Ex 19.1

Q1

$a_n = n^2 - n + 1$ ---(i) is the given sequence

Then, first 5 terms are a_1, a_2, a_3, a_4 and a_5

$$a_1 = (1)^2 - 1 + 1 = 1$$
$$a_2 = (2)^2 - 2 + 1 = 3$$
$$a_3 = (3)^2 - 3 + 1 = 7$$
$$a_4 = (4)^2 - 4 + 1 = 13$$
$$a_5 = (5)^2 - 5 + 1 = 21$$

First 5 terms 1, 3, 7, 13 and 21.

Q2

$a_n = n^3 - 6n^2 + 11n - 6 \qquad n \in N.$

The first three terms are a_1, a_2 and a_3

$$a_1 = (1)^3 - 6(1)^2 + 11(1) - 6 = 0$$
$$a_2 = (2)^3 - 6(2)^2 + 11(2) - 6 = 0$$
$$a_3 = (3)^3 - 6(3)^2 + 11(3) - 6 = 0$$

$\therefore$ The 1st 3 terms are zero.

and

$$a_n = n^3 - 6n^2 + 11n - 6$$
$$= (n-2)^3 - (n-2) \text{ is positive as } n \ge 4$$

$\therefore$ a_n is always positive.

Q3

$a_n = 3a_{n-1} + 2$ for $n > 1$

$$\therefore \quad a_2 = 3a_{2-1} + 2 = 3a_1 + 2$$
$$= 3(3) + 2 = 11 \qquad [\therefore a_1 = 3]$$
$$a_3 = 3a_{3-1} + 2 = 3a_2 + 2$$
$$= (11) + 2 = 35 \qquad [\therefore a_2 = 11]$$
$$a_4 = 3a_{4-1} + 2 = 3a_2 + 2$$
$$= 3(35) + 2 = 107 \qquad [\because a_3 = 35]$$

$\therefore$ The first four terms of A.P are 3, 11, 35, 107.

Q4

(i) $a_1 = 1,\ a_n = a_{n-1} + 2,\ n \geq 2$

$a_2 = a_{2-1} + 2 = a_{1+2} = 3$ $[\because a_1 = 1]$

$a_3 = a_{3-1} + 2 = a_2 + 2 = 5$ $[\because a_2 = 3]$

$a_4 = a_{4-1} + 2 = a_3 + 2 = 7$ $[\because a_3 = 5]$

$a_5 = a_{5-1} + 2 = a_4 + 2 = 9$ $[\because a_4 = 7]$

$\therefore$ The first 5 terms of series are 1, 3, 5, 7, 11.

(ii) $a_1 = a_2 = 1$

$a_n = a_{n-1} + a_{n-2}$ $n > 2$

$\Rightarrow$ $a_3 = a_{3-1} + a_{3-2}$

$= a_2 + a_1 = 1 + 1 = 2$

$\Rightarrow$ $a_4 = a_{4-1} + a_{4-2}$

$= a_3 + a_2 = 2 + 1 = 3$

$\Rightarrow$ $a_5 = a_{5-1} + a_{5-2}$

$= a_4 + a_3 = 5$

$\therefore$ The given sequence is 1, 1, 3, 5.

(iii) $a_1 = a_2 = 2$

$a_n = a_{n-1} - 1$ $n > 2$

$\Rightarrow$ $a_3 = a_{3-1} - 1$

$= a_2 - 1$

$= 2 - 1 = 1$

$\Rightarrow$ $a_4 = a_{4-1} - 1$

$= a_3 - 1 = 1 - 1 = 0$

$\Rightarrow$ $a_5 = a_{5-1} - 1$

$= 0 - 1 = -1$

$\therefore$ The first 5 terms of the sequence are 2, 2, 1, 0, −1.

Q5

$a_n = a_{n-1} + a_{n-2}$ for $n > 2$

$\Rightarrow$ $a_3 = a_{3-1} + a_{3-2} = a_2 + a_1 = 1 + 1 = 2$

$\Rightarrow$ $a_4 = a_{4-1} + a_{4-2} = a_3 + a_2 = 2 + 1 = 3$

$\Rightarrow$ $a_5 = a_{5-1} + a_{5-2} = a_4 + a_3 = 3 + 2 = 5$

$\Rightarrow$ $a_6 = a_{6-1} + a_{6-2} = a_5 + a_4 = 5 + 31 = 8$

$\therefore$ For $n = 1$

$\frac{a_{n+1}}{a_n} = \frac{a_2}{a_1} = \frac{1}{1} = 1$

For $n = 2$

$\frac{a^3}{a_2} = \frac{2}{1} = 2$

For $n = 3$

$\frac{a_4}{a_3} = \frac{3}{2} = 1.5$

For $n = 4$ and $n = 5$

$\frac{a_5}{a_4} = \frac{5}{3}$ and $\frac{a_6}{a_5} = \frac{8}{5}$

$\therefore$ The required series is $1, 2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \ldots$

Q6(i)

$3, -1, -5, -9\ldots$

$a_1 = 3,\ a_2 = -1,\ a_3 = -5,\ a_4 = -9$

$a_2 - a_1 = -1 - 3 = -4$

$a_3 - a_2 = -5 - (-1) = -4$

$a_4 - a_3 = -9(-5) = -4$

$\therefore$ Common difirence is $d = -4$

$a_4 - a_3 = a_3 - a_2 = a$

$\therefore$ The given sequence is a A.P

$\therefore$ $a_5 = 3 + (5 - 1)(-4) = -13$

$a_6 = 3 + (6 - 1)(-4) = -17$

$a_7 = 3 + (7 - 1)(-4) = -21$

Q6(ii)

$-1, \frac{1}{4}, \frac{3}{2}, \frac{11}{4} \ldots$

$$a_1 = -1, \; a_2 = \frac{1}{4}, \; a_3 = \frac{3}{2}, \; a_4 = \frac{11}{4}$$

$$a_4 - a_3 = a_3 - a_2 = a_2 - a_1 = \frac{5}{4}$$

$\therefore$ Common difference is $d = \frac{5}{4}$

$\therefore$ The given sequence is A.P

$$a_5 = -1 + (5-1)\frac{5}{4} = 4$$

$$a_6 = -1 + (6-1)\frac{5}{4} = \frac{21}{4}$$

$$a_7 = -1 + (7-1)\frac{5}{4} = \frac{26}{4} = \frac{13}{2}$$

Q6(iii)

(iii) $\sqrt{2}, \; 3\sqrt{2}, \; 5\sqrt{2}, \; 7\sqrt{2} \ldots$

$$a_1 = \sqrt{2}, \; a_2 = 3\sqrt{2}, \; a_3 = 5\sqrt{2}, \; a_4 = 7\sqrt{2}$$

$$a_4 - a_3 = a_3 - a_2 = a_2 - a_1 = 2\sqrt{2}$$

$\therefore$ The common difference is $2\sqrt{2}$
and the given sequence is A.P

$$a_5 = \sqrt{2} + 2\sqrt{2}(5-1) = 9\sqrt{2}$$

$$a_6 = \sqrt{2} + 2\sqrt{2}(6-1) = 11\sqrt{2}$$

$$a_7 = \sqrt{2} + 2\sqrt{2}(7-1) = 13\sqrt{2}$$

Q6(iv)

$9, \; 7, \; 5, \; 3, \; \ldots$

$$a_4 - a_3 = a_3 - a_2 = a_2 - a_1 = -2$$

$\therefore$ The common difference is -2
and the given sequence is A.P

$$a_5 = 9 + (-2)(5-1) = 1$$

$$a_6 = 9 + (-2)(6-1) = -1$$

$$a_7 = 9 + (-2)(7-1) = -3$$

Q7

$a_n = 2n + 7$

$$a_1 = 2(1) + 7 = 9$$
$$a_2 = 2(2) + 7 = 11$$
$$a_3 = 2(3) + 7 = 13$$

Here, $a_3 - a_2 = a_2 - a_1 = 2$

$\therefore$ The given sequence is A.P

$$a_7 = 2(7) + 7 = 21$$

7th term is 21.

Q8

$a_n = 2n^2 + n + 1$

$$a_1 = 2(1)^2 + (1) + 1 = 4$$
$$a_2 = 2(2)^2 + (2) + 1 = 11$$
$$a_3 = 2(3)^2 + (3) + 1 = 21$$

$$a_3 - a_2 \neq a_2 - a_1$$

$\therefore$ The given sequence is not as A.P as consequtive terms do not have a common difference.

Ex 19.2

Q1

(i) 10th term of A.P 1, 4, 7, 10, ...

Here, 1st term = $a_1 = 1$

and common difference $d = 4 - 1 = 3$

We know $a_n = a_1 + (n-1)d$

$\therefore \quad a_{10} = a_1 + (10-1)d$

$= 1 + (10-1)3 \Rightarrow 28$

(ii) To find 18th term of A.P $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \ldots$

Here, 1st term $a_1 = \sqrt{2}$

and $\quad d$ = common difference $= 2\sqrt{2}$

$\therefore \quad a_n = a_1 + (n-1)d$

$a_{18} = \sqrt{2} + 2\sqrt{2}(17) = 35\sqrt{2}$

(iii) Find nth term of A.P 13, 8, 3, −2

Here, $\quad a_1 = 13$

$d = -5$

$\therefore \quad a_n = a + (n-1)d$

$= 13 + (n-1)(-5)$

$= -5n + 18$

Q2

It is given that the sequence $< a_n >$ is an A.P

$\therefore \quad a_n = a + (n-1)d \qquad$ ---(i)

Similarly from (i)

$a_{m+n} = a + (m+n-1)d \qquad$ ---(ii)

$a_{m-n} = a + (m-n-1)d \qquad$ ---(iii)

Adding (ii) and (iii)

$a_{m+n} + a_{m-n} = (a + (m+n-1)d) + (a + (m-n-1)d)$

$= 2a + (m+n-1+m-n-1)d$

$= 2a + 2d(m-1)$

$= 2(a + (m-1)d)$

$= 2a_m$ Hence proved.

