bar{r}} = \frac{j\omega\mu}{j\bar{\beta}}$$

$$= \frac{\omega\mu}{\omega\sqrt{\mu \in}\sqrt{1-\left({}^{f_c}\!/_{f}\right)^2}}.$$

$$\eta_{TE} = \sqrt{\frac{\mu}{\in}} \cdot \frac{1}{\sqrt{1-\left(f_c/f\right)^2}}$$

$$\eta_{TE} = \cdot \frac{\eta}{\sqrt{1-\left(f_c/f\right)^2}}$$

$$\eta_{TE} . \eta_{TM} = \eta^2$$

$$\eta_{TE} . > \eta_{TEM} > \eta_{TM}$$

Wave propagation inside the guide is dispersion type.

$$\bar{\beta} = \beta\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

Group velocity is nothing but velocity of a dispersed wave

$$\text{Slope} = \frac{d\bar{\beta}}{d\omega}$$

$$V_g = \frac{1}{\left(\frac{d\bar{\beta}}{d\omega}\right)}$$

$$= \frac{d\omega}{d\bar{\beta}} = \frac{\sqrt{\mu \in}}{\sqrt{1-\left(\frac{\omega c}{\omega}\right)^2}}$$

$$V_g = v\left(\sqrt{1-\left(\frac{f_c}{f}\right)^2}\right)$$

$$\bar{V_p} = \frac{v}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$V_p > v > V_g$$

$$V_{g.} V_p = v^2 = c^2$$

V_p – velocity inside the wave guide

V_g – group velocity

$$V = \frac{1}{\sqrt{\mu \in}}$$

Example 11

An air-filled rectangular wave guide has inner dimensions of 3 cm × 2 cm. The wave impedance of TE_{20} mode of propagation in the wave guide at a frequency of 30 GHz is

(A) 300 Ω (B) 370 Ω
(C) 323 Ω (D) 377 Ω

Solution

$$f_c = \frac{3\times10^8}{2\times3\times10^{-2}\times2}$$

$$f_c = 2.5 \text{ GHz}$$

$$f = 30 \text{ GHz}$$

$$\eta_{TE} = \frac{\eta_o}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\eta_{TE} = \frac{\eta_o}{\sqrt{1-\left(\frac{1}{6}\right)^2}} = \frac{120\pi\times6}{\sqrt{35}} = 377\ \Omega$$

Example 12

The phase velocity of an electromagnetic wave propagating in a hollow metallic rectangular wave guide in the TE_{10} mode is

(A) equal to group velocity
(B) less than the velocity of light in free space
(C) equal to velocity of light in free space
(D) greater than the velocity of light in free space

Solution

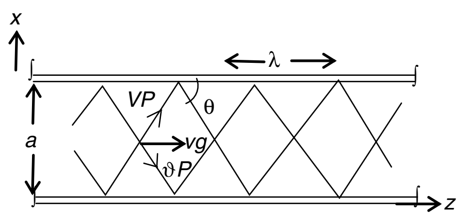
Velocity inside the wave guide is greater than the velocity of light in free space (c) according to

$$\bar{V_p} = \vartheta \Big/ \sqrt{1-\left(f_c/f\right)^2}$$

Group Velocity

It is the velocity of propagation of wave packet envelop of a group of frequencies. It is the energy propagation velocity in the guide.

The field in the wave guide may be depicted as sum of two-plane TEM waves propagating along zigzag paths between the guide walls. As a consequence of zigzag path, we have three types of velocity mentioned previously.



TE_{10} mode decomposition.

Non–Existence of TEM Waves in Wave guides

It is evident that TM and TE waves can propagate within hallow rectangular cylinders or circular wave guides. But astonishing is the fact that TEM waves for which there is no axial component possibly propagate within a single conductor wave guide. Suppose a TEM wave is assumed to exist within a hallow guide of cone shape.

Then, lines of H must lie entirely in the transverse plane.

Also, in non-magnetic material, $\nabla . \bar{H} = 0$, this requires that the lines of H can be closed loops. Thus, if a TEM exists inside the guide, the lines of H will be closed loops in plane perpendicular to the axis.

Now, by Maxwell's first equation, the m.m.f around each of these closed loops must equal the axial current (conduction or displacement) through the loop. In the case of a 'guide' with an inner conductor, for example, a coaxial transmission line, this axial current through the H loops is the inner conductor. However, for a hallow waveguide having no inner conductor, this axial current must be a displacement current,

But an axial displacement current requires an axial component of E which is not present in the TEM wave.

Hence, the TEM wave cannot exist in a single-conductor wave guide.

