Exercise 2.9

Chapter 2 Derivatives Exercise 2.9 1E

Consider the following function:

$$f(x) = x^4 + 3x^2$$

Find the linearization $L(x)$, of the above function at $a = -1$.

Recollect, that the linearization of f at *a* is as follows:

$$L(x) = f(a) + f'(a)(x - a)$$

Plug $x = -1$ in $f(x) = x^4 + 3x^2$, to obtain the following:

$$f(-1) = (-1)^4 + 3(-1)^2 \quad \text{Replace } x \text{ by } -1$$
$$= 1 + 3$$
$$= 4$$

Differentiate the function, $f(x) = x^4 + 3x^2$ on both sides with respect to *x*.

$$f'(x) = \frac{d}{dx} f(x)$$
$$= \frac{d}{dx}(x^4 + 3x^2)$$
$$= 4x^3 + 6x$$

Put $x = -1$ into $f'(x) = 4x^3 + 6x$, to obtain the following:

$$f'(-1) = 4(-1)^3 + 6(-1)$$
$$= -4 - 6$$
$$= -10$$

Put all the values into $L(x) = f(a) + f'(a)(x - a)$.

$$L(x) = f(-1) + f'(-1)\{x - (-1)\}$$
$$= 4 + (-10)(x + 1)$$
$$= 4 - 10x - 10$$
$$= -10x - 6$$

Therefore, the required linearization of f at *a* is

$$L(x) = \boxed{-10x - 6}.$$

Chapter 2 Derivatives Exercise 2.9 2E

Let $f(x) = \sin x$, $a = \frac{\pi}{6}$

Then $f'(x) = \cos x$ and $f'(a) = f'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\therefore$ The linearization of f at $a = \frac{\pi}{6}$ is

$$L(x) = f(a) + f'(a)(x-a)$$
$$= f\left(\frac{\pi}{6}\right) + f'\left(\frac{\pi}{6}\right)\left(x - \frac{\pi}{6}\right)$$
$$= \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$$
$$\boxed{\therefore\ L(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)}$$

Chapter 2 Derivatives Exercise 2.9 3E

Let $f(x) = \sqrt{x}$, $a = 4$

Then $f'(x) = \frac{1}{2\sqrt{x}}$ and

$$f'(a) = f'(4)$$
$$= \frac{1}{2\sqrt{4}}$$
$$= \frac{1}{4}$$

$\therefore$ The linearization of f at $x = 4$ is

$$L(x) = f(a) + f'(a)(x-a)$$
$$= f(4) + f'(4)(x-4)$$
$$= 2 + \frac{1}{4}(x-4)$$
$$= \frac{x}{4} - 1 + 2$$
$$= \frac{x}{4} + 1$$
$$\boxed{\therefore\ L(x) = \frac{x}{4} + 1}$$

Chapter 2 Derivatives Exercise 2.9 4E

Consider the function,

$$f(x) = x^{\frac{3}{4}}$$

The objective is to find the linearization $L(x)$ of the function at $a = 16$

The linearization of *f* at *a* is given by

$$L(x) = f(a) + f'(a)(x-a)$$

First calculate $f(a)$ at $a = 16$.

Plug $x = 16$ in to $f(x) = x^{\frac{3}{4}}$, to obtain

$$f(16) = (16)^{\frac{3}{4}}$$
$$= (2^4)^{\frac{3}{4}}$$
$$= 8$$

Differentiate the function $f(x)=x^{\frac{3}{4}}$ on both sides with respect to x, to get

$$\begin{aligned} f'(x) &= \frac{d}{dx}f(x) \\ &= \frac{d}{dx}\left(x^{\frac{3}{4}}\right) \\ &= \frac{3}{4}x^{-\frac{1}{4}} \end{aligned}$$

Substitute $x=16$ in to $f'(x)$, to obtain

$$\begin{aligned} f'(16) &= \frac{3}{4}(16)^{-\frac{1}{4}} \\ &= \frac{3}{4}\left(2^4\right)^{-\frac{1}{4}} \\ &= \frac{3}{4}\left(\frac{1}{2}\right) \\ &= \frac{3}{8} \end{aligned}$$

Plug all the values in to $L(x)=f(a)+f'(a)(x-a)$, to get

$$\begin{aligned} L(x) &= f(16)+f'(16)(x-16) \\ &= 8+\left(\frac{3}{8}\right)(x-16) \\ &= 8+\frac{3}{8}x-6 \\ &= 2+\frac{3}{8}x \end{aligned}$$

Therefore, the required linearization of f is $L(x)=\boxed{2+\frac{3}{8}x}$.

Chapter 2 Derivatives Exercise 2.9 5E

We have $f(x)=\sqrt{1-x}$

Then $f'(x)=-\frac{1}{2}(1-x)^{-1/2}$ [By Chain rule]

$$\Rightarrow f'(x)=\frac{-1}{2\sqrt{1-x}}$$

For $a=0$ we have $f(0)=\sqrt{1-0}=1$ and $f'(0)=\frac{-1}{2\sqrt{1-0}}=-\frac{1}{2}$

Then the linear approximation is

$$\begin{aligned} L(x) &= f(0)+f'(0)(x-0) \\ &= 1-\frac{1}{2}(x) \end{aligned}$$

$\boxed{L(x)=1-\frac{1}{2}x}$ The corresponding linear approximation is $\boxed{\sqrt{1-x}\approx 1-\frac{1}{2}x}$

We have $\sqrt{0.9}=\sqrt{1-0.1}\approx 1-\frac{1}{2}(0.1)$

$$= 1-0.05$$

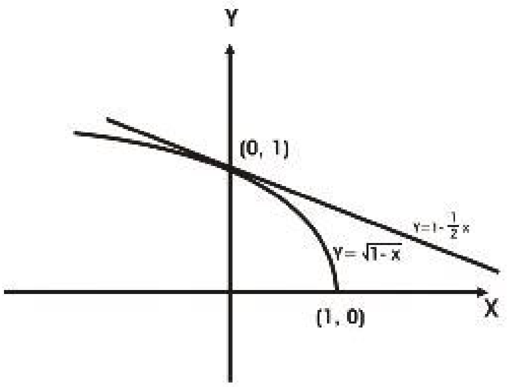
$$\boxed{\sqrt{0.9}\approx 0.95}$$

Now $\sqrt{0.99}=\sqrt{1-0.01}\approx 1-\frac{1}{2}(0.01)$

$$\approx 1-0.005$$

$$\Rightarrow \boxed{\sqrt{0.99}\approx 0.995}$$

Graph the function $f(x)$ and draw the tangent line



Chapter 2 Derivatives Exercise 2.9 6E

We have $g(x)=\sqrt[3]{1+x}$ or $g(x)=(1+x)^{1/3}$

Then $g'(x)=\frac{1}{3}(1+x)^{1/3-1}$

$$g'(x)=\frac{1}{3}(1+x)^{-2/3}$$

$$\Rightarrow g'(x)=\frac{1}{3\sqrt[3]{(1+x)^2}}$$

For a = 0 we have $g(0)=\sqrt[3]{1+0}=1$ and $g'(0)=\frac{1}{3\sqrt[3]{(1+0)^2}}=\frac{1}{3}$

Then the linear approximation is

$$L(x)=g(0)+g'(0)(x-0)$$

$$=1+\frac{1}{3}(x)$$

$$\boxed{L(x)=1+\frac{x}{3}}$$

The corresponding linear approximations is $\boxed{\sqrt[3]{1+x}=1+\frac{x}{3}}$

We have

$\sqrt[3]{0.95}=\sqrt[3]{1+(-0.05)}$

So $\sqrt[3]{0.95}\approx 1+\frac{1}{3}(-0.05)$

$\Rightarrow \boxed{\sqrt[3]{0.95}\approx 0.98\overline{3}}$

We have $\sqrt[3]{1.1}=\sqrt[3]{1+0.1}$

Thus $\sqrt[3]{1.1}\approx 1+\frac{1}{3}(0.1)$

$\Rightarrow \boxed{\sqrt[3]{1.1}\approx 1.03\overline{3}}$

Draw the graph $g(x)$ and tangent line

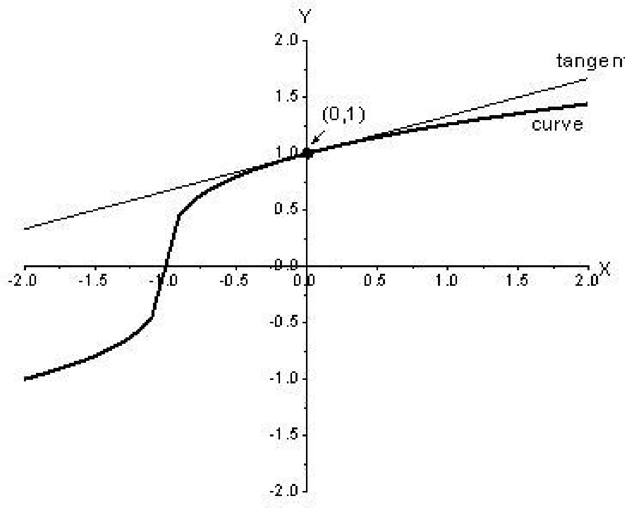


Fig.1

Chapter 2 Derivatives Exercise 2.9 7E

To verify the linear approximation

$$\sqrt[4]{1+2x} \approx 1+\frac{x}{2}$$

at $a=0$, first consider the derivative of $\sqrt[4]{1+2x}$.