Q3

(i) Let nth term of A.P = 248

$\therefore \quad a_n = 248 = a + (n-1)d$

$\Rightarrow \quad 248 = 3 + (n-1)5$

$\therefore \quad n = 50$

$\therefore$ 50th term of the given A.P is 248.

(ii) Which term of A.P 84, 80, 76 is 0?

Let nth term of A.P be 0

Then, $a_n = 0 = a + (n-1)d$

$0 = 84 + (n-1)(-4)$

$\therefore \quad n = 22$

$\therefore$ 22nd term of the given A.P is 0.

(iii) Which term of A.P is 4, 9, 14, ... is 254?

Let nth term of A.P be 254

$a_n = a + (n-1)d$

$254 = 4 + (n-1)5$

$\therefore \quad n = 51$

$\therefore$ 51st term of the given A.P is 254.

Q4

(i) Is 68 a term of A.P $7, 10, 13, ...?$

Here, $a = 7$

and $x = 10 - 7 = 3$

$\therefore$ a_n term is $= a + (n-1)d$

$= 7 + (n-1)3$

Let 68 be nth temr of A.P

Then,

$68 = 7 + 3(n-1)$

$\Rightarrow$ $68 = 7 + 3n - 3$

$\Rightarrow$ $68 - 4 = 3n$

$\Rightarrow$ $64 = 3n$

$\Rightarrow$ $n = \frac{64}{3}$

Which is not a natural number.

$\therefore$ 68 is nota term of given A.P.

(ii) Is 302 a term of A.P $3, 8, 13$

Let 302 be nth ter, pf tje given A.P

Here, $302 = 3 + (n-1)5$

$\frac{299}{5} = (n-1)$

$n = \frac{304}{5}$

Which is not a natural number.

$\therefore$ 302 is not a term of given A.P.

Q5

(i) The given sequence is $24, 23\frac{1}{4}, 22\frac{1}{2}, 21\frac{3}{4}, \ldots$

Here, $a = 24$

$$d = 23\frac{1}{4} - 24 = \frac{93-96}{4} = \frac{-3}{4}$$

Let nth term be the 1st negative term.

$$a_n < 0$$

$$a + (n-1)d < 0$$

$$24 - \frac{3}{4}(n-1) < 0$$

$$96 - 3n + 3 < 0$$

$$99 < 3n$$

$$33 < n \quad \text{or} \quad n > 33$$

$\therefore$ 34th term is 1st negative term.

(ii) The given sequence is $12 + 8i,\ 11 + 6i,\ 10 + 4i, \ldots$

Here, $a = 12 + 8i$

$$d = -1 - 2i$$

Then, $a_n = a + (n-1)d$

$$= 12 + 8i + (n-1)(-1-2i)$$

$$= (13 - n) + i(10 - 2n)$$

Let nth term be purely real the $(10 - 2n) = 0$ or $n = 5$

So, 5th term is purely real.

Let nth term be purely imaginary. Then, $13 - n = 0$

$\therefore \quad n = 13$

So, 13th term is purely imaginary.

Q6

(i) The given A.P is $7, 10, 13, \ldots 43.$

Let there be n terms,

then, n term = 43

or $\quad 43 = a_n = a + (n-1)d$

$\Rightarrow \quad 43 = 7 + (n-1)3$

$\Rightarrow \quad n = 13$

Thus, there are 13 terms in the given sequence.

(ii) The given A.P is $-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, \ldots, \frac{10}{3}?$

Let there be n terms

then, nth term = $\frac{10}{3}$

or $\quad \frac{10}{3} = a_n = a + (n-1)d$

$\Rightarrow \quad \frac{10}{3} = -1 + (n-1)\left(\frac{-5}{6} + 1\right)$

$\Rightarrow \quad n = 27$

Thus, there are 27 terms in the given sequence.

Q7

Given: $a = 5$

$d = 3$

a_n = last term = 80

Let there be n terms

$\therefore \quad a_n = 80 = a + (n-1)d$

$80 = 5 + (n-1)3$

$\Rightarrow \quad n = 26$

$\therefore$ Thus, thre are 26 terms in the given sequence.

Q8

Given that:

$a_6 = 19 = a + (6-1)d$ ---(i)

$a_{17} = 41 = a + (17-1)d$ ---(ii)

Solving (i) and (ii), we get

$a = 9$ and $d = 2$

$\therefore \quad a_{40} = a + (40-1)d$

$= 9 + (40-1)2$

$= 9 + 39(2)$

$= 87$

40th term of the given sequence is 87.

Q9

Given:

$a_9 = 0$

$\therefore \quad a + 8d = 0$

$a = -8d$ ---(i)

$a_{19} = a + (19-1)d$

$= a + 18d$

$= -8d + 18d$ $\quad [\because a = -8d$ from (i)$]$

$= 10d$ ---(ii)

$a_{29} = a + (29-1)d$

$= -8d + 28d$ $\quad [\because a = -8d$ from (i)$]$

$= 20d$ ---(iii)

From (ii) and (iii)

$a_{29} = 2a_{19}$ Hence proved.

Q10

Given:

$$10a_{10} = 15a_{15}$$

$\Rightarrow \quad 10\left(a + (10-1)d\right) = 15\left(a + (15-1)d\right)$

$\Rightarrow \quad 10a + 90d = 15a + 210d$

$\Rightarrow \quad 5a + 120d = 0$

$\Rightarrow \quad a + 24d = 0 \qquad$ ---(i)

$a_{25} = a + (25-1)d$

$= a + 24d$

$= 0 \qquad \left[\because \text{ from (i) } a + 24d = 0\right]$

Hence proved.

Q11

Given:

$a_{10} = 41 = a + 9d \qquad$ ---(i)

$a_{18} = 73 = a + 17d \qquad$ ---(ii)

Solving (i) and (ii)

$a + 9d = 41$

$a + 17d = 73$

We get $a = 5$ and $d = 4$

$\therefore \quad a_{26} = a + (26-1)d$

$= 5 + 25(4)$

$= 105$

26th term of the given A.P is 105.

Q12

Given:

$$a_{24} = 2a_{10}$$

$$\Rightarrow \quad a + 23d = 2(a + 9d)$$

$$\Rightarrow \quad a = 5d \qquad \text{---(i)}$$

$$a_{72} = a + (72 - 1)d$$

$$= a + 71d \qquad [\because a = 5d \text{ from (i)}]$$

$$\Rightarrow \quad = 76d \qquad \text{---(ii)}$$

$$a_{34} = a + (34 - 1)d$$

$$= 5d + 33d \qquad [\because a = 5d \text{ from (i)}]$$

$$= 38d \qquad \text{---(iii)}$$

From (ii) and (iii)

$a_{72} = 2a_{34}$ Hence proved.

Q13

Given:

$$a_{m+1} = 2a_{n+1}$$

$$\Rightarrow \quad a + (m + 1 - 1)d = 2(a + (n + 1 - 1)d)$$

$$\Rightarrow \quad a + md = 2a + 2nd$$

$$\Rightarrow \quad a = (m - 2n)d \qquad \text{---(i)}$$

Then,

$$a_{3m+1} = a + (3m + 1 - 1)d$$

$$= a + 3md$$

$$= 3d - 2nd + 3md$$

$$= 2(2m - n)d \qquad \text{---(ii)}$$

$$a_{m+n+1} = a + (m + n + 1 - 1)d$$

$$= md - 2nd + md + nd$$

$$= (2m - n)d \qquad \text{---(iii)}$$

From (ii) and (iii)

$a_{2m+1} = 2a_{m+n+1}$ Hence proved.

Q14

The given A.P is 9, 7, 5, ... and 15, 12, 9

Here,

$a = 9 \qquad A = 15$

$d = -2 \qquad D = 3$

Let $a_n = A_n$ for same n.

$\Rightarrow \quad a + (n-1)d = A + (n-1)d$

$\Rightarrow \quad 9 + (n-1)(-2) = 15 + (n-1)3$

$\Rightarrow \quad n = 7$

$\therefore$ 7th term of both the A.P is same.

Q15

(i) A.P is 3, 5, 7, 9, ..., 201.

Here, $a = 3$

$d = 2$

nth term from the end is $l - (n-1)d$

i.e. $201 - (n-1)2$ or $203 - 2n$ ---(i)

12th term from end is

$203 - 2(12) = 179$

(ii) A.P is 3, 8, 13, ..., 253.

Then, 12th term from end is $l - (n-1)d$ i.e.,

$= \quad 253 - (12-1)5$

$= \quad 253 - 55$

$= \quad 198$

(iii) A.P is 1, 4, 7, 10, ..., 88

Then, 12th term from end is $l - (n-1)d$

$= \quad 88 - (12-1)3$

$= \quad 88 - 33$

$= \quad 55$

Q16

Given,

$a = 3a_1$ ---(i)

$a_7 = 2a_3 + 1$ ---(ii)

Expanding (i) and (ii)

$a + 3d = 2a$

$\therefore$ $2a = 3d$ or $a = \frac{3d}{2}$ ---(iii)

$a + 6d = 2a + 4d + 1$

$a + 1 = 2d$ ---(iv)

From (iii) and (iv)

$a = 3$ and $d = 2$

$\therefore$ 1st term of the given A.P is 3, and common difference is 2.

Q17

$a_6 = a + 5d = 12$ ---(i)

$a_8 = a + 7d = 22$ ---(ii)

Solving (i) and (ii)

$a = -13$ and $d = 5$

Then,

$$
\begin{aligned}
a_n &= a + (n-1)d \\
&= -13 + (n-1)5 \\
&= 5n - 18
\end{aligned}
$$

and

$$
\begin{aligned}
a_2 &= a + (2-1)d \\
&= -13 + 5 \\
&= -8
\end{aligned}
$$

Q18

The first two digit number divisible by 3 is 12.
and last two digit number divisible by 3 is 99.