Circular Wave Guide

Case 1: For TE mode

TE_{nm}

Radius 'a'

$$f_c = \frac{\gamma_0 h^1{}_{nm}}{2\pi a}$$

$$h_{1,1}{}^1 = h^1 \text{nm (lowest)} = 1.841$$

$$f_c = \frac{\gamma_0 \times 1.841}{2\pi a}$$

$$f_c = \frac{\gamma_0}{a} \times \frac{1.841}{2\pi}$$

$$\lambda_c = \frac{\gamma_0}{f_c} = \frac{\gamma_0}{\frac{\gamma_0 \times 1.841}{2\pi a}}$$

$$\lambda_c = \frac{2\pi a}{1.841} = 3.41a$$

TE_{11} mode is dominant mode

$$\lambda_c = 3.41a$$

$$f_c = \frac{\gamma_0}{\lambda_c}$$

Case 2: TM_{nm} mode

$$f_c = \frac{\gamma_0 h_{nm}}{2\pi a}, \; h_{01} = h_{nm} \text{ (lowest)} = 2.405$$

$$\Rightarrow f_c = \frac{\gamma_0 \times 2.405}{2\pi a}, \; TM_{01} \text{ mode}$$

$$\Rightarrow \lambda_c = \frac{\gamma_0}{f_c} = \frac{2\pi a}{2.405}$$

$$TE_{nm}, f_c = \frac{1.841\gamma_0}{2\pi a}$$

$$\lambda_c = 3.41a$$

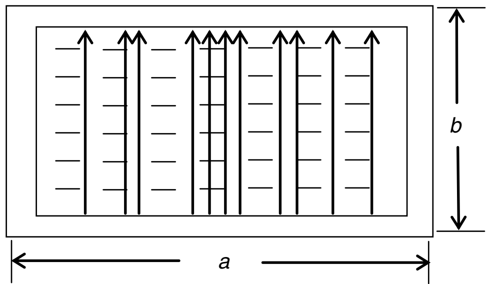
- TE_{11} mode (dominant mode)

TE_{mn},

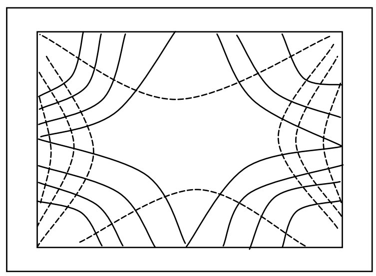
$$f_c = \frac{2.405\gamma_0}{2\pi a}$$

$$\lambda_c = \frac{2\pi a}{2.405}, \; TM_{01} \text{ mode}$$

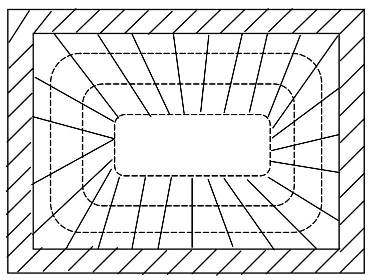
Field Configureation for Lower Order Modes in a Rectangular Wave Guide



TE10 Wave

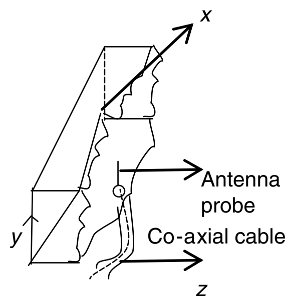


TE_{11} Wave

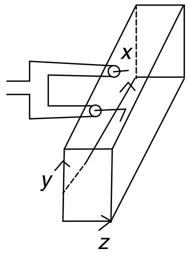


TM_{11} Wave

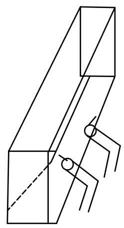
Pulse Excitation

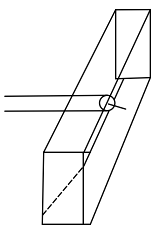


(a) TE_{10}



(b) TE_{11}


(c) TE_{20} wave


(d) TM_{11} wave

Possible methods for feeding rectangular wave guides are shown in the above figures.

In order to launch a particular mode, a type of probe is chosen which will produce lines of E and H that are roughly parallel to the lines of E and H for that mode. Thus, in Figure(a), the probe is parallel to y axis and so produces lines of E in the y direction and lines of H in x– z plane. This is the correct field configuration for TE_{10} mode. In (b), the parallel probes fed with opposite phase tend to setup TE_{20} mode.

In(d), the probe parallel to z axis produces magnetic lines in the x–y plane which is correct for TM modes.

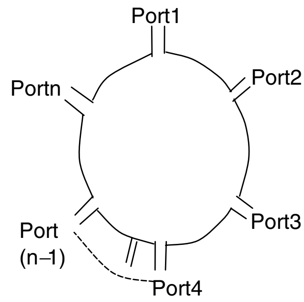
It is possible for several modes to exist simultaneously in a guide if the $f > f_c$ for those particular modes. However, the guide dimensions are often chosen so that only dominant mode exist.

S Parameters

The two port network parameters like z, y, ABCD parameters fails at microwave frequencies because the voltage and current does not have the linear relationship, and sometimes, it is even difficult to measure voltage and current. Hence, we use scattering parameters.

Scattering parameters are not measured. We are measuring reflection and transmission coefficient.

When the signal is excited, A part of it may reflect and remaining may scatter to all ports. Considering a n-port network.