The derivative of $f(x)=\sqrt[4]{1+2x}$ is

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left[(1+2x)^{\frac{1}{4}}\right] \\ &= \frac{1}{4}(1+2x)^{\frac{1}{4}-1}\frac{d}{dx}(1+2x) \quad \text{By } \frac{d}{dx}\left[(f(x))^n\right]=n(f(x))^{n-1}\frac{d}{dx}[f(x)] \\ &= \frac{1}{4}(1+2x)^{\frac{-3}{4}}\frac{d}{dx}(1+2x) \\ &= \frac{1}{4}(1+2x)^{\frac{-3}{4}}(2) \end{aligned}$$

$$=\frac{1}{2}(1+2x)^{\frac{-3}{4}}$$

Recall that, linear approximation of f at a is

$$f(x)\approx f(a)+f'(a)(x-a)$$

As $f(x)=\sqrt[4]{1+2x}$,

$$\begin{aligned} f(0) &= \sqrt[4]{1+2(0)} \\ &= \sqrt[4]{1+0} \\ &= 1 \end{aligned}$$

As $f'(x)=\frac{1}{2}(1+2x)^{\frac{-3}{4}}$,

$$\begin{aligned} f'(0) &= \frac{1}{2}(1+2(0))^{\frac{-3}{4}} \\ &= \frac{1}{2}(1+0)^{\frac{-3}{4}} \\ &= \frac{1}{2} \end{aligned}$$

Therefore, the linearization of f at $a=0$ is

$$f(0)+f'(0)(x-0)=1+\frac{1}{2}(x)$$
$$=1+\frac{x}{2}$$

So, linear approximation of f at $a=0$ is

$$1+\frac{x}{2}$$

Hence, the linear approximation

$$\sqrt[4]{1+2x}\approx 1+\frac{x}{2}$$

at $a=0$, is verified.

Accuracy to within 0.1 means that the functions should differ by less than 0.1

$$\left|\sqrt[4]{1+2x}-\left(1+\frac{x}{2}\right)\right|<0.1$$

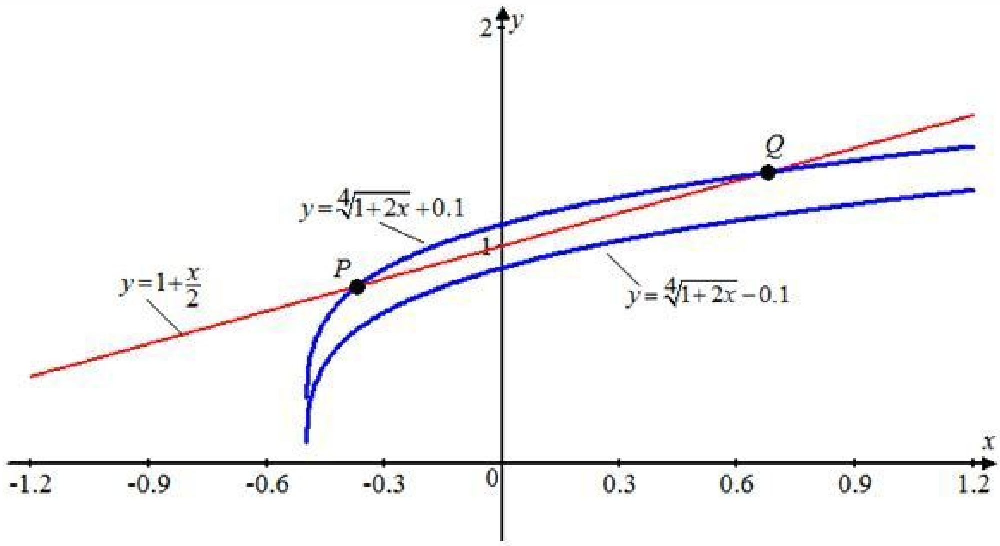
$$-0.1<\sqrt[4]{1+2x}-\left(1+\frac{x}{2}\right)<0.1 \qquad \text{since } |x|<a \text{ means } -a<x<a$$

$$-0.1<\left(1+\frac{x}{2}\right)-\sqrt[4]{1+2x}<0.1 \qquad \text{By multiplying with } -\text{ve sign}$$

$$\sqrt[4]{1+2x}-0.1<1+\frac{x}{2}<\sqrt[4]{1+2x}+0.1$$

This says that the linear approximation should lie between the curves obtained by shifting the curve $y=\sqrt[4]{1+2x}$ upward and downward by an amount $0.1.$

Below figure shows the tangent line $y=1+\frac{x}{2}$ intersecting the upper curve $y=\sqrt[4]{1+2x}+0.1$ at P and $Q.$



Zooming in, we estimate that the $x-$coordinate of P is about -0.368 and the $x-$coordinate of Q is about 0.677. thus we see from the graph that the approximation

$$\sqrt[4]{1+2x}\approx 1+\frac{x}{2}$$

is accurate to within 0.1 when $\boxed{-0.368<x<0.677}$.

Chapter 2 Derivatives Exercise 2.9 8E

Consider the linear approximation,

$$(1+x)^{-3} \approx 1-3x$$

The objective is to verify the above linear approximation at $a=0$.

In general, it is given by the formula,

$$f(x) \approx f(a)+f'(a)(x-a).$$

In this case,

$$f(x)=(1+x)^{-3}$$
$$f'(x)=-3(1+x)^{-4}$$

So, at the point $a=0$,

$$f(0)=1$$
$$f'(0)=-3$$

Therefore, the linear approximation is then given by,

$$\begin{aligned} f(x) &\approx 1+(-3)(x-0) \\ &= 1-3x \end{aligned}$$

Thus, the linear approximation at $a=0$ is $(1+x)^{-3} \approx \boxed{1-3x}$.

The objective is to find the values of *x* for which the linear approximation is accurate to within 0.1.

In other words we want to know which numbers satisfy the inequality,

$$|f(x)-L(x)|<0.1$$

Recollect that, $|a|<b \Leftrightarrow -b<a<b$

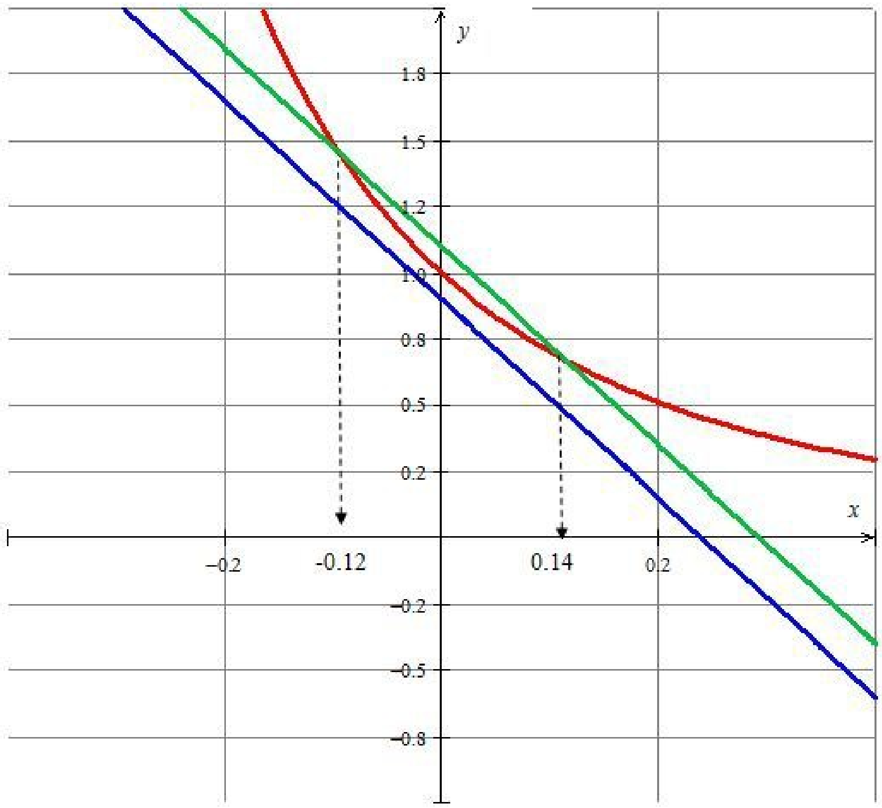
Hence use the above fact to obtain that,

$$\begin{aligned} -0.1 &< f(x)-L(x) < 0.1 \\ L(x)-0.1 &< f(x) < L(x)+0.1 \\ 1-3x-0.1 &< (1+x)^{-3} < 1-3x+0.1 \\ -3x+0.9 &< (1+x)^{-3} < -3x+1.1 \end{aligned}$$

Find the value of *x* that satisfy the compound inequality,

$$-3x+0.9<(1+x)^{-3}<-3x+1.1$$

Use a CAS to graph three functions $y=-3x+0.9$ (blue), $y=(1+x)^{-3}$ (red) and $y=-3x+1.1$ (green).



Clearly it is seen that the inequality is satisfied for $-0.12<x<0.14$.

Thus, the linear approximation is accurate to within 0.1 in the interval $\boxed{-0.12<x<0.14}$.

Chapter 2 Derivatives Exercise 2.9 9E

We have $f(x)=\dfrac{1}{(1+2x)^4}=(1+2x)^{-4}$

Then $f'(x)=-4(1+2x)^{-5}.2$

$=-8(1+2x)^{-5}$

For $x=a$ we have

$f(a)=(1+2x)^{-4}$ and $f'(a)=-8(1+2a)^{-5}$

The linear approximation of a is

$L(x)=f(a)+f'(a)(x-a)$

$=(1+2a)^{-4}-8(1+2a)^{-5}(x-a)$

For $a=0$

$L(x)=(1+0)^{-4}-8(1+0)^{-5}(x-0)$

$=1-8x$

Thus at $a=0$ the linear approximation is

$$\boxed{\frac{1}{(1+2x)^4}\approx 1-8x}$$

Accuracy to within 0.1 means

$$\left|\frac{1}{(1+2x)^4}-(1-8x)\right|<0.1$$

Or $\dfrac{1}{(1+2x)^4}-0.1<1-8x<\dfrac{1}{(1+2x)^4}+0.1$

Draw the graph of the curve $y = \dfrac{1}{(1+2x)^4}$ and shift it 0.1 units upward

And downward to get the graph of the curve $\dfrac{1}{(1+2x)^4} + 0.1$ and

$\dfrac{1}{(1+2x)^4} - 0.1$ Respectively. Draw the tangent line $y = 1 - 8x$ on the same axis

[Figure 1] the tangent line intersects the lower curve at P & Q.