So, the required series is 12, 15, 18, ...99.
Let there be n terms then nth term = 99

$\Rightarrow \quad 99 = a + (n-1)d$

$\Rightarrow \quad 99 = 12 + (n-1)3$

$\Rightarrow \quad n = 30$

30 two digit numbers are divisible by 3.

Q19

Given,

$n = 60$

$a = 7$

$l = 125$

$\therefore \quad a + (n-1)d = 125$

$7 + (59)d = 125$

$d = 2$

$\therefore \quad a_{32} = a + (32-1)d$

$= 7 + (31)2$

$= 69$

32nd term is 69.

Q20

$$a_4 + a_8 = 24 \qquad [\text{Given}]$$

$$\Rightarrow \quad (a + 3d) + (a + 7d) = 24$$

$$\Rightarrow \quad a + 5d = 12 \qquad \text{---(i)}$$

$$a_6 + a_{10} = 34$$

$$\Rightarrow \quad (a + 5d) + (a + 9d) = 34$$

$$\Rightarrow \quad a + 7d = 17 \qquad \text{---(ii)}$$

From (i) and (ii)

$$a = \frac{-1}{2} \text{ and } d = \frac{5}{2}$$

$\therefore$ 1st term is $\frac{-1}{2}$ and common difference is $\frac{5}{2}$.

Q21

The nth term from starting

$$= a_n = aa + (n - 1)d \qquad \text{---(i)}$$

The nth term from end

$$= l - (n - 1)d \qquad \text{---(ii)}$$

Adding (i) and (ii), we get

Sum of nth term from begining and nth term from the end

$$= a + (n - 1)d + l - (n - 1)d$$

$$= a + l$$ Hence proved.

Q22

$$\frac{a_4}{a_7} = \frac{2}{3} \qquad [\text{Given}]$$

$$\Rightarrow \quad \frac{a+3d}{a+6d} = \frac{2}{3} \qquad =$$

$$\Rightarrow \quad 3a + 9d = 2a + 12d$$

$$\Rightarrow \quad a = 3d \qquad \text{---(i)}$$

$$\frac{a_6}{a_8} = \frac{a+5d}{a+7d}$$

$$\Rightarrow \quad = \frac{3d+5d}{3d+7d} \qquad [\because 3d \text{ from (i)}]$$

$$\Rightarrow \quad = \frac{8d}{10d}$$

$$\Rightarrow \quad = \frac{4}{5}$$

$$\frac{a_6}{a_8} = \frac{4}{5}$$

Q23

$$\sec\theta_1\sec\theta_2+\sec\theta_2\sec\theta_3+....+\sec\theta_{n-1}\sec\theta_n=\frac{\tan\theta_n-\tan\theta_1}{\sin d}$$

$$\theta_2-\theta_1=\theta_3-\theta_2=..........=d$$

$$\sec\theta_1\sec\theta_2=\frac{1}{\cos\theta_1\cos\theta_2}=\frac{\sin d}{\sin d\left(\cos\theta_1\cos\theta_2\right)}$$

$$=\frac{\sin\left(\theta_2-\theta_1\right)}{\sin d\left(\cos\theta_1\cos\theta_2\right)}$$

$$=\frac{\sin\theta_2\cos\theta_1-\cos\theta_2\sin\theta_1}{\sin d\left(\cos\theta_1\cos\theta_2\right)}$$

$$=\frac{1}{\sin d}\left[\frac{\sin\theta_2\cos\theta_1}{\left(\cos\theta_1\cos\theta_2\right)}-\frac{\cos\theta_2\sin\theta_1}{\left(\cos\theta_1\cos\theta_2\right)}\right]$$

$$=\frac{1}{\sin d}\left[Tan\theta_2-Tan\theta_1\right]$$

Similarly, $\sec\theta_2\sec\theta_3=\frac{1}{\sin d}\left[Tan\theta_3-Tan\theta_2\right]$

If we add up all terms, we get

$$=\frac{1}{\sin d}\left[Tan\theta_2-Tan\theta_1+Tan\theta_3-Tan\theta_2+............+Tan\theta_n-Tan\theta_{n-1}\right]$$

$$=\frac{1}{\sin d}\left[Tan\theta_n-Tan\theta_1\right]$$

Hence Proved

Ex 19.3

Q1

Let the 3rd term of A.P be

$a-d, a, a+d$

Then,

$a-d+a+a+d = 21$

$3a = 21$

$\therefore \quad a = 7$

and

$(a-d)(a+d) = a+6$

$a^2 - d^2 = a+6$

$7^2 - d^2 = 7+6 \qquad [\because a = 7]$

$d^2 = 36$

$d = \pm 6$

Since d can't be negative, therefore

$\therefore$ The A.P is 1, 7, 13.

Q2

Let the 3 numbers in A.P are

$a-d, a, a+d$

Then,

$a-d+a+a+d = 27$

$3a = 27$

$\therefore \quad a = 9 \qquad$ ---(i)

and

$(a-d)(a)(a+d) = 648$

$(9-d)\,9\,(9-d) = 648 \qquad [\because a = 9]$

$9^2 - d^2 = 72$

$\therefore \quad d = 3 \qquad$ ---(ii)

$\therefore$ The given sequence is 6, 9, 12.

Q3

Let the four numbers in A.P be

$$a-3d, a-d, a+d, a+3d$$

$$(a-3d)+(a-d)+(a+d)+(a+3d)=50$$

$$4a=50$$

$$a=\frac{25}{2} \qquad \text{---(i)}$$

and

$$(a+3d)=4(a-3d)$$

$$\frac{25+6d}{2}=50-12d$$

$$30d=75$$

$$d=\frac{25}{10}=\frac{5}{2} \qquad \text{---(ii)}$$

$\therefore$ The required sequence is 5, 10, 15, 20.

Q4

Let three numbers be $a-d, a, a+d$

Then,

$$a-d+a+a+d=12$$

$$3a=12$$

$$a=4$$

and

$$(a-d)^3+a^3+(a+d)^3=\pm 288$$

$$a^3+d^3+3ad(a+d)+a^3+a^3-a^3-3ad(a-d)-288$$

$\Rightarrow$ $2a^3+3a^2d+3ad^2-3a^2d+3ad^2=288$

$\Rightarrow$ $2a^3+3a^2d^2=288$

$\Rightarrow$ $128+48d^2=288$

$\therefore$ $d=\pm 2$

$\therefore$ The required sequence is 2, 4, 6 or 6, 4, 2.

Q5

Let 3 numbers in A.P be

$a-d, a$ and $a+d$

$\Rightarrow \quad (a-d)+(a)+(a+d)=24$

$3a=24$

$a=8$

and

$(a-d)(a)(a+d)=440$

$8^2-d^2=55$

$d=3$

$\therefore$ The required sequence is $5, 8, 11$.

Q6

Let the four angle be

$a-3d, a-d, a+d, a+3d$

Then,

sum of alll angles = $360°$

$a-3d+a-d+a+d+a+3d=360°$

$4a=360°$

$a=90° \qquad$ ---(i)

and

$(a-d)-(a-3d)=10$

$2d=10$

$d=5$

$\therefore$ The angle of the given quadrilateral are $75°, 85°, 95°$ and $105°$.

Ex 19.4

Q1

(i) $50, 46, 42, \ldots, 10$ terms

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2 \times 50 + (10-1)(-4)\right]$$

$$= 320$$

(ii) $13, 5, \ldots, 12$ terms

$$S_{12} = \frac{12}{2}\left[2 \times 1 + (12-1)2\right]$$

$$= 6 \times 24 = 144$$

(iii) $3, \frac{9}{2}, 6, \frac{15}{2}, \ldots, 25$ terms

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{25} = \frac{25}{2}\left(2 \times 3 + 24 \times \frac{3}{2}\right)$$

$$= 525$$

(iv) $41, 36, 31, \ldots, 12$ terms

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{25} = \frac{25}{2}\left[2 \times 41 + (11)(-5)\right]$$

$$= 162$$

(v) $a + b, a - b, a - 3b, \ldots$ to 22 terms

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{22} = \frac{22}{2}\left[2a + 2b + 21(-2b)\right]$$

$$= 22a - 440b$$

(vi) $(x-y)^2, \left(x^2 + y^2\right), (x+y)^2, \ldots, x$ terms

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2\left(x^2 + y^2 - 2xy\right) + (x-1)(-2xy)\right]$$

$$= n\left[(x-y)^2 + (n-1)xy\right]$$

$\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}$,.....to n terms

nth term in above sequence is $\frac{(2n-1)x-ny}{x+y}$

Sum of n terms is given by

$$\frac{1}{x+y}\left[x+3x+5x+.....+(2n-1)x-(y+2y+3y...+ny)\right]$$

$$=\frac{1}{x+y}\left[\frac{n}{2}(2x+(n-1)2x)-\frac{n(n+1)y}{2}\right]$$

$$=\frac{1}{2(x+y)}\left[2n^2x-2n^2y-ny\right]$$

Q2

(i) $2+5+8+\ldots+182.$

a_n term of given A.P is 182

$a_n = a+(n-1)d = 182$

$\Rightarrow$ $182 = 2+(n-1)3$

or $n = 61$

Then,

$$S_n = \frac{n}{2}[a+l]$$

$$= \frac{61}{2}[2+182]$$

$$= 61\times 92$$

$$= 5612$$

(ii) $101+99+97+\ldots+47$

a_n term of A.P of n terms is 47.