$b_1 = s_{11}\,a_1 + s_{12}\,a_2 + \ldots\ldots\ s_{1n}\,a_n$
$b_2 = s_{21}\,a_1 + s_{22}\,a_2 + \ldots\ldots\ s_{2n}\,a_n$
$b_n = s_{n1}\,a_1 + s_{n2}\,a_2 + \ldots\ldots\ s_{nn}\,a_4$
s_{ij} – scattering co-efficient
If $i = j$ – reflection coefficient
$i \neq j$ – transmission coefficient
In matrix for

$$\begin{bmatrix} b_1 \\ b_2 \\ ; \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13}\ldots & .S_{1n} \\ S_{21} & S_{22} & S_{23}\ldots & S_{2n} \\ ; & ; & ; & ; \\ S_{n1} & S_{n2} & S_{n3}\ldots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ ; \\ a_n \end{bmatrix}$$

$$[b]_{nx1} = [S]_{nxn}\,[a]_{nx1}$$

Properties of *s* Matrix

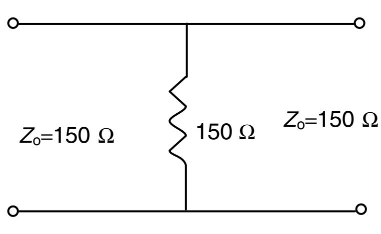
(i) s matrix is a square matrix
(ii) s matrix is a symmetrical matrix, i.e., $[S^T] = [S]$ when network is reciprocal
(iii) s matrix is unitary.
That is., $[S] \times [S^T]^* = I$ (or) $[S^T]^* = S^H$ is called Hermitian matrix of $[S]$

Example 13

A load of 150 Ω is connected in shunt in a 2-wire transmission line of $z_o = 150\ \Omega$ as shown in the figure. The 2–port scattering parameters (s-matrix) of the shunt element is

(A) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{4} & -\frac{3}{4} \\ -\frac{3}{4} & \frac{1}{4} \end{bmatrix}$



Solution

$$\bar{B} = \bar{S}.\bar{A}$$

$\bar{B}$ – Scattered case matrix

$\bar{A}$ = Incident matrix

$\bar{S}.$ – Scattered matrix

Shunt impedance = characteristic impedance.
∴ Matched condition. No reflection occurs,
i.e., $S_{11} = S_{22} = 0$ complete power transfer

$$\therefore S_{12} = S_{21} = 1$$

Example 14

Scattering parameters are more suited than impendence parameters to describe a wave guide junction because

(A) The scattering parameters are frequency invariant whereas the impedance parameters are not so.
(B) Scattering matrix is always unitary
(C) impedence parameters vary over unacceptably wide ranges.
(D) Scattering parameters are directly measurable, but impedance parameters are not so.

Solution

Theorems:

(i) A reciprocal 3-port microwave junction does not exhibit matched condition at all the 3-port simultaneously.
(ii) In Any 4-port microwave reciprocal (or) non-reciprocal network. If two out of four ports are matched, the remaining two ports also matched.

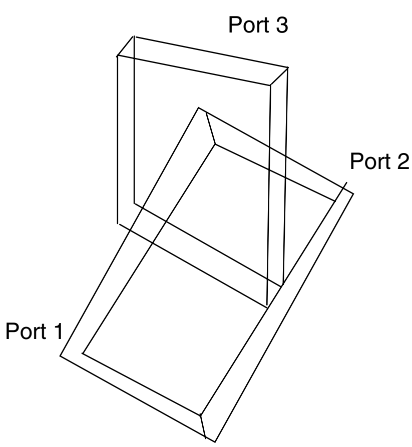
Microwave Networks

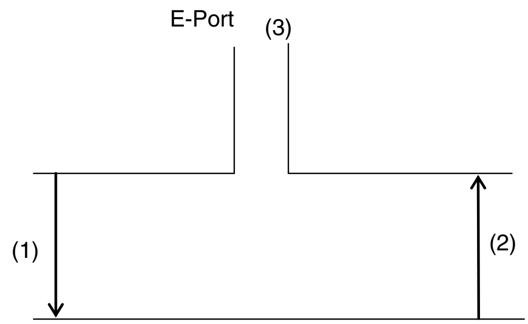
I:-Reciprocal networks

$$(S_{ij} = S_{ji})$$

2-port: Wave guide
3 Port

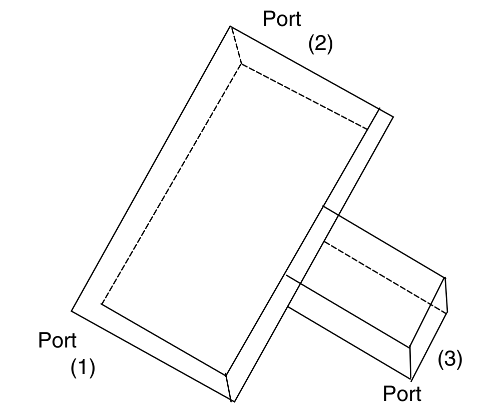
(i) E-Plane 'T'
It is also called as shunt 'T'. It is called E-Plane 'T' because the 'T' junction is parallel to E-field.