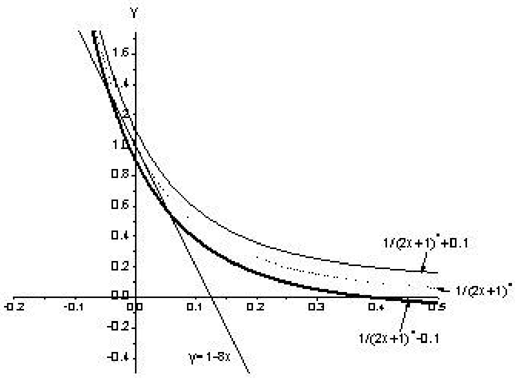


Fig.1

Now zoom the scale to get the co-ordinates of P. The x-co-ordinate of P is -0.045 [Figure 2]

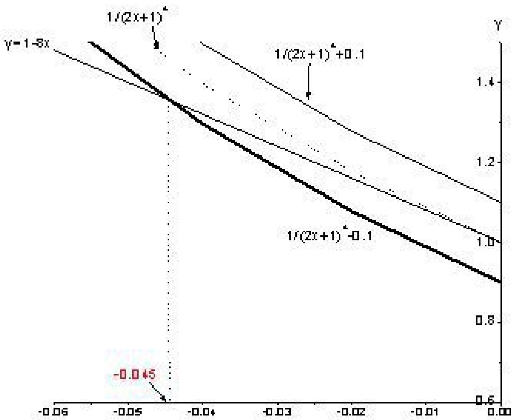


Fig.2

Again zoom the scale to get the co-ordinates of Q the x-co-ordinates of Q is 0.055 [Figure 3]

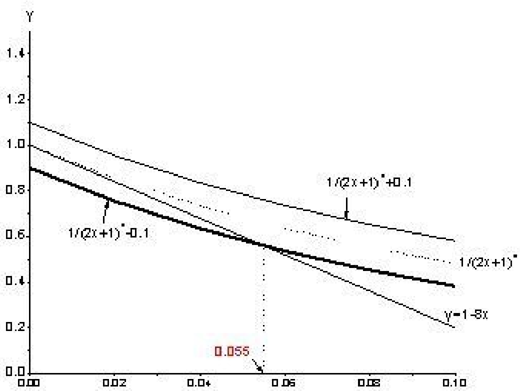


Fig.3

Then the linear approximation $\dfrac{1}{(1+2x)^4} \approx 1 - 8x$ is accurate to within 0.1

When $\boxed{-0.045 < x < 0.055}$

Chapter 2 Derivatives Exercise 2.9 10E

We have $f(x) = \tan x$

Then $f'(x) = \sec^2 x$

For $x = a$

We have $f(a) = \tan a$

And $f'(a) = \sec^2 a$

Then linear approximation at a is

$$L(x) = f(a) + f'(a)(x-a)$$
$$= \tan a + \sec^2 a.(x-a)$$

For $a = 0$

$$\Rightarrow L(x) = \tan 0 + \sec^2 0.(x-0)$$
$$= 0 + 1.(x)$$
$$= x$$

Hence at a = 0 the linear approximation is $\boxed{\tan x \approx x}$

Accuracy to within 0.1 means

$|\tan - x| < 0.1$ Or $\tan x - 0.1 < x < \tan x + 0.1$

We draw the graph of the curves $y = \tan x$ and shift it 0.1 point upward And downward for getting the curves $y = \tan x + 0.1$ and $y = \tan x - 0.1$ respectively Draw the tangent line $y = x$ on the same axis. So we see that the tangent line intersects the upper curve $y = \tan x + 0.1$ at P and the lower curve $y = \tan x - 0.1$ at Q [Figure 1]

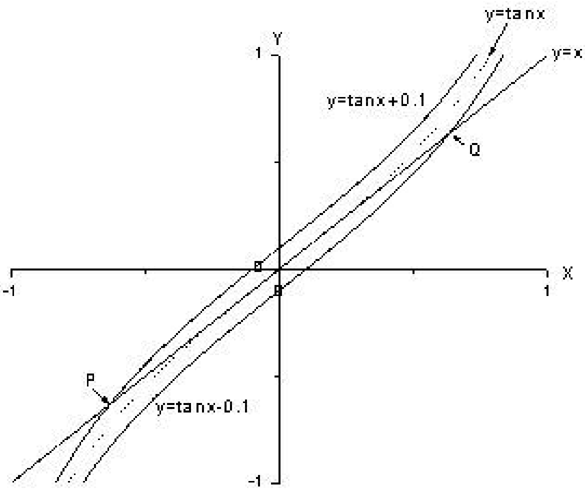


Fig.1

Now zoom the graph to get the co-ordinates of P (figure 2) the x-coordinate of P is -0.633

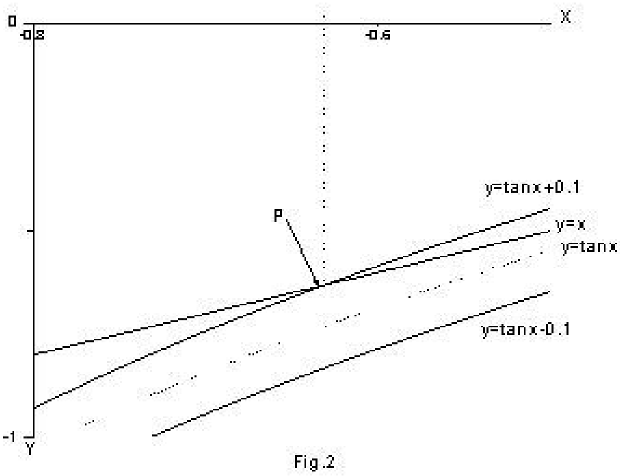


Fig.2

Again zoom the scale to get the co-ordinates of Q (figure 3). The x- coordinates of Q is 0.633

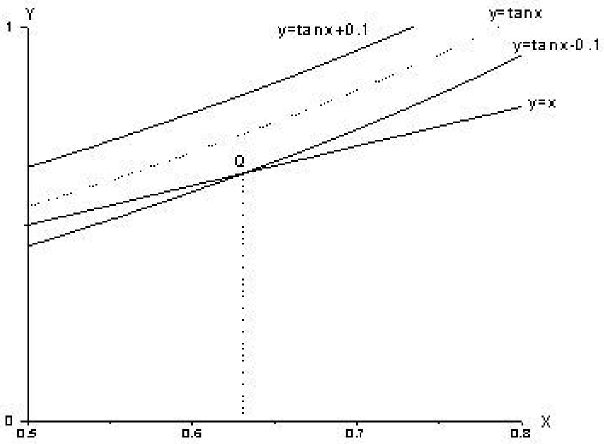


Fig.3

We see that the linear approximation $\tan x \approx x$ is accurate to within 0.1 when $\boxed{-0.633 < x < 0.633}$

Chapter 2 Derivatives Exercise 2.9 11E

a) y = x2sin2x

use product rule : $dy = \left[x^2 \frac{d}{dx}\sin 2x + \left(\sin 2x\right)\frac{d}{dx}\left(x^2\right)\right]dx$

use chain rule to differentiate sin2x :

dy = [(x2)(cos2x * 2)+(sin2x)(2x)] * dx

multiply parenthesis : dy = [2x2cos2x + 2xsin2x]* dx

factor out a 2x,

Answer : **dy = 2x(xcos2x + sin2x)dx**

b) y = √1+t2

change square root to exponent : y= (1+t2)1/2

use chain rule : dy = (1/2)(1+t2)-1/2 *2t * dt

multiply the 2t times the 1/2 : dy = (t)(1+t2)-1/2 * dt

change back the exponent to square root,

Answer : **dy = [t/ √1+t2] dt**

Chapter 2 Derivatives Exercise 2.9 12E

For a function $y = f(s)$, the differential ds is an independent variable which represents a change in the variable s. The differential dy approximates the corresponding change in y, and is given by the formula

$$dy = f'(s)ds$$

(a)

In this case, for $y = s/(1+2s)$, finding the differential involves using the quotient rule. The **quotient rule** says that for a quotient of functions,

$$\left(\frac{u}{v}\right)'(s) = \frac{v(s)u'(s) - u(s)v'(s)}{(v(s))^2}$$

Since y is a quotient of $u(s) = s$ and $v(s) = 1+2s$, to get

$$dy = \frac{(1+2s)(1) - s(2)}{(1+2s)^2}ds$$
$$= \frac{1+2s-2s}{(1+2s)^2}ds$$
$$= \boxed{\frac{1}{(1+2s)^2}ds}$$

(b)

In this case, for $y = u\cos u$, finding the derivative involves using the product rule. The **product rule** says that for a product of functions,

$$(fg)'(u) = f(u)g'(u) + g(u)f'(u)$$

Since y is a product of $f(u) = u$ and $g(u) = \cos u$, to get

$$dy = [u(-\sin u) + \cos u]du$$
$$= (-u\sin u + \cos u)du$$
$$= \boxed{(\cos u - u\sin u)du}$$

Chapter 2 Derivatives Exercise 2.9 13E

(a)

Consider the function,

$$y = \tan\sqrt{t}$$

To find the differential of the function

Differentials:

If $y = f(x)$, where f is a differentiable function, then the **differential** dx is an independent; that is, dx can be given the value of any real number. The differential dy is then defined in terms of dx by the equation

$$dy = f'(x)dx$$

Now,

$$dy = \left[\frac{d}{dt}\tan\sqrt{t}\right]dt$$

$= \left[\sec^2\sqrt{t}\frac{d}{dt}\sqrt{t}\right]dt$ Since $\frac{d}{dx}(\tan x) = \sec^2 x$ and $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

$$= \sec^2\sqrt{t}\frac{1}{2\sqrt{t}}dt$$

$$= \frac{1}{2\sqrt{t}}\sec^2\sqrt{t}dt$$

Therefore,

$$dy = \boxed{\frac{\sec^2\sqrt{t}}{2\sqrt{t}}dt}.$$

(b)