$\therefore$ $47 = a+(n-1)d$

$47 = 101+(n-1)(-2)$

or $n = 28$

Then,

$$S_n = \frac{n}{2}[a+l]$$

$$= \frac{28}{2}[101+47]$$

$$= 14\times 148$$

$$= 2072$$

(iii) $(a-b)^2+(a^2+b^2)+(a+b)^2+\ldots+\left[(a+b)^2+6ab\right]$

Let number of terms be n

Then,

$a_n = (a+b)^2+6ab$

$\Rightarrow$ $(a-b)^2+(n-1)(2ab) = (a+b)^2+6ab$

$\Rightarrow$ $a^2+b^2-2ab+2abn-2ab = a^2+b^2+2ab+6ab$

$\Rightarrow$ $n = 6$

Then,

$$S_n = \frac{n}{2}[a+l]$$

$$S_6 = \frac{6}{2}\left[a^2+b^2-2ab+a^2+b^2+2ab+6ab\right]$$

$$= 6\left[a^2+b^2+3ab\right]$$

Q3

A.P formed is $1, 2, 3, 4, \ldots, n$.
Here,

$a = 1$
$d = 1$
$l = n$

So sum of n terms $= S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{n}{2}\left[2 + (n-1)1\right]$

$= \frac{n(n+1)}{2}$ is the sum of first n natural numbers.

Q4

The natural numbers which are divisible by 2 or 5 are:
$2+4+5+6+8+10+\cdots+100 = (2+4+6+\cdots+100)+(5+15+25+\cdots+95)$ Now
$(2+4+6+\cdots+100)$ and $(5+15+25+\cdots+95)$ are AP with common difference 2 and 10 respectively.
Therefore

$$2+4+6+\cdots+100 = 2\frac{50}{2}(1+50)$$
$$= 2550$$

Again

$$5+15+25+\cdots+95 = 5(1+3+5+\cdots+19)$$
$$= 5\left(\frac{10}{2}\right)(1+19)$$
$$= 500$$

Therefore the sum of the numbers divisible by 2 or 5 is:

$$2+4+5+6+8+10+\cdots+100 = 2550+500$$
$$= 3050$$

Q5

The series of n odd natural numbers are $1, 3, 5, \ldots, n$

Where n is odd natural number
Then, sum of n terms is

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$= \frac{n}{2}\left[2(1) + (n-1)(2)\right]$$
$$= n^2$$

The sum of n odd natural numbers is n^2.

Q6

The series so formed is $101, 103, 105, \ldots, 199$

Let number of terms be n

Then,

$$a_n = a + (n-1)d = 199$$

$$\Rightarrow \quad 199 = 101 + (n-1)2$$

$$\Rightarrow \quad n = 50$$

The sum of n terms $= S_n = \frac{n}{2}[a + l]$

$$S_{50} = \frac{50}{2}[101 + 199]$$

$$= 7500$$

The sum of odd numbers between 100 and 200 is 7500.

Q7

The odd numbers betwwen 1 and 100 divisible by 3 are $3, 9, 15, \ldots, 999$

Let the number of terms be n then, nth term is 999.

$$a_n = a(n-1)d$$

$$999 = 3 + (n-1)6$$

$$\Rightarrow \quad n = 167$$

The sum of n terms

$$S_n = \frac{n}{2}[a + l]$$

$$\Rightarrow \quad S_{167} = \frac{167}{2}[3 + 999]$$

$$= 83667$$ Hence proved.

Q8

The required series is 85, 90, 95, ..., 715

Let there be n terms in the A.P

Then,

$$n\text{th term} = 715$$
$$715 = 85 + (n-1)5$$
$$n = 127$$

Then,

$$S_n = \frac{n}{2}[a + l]$$
$$S_{127} = \frac{127}{2}[85 + 715]$$
$$= 50800$$

Q9

The series of integers divisble by 7 between 50 and 500 are

56, 63, 70, ..., 497

Let the number of terms be n then, nth term = 497

$$a_n = a + (n-1)d$$
$$\Rightarrow \quad 497 = 56 + (n-1)7$$
$$\Rightarrow \quad n = 64$$

The sum $S_n = \frac{n}{2}[a + l]$

$$\Rightarrow \quad S_{64} = \frac{64}{2}[56 + 497]$$
$$= 32 \times 553$$
$$= 17696$$

Q10

All even integers will have common difference = 2

$\therefore$ A.P is 102, 104, 106, ..., 998

$$t_n = a + (n-1)d$$
$$t_n = 998, a = 102, d = 2$$
$$998 = 102 + (n-1)(2)$$
$$998 = 102 + 2n - 2$$
$$998 - 100 = 2n$$
$$2n = 898$$
$$n = 449$$

S_{449} can be calculated by

$$S_n = \frac{n}{2}[a + l]$$
$$= \frac{449}{2}[102 + 998]$$
$$= \frac{449}{2} \times 1100$$
$$= 449 \times 550$$
$$= 246950$$

Q11

The series formed by all the integers between 100 and 550 which are divisible by 9 is

108, 117, 123, ..., 549

Let there be n terms in the A.P then, the nth term is 549

$$549 = a + (n-1)d$$
$$549 = 108 + (n-1)9$$
$$\Rightarrow \quad n = 50$$

Then,

$$S_n = \frac{n}{2}[a + l]$$
$$S_{50} = \frac{50}{2}[108 + 549]$$
$$= 16425$$

Q12

In the given series $3 + 5 + 7 + 9 + \ldots$ to $3n$

Here,

$a = 3$

$d = 2$

Number of terms = $3n$

The sum of n term is

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow \quad S_{3n} = \frac{3n}{2}\left[6 + (3n-1)2\right]$$

$$= 3n(2n+3)$$

Q13

The first number between 100 and 800 which on division by 16 leaves the remainder 7 is 112 and last number is 791.

Thus, the series so formed is 103, 119, ..., 791

Let number of terms be n, then

nth term = 791

Then,

$$a_n = a + (n-1)d$$

$$\Rightarrow \quad 791 = 103 + (n-1)16$$

$$\Rightarrow \quad n = 44$$

Then, sum of all terms of the given series is

$$S_{43} = \frac{44}{2}\left[103 + 791\right]$$

$$= \frac{44 \times 894}{2}$$

$$= 19668$$

Q14

(i) $25 + 22 + 19 + 16 + \ldots + x = 115$

Here, sum of the given series of say n terms is 115

So, the nth term = x

Here, $a = 25$ and $d = 22 - 25 = -3$

$\therefore \quad a_n = a + (n - 1)d$

$\Rightarrow \quad x = 25 - 3(n - 1)$

$\Rightarrow \quad x = 28 - 3n \qquad$ ---(i)

The sum of n terms

$$S_n = \frac{n}{2}[a + l]$$

$\Rightarrow \quad 115 = \frac{n}{2}[25 + 28 - 3n]$

$\Rightarrow \quad 230 = 53n - 3n^2$

$\Rightarrow \quad 3n^2 - 53n - 230 = 0$

$\Rightarrow \quad 3n^2 - 30n - 23n - 230 = 0$

$\Rightarrow \quad n = 10$ or $\frac{23}{3}$

But n can't be function

$\therefore \quad n = 10 \qquad$ ---(ii)

From (i) and (ii)

$$\begin{aligned} x &= 28 - 3n \\ &= 28 - 3(10) \\ &= -2 \end{aligned}$$

$x = -2$

Q15

Sum first n terms of the given AP is

$S_n = 3n^2 + 2n$

$S_{n-1} = 3(n-1)^2 + 2(n-1)$

$a_n = S_n - S_{n-1}$

$a_n = 3n^2 + 2n - \ 3(n-1)^2 - 2(n-1)$

$a_n = 6n - 1$

$a_r = 6r - 1$

r^{th} term is $6r - 1$.

Q16

Given,

$a_1 = -14 = a + 0d$ ---(i)

$a_5 = 2 = a + 4d$ ---(ii)

Solving (i) and (ii)

$a_1 = a = -14$ and $d = 4$

Let ther be n terms then sum of there n terms = 40

$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad 40 = \frac{n}{2}[-28 + (n-1)4]$

$\Rightarrow \quad 4n^2 - 32n - 80 = 0$

or $\quad n = 10$ or -2

But n can't be negative

$\therefore \quad n = 10$

The given A.P has 10 terms.

Q17

Given,

$a_7 = 10$

$S_{14} - S_7 = 17$ ---(i)

$\therefore \quad S_{14} = 17 + S_7 = 17 + 10 = 27$ ---(ii)

From (i) and (ii)

$S_7 = \frac{7}{2}\left[2a + (7-1)d\right]$ $\left[\text{Using } S_n = \frac{n}{2}\left[2a + (n-1)d\right]\right]$

$\Rightarrow \quad 10 = 7a + 21d$ ---(iii)

and

$S_{14} = \frac{14}{2}\left[2a + 13d\right]$

$\Rightarrow \quad 27 = 28a + 182d$ ---(iv)

Solving (iii) and (iv)

$a = 1 \text{ and } d = \frac{1}{7}$

$\therefore$ The required A.P is

$1, 1 + \frac{1}{7}, 1 + \frac{2}{7}, 1 + \frac{3}{7}, \ldots, +\infty$

or $\quad 1, \frac{8}{7}, \frac{9}{7}, \frac{10}{7}, \frac{11}{7}, \ldots, \infty$

Q18

Given,

$a_3 = 7 = a + 2d$ ---(i)

$a_7 = 3a_3 + 2$

$\therefore \quad a_7 = 3(7) + 2$ $\left[\because a_3 = 7\right]$

$= 23 = a + 6d$ ---(ii)

solving (i) and (ii)

$a = -1, \; d = 4$

Then, sum of 20 terms of this A.P

$\Rightarrow \quad S_{20} = \frac{20}{2}\left[2 + (20-1)4\right]$ $\left[\text{Using } S_n = \frac{n}{2}\left[2a + (n-1)d\right]\right]$

$= 10 \times 74$

$= 740$

First term is -1 common defference = 4, sum of 20 terms = 740.