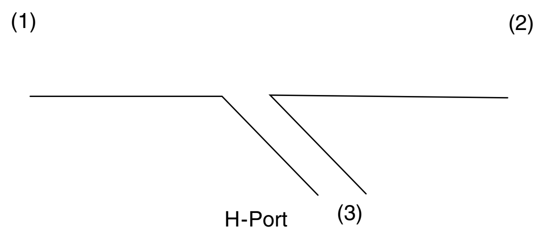




$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & -1 & S_{33} \end{bmatrix}$$

(ii) H-Plane T
It is also called as series 'T'

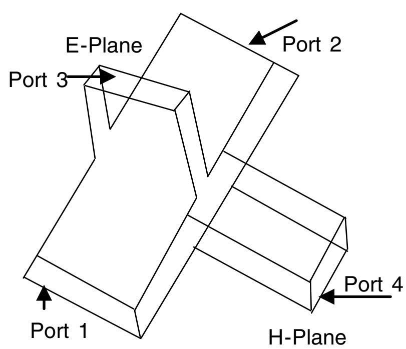


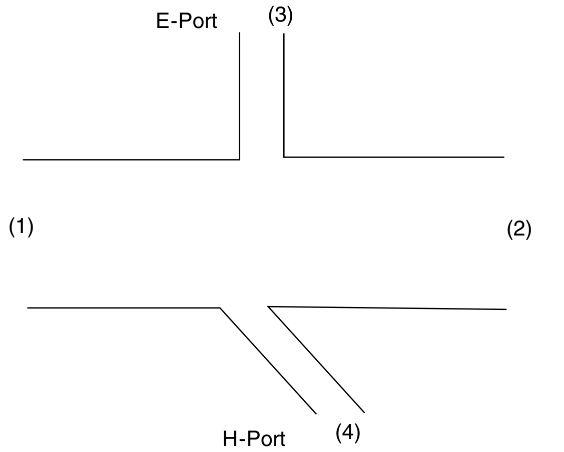


$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & S_{33} \end{bmatrix}$$

4 port

(i) E–H plane T (or) magic T

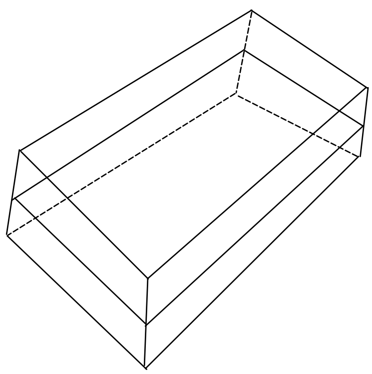




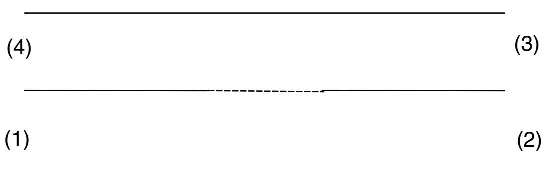
If both of the junctions are symmetrical then EH plane T is called magic T.

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

(ii) Direction coupler



Directional coupler



Two (or) multihole direction coupler

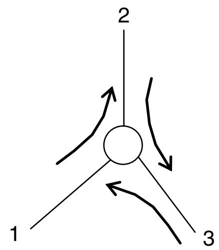
$$[S] = \begin{bmatrix} 0 & jb & a & 0 \\ jb & 0 & 0 & a \\ a & 0 & 0 & jb \\ 0 & a & jb & 0 \end{bmatrix}$$

Direction coupler couples the power from any of the port to the opposite ports only such that power coupled to the diagonal port always is with phase shift of 90° with respect to direct opposite port, and therefore, directional coupler is used not only as power sampler also as phase shifter.

Non-Reciprocal Networks

Circulator

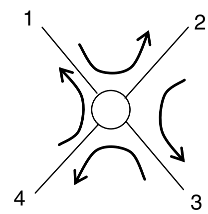
(i) 3 port



Power coupled from n^{th} to $(n + 1)^{th}$ port, but in opposite direction no coupling.

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

4- port:-



$$[S] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Example 15

As a result of reflection from a plane conducting wall, electromagnetic waves acquire an apparent velocity greater than the velocity of light in space. This is called

(A) Velocity of propagation (B) Normal velocity
(C) Group velocity (D) Phase velocity

Solution: (D)

Example 16

The cut-off wavelength λ_c for TE_{20} mode for a standard rectangular wave guide is

(A) $\frac{2}{a}$ (B) $2a$ (C) a (D) $2a^2$

Solution

$$f_c = \frac{c}{2}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

For TE_{20} mode, $m = 2$, $n = 0$

then $f_c = \frac{c}{a}$

or $\lambda_c = a$

Example 17

In an airline, adjacent maxima are found at 12.5 cm and 37.5 cm. The operating frequency is

(A) 1.5 GHz (B) 600 MHz
(C) 300 MHz (D) 1.2 GHz

Solution

Here, $\frac{\lambda}{2} = 37.5 - 12.5 = 25$ cm

Then $f = \frac{v}{\lambda} = \frac{3\times10^8}{50\times10^{-2}}$

$f = 600$ Hz

Practice Problems I

Direction for questions 1 to 19: Select the correct alternative from the given choices.