Consider the function,

$$y = \frac{1-v^2}{1+v^2}$$

To find the differential of the function

Differentials:

If $y = f(v)$, where f is a differentiable function, then the **differential** dv is an independent; that is, dv can be given the value of any real number. The differential dy is then defined in terms of dv by the equation

$$dy = f'(v)dv$$

Now,

$$dy = \left[\frac{d}{dv}\left(\frac{1-v^2}{1+v^2}\right)\right]dv$$

$$= \frac{(1+v^2)\frac{d}{dv}(1-v^2)-(1-v^2)\frac{d}{dv}(1+v^2)}{(1+v^2)^2}dv$$

Using $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x))-f(x)\frac{d}{dx}(g(x))}{[g(x)]^2}$ and $\frac{d}{dx}(x^n) = nx^{n-1}$

$$= \frac{(1+v^2)(-2v)-(1-v^2)2v}{(1+v^2)^2}dv$$

$$= \frac{-2v-2v^3-2v+2v^3}{(1+v^2)^2}dv$$

$$= \frac{-4v}{(1+v^2)^2}dv$$

Therefore,

$$dy = \boxed{\frac{-4v}{(1+v^2)^2}dv}.$$

Chapter 2 Derivatives Exercise 2.9 14E

(a)

We are given that

$$y = (t+\tan t)^5$$

Let

$$y = (t+\tan t)^5 \quad \text{.................................} \quad (1)$$

By the differentiation of the equation (1) with respect to t, we get

$$\frac{dy}{dt} = \frac{d}{dt}(t+\tan t)^5$$

$$= 5(t+\tan t)^4 \cdot \frac{d}{dt}\left(t+\tan t\right)$$

$$= 5(t+\tan t)^4 \cdot \left[\frac{d}{dt}\left(t\right) + \frac{d}{dt}\left(\tan t\right)\right]$$

$$= 5(t+\tan t)^4 \cdot \left[1+\sec^2 t\right]$$

$$\Rightarrow \frac{dy}{dt} = 5(t+\tan t)^4\left[1+\sec^2 t\right]$$

(b)

We are given that

$$y = \sqrt{z+\frac{1}{z}}$$

By the differentiating of the equation $y = \sqrt{z+\frac{1}{z}}$ with respect to z, we get

$$\frac{dy}{dz} = \frac{d}{dz}\left[\sqrt{z+\frac{1}{z}}\right]$$

$$= \frac{1}{2\sqrt{z+\frac{1}{z}}} \cdot \frac{d}{dz}\left[z+\frac{1}{z}\right]$$

$$= \frac{1}{2\sqrt{z+\frac{1}{z}}}\left[\frac{d}{dz}\left(z\right) + \frac{d}{dz}\left(\frac{1}{z}\right)\right]$$

$$= \frac{1}{2\sqrt{z+\frac{1}{z}}}\left[1-\frac{1}{z^2}\right]$$

$$\Rightarrow \frac{dy}{dz} = \frac{1}{2\sqrt{z+\frac{1}{z}}}\left[1-\frac{1}{z^2}\right]$$

Chapter 2 Derivatives Exercise 2.9 15E

(A)

We have $y = \tan x$

Here $f(x) = \tan x$

Then $f'(x) = \sec^2 x$

The differential is $dy = f'(x).dx$

$\Rightarrow \boxed{dy = \sec^2 x.dx}$

(B)

We have $x = \frac{\pi}{4}$ and $dx = -0.1$

Then we have

$$dy = \left[\sec^2\left(\frac{\pi}{4}\right)\right].(-0.1)$$

$$\Rightarrow dy = \left(\sqrt{2}\right)^2.(-0.1)$$

$$\Rightarrow dy = 2.(-0.1)$$

$$\Rightarrow \boxed{dy = -0.2}$$

Chapter 2 Derivatives Exercise 2.9 16E

(a) Let $y = \cos \pi x$, $x = \frac{1}{3}$, $dx = -0.02$

Then $dy = \left[\frac{d}{dx}\cos \pi x\right]dx$

$$= \left[-\sin \pi x \frac{d}{dx}\pi x\right]dx$$

$$= -\sin \pi x \pi dx$$

$$= -\pi \sin \pi x dx$$

$$\boxed{\therefore\ dy = -\pi \sin \pi x dx}$$

(b) If $x = \frac{1}{3}$ and $dx = -0.02$ then

$$dy = -\pi\left[\sin\frac{\pi}{3}\right](-0.02)$$

$$= -\pi \sin 60^0 (-0.02)$$

$$= -\pi\frac{\sqrt{3}}{2}(-0.02)$$

$$= \frac{\pi\sqrt{3}(0.02)}{2}$$

$$\approx 0.054$$

$$\boxed{\therefore\ dy = 0.054}$$

Chapter 2 Derivatives Exercise 2.9 17E

(a) Let $y = \sqrt{3+x^2}$, $x = 1$, $dx = -0.1$

Then $dy = \left[\frac{d}{dx}\sqrt{3+x^2}\right]dx$

$$= \left[\frac{1}{2\sqrt{3+x^2}}\frac{d}{dx}\left(3+x^2\right)\right]dx$$

$$= \left[\frac{2x}{2\sqrt{3+x^2}}\right]dx$$

$$= \frac{x}{\sqrt{3+x^2}}dx$$

$$\boxed{\therefore\ dy = \frac{x}{\sqrt{3+x^2}}dx}$$

(b) If $x = 1$ and $dx = -0.1$ then

$$dy = \frac{1}{\sqrt{3+1}}(-0.1)$$

$$= \frac{-0.1}{2}$$

$$= -0.05$$

$$\boxed{\therefore\ dy = -0.05}$$

Chapter 2 Derivatives Exercise 2.9 18E

(a) Let $y = \frac{x+1}{x-1}$, $x = 2$, $dx = 0.05$

Then $dy = \left[\frac{d}{dx}\left(\frac{x+1}{x-1}\right)\right]dx$

$$= \left[\frac{(x-1)(1)-(x+1)(1)}{(x-1)^2}\right]dx$$

$$= \left[\frac{x-1-x-1}{(x-1)^2}\right]dx$$

$$= \left[\frac{-2}{(x-1)^2}\right]dx$$

$$\boxed{\therefore\ dy = \left[\frac{-2}{(x-1)^2}\right]dx}$$

(b) If $x = 2$ and $dx = 0.05$ then

$$dy = \frac{-2}{(2-1)^2}(0.05)$$

$$= -2(0.05)$$

$$= -0.1$$

$$\boxed{\therefore\ dy = -0.1}$$

Chapter 2 Derivatives Exercise 2.9 19E

Consider the following curve:

$$y = 2x - x^2$$

The objective is to compute Δy and dy for the values of $x = 2$ and $\Delta x = dx = -0.4$.

Let $y = f(x) = 2x - x^2$

Use the formula, $\Delta y = f(x+\Delta x) - f(x)$

So, Δy at $x = 2$ will be:

$$\Delta y = f(2+\Delta x) - f(2) \quad \text{...... (1)}$$

Find $f(2)$ and $f(2+\Delta x)$ as follows:

$$f(2) = 2(2) - (2)^2$$

$$= 4 - 4$$

$$= 0$$

And,

$$f(2+\Delta x) = f(2-0.4) \qquad \text{Since } \Delta x = -0.4$$

$$= f(1.6)$$

$$= 2(1.6) - (1.6)^2$$

$$= 0.64$$

Therefore, Plug in these values in (1), then

$$\Delta y = f(2+\Delta x) - f(2)$$

$$= 0.64 - 0$$

$$= \boxed{0.64}$$

If $y = f(x)$, then the differential is given by

$dy = f'(x)dx$

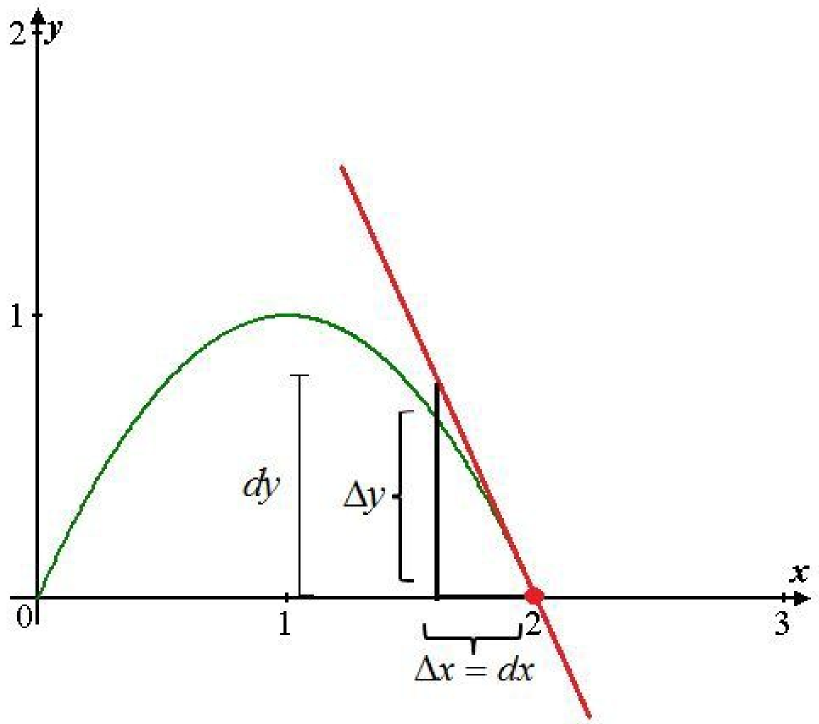
The differential of $y = 2x - x^2$ is

$dy = (2 - 2x)dx$
$dy = 2(1 - x)dx$

Substitute the values of $x = 2$ and $dx = -0.4$, we obtain

$dy = 2(1 - x)dx$
$dy = 2(1 - 2)(-0.4)$
$dy = 2(-1)(-0.4)$
$dy = \boxed{0.8}$

The figure is as shown below.