Q19

Given,

$$a = 2$$

$$l = 50$$

$$\therefore \quad l = a + (n-1)d$$

$$50 = 2 + (n-1)d$$

$$(n-1)d = 48 \qquad \text{---(i)}$$

S_n of all n terms is given 442

$$\therefore \quad S_n = \frac{n}{2}[a + l]$$

$$442 = \frac{n}{2}[2 + 50]$$

or $\quad n = 17 \qquad$ ---(ii)

From (i) and (ii)

$$d = \frac{48}{n-1} = \frac{48}{16} = 3$$

The common difference is 3.

Q20

Let no. of terms be 2n

Odd terms sum=24=$T_1+T_3+...+T_{2n-1}$

Even terms sum=30=$T_2+T_4+...+T_{2n}$

Subtract above two equations

nd=6

$$T_{2n}=T_1+\frac{21}{2}$$

$$T_{2n}-a=\frac{21}{2}$$

$$(2n-1)d=\frac{21}{2}$$

$$12-\frac{21}{2}=d=\frac{3}{2}$$

$$\Rightarrow n=6\times\frac{2}{3}=4$$

$$Total\ terms=2n=8$$

Subtitute above values in equation of sum of even terms or odd terms, we get

a=$\frac{3}{2}$

So series is $\frac{3}{2},3,\frac{9}{2}........$

Q21

Let a be the first term of the AP and d is the common difference. Then

$$S_n = \frac{n}{2}\left(2a+(n-1)d\right)$$

$$n^2 p = \frac{n}{2}\left(2a+(n-1)d\right)$$

$$np = \frac{1}{2}\left[2a+(n-1)d\right]$$

$$2np = 2a+(n-1)d \qquad \text{......(1)}$$

Again

$$S_m = \frac{m}{2}\left(2a+(m-1)d\right)$$

$$m^2 p = \frac{m}{2}\left(2a+(m-1)d\right)$$

$$mp = \frac{1}{2}\left[2a+(m-1)d\right]$$

$$2mp = 2a+(m-1)d \qquad \text{......(2)}$$

Now subtract (1) from (2)

$$2p(m-n) = (m-n)d$$

$$d = 2p$$

Therefore

$$2mp = 2a+(m-1)\cdot 2p$$

$$2a = 2p$$

$$a = p$$

The sum up to p terms will be:

$$S_p = \frac{p}{2}\left(2a+(p-1)d\right)$$

$$= \frac{p}{2}\left(2p+(p-1)\cdot 2p\right)$$

$$= \frac{p}{2}\left(2p+2p^2-2p\right)$$

$$= p^3$$

Hence it is shown.

Q22

$a_{12} = a + 11d = -13$ ---(i) [Given]

$s_4 = \frac{4}{2}(2a + 3d) = 24$ ---(ii) [Given]

From (i) and (ii)

$d = -2$ and $a = 9$

Then,

Sum of irst 10 terms is

$S_{10} = \frac{10}{2}[2 \times 9 + (9)(-2)]$ $\left[\text{Using } S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$= 0$

Sum of first 10 terms is zero.

Q23

$a_5 = a + 4d = 30$ ---(i) [Given]

$a_{12} = a + 11d = 65$ ---(ii) [Given]

From (i) and (ii)

$d = 5$ and $a = 10$

Then,

Sum of irst 20 terms is

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow$ $S_{20} = \frac{20}{2}[2 \times 10 + (20-1)5]$

$= 1150$

Sum of first 20 terms is 1150.

Q24

Here,

$$a_k = 5k + 1$$
$$a_1 = 5 + 1 = 6$$
$$a_2 = 5(2) + 1 = 11$$
$$a_3 = 5(3) + 1 = 16$$
$$d = 11 - 6 = 16 - 11 = 5$$
$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$= \frac{n}{2}\left[2(6) + (n-1)(5)\right]$$
$$= \frac{n}{2}\left[12 + 5n - 5\right]$$
$$S_n = \frac{n}{2}(5n + 7)$$

Q25

sum of all two digit numbers which when divided by 4,

yields 1 as remainder, $\Rightarrow$ all 4n+1 terms with $n \geq 3$

13,17,21,............97

$n = 22, a = 13, d = 4$

$$sum of\ terms = \frac{22}{2}\left[26 + 21 \times 4\right] = 11 \times 110 = 1210$$

Q26

Sum of terms 25, 22, 19,......, is 116

$$\frac{n}{2}[50+(n-1)(-3)]=116$$

$$\frac{n}{2}[53-3n]=116$$

$$53n-3n^2=232$$

$$3n^2-53n+232=0$$

$$3n^2-29n-24n+232=0$$

$$n(3n-29)-8(3n-29)=0$$

$$(3n-29)(n-8)=0$$

$$\Rightarrow n=8 or \frac{29}{3}$$

n cannot be in fraction, so n=8

last term=25-7×3=4

Q27

Let the number of terms is n.
Now the sum of the series is:

$$1+3+5+\cdots+2001$$

Here $l=2001$ and $d=2$.
Therefore

$$l=a+(n-1)d$$

$$2001=1+(n-1)\cdot 2$$

$$2(n-1)=2000$$

$$n-1=1000$$

$$n=1001$$

Therefore the sum of the series is:

$$S=\frac{1001}{2}[2+(1001-1)2]$$

$$=1001^2$$

$$=1002001$$

Q28

Let the number of terms to be added to the series is n.
Now $a=-6$ and $d=0.5$.
Therefore

$$-25=\frac{n}{2}\left[2(-6)+(n-1)(0.5)\right]$$

$$-50=n\left[-12+0.5n-0.5\right]$$

$$-12.5n+0.5n^2+50=0$$

$$n^2-25n+100=0$$

$$n=20,5$$

Therefore the value of n will be either 20 or 5.

Q29

Here the first term $a=2$. Let the common difference is d.
Now

$$\frac{5}{2}\left[2a+(5-1)d\right]=\frac{1}{4}\left[\frac{5}{2}\left[2(a+5d)+(5-1)d\right]\right]$$

$$\frac{5}{2}\left[2\cdot 2+4d\right]=\frac{5}{8}\left[2\cdot 2+14d\right]$$

$$10+10d=\frac{5}{2}+\frac{35}{4}d$$

$$\frac{5}{4}d=-7.5$$

$$d=-6$$

The 20^{th} term will be:

$$a+(n-1)d=2+(20-1)(-6)$$

$$=-112$$

Hence it is shown.

Q30

$$S_{(2n+1)} = S_1 = \frac{(2n+1)}{2}\left[2a+(2n+1-1)d\right]$$

$$S_1 = \frac{(2n+1)}{2}\left[2a+2nd\right]$$

$$= (2n+1)(a+nd) \qquad \text{---(i)}$$

Sum of odd terms $= S_2$

$$S_2 = \frac{(n+1)}{2}\left[2a+(n+1-1)(2d)\right]$$

$$= \frac{(n+1)}{2}\left[2a+2nd\right]$$

$$S_2 = (n+1)(a+nd) \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$S_1 : S_2 = (2n+1)(a+nd) : (n+1)(a+nd)$$

$$S_1 : S_2 = (2n+1) ; (n+1)$$

Q31

Here,

$$S_n = 3n^2 \qquad \text{---(i)} \qquad [\text{Given}]$$

Where n is number of term

$$\therefore \quad S_n = \frac{n}{2}\left[2a+(n-1)d\right] \qquad \text{---(ii)}$$

From (i) and (ii)

$$3n^2 = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$6n = 2a + nd - d$$

Equating both sides

$$6n = nd$$

$$\therefore \quad d = 6 \qquad \text{---(iii)}$$

and

$$0 = 2a - d$$

or $\quad d = 2a \qquad$ ---(iv)

From (iii) and (iv)

$$a = 3 \text{ and } d = 6$$

$\therefore$ The required A.P is $3, 9, 15, 21, \ldots, \infty$

Q32

$$S_n = nP + \frac{1}{2}n(n-1)Q \qquad [\text{Given}]$$

$$S_n = \frac{n}{2}[2P + (n-1)Q] \qquad \text{---(i)}$$

We know

$$S_n = \frac{n}{2}[2a + (n-1)d] \qquad \text{---(ii)}$$

Where a = first term and d = common difference comparing (i) and (ii)

$$d = Q$$

$\therefore$ The common difference is Q.

Q33

Let sum of n terms of two A.P be S_n and $S'n$.

Then, $S_n = 5n + 4$ and $S'_n = 9n + 16$ respectively.

Then, if ratio of sum of n terms of 2A.P is giben, then the ratio of there nth ther is obtained by replacing n by $(2n-1)$.

$$\frac{a_n}{a_n'} = \frac{5(2n-1)+4}{9(2n-1)+16}$$

$\therefore$ Ratio of there 18th term is

$$\frac{a_{18}}{a'_{18}} = \frac{5(2\times18-1)+4}{9(2\times18-1)+16}$$

$$= \frac{5\times35+4}{9\times35+16}$$

$$= \frac{179}{321}$$

Q34

Let sum of n term of 1 A.P series be S_n are other S_n

The, $S_n = 7n + 2$ ---(i).

$S_n = n + 4$ ---(ii)

If the ratio of sum of n terms of 2 A.P is given, then the ratio of there nth term is obtained by replacing n by $(2n - 1)$.

$$\frac{a_n}{a_{n'}} = \frac{7(2n-1)+2}{(2n-1)+4}$$

Putting $n = 5$ to get the ratio of 5th term, we get

$$\frac{a_5}{a'5} = \frac{7(2\times5-1)+2}{(2\times5-1)+4} = \frac{65}{13} = \frac{5}{1}$$

The ratio is 5 : 1.