1. A particular mode is excited in a wave guide, there appears an extra electric field component in the direction of propagation In what mode is the wave propagating?
(A) Transverse electric
(B) Transverse Magnetic
(C) Transverse electromagnetic
(D) Longitudinal

2. Which of the following components exist for TE_{11} wave (Assume wave along z direction)
(A) All are present except E_z (B) E_x, E_y, H_x, H_y
(C) E_x, H_y, E_z (D) E_x, H_y, H_z

3. If in a Rectangular wave guide for which $a = 2b$, the cut-off frequency for TE_{02} mode is 12GHz, the cut-off frequency for TM_{11} mode is
(A) 3 GHz (B) $3\sqrt{5}$ GHz
(C) 12 GHz (D) $6\sqrt{5}$ GHz

4. For TE or TM modes of propagation in bounded media, the phase velocity
(A) is independent of frequency
(B) is linear function of frequency
(C) is non-linear function of frequency
(D) can be frequency dependent or independent depending on the source.

5. The cut-off frequency of the dominant mode of a rectangular wave guide having aspect ratio more than 4 is 5GHz. The inner broad wall dimension is given by
(A) 3 cm (B) 1.5 cm
(C) 2 cm (D) 2.5 cm

6. Which of the following is the second dominant mode if $b < a/2$ in rectangular wave guide
(A) TE_{10} (B) TE_{20} (C) TE_{01} (D) TE_{11}

7. A rectangular wave guide of internal dimensions a = 8 cm, b = 6 cm is to be operated in TE_{11} mode. The minimum operating frequency is
(A) 3.125 GHz (B) 625 GHz
(C) 9.375 GHz (D) 1.875 GHz

8. A rectangular metal wave guide filled with a material of $\mu_r = 4, \in_r = 9$, has the internal dimensions of 2.0 cm × 1.2 cm the maximum frequency for which, no propagation through wave guide.
(A) 2.5 GHz (B) 0.75 GHz
(C) 1.25 GHz (D) 7.5 GHz

9. A rectangular wave guide has dimensions of a = 4 cm, b = 3 cm. The modes that propagate through wave guide at a frequency of 6.5 GHz
(A) TE_{10}, TE_{01}, TM_{01}, TM_{10}, TE_{11}
(B) TE_{10}, TE_{01}, TE_{11}, TE_{11},
(C) TE_{10}, TE_{20}, TM_{11}, TE_{11},
(D) TE_{10}, TE_{01}, TE_{11}, TE_{20}

10. In problem 9, the dominant mode frequency is
(A) 3.75 GHZ (B) 5 GHZ
(C) 6.25 GHZ (D) 7.5 GHZ

11. An air-filled rectangular wave guide has inner dimensions of 8 cm × 6 cm. The wave impedance of TE_{11}, TM_{11} modes of propagation at a frequency of 6.25 GHZ respectively in ohms are
(A) $60\sqrt{3}\pi, 80\sqrt{3}\pi$ (B) $\frac{60}{\sqrt{3}}\pi, \frac{80}{\sqrt{3}}\pi$
(C) $80\sqrt{3}\pi, 60\sqrt{3}\pi$ (D) $\frac{80}{\sqrt{3}}\pi, \frac{60}{\sqrt{3}}\pi$

12. In problem 11, the degenerate modes are
(A) TE_{11}, TM_{11} (B) TE_{22}, TM_{22}
(C) both a and b (D) only (a)

13. A rectangular wave guide of dimensions $a \times b$ has $E = \frac{\omega\mu}{h^2}\left(\frac{\pi}{a}\right)\sin\left(\frac{2\pi x}{a}\right).e^{j(\omega t-\beta z)}a_y$ The mode of operation of the guide
(A) TM_{11} (B) TM_{10} (C) TE_{10} (D) TM_{20}

14. A wave guide has a separation of 6 cm for the broader dimension and carries the dominant mode at an unknown frequency if the wave impedance is 754 Ω the unknown frequency is
(A) $\frac{5}{\sqrt{3}}$ GHz (B) $5\sqrt{3}$ GHz
(C) 8.0 GHz (D) 0.29 GHz

15. Determine the phase velocity of the wave of rectangular wave guide of dimensions 7 cm × 3·5 cm operating in dominant shade at a frequency of 3·5GHz
(A) 3×10^8 m/s (B) 3.78×10^8 m/s
(C) 3.1×10^8 m/s (D) 3.6×10^8 m/s

Direction for questions 16 and 17:

An air-filled rectangular wave guide of dimensions a = 4 cm, b = 2 cm, transports energy in the dominant mode at a rate of 2 mW. If the frequency of operation is 10 GHz.