Chapter 2 Derivatives Exercise 2.9 20E

Given $y = f(x) = \sqrt{x}$

Then $f'(x) = \dfrac{1}{2\sqrt{x}}$

We have $f(1) = \sqrt{1} = 1$

$f(2) = \sqrt{2} = 1.414$

Then $\Delta y = f(2) - f(1) = 1.414 - 1 = 0.414$

$\Rightarrow \boxed{\Delta y \approx 0.414}$

We have $\Rightarrow dy = f'(x)dx$

$= \dfrac{1}{2\sqrt{x}}.dx$

We have $x = 1$ and $dx = \Delta x = 1$

$dy = \dfrac{1}{2\sqrt{1}}.1$

$= \dfrac{1}{2}$

$\Rightarrow \boxed{dy = 0.5}$

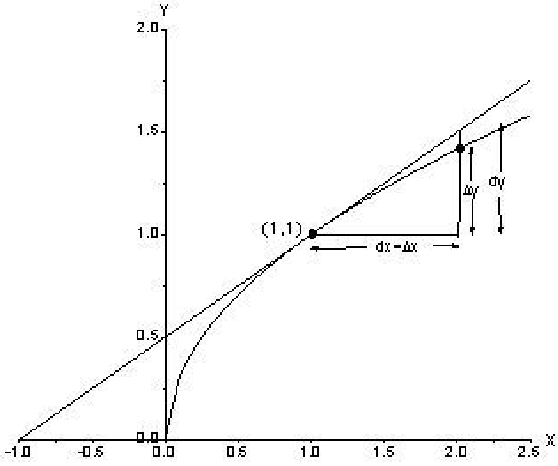


Fig.1

Chapter 2 Derivatives Exercise 2.9 21E

Consider the function $y = f(x) = \frac{2}{x}$.

And $x = 4, \quad dx = \Delta x = 1$

The objective is to find Δy and dy for the given values and sketch a diagram.

Differentiate the function $f(x)$ with respect to *x* to get,

$$f'(x) = -\frac{2}{x^2}$$

Recollect that, the change in *y* with respect to corresponding change in *x* is given by,

$$\Delta y = f(x+\Delta x) - f(x)$$

And,

$$dy = f'(x)dx$$

Find $f(x+\Delta x)$, at $x = 4$ and $dx = \Delta x = 1$.

$$\begin{aligned} f(x+\Delta x) &= f(4+1) \\ &= f(5) \\ &= \frac{2}{5} \end{aligned}$$

And,

$$\begin{aligned} f(x) &= f(4) \\ &= \frac{2}{4} \\ &= \frac{1}{2} \end{aligned}$$

Therefore, the value of Δy is,

$$\begin{aligned} \Delta y &= f(x+\Delta x) - f(x) \\ &= f(5) - f(4) \\ &= \frac{2}{5} - \frac{1}{2} \\ &= -\frac{1}{10} \end{aligned}$$

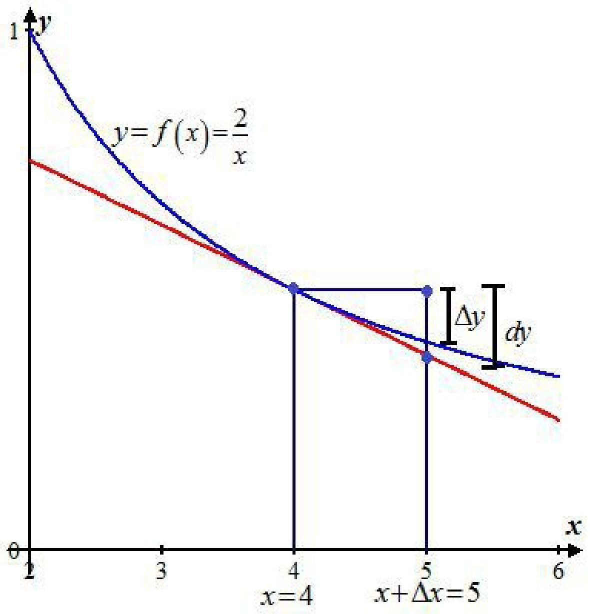
$$= \boxed{-0.1}.$$

Find the value of dy.

$$\begin{aligned} dy &= f'(x)dx \\ &= -\frac{2}{x^2}\cdot dx \\ &= -\frac{2}{4^2}\cdot 1 \\ &= -\frac{1}{8} \\ &= -0.125 \end{aligned}$$

Thus, the value of dy is $dy = \boxed{-0.125}$.

Sketch the diagram of the line segments with lengths dx, dy and Δy as follows:



Chapter 2 Derivatives Exercise 2.9 22E

Consider the function,

$y = x^3$.

Compute the values Δy and dy for the given values of $x = 1$ and $dx = \Delta x = 0.5$.

The corresponding change in *y* Δy is the amount that the curve $y = f(x)$ rises or falls when *x* changes by an amount dx. That is,

$\Delta y = f(x+\Delta x) - f(x)$.

So, calculate the value of $f(x+\Delta x)$:

$$\begin{aligned} f(x+\Delta x) &= f(1+0.5) \\ &= f(1.5) \\ &= (1.5)^3 \\ &= 3.375 \end{aligned}$$

And, calculate the value of $f(x)$:

$$\begin{aligned} f(x) &= f(1) \\ &= (1)^3 \\ &= 1 \end{aligned}$$

Thus, the corresponding change in *y* is

$$\begin{aligned} \Delta y &= f(x+\Delta x) - f(x) \\ &= 3.375 - 1 \\ &= 2.375 \end{aligned}$$

Therefore, the corresponding change in *y* is $\boxed{\Delta y = 2.375}$.

The **differential** dy is then defined in terms of dx by the equation

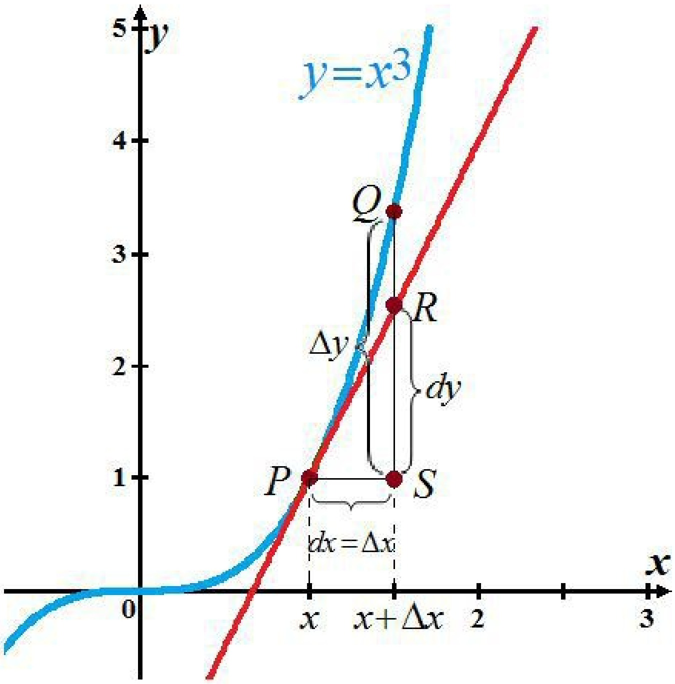
$$dy = f'(x)dx$$
$$= (x^3)' dx \qquad \text{Since, } f(x) = x^3$$
$$= 3x^2 dx$$

When $x = 1$ and $dx = \Delta x = 0.5$, the **differential** dy becomes

$$dy = 3(1)^2 \cdot (0.5)$$
$$= 3 \cdot (0.5)$$
$$= 1.5$$

Therefore, the value for the **differential** dy is $\boxed{dy = 1.5}$.

Sketch a diagram showing the line segments with lengths dx, dy, and Δy as follows:



Chapter 2 Derivatives Exercise 2.9 23E

Let $f(x) = x^4$

Then $f'(x) = 4x^3$

The linearization of f at $x = 2$ is

$$L(x) = f(2) + f'(2)(x-2)$$
$$= 16 + 32(x-2)$$
$$= 16 + 32x - 64$$
$$= 32x - 48$$
$$\boxed{\therefore\ L(x) = 32x - 48}$$

Suppose $x = 1.999$

Then $(1.999)^4 \approx 32(1.999) - 48$

$$= 15.968$$

$$\boxed{\therefore\ (1.999)^4 \approx 15.968}$$

Chapter 2 Derivatives Exercise 2.9 24E

Consider the number $\sin 1°$.

The objective is to estimate the above number using a linear approximation.

Note that, linear approximation (or differentials) can be written as,

$$f(a+\Delta x) = f(a) + f'(a)\Delta x.$$

Let $f(x)=\sin x$, then $f'(x)=\cos x$.

Take $a=0, \Delta x=1°$

Use the linear approximation to get,

$$f(a+\Delta x)=f(a)+f'(a)\Delta x$$
$$f(0+1)=f(0)+f'(0)\Delta x$$
$$f(1)=\sin 0°+(\cos 0°)\cdot(1°)$$

$$\sin 1°=0+1\cdot\left(\frac{\pi}{180}\text{radians}\right)$$
$$\approx 0+0.01745$$
$$=0.01745$$

Thus by the linear approximation (or differentials), $\sin 1°=\boxed{0.01745}$.