Ex 19.5

Q1(i)

$\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ will be in A.P if $\frac{c+a}{b} - \frac{b+c}{a} = \frac{a+b}{c} - \frac{c+a}{b}$

if $\quad \frac{ca+a^2-b^2-cb}{ab} = \frac{ab+b^2-c^2-ac}{bc}$

LHS $\quad \Rightarrow \frac{ca+a^2-b^2-cb}{ab}$

$\Rightarrow \quad \frac{c^2a+a^2c-b^2c-c^2b}{abc}$

$\Rightarrow \quad \frac{c(a-b)[a+b+c]}{abc}$ ---(i)

RHS $\quad \Rightarrow \frac{ab+b^2-c^2-ac}{bc}$

$\Rightarrow \quad \frac{a^2b+ab^2-ac^2-a^2c}{abc}$

$\Rightarrow \quad \frac{a(b-c)[a+b+c]}{abc}$ ---(ii)

and since $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$

$c(b-a) = a(b-c)$ ---(iii)

$\therefore \quad$ LHS = RHS and the given terms are in A.P.

Q1(ii)

$a(b+c), b(c+a), c(a+b)$ are in A.P if $b(c+a) - a(b+c) = c(a+b) - b(c+a)$

LHS $\quad = b(c+a) - a(b+c)$

$= bc + ab - ab - ac$

$= c(b-a)$ ---(i)

RHS $\quad = c(a+b) - b(c+a)$

$= ca + cb - bc - ba$

$= a(c-b)$ ---(ii)

and $\quad \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$\therefore \quad \frac{1}{a} - \frac{1}{b} = \frac{1}{b} - \frac{1}{c}$

or $\quad c(b-a) = a(c-b)$ ---(iii)

From (i), (ii) and (iii)

$a(b+c), b(c+a), c(a+b)$ are in A.P

Q2

$\frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b}$ are in A.P if $\frac{b}{a+c} - \frac{a}{b+c} = \frac{c}{a+b} - \frac{b}{a+c}$

LHS $= \frac{b}{a+c} - \frac{a}{b+c}$

$\Rightarrow \quad \frac{b^2+bc-a^2-ac}{(a+c)(b+c)}$

$\Rightarrow \quad \frac{(b-a)(a+b+c)}{(a+c)(b+c)}$ ---(i)

RHS $= \frac{a}{a+b} - \frac{b}{a+c}$

$\Rightarrow \quad \frac{ca+c^2-b^2-ab}{(a+b)(b+c)}$

$\Rightarrow \quad \frac{(c-b)(a+b+c)}{(a+b)(b+c)}$ ---(ii)

and $\quad a^2, b^2, c^2$ are in A.P

$\therefore \quad b^2-a^2 = c^2-b^2$ ---(iii)

Substituting b^2-a^2 with c^2-b^2

(i) = (ii)

$\therefore \quad \frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b}$ are in A.P

Q3(i)

$a^2(b+c), b^2(c+a), c^2(a+b)$ are in A.P.

If $b^2(c+a) - a^2(b+c) = c^2(a+b) - b^2(a+c)$

$\Rightarrow \quad b^2c + b^2a - a^2b - a^2c = c^2a + c^2b - b^2a - b^2c$

Given, $b-a = c-b$ [a, b, c are in A.P]

$c\left(b^2-a^2\right) + ab(b-a) = a\left(c^2-b^2\right) + bc(c-b)$

$(b-a)(ab+bc+ca) = (c-b)(ab+bc+ca)$

Cancelling $ab+bc+ca$ from both sides

$b-a = c-b$

$2b = c+a$ which is true

Hence, $a^2(b+c), (c+a)b^2$ and $c^2(a+b)$ are also in A.P.

Q3(ii)

(ii) T.P $b+c-a, c+a-b, a+b-c$ are in A.P.

$b+c-a, c+a-b, a+b-c$ are in A.P only if $(c+a-b)-(b+c-a)=(a+b-c)-(c+a-b)$

LHS $\Rightarrow (c+a-b)-(b+c-a)$

$\Rightarrow \quad 2a-2b \qquad$ ---(i)

RHS $\Rightarrow (a+b-c)-(c+a-b)$

$\Rightarrow \quad 2b-2c \qquad$ ---(ii)

Since,

a, b, c are in A.P

$\therefore \quad b-a=c-b$

or $\quad a-b=b-c \qquad$ ---(iii)

From (i), (ii) and (iii)

LHS = RHS

Thus, given numbers

$b+c-a, c+a-b, a+b-c$ are in A.P

Q3(iii)

To prove $bc-a^2, ca-b^2, ab-c^2$ are in A.P

$\left(ca-b^2\right)-\left(bc-a^2\right)=\left(ab-c^2\right)-\left(ca-b^2\right)$

LHS $=\left(a-b^2-bc+a^2\right)$

$=(a-b)[a+b+c] \qquad$ ---(i)

RHS $=ab-c^2-ca+b^2$

$=(b-c)[a+b+c] \qquad$ ---(ii)

and since a, b, c are in ab

$b-c=a-b$

$\therefore \quad$ LHS = RHS

and

Thus, $bc-a^2, ca-b^2, ab-c^2$ are in A.P

Q4

(i) If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

LHS $= \frac{1}{b} - \frac{1}{a}$

$$= \frac{a-b}{ab} = \frac{c(a-b)}{abc} \qquad \text{---(i)}$$

RHS $= \frac{1}{c} - \frac{1}{b}$

$$= \frac{a(b-c)}{abc} \qquad \text{---(ii)}$$

Since, $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P

$$\frac{b+c}{a} - \frac{c+a}{b} = \frac{c+a}{b} - \frac{a+b}{c}$$

$$\frac{b^2+cb-ac-a^2}{ab} = \frac{c^2+ac-ab-b^2}{bc}$$

$$\Rightarrow \quad \frac{(b-a)(a+b+c)}{ab} = \frac{(c-b)(a+b+c)}{bc}$$

or $\quad \frac{a(b-c)}{abc} = \frac{c(a-b)}{abc} \qquad \text{---(iii)}$

From (i), (ii) and (iii)

LHS = RHS

Hence, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

(ii) If bc, ca, ab are in A.P

Then,

$$ca - bc = ab - ca$$

$$c(a-b) = a(b-c) \qquad \text{---(i)}$$

If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\Rightarrow \quad c(a-b) = a(b-c) \qquad \text{---(ii)}$$

Thus, the condition necessary to prove bc, ca, ab in A.P is fullfilled.

Thus, bc, ca, ab, are in A.P.

Q5

(i) If $(a-c)^2 = 4(a-b)(b-c)$

Then,

$a^2 + c^2 - 2ac = 4\left(ab - b^2 - ac + bc\right)$

$\Rightarrow \quad a^2 + c^2 4b^2 + 2ac - 4ab - 4bc = 0$

$\Rightarrow \quad (a+c-2b)^2 = 0$ $\quad$ $\left[\text{Using } (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc\right]$

$\therefore \quad a + c - 2b = 0$

or $\quad a + c = 2b$

and since,

a, b, c are in A.P $\quad$ [Given]

$a + c = 2b$

Hence proved.

$(a-c)^2 = 4(a-b)(b-c)$

(ii) If $a^2 + c^2 + 4ac = 2(ab + bc + ca)$

Then,

$a^2 + c^2 + 2ac - 2ab - 2bc = 0$

or $\quad (a+c-b)^2 - b^2 = 0$ $\quad$ $\left[\because (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc\right]$

or $\quad b = a + c - b$

or $\quad 2b = a + c$

$b = \dfrac{a+c}{2}$

and since,

a, b, c are in A.P

$b = \dfrac{a+c}{2}$

Thus, $a^2 + c^2 + 4ac = 2(ab + bc + ca)$Hence proved.

(iii) If $a^3 + c^3 + 6abc = 8b^3$

or $\quad a^3 + c^3 - (2b)^3 + 6abc = 0$

or $\quad a^3 + (-2b)^3 + c^3 + 3 \times a \times (-2b) \times c = 0$

$\therefore \quad (a - 2b + c) = 0$ $\quad$ $\left[\begin{array}{l}\because x^3 + y^3 + z^3 + 3xyz = 0 \\ \text{or if } x + y + z = 0\end{array}\right]$

or $\quad a + c = 2b$

$a - b = c - b$

and since, a, b, c are in A.P

Thus, $a - b = c - b$

Hence proved.$a^3 + c^3 + 6abc = 8b^3$

Q6

Here,

$a\left(\frac{1}{b}+\frac{1}{c}\right),\ b\left(\frac{1}{c}+\frac{1}{a}\right),\ c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P.

$\Rightarrow \quad a\left(\frac{1}{b}+\frac{1}{c}\right)+1,\ b\left(\frac{1}{c}+\frac{1}{a}\right)+1,\ c\left(\frac{1}{a}+\frac{1}{b}\right)+1$ are in A.P.

$\Rightarrow \quad \left(\frac{ac+ab+bc}{bc}\right),\ \left(\frac{ab+bc+ac}{ac}\right),\ \left(\frac{cb+ac+ab}{ab}\right)$ are in A.P.

$\Rightarrow \quad \frac{1}{bc},\ \frac{1}{ac},\ \frac{1}{ab}$ are in A.P.

$\Rightarrow \quad \frac{abc}{bc},\ \frac{abc}{ac},\ \frac{abc}{ab}$ are in A.P.

$\Rightarrow \quad a,\ b,\ c$ are in A.P.

Q7

x, y and z are in AP.
Let d be the common difference then,
$y = x+d$ and $z = x+2d$

To show $x^2 + xy + y^2$, $z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P., it is enough to show that,

$\left(z^2 + zx + x^2\right) - \left(x^2 + xy + y^2\right) = \left(y^2 + yz + z^2\right) - \left(z^2 + zx + x^2\right)$

LHS $= \left(z^2 + zx + x^2\right) - \left(x^2 + xy + y^2\right)$

$= z^2 + zx - xy - y^2$

$= (x+2d)^2 + (x+2d)x - x(x+d) - (x+d)^2$

$= x^2 + 4xd + 4d^2 + x^2 + 2xd - x^2 - xd - x^2 - 2xd - d^2$

$= 3xd + 3d^2$

RHS $= \left(y^2 + yz + z^2\right) - \left(z^2 + zx + x^2\right)$

$= y^2 + yz - zx - x^2$

$= (x+d)^2 + (x+d)(x+2d) - (x+2d)x - x^2$

$= x^2 + 2dx + d^2 + x^2 + 2dx + xd + 2d^2 - x^2 - 2dx - x^2$

$= 3xd + 3d^2$

$\therefore$ LHS = RHS

$\therefore$ $x^2 + xy + y^2$, $z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P.

