Class-XII

Session - 2022-23

Subject - Mathematics (041)

Sample Question Paper - 24

With Solution

BLUE PRINT

Ch. No.	Chapter Name	Per Unit Marks	Section-A (1 Mark)		Section-B (2 Marks)	Section-C (3 Marks)	Section-D (5 Marks)	Section-E (4 Marks)	Total Marks
			MCQ	A/R	VSA	SA	LA	Case-Study	
1	Relations and Functions	8					Q.32		5
2	Inverse Trigonometry Functions			Q.20	Q.21				3
3	Matrices	10	Q.1,4						2
4	Determinants		Q.7,10,14				Q.33		8
5	Continuity and Differentiability	35	Q.2,5		Q.23	Q.27			8
6	Applications of Derivatives		Q.17		Q.22	Q.28		Q.38	9
7	Integrals		Q.6,18			Q.26,29			8
8	Applications of Integrals						Q.34		5
9	Differential Equations		Q.12,16			Q.31			5
10	Vector Algebra	14	Q.3,8,13		Q.25				5
11	Three Dimensional Geometry		Q.9	Q.19	Q.24		Q.35		9
12	Linear Programming	5	Q.15					Q.36	5
13	Probability	8	Q.11			Q.30		Q.37	8
	Total Marks (Total Questions)		18(18)	2(2)	10(5)	18(6)	20(4)	12(3)	80(38)

Note : The number given inside the bracket denotes question number, asked in the sample paper, while the number given outside the bracket are the number of questions from that particular chapter.

Time : 3 Hours **Max. Marks : 80**

General Instructions

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.*

SECTION-A (Multiple Choice Questions)

Each question carries 1 mark.

1. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(a) A (b) I – A (c) I (d) 3 A

2. Let $f(x) = \begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x}, & x < \dfrac{\pi}{2} \\ p, & x = \dfrac{\pi}{2} \\ \dfrac{q(1-\sin x)}{(\pi - 2x)^2}, & x > \dfrac{\pi}{2} \end{cases}$

If f(x) is continuous at $x = \dfrac{\pi}{2}$, (p, q) =

(a) (1,4) (b) $\left(\dfrac{1}{2}, 2\right)$ (c) $\left(\dfrac{1}{2}, 4\right)$ (d) None of these

3. If $\bar{a}$ is a non-zero vector of magnitude a and λ a non-zero scalar, then λ $\bar{a}$ is a unit vector if.

(a) $\lambda = 1$ (b) $\lambda = -1$ (c) $a = |\lambda|$ (d) $a = \dfrac{1}{|\lambda|}$

4. If $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$, then $(AB)'$ is equal to,

(a) $\begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & 4 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 4 & -3 \\ 2 & 8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$

5. The value of $\dfrac{d}{dx}\left[\tan^{-1}\left(\dfrac{a-x}{1+ax}\right)\right]$ is :

(a) $-\dfrac{1}{1+x^2}$ (b) $\dfrac{1}{1+a^2} - \dfrac{1}{1+x^2}$ (c) $\dfrac{1}{1+x^2}$ (d) None of these

6. The integral $\int_0^{\pi/2} |\sin x - \cos x|\, dx$ is equal to :

(a) $2\sqrt{2}$ (b) $2(\sqrt{2}-1)$ (c) $\sqrt{2}+1$ (d) None of these

7. The equations $2x + 3y + 4 = 0$; $3x + 4y + 6 = 0$ and $4x + 5y + 8 = 0$ are

(a) consistent with unique solution (b) inconsistent

(c) consistent with infinitely many solutions (d) None of the above

8. If the vectors $a\hat{i}+2\hat{j}+3\hat{k}$ and $-\hat{i}+5\hat{j}+a\hat{k}$ are perpendicular to each other then a is equal to:
(a) 5 (b) -6 (c) -5 (d) 6