Chapter 2 Derivatives Exercise 2.9 25E

Let $f(x)=\sqrt[3]{x}=x^{\frac{1}{3}}$

Then $f'(x)=\frac{1}{3}x^{\frac{-2}{3}}$

The linearization of f at $x=1000$ is

$$L(x)=f(1000)+f'(1000)(x-1000)$$
$$=10+\frac{1}{300}(x-1000)$$
$$=10+\frac{x}{300}-\frac{10}{3}$$
$$=\frac{x}{300}+\frac{20}{3}$$
$$\boxed{\therefore L(x)=\frac{x}{300}+\frac{20}{3}}$$

Suppose $x=1001$

Then $\sqrt[3]{1001}\approx\frac{1001}{300}+\frac{20}{3}$
$$\approx 10.003333....$$
$$=10.00\bar{3}$$
$$\boxed{\therefore \sqrt[3]{1001}\approx 10.00\bar{3}}$$

Chapter 2 Derivatives Exercise 2.9 26E

Let $f(x)=\frac{1}{x}$

Then $f'(x)=\frac{-1}{x^2}$

The linearization of f at $x=4$ is

$$L(x)=f(4)+f'(4)(x-4)$$
$$=\frac{1}{4}-\frac{1}{16}(x-4)$$
$$=\frac{1}{4}-\frac{x}{16}+\frac{4}{16}$$
$$=\frac{-x}{16}+\frac{1}{2}$$
$$\boxed{\therefore L(x)=\frac{-x}{16}+\frac{1}{2}}$$

Then $\frac{1}{4.002}\approx-\frac{4.002}{16}+\frac{1}{2}$
$$=0.249875$$
$$\approx 0.249875$$
$$\boxed{\therefore \frac{1}{4.002}\approx 0.249875}$$

Chapter 2 Derivatives Exercise 2.9 27E

We have $1° = \frac{\pi}{180}$ radians

Then $\tan 44° = \tan\left(44.\frac{\pi}{180}\right)$ (radians)

Then

$$= \tan\frac{\pi}{180}(45-1)$$
$$= \tan\left(\frac{\pi}{4}-\frac{\pi}{180}\right)$$

Now we can consider the function $f(x) = \tan\left(\frac{\pi}{4}-x\right)$

Then $f'(x) = -\sec^2\left(\frac{\pi}{4}-x\right)$

For x = 0, we have

$f(0) = \tan\left(\frac{\pi}{4}-0\right) = 1$

And $f'(0) = -\sec^2\left(\frac{\pi}{4}-0\right) = -2$

Now the linear approximation is

$L(x) = f(0) + f'(0)(x-0)$

$\Rightarrow L(x) = 1-2x$

Thus corresponding approximation is $\tan\left(\frac{\pi}{4}-x\right) \approx 1-2x$

We have $\tan 44° = \tan\left(\frac{\pi}{4}-\frac{\pi}{180}\right)$

$$\approx 1-2.\frac{\pi}{180}$$
$$\approx 1-\frac{\pi}{90}$$
$$\approx 0.965$$

Thus $\boxed{\tan 44° \approx 0.965}$

Chapter 2 Derivatives Exercise 2.9 28E

We have $\sqrt{99.8} = \sqrt{100-x}$

Here $x = 0.2$

We can consider a function $f(x) = \sqrt{(100-x)}$

Then $f'(x) = \frac{-1}{2\sqrt{100-x}}$

For $x = 0$, we have

$$f(0) = \sqrt{100} = 10$$

And $f'(0) = \frac{-1}{2\sqrt{100-0}} = \frac{-1}{20} = -0.05$

Linear approximation is

$L(x) = f(0) + f'(0)(x-0)$

$L(x) = 10 + (-0.05).(x)$

$L(x) = 10-(0.05)x$

Thus the corresponding linear approximation is as follows:

$\boxed{\sqrt{100-x} \approx 10-(0.05)x}$

Then we have

$\sqrt{99.8} = \sqrt{100-0.2}$

$\approx 10-(0.05)(0.2)$

$= 10-0.01$

$\Rightarrow \boxed{\sqrt{99.8} \approx 9.99}$

Chapter 2 Derivatives Exercise 2.9 29E

W have $\sec 0.08 = \sec(0+0.08)$

We can consider a function $f(x)=\sec(x)$

Then $f'(x)=\sec(x).\tan(x)$

For $x=0$

$f(0)=\sec 0 = 1$

And $f'(0)=\sec 0.\tan 0 = 0$

The linear approximation is

$L(x)=f(0)+f'(0).(x-0)$

$\Rightarrow L(x)=1+0=1$

Thus corresponding approximation (at x =0) is

$$\boxed{\sec(x)\approx 1}$$

We have,

$$\sec(0.08)\approx f(0)+f'(0)(0.08-0)$$

$$\sec(0.08)\approx 1+0.(0.08)$$

$$\sec(0.08)\approx 1$$

Thus $\boxed{\sec(0.08)\approx 1}$

So this approximation is reasonable.

Chapter 2 Derivatives Exercise 2.9 30E

We have $(1.01)^6=(1+0.01)^6$

So we can consider a function

$$f(x)=(1+x)^6$$

Then $f'(x)=6(1+x)^5$

For $x=0$

$$f(0)=(1+0)^6=1$$

And $f'(0)=6(1+0)^5=6$

Linear approximation is

$L(x)=f(0)+f'(0)(x-0)$

$L(x)=1+6x$

Then corresponding approximation is $(1+x)^6\approx 1+6x$

$(1.01)^6=(1+0.01)^6$

$\approx 1+6.(0.01)$

$\approx 1+0.06$

$\Rightarrow \boxed{(1.01)^6\approx 1.06}$

Thus this approximation is reasonable

Chapter 2 Derivatives Exercise 2.9 31E

(A)

Let the edge of a cube is a ,

then we have a = 30cm and da = 0.1cm.

The volume of the cube is $V=a^3$

$$\Rightarrow V=a^3$$

The corresponding error in the calculated value of V is ΔV

We can approximate ΔV by the differential

$$dV = 3a^2.da$$

When $a = 30$ cm and $da = 0.1$ cm

Then $dV = 3(30)^2.(0.1)$

$$= 3\times 900\times 0.1$$

$$\Rightarrow dV = 270.0 \quad cm^3$$

The maximum possible error in the volume of the cube is about $\boxed{270.0 \quad cm^3}$

Relative error $= \frac{\Delta V}{V} \approx \frac{dV}{V} = \frac{3a^2.da}{a^3}$

$$= 3.\frac{da}{a}$$

Means relative error in volume is three times the relative error in the edge,

relative error in edge is about $\frac{da}{a} = \frac{0.1}{30}$

Then relative error in volume $\approx 3.\left(\frac{0.1}{30}\right)$

Relative error $\boxed{\approx 0.01}$

And the percentage error in volume $\approx$ relative error $\times 100\%$

$$\approx 0.01\times 100\%$$

Percentage error $\boxed{= 1\%}$

Chapter 2 Derivatives Exercise 2.9 32E

(A)

The radius of the circle is r = 24cm

The maximum possible error in radius dr =0.2cm

The area of the disk is given by

$$A = \pi r^2$$

Then differential $dA = 2\pi r.dr$

Then maximum possible error in area of the disk

$$dA = 2\pi(24).(0.2)\text{cm}^2$$

The maximum possible error in area is about $\boxed{9.6\pi \quad \text{cm}^2}$ or about $\boxed{30.16 \quad \text{cm}^2}$

(B) Relative error in the area $= \frac{\Delta A}{A} \approx \frac{dA}{A}$

$$= \frac{2\pi r.dr}{\pi r^2} = 2.\frac{dr}{r}$$

Means relative error in the area is two times the relative error in the radius then

relative error in the area $= 2.\frac{(0.2)}{24}$

$\Rightarrow$Relative error $\boxed{\approx 0.016}$

And the percentage error in area

$$= \text{Relative error} \times 100\%$$

$$= 0.016\times 100\%$$

$\Rightarrow$ Percentage error $\boxed{= 1.6\%}$

Chapter 2 Derivatives Exercise 2.9 33E

(A) Given $C = 2\pi r = 84$ cm

Where r is the radius of the sphere and C is the circumference of the sphere. Then

$$C = 84 \quad \text{cm}$$

$$\Rightarrow 2\pi r = 84 \quad \text{cm}$$

$$\Rightarrow \boxed{r = \frac{42}{\pi}} \quad \text{cm}$$

Now $C = 2\pi r$

$$\Rightarrow dc = 2\pi dr = 0.5 \text{ (Given)}$$

$$\Rightarrow \boxed{dr = \frac{0.5}{2\pi}}$$

The surface area of the sphere is $S = 4\pi r^2$

Then maximum error in the calculated surface area is about

$$dS = 8\pi r dr \quad \text{cm}^2$$

When $r = \dfrac{42}{\pi}$ and $dr = \dfrac{0.5}{2\pi}\text{cm}^2$

Then maximum possible error $\Rightarrow dS = 8\pi . \dfrac{42}{\pi} . \dfrac{0.5}{2\pi}$

$$\Rightarrow \boxed{dS = \frac{84}{\pi}} \approx 27 \ \text{cm}^2$$

The relative error in the surface area $= \dfrac{\Delta S}{S}$

$$\approx \frac{dS}{S} = \frac{8\pi r dr}{4\pi r^2}$$

$$= 2\frac{dr}{r}$$

When $r = \dfrac{42}{\pi}$ and $dr = \dfrac{0.5}{2\pi}$

Then relative error in the surface area

$$= 2.\frac{0.5/2\pi}{42/\pi}$$

$$\boxed{= \frac{1}{84} \approx 0.0119} \approx 0.012$$

(B) The volume of the sphere is $V = \dfrac{4}{3}\pi r^3$

The maximum possible error in the volume is

$\Rightarrow dV = 4\pi r^2 . dr$

When $r = \dfrac{42}{\pi}$ and $dr = \dfrac{0.5}{2\pi}$,

the maximum possible error in the volume is

$$dV = 4\pi . \left(\frac{42}{\pi}\right)^2 . \left(\frac{0.5}{2\pi}\right)$$

$$= 4\pi . \frac{(42)^2}{\pi^2} . \frac{0.5}{2\pi}$$

$$\Rightarrow \boxed{dV = \frac{1764}{\pi^2}} \approx 179 \ \text{cm}^3$$

The relative error in the volume is about $= \dfrac{\Delta V}{V}$

$$\approx \frac{\Delta V}{V} = \frac{4\pi r^2 . dr}{\frac{4}{3}\pi r^3}$$

$$= 3.\frac{dr}{r}$$

When $r = \dfrac{42}{\pi}$ and $dr = \dfrac{0.5}{2\pi}$

Then relative error in the volume

$$= 3.\frac{0.5/2\pi}{42/\pi}$$

$$= \frac{1.5}{84}$$

Relative error $\Rightarrow \boxed{\approx 0.0179} \approx 0.018$

Chapter 2 Derivatives Exercise 2.9 34E

Consider a hemi sphere of diameter 50 m.