Ex 19.6

Q1

(i) 7 and 13

Let A be the arithematic mean of 7 and 13.

Then,

$7, A, 13$ ar ein A.P

$\Rightarrow \quad A - 7 = 13 - A$

$\Rightarrow \quad A = \dfrac{13+7}{2} = 10$

$\therefore \quad$ A.M is 10.

(ii) 12 and -8

Let A be the arithematic mean of 12 and -8

Then,

$12, A, -8$ are in A.P

$\Rightarrow \quad A - 12 = -8 - A$

$\Rightarrow \quad A = \dfrac{12+(-8)}{2} = 2$

$\therefore \quad$ A.M is 2.

(iii) $(x - y)$ and $(x + y)$

Let A be the arithematic mean of $(x - y)$ and $(x + y)$

Then,

$(x - y), A, (x + y)$ are in A.P

$\Rightarrow \quad A - (x - y) = (x + y) - A$

$\Rightarrow \quad A = \dfrac{(x-y)+(x+y)}{2} = \dfrac{2x}{2} = x$

$\therefore \quad$ A.M is x.

Q2

Let A_1, A_2, A_3, A_4 be the 4 A.M.s between 4 and 19

Then,

$4, A_1, A_2, A_3, A_4, 19$ are in A.P of 6 terms

$A_n = a + (n-1)d$

$a_6 = 19 = 4 + (6-1)d$

or $d = 3$ ---(i)

Now,

$A_1 = a + d = 4 + 3 = 7$

$A_2 = A_1 + d = 7 + 3 = 10$

$A_3 = A_2 + d = 10 + 3 = 13$

$A_4 = A_3 + d = 13 + 3 = 16$

The 4 A.M.s between 4 and 19 are 7, 10, 13, 16.

Q3

$2, a_1, a_2, a_3, a_4, a_5, a_6, a_7, 17$

$17 = a + 8d$

$a = 2 \Rightarrow d = \frac{15}{8}$

$a_1 = 2 + \frac{15}{8} = \frac{31}{8}$

$a_2 = \frac{31}{8} + \frac{15}{8} = \frac{46}{8}$

so we get our final series as

$2, \frac{31}{8}, \frac{46}{8}, \frac{61}{8}, \frac{76}{8}, \frac{91}{8}, \frac{106}{8}, \frac{121}{8}, \frac{136}{8} = 17$

Q4

Let $A_1, A_2, A_3, A_4, A_5, A_6$ be the 6 AM's between 15 and -13

Then,

$15, A_1, A_2, A_3, A_4, A_5, A_6, -13$ are in A.P of 8 terms

Here, $-13 = a_8 = a + 7d$

$\Rightarrow \quad -13 = 15 + 7d$

or $\quad d = -4 \qquad$ ---(i)

$\therefore \quad A_1 = a + d = 15 - 4 = 11$

$A_2 = a + 2d = 15 - 2(4) = 7$

$A_3 = a + 3d = 15 - 4(3) = 3$

$A_4 = a + 4d = 15 - 4(4) = -1$

$A_5 = a + 5d = 15 - 4(5) = -5$

$A_6 = a + 6d = 15 - 4(6) = -9$

The 6 A.M.s between 15 and -13 are $11, 7, 3, -1, -5$ and -9.

Q5

Let the n A.M's between 3 and 17 be $A_1, A_2, A_3, \ldots, A_n$

Then,

ATQ

$$\frac{A_n}{A_1} = \frac{3}{1} \qquad \text{---(i)}$$

We know that

$3, A_1, A_2, A_3, \ldots, A_n, 17$ are in A.P of $n+2$ terms

So, 17 is the $(n+2)$ th terms.

i.e. $\quad 17 = 3 + (n + 2 - 1)d \qquad [\text{Using } a_n = a + (n-1)d]$

or $\quad d = \dfrac{14}{(n+1)} \qquad$ ---(ii)

$\therefore \quad A_n = 3 + (n + 1 - 1)d$

$$= 3 + \frac{14n}{n+1} = \frac{17n+3}{n+1} \qquad \text{---(iii)}$$

$$A_1 = 3 + d = \frac{3n+17}{n+1} \qquad \text{---(iv)}$$

From (i), (iii) and iv

$$\frac{A_n}{A_1} = \frac{17n+3}{3n+17} = \frac{3}{1}$$

$\therefore \quad n = 6$

There are 6 A'M between 3 and 17.

Q6

Let there be n A.M between 7 and 71 and let the A.M's be $A_1, A_2, A_3, \ldots, A_n$.
So,

$7, A_1, A_2, A_3, \ldots, A_n, 71$ are in A.P of $(n+2)$ terms

$A_5 = a_6 = a + 5d = 27$ [Given]

$\Rightarrow \quad a + 5d = 27$

$\Rightarrow \quad d = 4$ $[\because a = 7]$ ---(i)

The $(n+2)$ th term of A.P is 71

$\therefore \quad a_{n+2} = 7 = a + (n + 2 - 1)d$

or $\quad n = 15$

There are 15 AM's between 7 and 71.

Q7

Let $A_1, A_2, A_3, A_4, \ldots, A_n$ be the n AMs inserted between two number a and b.
Then,

$a, A_1, A_2, A_3, A_4, \ldots, A_n, b$ are in A.P

So, the mean of a and b

$$\text{A.M} = \frac{a+b}{2}$$

The mean of A_1 and A_n

$$\text{A.M} = \frac{a+d+b-d}{2} = \frac{a+b}{2}$$

Similarly mean of A_2 and A_{n-1}

$$\text{A.M} = \frac{a+2d+b-2d}{2} = \frac{a+b}{2}$$

Similarly we observe the means is equidistant from begining and the end is constant $\frac{a+b}{2}$.

The AM is $\frac{a+b}{2}$.

Q8

Here,

A_1 is the A.M of x and y,

and A_2 is the A.M of y and z.

Then,

$$A_1 = \frac{x+y}{2} \qquad \text{---(i)} \qquad \left[\because \text{AM} = \frac{a+b}{2}\right]$$

$$A_2 = \frac{y+z}{2} \qquad \text{---(ii)}$$

Let A.M be the arithematic mean of A_1 and A_2

Then,

$$\text{A.M} = \frac{A_1 + A_2}{4}$$

$$= \frac{x+y+y+z}{4}$$

$$= \frac{x+2y+z}{4} \qquad \text{---(iii)}$$

Since, 4, y, z are in A.P [Given]

$$y = \frac{x+a}{2} \qquad \text{---(iv)}$$

From (iii) and (iv)

$$\text{A.M} = \frac{\left(\frac{x+a}{2}\right) + \left(\frac{2y}{2}\right)}{2} = \frac{y+y}{2} = y$$

Hence, proved A.M between A_1 and A_2 is y.

Q9

$8, a_1, a_2, a_3, a_4, a_5, 26$

$a = 8$

$a + 6d = 26$

$\Rightarrow d = \frac{18}{6} = 3$

So series is 8, 11, 14, 17, 20, 23, 26

Ex 19.7

Q1

Let the amount saved by the man in first year be x.
Then,

ATQ

$$x+(x+100)+(x+200)+\ldots+(x+900)=16500$$

As his saving increased by Rs 100 every year.

$$\therefore \quad 10x+100+200+\ldots+900=16500 \qquad \text{---(i)}$$

Here,

$100+200+300+\ldots+900$ form a seried of

$a=100,\ d=100$ and $n=9$

So,

$$S_n=\frac{n}{2}[a+l]$$

$$S_9=\frac{9}{2}[100+900]=4500 \qquad \text{---(ii)}$$

From (i) and (ii)

$$10x+(4500)=16500$$

$$10x=12000$$

or $\quad x=1200$

The man saved Rs 1200 in the first year.

Q2

Let the man save Rs 200 in n numbers of years.
Then,

ATQ

$$32+36+40+\ldots=200$$

It rorms a series of n terms, with $a=32$ and $d=4$

$$\therefore \quad S_n=\frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow \quad 200=\frac{n}{2}[2(32)+(n-1)4]$$

$$\Rightarrow \quad 400=60n+4n^2$$

$$\Rightarrow \quad n^2+15n-100=0$$

$$\Rightarrow \quad n=5 \text{ or } -20$$

But, $\quad n\neq -20 \qquad$ [It can't be negative]

$$\therefore \quad n=5$$

The man will save Rs 200 in 5 years.

Q3

Let the 40 annual instalments form an alithmetic series of common diference d and first instalment a
Then, series so firmed is

$$a+(a+d)+(a+2d)+\ldots=3600$$

or $\quad s_n=\frac{n}{2}[2a+(n-1)d]$

or $\quad 3600=20[2a+39d]$

$$2a+39d=180 \qquad \text{---(i)}$$

and sum of first 30 terms is $\frac{2}{3}$ of 3600

$$=2400$$

$\Rightarrow \quad 2400=\frac{30}{2}[2a+(29)d]$

or $\quad 2a+29d=160 \qquad \text{---(ii)}$

From (i) and (ii)

$$a=51$$

The first installment paid by this man is Rs 51.

Q4

Let the number of Radio manufactured increase by x each year and number of radio manufacture in first year be a. So, A.P formed ATQ is

$$a, a+x, a+2x, \ldots$$

Here,

$$a_3=a+2x=600 \qquad \text{---(i)}$$

$$a_7=a+6x=700 \qquad \text{---(ii)}$$

From (i) and (ii)

$$a=550,\ x=25$$

(i) 550 Radio's were manufactured in the first year.