16. Determine η_{TE}?
(A) 406.7 Ω (B) 40.67 Ω
(C) 4.067 Ω (D) 0.4067 Ω

17. Determine peak value of the electric field in the guide
(A) 63.77 v/m (B) 637.7 v/m
(C) 6377 v/m (D) 6.377 v/m

Direction for questions 18 and 19:

A certain microstrip line has fused Quartz (ε_r = 3.8) as a substrate. The ratio of the line width to substrate thickness $\frac{w}{t} = 4.5$.

18. Determine effective relative permitivity of the substrate.
(A) 31.31 (B) 0.3131
(C) 3.131 (D) 313.1

19. Find the wave length of the line at 10 GHz
(A) 169 mm (B) 1.69 mm
(C) 0.169 mm (D) 16.9 mm

Practice Problems 2

Direction for questions 1 to 15: Select the correct alternative from the given choices.

1. When a particular mode is excited in a wave guide, there appears an extra magnetic field component in the direction of propagation. In what is the wave propagating
(A) Transverse magnetic
(B) Transverse electric
(C) Transverse electromagnetic
(D) None of these

2. Which of the following is the dominant mode in rectangular wave guides?
(A) TE_{10} (B) TE_{11} (C) TE_{01} (D) TE_{11}

3. Which of the following components present in TE_{30} (assume wave along z-direction)
(A) E_y, H_x, H_z (B) E_x, H_y, H_z
(C) E_x, H_y, H_z (D) E_x, E_y, H_x, H_y, H_z

4. For a wave propagating in an air-filled rectangular wave guide
(A) Guide wavelength is never less than free-space wavelength
(B) Wave impedance is never less than free-space impedance
(C) TEM mode is positive if the dimensions of the guide are properly chosen.
(D) Propagation constant is always a real quantity.

5. The cut-off wave length λc for TE_{20} mode for a rectangular wave guide is
(A) a (B) $2a$ (C) $2/a$ (D) $2a^2$

6. Which of the following is true about degenerate modes
(A) The modes having the same resonant frequency is called degenerate modes.
(B) The modes having the frequencies in integral multiples of other.
(C) The modes having the cut-off frequencies in multiples of dominant mode frequency.
(D) None of the above.

7. A rectangular wave guide having a cut-off frequency of 9 GHz for TE_{03} mode and Then, the inner narrow wall dimension of wave guide is
(A) 1.5 cm (B) 5 cm
(C) 2.5 cm (D) 3.75 cm

8. An air-filled rectangular wave guide has dimension as 4 cm × 4 cm. The ratio of dominant mode cut-off frequencies in TE to TM mode is
(A) 1:1 (B) 1:2 (C) 2:1 (D) $1:\sqrt{2}$

9. The guide wave length (λ_g) cut-off wavelength (λ_c) and free-space wave length (λ_o) related as

(A) $\frac{1}{\lambda_o^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_g^2}$ (B) $\frac{1}{\lambda_c^2} = \frac{1}{\lambda_0^2} + \frac{1}{\lambda_g^2}$

(C) $\frac{1}{\lambda_g^2} = \frac{1}{\lambda_0^2} + \frac{1}{\lambda_c^2}$ (D) $\frac{1}{\lambda_o} = \frac{1}{\lambda_g} + \frac{1}{\lambda_c}$

10. Which of the following have the least cut-off frequency for a rectangular wave guide of $a \times b$ sides with $\left(b \le \frac{a}{2}\right)$?
(A) TE_{11} (B) TE_{02} (C) TE_{20} (D) TE_{12}

11. Relation that exists between guide wave length (λ_g) free-space wave length (λ_0) and cut-off wave length (λ_c) a rectangular wave guide is

(A) $\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}}$ (B) $\lambda_g = \lambda_0\sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}$

(C) $\lambda_g = \frac{\lambda_0}{\sqrt{1+\left(\frac{\lambda_o}{\lambda_c}\right)^2}}$ (D) $\lambda_g = \lambda_0\sqrt{1+\left(\frac{\lambda_o}{\lambda_c}\right)^2}$

12. A wave guide has internal breadth of a = 3 cm and carries a dominant mode of signal unknown frequency. If the characteristic wave impedance is 500 Ω, what is the frequency?
(A) 7.621 GHz (B) 76.21 GHz
(C) 0.7621 GHz (D) 762.1 GHz

13. An air dielectric L band wave guide has $\frac{a}{b} = 2$ and a dominant-mode cut-off frequency of 0.908 GHz. If the measured guide wave length is 40 cm, find the operating frequency

(A) 1.18 GHz (B) 2.18 GHz
(C) 3.18 GHz (D) 4.18 GHz

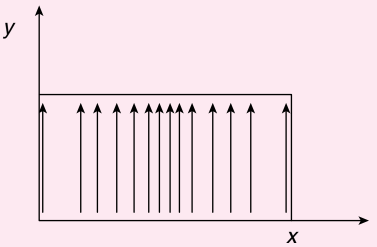
14. A TE_{10} rectangular wave guide is to be designed for operation over 25–35 GHz and the band centre is 1.5 times the cut-off frequency, what should be the dimensions of the broadside
(A) 15 mm (B) 10 mm
(C) 9 mm (D) 7.5 mm