9. The shortest distance between the lines $x = y + 2 = 6z - 6$ and $x + 1 = 2y = -12z$ is
(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\frac{3}{2}$

10. If $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$, then the value of $|\text{adj}(\text{adj } A)|$ is
(a) 14 (b) 16 (c) 15 (d) 12

11. Three persons P, Q and R independently try to hit a target. If the probabilities of their hitting the target are $\frac{3}{4}, \frac{1}{2}$ and $\frac{5}{8}$ respectively, then the probability that the target is hit by P or Q but not by R is :
(a) $\frac{21}{64}$ (b) $\frac{9}{64}$ (c) $\frac{15}{64}$ (d) $\frac{39}{64}$

12. The order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/3}+x^{1/4}=0$ is
(a) order = 3, degree = 2 (b) order = 2, degree = 3 (c) order = 2, degree = 2 (d) order = 3, degree = 3

13. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{a}.\vec{b}=1$ and $\vec{a}\times\vec{b}=\hat{j}-\hat{k}$, then $\vec{b}$ is
(a) $\hat{i}-\hat{j}+\hat{k}$ (b) $2\hat{j}-\hat{k}$ (c) $2\hat{i}$ (d) $\hat{i}$

14. If the area of a triangle ABC, with vertices A(1, 3), B(0, 0) and C(k, 0) is 3 sq. units, then the value of k is
(a) 2 (b) 3 (c) 4 (d) 5

15. The maximum vale of $P = x + 3y$ such that $2x + y \le 20, x + 2y \le 20$ $x \ge 0, y \ge 0$ is
(a) 10 (b) 60 (c) 30 (d) None

16. If the I.F. of the differential equation $\frac{dy}{dx}+5y=\cos x$ is $\int e^{Adx}$, then A =
(a) 0 (b) 1 (c) 3 (d) 5

17. The radius of a sphere initially at zero increases at the rate of 5 cm/sec. Then its volume after 1 sec is increasing at the rate of :
(a) 50π (b) 5π (c) 500π (d) None of these

18. $\int \frac{x^2 \tan^{-1} x^3}{1+x^6} dx$ is equal to
(a) $\tan^{-1}x^3 + C$ (b) $\frac{1}{6}(\tan^{-1}x^3)^2 + C$ (c) $-\frac{1}{2}(\tan^{-1}x^3)^2 + C$ (d) $\frac{1}{2}(\tan^{-1}x^2)^3 + C$

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) ***Both A and R are true and R is the correct explanation of A.***

(b) ***Both A and R are true but R is not the correct explanation of A.***

(c) ***A is true but R is false.***

(d) ***A is false but R is true.***

19. Assertion: Lines $\frac{x+2}{-2}=\frac{y-1}{3}=\frac{z-2}{1}$ and $\frac{x-3}{-3}=\frac{y}{-2}=\frac{z+1}{2}$ are perpendicular.

Reason: Two lines $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ are perpendicular if $l_1l_2+m_1m_2+n_1n_2=0$

20. Assertion: The value of $\sin\left[\tan^{-1}(-\sqrt{3})+\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ is 1.

Reason: $\tan^{-1}(-x)=\tan x$ and $\cos^{-1}(-x)=\cos^{-1}x$.

SECTION-B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Find the principal value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.

OR

Find the principal value of $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$.

22. Find points at which marginal cost $c(x)=x^3-3x^2-9x+7$ is zero.

23. Show that cot x is a continuous function in its domain.

OR

Examine the continuity of f, where f is defined by $f(x)=\begin{cases}\sin x-\cos x, & \text{if } x\neq 0\\ -1, & \text{if } x=0\end{cases}$

24. The coordinates of two points A and B are respectively (–2, 2, 3) and (13, –3, 13). A point P moves in the space such that 3PA = 2PB. find the locus of P.