(ii) The total produce in 7 years is sum of produce in the first 7 years.

$$S_7=\frac{7}{2}[550+700] \qquad \left[\because S_n=\frac{n}{2}[a+l]\right]$$

$$=4375$$

4375 Radio's were manufactured in first 7 years.

(iii) The product in 10th year

$$a_{10}=a+9d$$

$$=550+9(25)=775$$

775 Radio's were manufactured in the 10th year.

Q5

There are 25 trees at equal distance of 5 m in a line with a well(w), and the distance of the well from the nearesst tree = 10 m.

Thus,

The total distance travelled by gardener to tree 1 and back is 2×10 m = 20 m

Similarly for all the 25 trees.

The distance covered by gardener is

$$= 2\left[10 + (10+5) + (10 + 2\times 5) + (10 + 3\times 5) + \ldots + (10 + 23\times 5)\right] \qquad \text{---(i)}$$

This forms a series of 1st term $a = 10$, common difference $d = 5$ and $n = 25$

$$\therefore \quad 10 + (10+5) + (10 + 2\times 5) + \ldots + (10 + 24 \times 5)$$

$$\Rightarrow \quad S_{25} = \frac{25}{2}\left[2\times 10 + (24)5\right] = 25\left[10 + 60\right] = 1750 \text{ m} \qquad \text{---(ii)}$$

From (i) and (ii)

Total distance = 2×1750 m = 3500 m.

Q6

The man counts at the rate of Rs 180 per minute for half an hour. After this he counts at the rate of Rs 3 less every minute than preceding minute.

Then, the amount counted in first 30 minute

$$= \text{Rs } 180 \times 30 = \text{Rs } 5400 \qquad \text{---(i)}$$

The amount left to be counted after 30 minute

$$= \text{Rs } 10710 - 5400 = \text{Rs } 5310 \qquad \text{---(ii)}$$

ATQ

A.P formed is $(180 - 3) + (180 - 2\times 3) + \ldots = 5310$

Let time taken to count 5310 be t

Then,

$$S_t = \frac{t}{2}\left[(180 - 3) + (t - 1)(-3)\right]$$

$$5310 = \frac{t}{2}\left[200 - 3t\right]$$

or $\quad t = 59$ minute

Thus, the total time taken by the man to count Rs 10710 is $(59 + 30) = 89$ minutes.

Q7

The piece of equipment deprecites 15% in first year i.e., $\frac{15}{100} \times 600,000 = \text{Rs } 90,000$

$\therefore$ Value after 1st year $= 600,000 - 90,000$

$= \text{Rs } 510,000$ ---(i)

The equipment deprecites at the rate 135% in 2nd year i.e., $\frac{135}{1000} \times 600,000 = 81000$

$\therefore$ Value after 2nd year $= 81000$

The value after 3rd year $= \frac{12}{100} \times 600000 = 72000$

The total depreciation in 10 years

$\Rightarrow$ $S_{10} = \frac{10}{2}\left[2 \times 81000 + (9)(-9000)\right]$

$= 5[81000]$ $\left[\text{Using } S_n = \frac{n}{2}\left[2a + (n-1)d\right]\right]$

$= 405000$

$\therefore$ The cost of machine after 10 years $= \text{Rs } 600000 - 405000$

$= 105000.$

Q8

Total cost of tractor

$= 6000 + \left[(500 + 12\% \text{ of } 6000 \text{ for 1 year}) + (500 + 12\% \text{ of } 5500 \text{ 1 year}) + \ldots\ldots + 12 \text{ times}\right]$

$= 6000 + 6000 + \frac{12}{100}(6000 + 5500 + \ldots\ldots + 12 \text{ times})$

$= 12000 + \frac{12}{100}\left[\frac{12}{2}(6000 + 5000)\right]$

$= 12000 + \frac{12}{100} \times \frac{12}{2} \times 6500$

$= 12000 + (72 \times 65)$

$= 12000 + 4680$

$= 16680$

Total cost of tractor = Rs. 16680

Q9

Total cost of Scooter

$$= Rs4000 + \begin{bmatrix} \{Rs\ 1000 + S.I.\ on\ Rs\ Rs\ 18000\ for\ 1\ year\} \\ +\{Rs\ 1000 + S.I.\ on\ Rs\ Rs\ 17000\ for\ 1\ year\} \\ +.....+18\ times \end{bmatrix}$$

$$= (4000 + 18000) + S.I.\ for\ 1\ year\ on\ (18000 + 17000 + ...to\ 18\ times)$$

$$= 22000 + S.I.\ for\ 1\ year\ on\ \left\{\frac{18}{2}(18000 + 1000)\right\}$$

$$= 22000 + 9(19000) \times \frac{10}{100}$$

$$= 22000 + 17100$$

$$= Rs\ 39100$$

Total cost of Scooter = Rs. 39100

Q10

First year the person income is: 300,000

Second year his income will be: $300,000 + 10,000 = 310,000$

⋮

⋮

This way he receives the amount after 20 years will be:

$$300,000 + 310,000 + \cdots + 490,000$$

This is an AP with first term $a = 300000$ and common difference $d = 10,000$.

Therefore

$$S = \frac{20}{2}\left[2 \cdot 300000 + (20-1)10000\right]$$

$$= 10[600000 + 190000]$$

$$= 7900000$$

Q11

In 1^{st} installment the man paid 100 rupees.

In 2^{nd} installment the man paid $(100+5) = 105$ rupees.

⋮

⋮

Likewise he pays up to the 30^{th} installment as follows:

$$100 + 105 + \cdots + (100 + 5 \times 29)$$

This is an AP with $a = 100$ and common difference $d = 5$.

Therefore at the 30^{th} installment the amount he will pay

$$T_{30} = 100 + (30-1)(5)$$

$$= 100 + 145$$

$$= 245$$

Q12

Suppose carpenter took n days to finish his job.

First day carpenter made five frames

$a_1 = 5$

Each day after first day he made two more frames

$d=2$

$\therefore$ On n^{th} day frames made by carpenter are,

$a_n = a_1 + (n-1)d$

$\Rightarrow a_n = 5 + (n-1)2$

Sum of all the frames till n^{th} day is

$S = \frac{n}{2}[a_1 + a_n]$

$192 = \frac{n}{2}[5 + 5 + (n-1)2]$

$192 = 5n + n^2 - n$

$n^2 + 4n - 192 = 0$

$(n+16)(n-12) = 0$

$n = -16$ or $n = 12$

But number of days cannot be negative hence $n = 12$.

The carpenter took 12 days to finish his job.

Q13

We know that sum of interior angles of a polygon with n sides is given by,
$a_n = 180°(n - 2)$

Sum of interior angles of a polygon with 3 sides is given by,
$a_3 = 180°(3 - 2) = 180°$............(i)

Sum of interior angles of a polygon with 7 sides is given by,
$a_4 = 180°(4 - 2) = 360°$............(ii)

Sum of interior angles of a polygon with 5 sides is given by,
$a_5 = 180°(5 - 2) = 540°$............(iii)

From eq^n (i), eq^n (ii) and eq^n (iii) we get,
$a_4 = 360° = 180° + 180° = a_3 + 180° = a_3 + d$
$a_5 = 540° = 180° + 360° = a_3 + 2d$

Hence the sums of the interior angles of polygons with 3, 4, 5, 6,...
sides form an arithmetic progression.

Sum of interior angles of 21 sided polygon
$= 180°(21 - 2)$
$= 3420°$

Q14

20 potatoes are placed in a line at intervals of 4 meters.
$\therefore$ n = 20 and d = 4

The first potato 24 meters from the starting point.
$a_1 = 24$
$a_2 = a_1 + d = 24 + 8 = 32$
.
.
.
$a_n = a_1 + (n - 1)d$
$a_{20} = 24 + 19 \times 4 = 24 + 76 = 100$

$S = \frac{20}{2}[a_1 + a_{20}] = 10[24 + 100] = 1240$

As contestant is required to bring the potatos back to the starting point.
The distanced contestant would run
= 1240+1240
= 2480 m.

Q15(i)

A man accepts a position with an initial salary of Rs.5200 per month.

$a_1 = 5200$

Man will receive an automatic increase of Rs.320.

$d = 320$

We need to find his salary for the n^{th} month is given by,

$a_n = a_1 + (n-1)d$

$a_{10} = 5200 + 9 \times 320 = 8080$

The salary of that man for tenth month is Rs.8080.

Q15(ii)

A man accepts a position with an initial salary of Rs.5200 per month.

$a_1 = 5200$

Man will receive an automatic increase of Rs.320.

$d = 320$

Man's salary for the n^{th} month is given by,

$a_n = a_1 + (n-1)d$

Total earnig of the man for the first year

$= \frac{12}{2}[a_1 + a_{12}]$

$= 6[5200 + 5200 + (12-1)320]$

$= 83520$

Total earnig of the man for the first year is Rs. 83,520.

Q16

Suppose the man saved Rs. x in the first year

$a_1 = x$

In each succeeding year after the first year man saved Rs 200 more then what he saved in the previous year.

$d = 200$

Man saved Rs. 66000 in 20 years.

$S = 66000$

$$\frac{20}{2}\left[a_1 + a_1 + (20-1)200\right] = 66000$$

$$a_1 + 1900 = 3300$$

$$a_1 = 1400$$

Man saved Rs 1400 in the first year.

Q17

Suppose the award increases by Rs. x.

$d = x$

In cricket team tournament 16 teams participated.

$n = 16$

The last place team is awarded Rs. 275 in prize money

$a_1 = 275$

Sum of Rs. 8000 is to be awarded as prize money

$S = 8000$

$$\frac{16}{2}\left[a_1 + a_1 + (16-1)x\right] = 8000$$

$$2a_1 + 15x = 1000$$

$$550 + 15x = 1000$$

$$15x = 450$$

$$x = 30$$

The amount received by first place team

$= a_{16}$

$= a_1 + (16-1)d$

$= 275 + 15 \times 30$

$= 275 + 450$

$= 725$

The amount received by first place team is Rs. 725.