So, the radius of the hemi sphere is 25 m.

The objective is to estimate the amount of paint needed to apply a coat of paint 0.05 cm thick to the dome.

The amount of paint needed is the extra volume which would be necessary to increase the radius of the dome by 0.05 cm. The relationship between the volume and radius is given by

$$V(r) = \frac{2}{3}\pi r^3$$

To estimate the extra volume, we can use the differential

$$\begin{aligned} dV &= V'(r)dr \\ &= 2\pi r^2 dr \end{aligned}$$

Now, plug in the given values of r and dr, making sure to either convert all units to meters or convert all units to centimeters. The radius is half of the diameter, and dr is the small change in the radius.

$$\begin{aligned} dV &= 2\pi(25)^2(0.0005) \\ &= 0.625\pi \end{aligned}$$

So, the amount of paint needed is approximately $\boxed{0.625\pi \text{ cubic meters}}$.

Chapter 2 Derivatives Exercise 2.9 35E

(A) The height of cylindrical shell is = h

Inner radius = r and thickness $= \Delta r = dr$

The volume of the cylinder is $V = \pi r^2 h$

Then we approximate the volume by the differential

$$dV = 2\pi rh.dr \quad \text{Where h is constant}$$

Hence $\boxed{\Delta V \approx dV = 2\pi rh.\Delta r}$

(B) The thickness of the shell is Δr .this means outer radius is $r + \Delta r$

Then error is

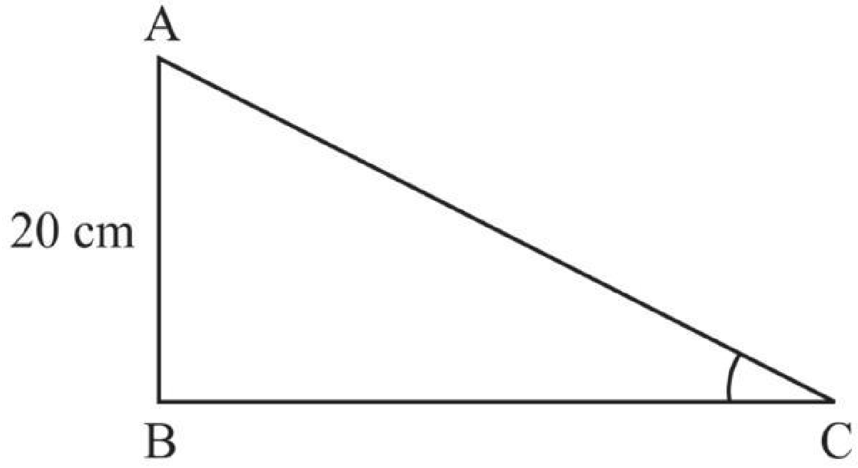
$$\begin{aligned} \Delta V - dV &= \pi\left[(r+\Delta r)^2 - r^2\right]h - 2\pi rh\Delta r \\ &= \pi\left[r^2 + (\Delta r)^2 + 2r.\Delta r - r^2\right]h - 2\pi rh\Delta r \\ &= \pi(\Delta r)^2 h + 2\pi rh.\Delta r - 2\pi rh\Delta r \\ &= \pi(\Delta r)^2 h \end{aligned}$$

So the error $\boxed{\pi(\Delta r)^2 h}$ is involved in the formula.

(a)

Consider a right triangle ABC such that,

$$\angle ABC = 90^\circ$$
$$AB = 20 \text{ cm}$$
$$\text{and, } \angle ACB = x$$



Apply trigonometric identity in right-angled triangle ABC,

$$\sin(\angle ACB) = \frac{AB}{AC}$$
$$AC = \frac{AB}{\sin(\angle ACB)}$$
$$h = \frac{AB}{\sin x}$$
$$h = 20\text{cosec}x$$

If the error in the measured value of *x* is denoted by $dx = \Delta x$, then the corresponding error in the calculated value of *h* is Δh, which can be approximated by the differential,

$$dh = -20\text{cosec}(x) \times \cot(x) \times dx$$

When $x = 30^\circ$ and $dx = +1^\circ$, then

$$\begin{aligned} dh &= -20\text{cosec}(30^\circ) \times \cot(30^\circ) \times (+1^\circ) \\ &= -20 \times 2 \times \sqrt{3} \times (+0.0174533) \\ &= -1.2092000943896\ldots \\ &\approx -1.21 \text{ cm} \end{aligned}$$

When $x = 30^\circ$ and $dx = -1^\circ$, then

$$\begin{aligned} dh &= -20\text{cosec}(30^\circ) \times \cot(30^\circ) \times (-1^\circ) \\ &= -20 \times 2 \times \sqrt{3} \times (-0.0174533) \\ &= +1.2092000943896\ldots \\ &\approx +1.21 \text{ cm} \end{aligned}$$

Therefore, the error in computing the length of the hypotenuse will be approximately,

$$\boxed{\pm 1.21 \text{ cm}}$$

(b)

The percentage error in the measurement of hypotenuse will be given by,

$$\frac{dh}{h} \times 100\%$$

Substitute $dh = -20\text{cosec}(x) \times \cot(x) \times dx$ and $h = -20\text{cosec}(x)$,

$$\begin{aligned} \frac{dh}{h} \times 100\% &= \frac{(-20\text{cosec}(x) \times \cot(x) \times dx)}{(-20\text{cosec}(x))} \times 100\% \\ &= \cot x \times dx \times 100\% \end{aligned}$$

When $x = 30°$ and $dx = +1°$, then

$$\frac{dh}{h}\times 100\% = \cot(30°)\times(+1°)\times 100\%$$
$$= \sqrt{3}\times(0.0174533)\times 100\%$$
$$= 3.0230002359\ldots\%$$
$$\approx 3.023\%$$

When $x = 30°$ and $dx = -1°$,

$$\frac{dh}{h}\times 100\% = \cot(30°)\times(-1°)\times 100\%$$
$$= \sqrt{3}\times(-0.0174533)\times 100\%$$
$$= -3.0230002359\ldots\%$$
$$\approx -3.023\%$$

Therefore, the percentage error will be approximately $\boxed{3.023\%}$.

Chapter 2 Derivatives Exercise 2.9 37E

Consider the equation for the voltage drop.

$V = RI$.

Here, *I* is the current that passes through a resistor with a resistance *R*.

Consider the voltage *V* as a constant, and the resistance *R* is measured with a certain error, by using differentials.

Show that the relative error in calculating *I* is approximately the same (in magnitude) as the relative error in *R*.

So, rewrite the equation, $V = RI$ as follows:

$$I = \frac{V}{R}$$

The **differential** dy is then defined in terms of dx by the equation.

$$dy = f'(x)dx$$

Thus, the differential dI in terms of dR is expressed as follows:

$dI = -\frac{V}{R^2}dR$ Since, *V* is a constant, and $\frac{d}{dx}(x^n) = nx^{n+1}$

The relative error in calculating *I* is defined as follows:

$$\frac{\Delta I}{I} \approx \frac{dI}{I}.$$

Substitute the values $dI = -\frac{V}{R^2}dR$ and $I = \frac{V}{R}$ in the above formula, to calculate the relative error in *I*:

$$\frac{\Delta I}{I} \approx \frac{dI}{I} = \frac{-(V/R^2)dR}{V/R}$$
$$= -\frac{dR}{R}$$

Therefore, the relative error in calculating *I* is approximately the same (in magnitude) as the relative error in *R*.