25. If $(\vec{a}\times\vec{b})^2+(\vec{a}.\vec{b})^2=676$ and $|\vec{b}|=2$, then find $|\vec{a}|$.

SECTION-C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Evaluate : $\int\frac{1}{\cos(x-a)\cos(x-b)}dx$.

27. If $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$, show that $y^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0$.

28. A window is in the form of rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimension of the window to admit maximum sunlight through the whole opening.

29. Evaluate : $\int x\cos^{-1}x\,dx$

OR

$\int_0^{\pi}\log(1+\cos x)\,dx$

30. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15, respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

OR

Events A and B are such that $P(A)=\frac{1}{2}, P(B)=\frac{7}{12}$ and P (not A or not B) = $\frac{1}{4}$. State whether A and B are independent.

31. Solve the following differential equation: $ye^{x/y}dx=(xe^{x/y}+y)dy$.

OR

Solve the following differential equation: $(1+y+x^2y)dx+(x+x^3)dy=0$, where $y=0$ when $x=1$.

SECTION-D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Let $A = R - \{3\}$ and $B = R - \{1\}$. Let $f: A \to B$ defined as $f(x) = \frac{x-2}{x-3}\ \forall x \in A$. Then show that f is bijective.

33. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find $\mathbf{A^{-1}}$ and hence prove that $A^2 - 4A - 5I = 0$

OR

Given that $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$. Find AB, use this to solve the following system of equations:
$x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7.$

34. Find area of the region $\{x, y\}; x^2 + y^2 \le 4, x + y \ge 2\}$.

35. Find the foot of the perpendicular drawn from the point $2\hat{i} - \hat{j} + 5\hat{k}$ to the line

$\vec{r} = \left(11\hat{i} - 2\hat{j} - 8\hat{k}\right) + \lambda\left(10\hat{i} - 4\hat{j} - 11\hat{k}\right)$. Also, find the length of the perpendicular.

OR

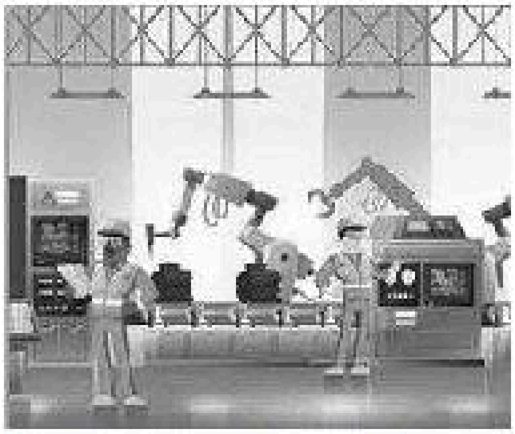
Find the length intercepted by a line with direction ratios 2, 7, –5 between the lines

$\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1}$ and $\frac{x+3}{-3} = \frac{y-3}{2} = \frac{z-6}{4}$

SECTION-E

This section comprises of 3 case study/passage - based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. Case - Study 1: Read the following passage and answer the questions given below.
Students of class-XII are given a practical problem to convert into mathematical problem and then to find the solution.
Manufacturing problem : A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of ₹8000 on each piece of model A and ₹12000 on each piece of model B.



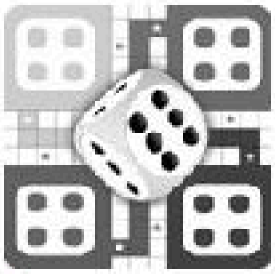
Let x be the number of pieces of model A and y be the piece of model B.
(i) Find the objective function.
(ii) What is the fabricating and finishing constraints?
(iii) How many pieces of model A and model B should be manufactured per week to realise maximum profit?

OR

What is the maximum profit per week?

37. Case - Study 2: Read the following passage and answer the questions given below.

Deepa and Radhika playing ludo. Deepa is known to speak truth 3 out of 4 times and Radhika is known to speak truth 2 out of 3 times. Both throws a die one by one and report the number occurs.