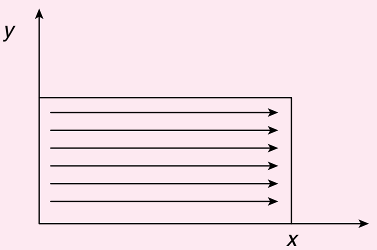
15. A TE_{10} mode of propagation operating at 3 GHz is impressed on a hollow rectangular wave guide of dimensions 6 cm × 4 cm. Determine cut-off wave length, find wave impedance?
(A) 67.648 Ω
(B) 676.48 Ω
(C) 6.7648 Ω
(D) 0.67648 Ω

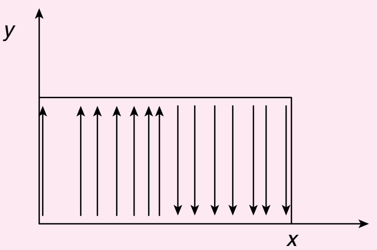
Previous Years' Questions

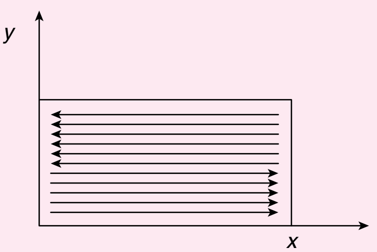
1. In a microwave test bench, why is the microwave signal amplitude modulated at 1 kHz. **[2004]**
(A) To increase the sensitivity of measurement
(B) To transmit the signal to a far-off place
(C) To study amplitude modulation
(D) Because crystal detector fails at microwave frequencies.

2. Which one of the following does represent the electric field lines for the TE_{02} mode in the cross-section of a hollow rectangular metallic wave guide? **[2005]**

(A)


(B)


(C)


(D)


3. A rectangular wave guide having TE_{10} mode as dominant mode is having a cut-off frequency of 18 GHz for the TE_{30} mode. The inner broad wall dimension of the rectangular wave guide is: **[2006]**
(A) $\frac{5}{3}$cms (B) 5 cm
(C) $\frac{5}{2}$cms (D) 10cms

4. An air-filled rectangular wave guide has inner dimensions of 3 × 2 cm. the wave impedance of the TE_{20} mode of propagation in the wave guide at a frequency of 30 GHz is (free space impedance η_0 = 377 Ω) **[2007]**
(A) 308 Ω (B) 355 Ω
(C) 400 Ω (D) 461 Ω

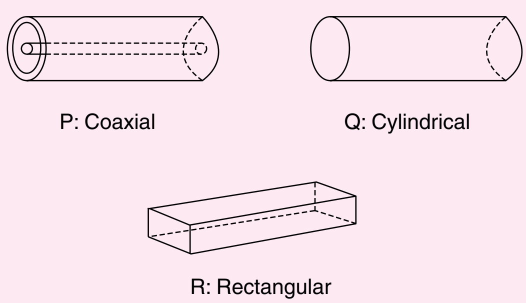
5. The $\vec{E}$ field in a rectangular wave guide of inner dimensions $a \times b$ is given by

$$\vec{E} = \frac{\omega\mu}{h^2}\left(\frac{\pi}{a}\right)H_0 \sin\left(\frac{2\pi x}{a}\right)^2 \sin(\omega t - \beta z)y \text{ V/m}$$

Where H_0 is a constant, and a and b are the dimensions along the x-axis and the y-axis, respectively. The mode of propagation in the wave guide is: **[2007]**
(A) TE_{20} (B) TM_{11} (C) TM_{20} (D) TE_{10}

6. A rectangular wave guide of internal dimensions (a = 4 cm and b = 3) is to be operated in TE_{11} mode. The minimum operating frequency is **[2008]**
(A) 6.25 GHz (B) 6.0 GHz
(C) 5.0 GHz (D) 3.75 GHz

7. Which of the following statements is true regarding the fundamental mode of the metallic wave guides shown? **[2009]**

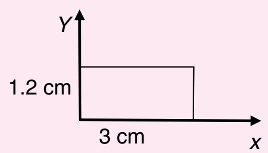


(A) Only P has no cut-off-frequency
(B) Only Q has no cut-off-frequency
(C) Only R has no cut-off-frequency
(D) all three have cut-off-frequency

8. If the scattering matrix [S] of a two port network is $[S]=\begin{bmatrix}0.2\angle 0^\circ & 0.9\angle 90^\circ\\ 0.9\angle 90^\circ & 0.1\angle 90^\circ\end{bmatrix}$ then the network is **[2010]**
(A) lossless and reciprocal
(B) lossless but not reciprocal
(C) not lossless but reciprocal
(D) neither lossless nor reciprocal

9. The modes in a rectangular wave guide are denoted by TE_{mn}/TM_{mn} where m and n are the eigen numbers along the larger and smaller dimensions of the wave guide respectively. Which one of the following statements is **TRUE**? **[2011]**
(A) The TM_{10} mode of the wave guide does not exist.
(B) The TE_{10} mode of the wave guide does not exist.
(C) The TM_{10} and the TE_{10} modes both exist and have the same cut-off frequencies.
(D) The TM_{10} and the TM_{01} modes both exist and have the same cut-off frequencies.