Chapter 2 Derivatives Exercise 2.9 38E

We have $V = F = kR^4$ --- (1)
Where F is flux of R is the radius of blood vessel
Now differentiate the equation (1)

$$dF = 4kR^3.dR \quad \text{--- (2)}$$

Where differential dF is maximum error in the calculated flux.
Now the relative change in flux is computed by dividing dF by F

$$\Rightarrow \frac{\Delta F}{F} \approx \frac{dF}{F} = \frac{4kR^3.dR}{kR^4}$$
$$= \frac{4.dR}{R}$$

$\frac{dR}{R}$ is relative change in radius, so we have the relative change in F is about 4 times the relative change in R

Now the change in R is 5%

Means $dR = 5\%\ of\ R$ or $\frac{dR}{R} = 0.05$

Then $\frac{dR}{R} = 4.(0.05) = 0.20$

This means F will also increase about $\boxed{20\%}$

Chapter 2 Derivatives Exercise 2.9 39E

(A) Let $C = f(x)$ or $f(x) = C$

Then $f'(x) = 0$ [C is a constant]

Then differential $dc = f'(x).dx$

$$\Rightarrow dc = 0..dx$$
$$\Rightarrow \boxed{dc = 0}$$

(B) Let $y = cu$

Then

$$\frac{dy}{dx} = c.\frac{du}{dx} \quad \text{[C is a constant]}$$
$$\Rightarrow dy = c\,du$$

Hence $\boxed{d(cu) = c.du}$

(C) Let $y = u + v$ [u and v are function of x]

Then

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{Since } \left[(f+g)' = f' + g'\right]$$
$$\Rightarrow dy = du + dv$$

Then we have $\boxed{d(u+v) = du + dv}$

(D) Let $y = uv$ where u and v are the function of x

They by using product rule

$$\frac{dy}{dx} = u.\frac{dv}{dx} + v.\frac{du}{dx}$$
$$\Rightarrow dy = udv + vdu$$
$$\Rightarrow \boxed{d(u.v) = udv + vdu}$$

(E) Let $y = \frac{u}{v}$ where u and v are the function x

Then by the Quotient rule.

$$\frac{dy}{dx} = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$$

$$\text{or } dy = \frac{vdu - u.dv}{v^2}$$

$$\Rightarrow \boxed{d\left(\frac{u}{v}\right) = \frac{vdu - u.dv}{v^2}}$$

(F) Let $f(x)=x^n$

Then $f'(x)=nx^{n-1}$

Again $x^n=f(x)$

Then by the differential rule

$$d(x^n)=f'(x).dx$$

$$\boxed{d(x^n)=nx^{n-1}.dx}$$

Chapter 2 Derivatives Exercise 2.9 40E

(A) $f(x)=\sin x$

Then $f'(x)=\cos x$

For x = 0

$$f(0)=\sin 0=0$$

And $f'(0)=\cos 0=1$

Then linear approximation at 0 is

$$L(x)=f(0)+f'(0)(x-0)$$

$$=0+1.(x)$$

$$=x$$

Hence at 0 the linear approximation is $\boxed{\sin x\approx x}$

(B) We want to know the values of x for which sinx and x differ by less than 2% = 0.02.

That is

$$\left|\frac{x-\sin x}{\sin x}\right|<0.02$$

$$\Rightarrow -0.02<\frac{x-\sin x}{\sin x}<0.02$$

$$\Rightarrow\begin{Bmatrix}-0.02\sin x<x-\sin x<0.02\sin x & \text{if } \sin x>0\\ -0.02\sin x>x-\sin x>0.02\sin x & \text{if } \sin x<0\end{Bmatrix}$$

$$\Rightarrow\begin{Bmatrix}0.98\sin x<x<1.02\sin x & \text{if } \sin x>0\\ 1.02\sin x<x<0.98\sin x & \text{if } \sin x<0\end{Bmatrix}$$

Now we sketch the graph of $y=0.98\sin x$, $y=1.02\sin x$, and $y=x$ on the same screen in figure 1.

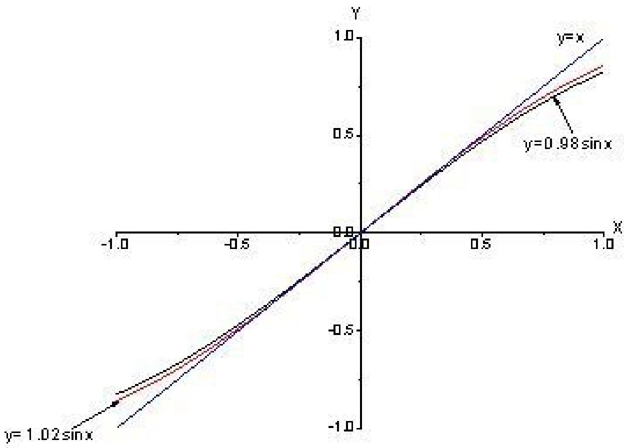


Fig.1

We see that the graphs are very close to ach other near $x = 0$. now we zoom the scale and see that $y = x$ intersects $y = 1.02\sin x$ at $x \approx 0.344$ and by symmetry at $x \approx -0.344$. (figure2)

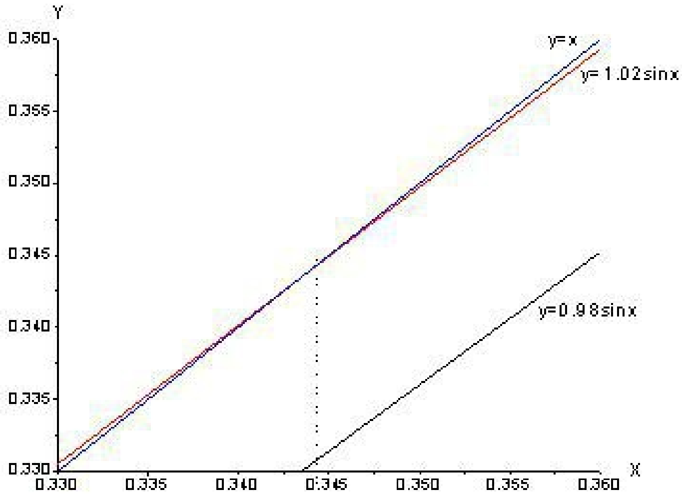


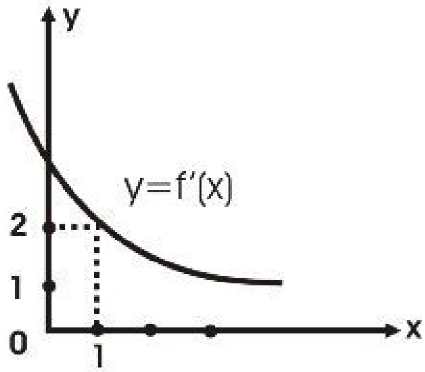
Fig.2

Converting 0.344 radians into degrees.

$$0.344\left(\frac{180°}{\pi}\right) \approx 19.7° \approx \boxed{20°}$$

This verifies the statement.

Chapter 2 Derivatives Exercise 2.9 41E



We have $f(1) = 5$ and from the graph we have $f'(1) = 2$.

Then linear approximation $L(x) = f(1) + f'(1)(x-1)$.

(A)
$$\begin{aligned} f(0.9) &= f(1-0.1) \\ &\approx 5 + 2.(0.9-1) \\ &\approx 5 + 2.(-0.1) \\ &\approx 5 + (-0.2) \end{aligned}$$

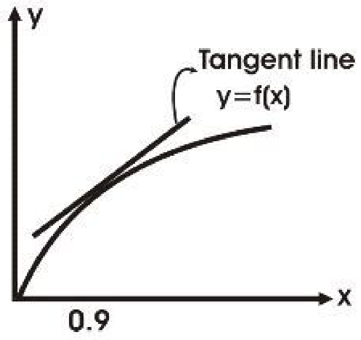
$$\boxed{f(0.9) \approx 4.8}$$

$$\begin{aligned} f(1.1) &\approx f(1) + f'(1)(1.1-1) \\ &\approx 5 + 2(0.1) \\ &\approx 5 + 0.2 \end{aligned}$$

$$\boxed{f(1.1) \approx 5.2}$$

(B)

If we draw the graph of function f, then we see that the tangent line lies above the curve, so our approximations are $\boxed{\text{too large}}$.



Chapter 2 Derivatives Exercise 2.9 42E

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Then $f'(x)=\cos x$

For $x=0$

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Then linear approximation at 0 is

$$L(x)=f(0)+f'(0)(x-0)$$

$$=0+1.(x)$$
$$=x$$

Hence, at 0 the linear approximation is $\boxed{\sin x\approx x}$.

(B) We want to know the values of x for which $\sin x$ and x differ by less than $2\%=0.02$.

That is,

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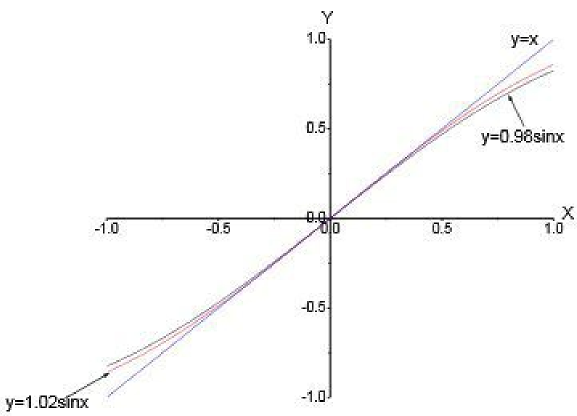

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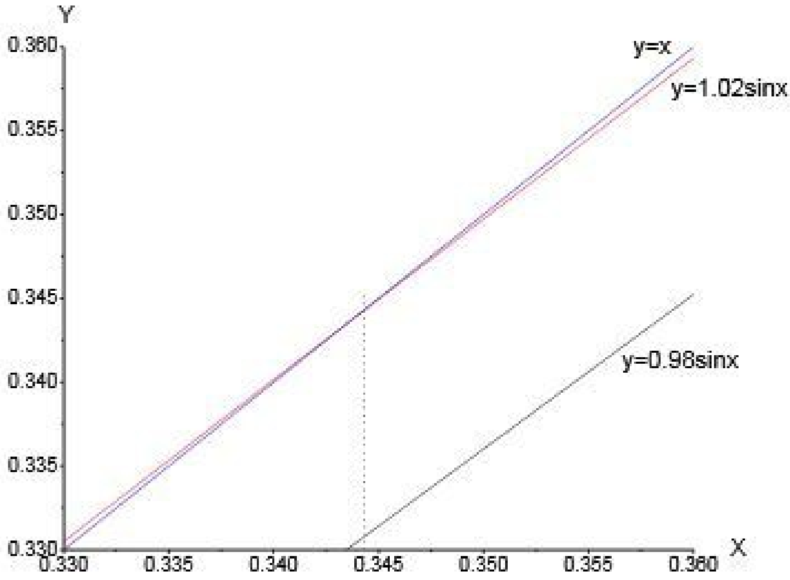

Fig.2

Converting 0.344 radians into degrees, we have

$$0.344\left(\frac{180^\circ}{\pi}\right) \approx 19.7^\circ \approx \boxed{20^\circ}$$

This verifies the statement.