10. The magnetic field along the propagation direction inside a rectangular wave guide with the cross section shown in the figure is $H_z = 3\cos(2.094\times 10^2 x)\cos(2.618\times 10^2 y)\cos(6.283\times 10^{10} t - \beta z)$



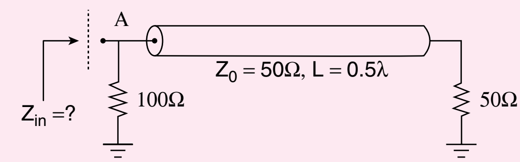
The phase velocity V_p of the wave inside the wave guide satisfies **[2012]**
(A) $V_p > c$
(B) $V_p = c$
(C) $0 < V_p , c$
(D) $V_p = 0$

11. The return loss of a device is found to be 20 dB. The voltage standing wave ratio (VSWR) and magnitude of reflection coefficient are, respectively **[2013]**
(A) 1.22 and 0.1
(B) 0.81 and 0.1
(C) −1.22 and 0.1
(D) 2.44 and 0.2

12. A two-port network has scattering parameters given by $[S]=\begin{bmatrix}s_{11} & s_{12}\\ s_{21} & s_{22}\end{bmatrix}$. If the port-2 of the two port is short-circuited, the s_{11} parameter for the resultant one-port network is **[2014]**
(A) $\dfrac{s_{11}-s_{11}s_{22}+s_{12}s_{21}}{1+s_{22}}$
(B) $\dfrac{s_{11}+s_{11}s_{22}-s_{12}s_{21}}{1+s_{22}}$
(C) $\dfrac{s_{11}+s_{11}s_{22}+s_{12}s_{21}}{1-s_{22}}$
(D) $\dfrac{s_{11}-s_{11}s_{22}+s_{12}s_{21}}{1-s_{22}}$

13. Which one of the following field patterns represents a TEM wave travelling in the positive × direction? **[2014]**
(A) $E=+8\,\hat{y}$, $H=-4\,\hat{z}$
(B) $E=-2\,\hat{y}$, $H=-3\,\hat{z}$
(C) $E=+2\,\hat{z}$, $H=+2\,\hat{y}$
(B) $E=-3\,\hat{y}$, $H=+4\,\hat{z}$

14. In the transmission line shown, the impedance Z_{in} (in ohms) between node A and the ground is ____________. **[2014]**



15. For a rectangular wave guide of internal dimensions $a\times b$ ($a > b$), the cut-off frequency for the TE_{11} mode is the arithmetic mean of the cut-off frequencies for TE_{10} mode and TE_{20} mode. If $a=\sqrt{5}$ cm, the value of b (in cm) is________. **[2014]**

16. Consider an air-filled rectangular wave guide with a cross-section of 5 cm × 3 cm. For this wave guide, the cut-off frequency (in MHz) of TE_{21} mode is __________. **[2014]**

17. Consider an air filled rectangular waveguide with dimensions a = 2.286 cm and b = 1.016 cm. At 10 GHz operating frequency, the value of the propagation constant (per meter) of the corresponding propagating mode is ___________. **[2016]**

18. Consider an air filled rectangular waveguide with dimensions a = 2.286 and b = 1.016 cm. The increasing order of the cut off frequencies for different modes is **[2016]**
(A) $TE_{01} < TE_{10} < TE_{11} < TE_{20}$
(B) $TE_{20} < TE_{11} < TE_{10} < TE_{01}$
(C) $TE_{10} < TE_{20} < TE_{01} < TE_{11}$
(D) TE_{10} ,< $TE_{11} < TE_{20} < TE_{01}$

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** A	**3.** B	**4.** C	**5.** C	**6.** B	**7.** A	**8.** C	**9.** B	**10.** A
11. C	**12.** C	**13.** D	**14.** A	**15.** B	**16.** A	**17.** A	**18.** C	**19.** D	

Practice Problems 2

1. B	**2.** A	**3.** A	**4.** A	**5.** A	**6.** A	**7.** B	**8.** D	**9.** A	**10.** C
11. A	**12.** A	**13.** A	**14.** D	**15.** B					

Previous Years' Questions

1. D	**2.** D	**3.** C	**4.** C	**5.** A	**6.** A	**7.** A	**8.** C	**9.** A	**10.** D
11. A	**12.** B	**13.** B	**14.** 32.99 to 34.01		**15.** 1.9 to 2.1		**16.** 7810 to 7815		
17. 157 m^{-1}		**18.** C							