(i) When Deepa reports that it is a six, then find the probability of getting six and speak truth.

(ii) When Deepa reports that it is a six, then find the probability that it is actually a six.

(iii) When Radhika reports that it is even prime number, then find the probability of getting even prime number and speak truth.

OR

When Radhika report that is a six, then find the probability that it is actually a six.

38. Case - Study 3: Read the following passage and answer the questions given below.

Pollution in the Delhi city increases with the increase in number of vehicles. If the amount of pollution content in air in city due to x vehicles is given by

$P(x) = 0.003x^2 - 0.006x + 100$

(i) Find the marginal increase in pollution content when 10 vehicles are increases.

(ii) Find the number of vehicles when pollution level is minimum.

Solutions

SAMPLE PAPER-4

1. **(c)** We have, $A^2 = A$ ...(i)

Now, $(I + A)^3 - 7A = I^3 + A^3 + 3A^2I + 3AI^2 - 7A$

$= I + A^2A + 3A^2I + 3AI - 7A$

$= I + AA + 3A + 3A - 7A$ {using (i)}

$= I + A^2 - A = I + A - A$ {using (i)}

2. **(c)** $f[(\pi/2)^-] = \lim_{h\to 0} \frac{1-\sin^3[(\pi/2)-h]}{3\cos^2[(\pi/2)-h]} = \lim_{h\to 0} \frac{1-\cos^3 h}{3\sin^2 h} = \frac{1}{2}$

$f[(\pi/2)^+] = \lim_{h\to 0} \frac{q[1-\sin\{(\pi/2)+h\}]}{[\pi-2\{(\pi/2)+h\}]^2} = \lim_{h\to 0} \frac{q(1-\cosh)}{4h^2} = \frac{q}{8}$

$\therefore p = \frac{1}{2} = \frac{q}{8} \Rightarrow p = \frac{1}{2}, q = 4.$

3. **(d)** $\vec{a}$ is a non-zero, vector of magnitude a $\Rightarrow |\vec{a}| = a$

Now, $\lambda\, \vec{a}$ is a unit vector if $|\lambda\, \vec{a}| = 1$ or $|\lambda||\vec{a}| = 1$

or $|\lambda| a = 1 \Rightarrow a = \frac{1}{|\lambda|}$

4. **(a)** $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$

$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}\begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$

$\therefore (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$

5. **(a)** $\frac{d}{dx}\left[\tan^{-1}\left(\frac{a-x}{1+ax}\right)\right]$

$= \frac{d}{dx}(\tan^{-1} a - \tan^{-1} x) = 0 - \frac{1}{1+x^2} = -\frac{1}{1+x^2}$

6. **(b)** We have $\cos x \ge \sin x$ for $0 \le x \le \frac{\pi}{4}$

and $\sin x \ge \cos x$ for $\frac{\pi}{4} \le x \le \frac{\pi}{2}$

$\therefore \int_0^{\frac{\pi}{2}} |\sin x - \cos x|\, dx$

$= \int_0^{\frac{\pi}{4}} (\cos x - \sin x)dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x)dx$

$= [\sin x + \cos x]_0^{\frac{\pi}{4}} + [-\cos x - \sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 0 - 1\right] - \left[0 + 1 - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right]$

$= \sqrt{2} - 1 - 1 + \sqrt{2} = 2\sqrt{2} - 2$

7. **(a)** Consider first two equations :

$2x + 3y = -4$ and $3x + 4y = -6$

We have $\Delta = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = -1 \ne 0$

$\Delta_x = \begin{vmatrix} -4 & 3 \\ -6 & 4 \end{vmatrix} = 2$ and $\Delta_y = \begin{vmatrix} 2 & -4 \\ 3 & -6 \end{vmatrix} = 0$

$\therefore x = -2$ and $y = 0$

Now this solution satisfies all the equations, so the equations are consistent with unique solution.

8. **(c)** Let $\vec{X} = a\hat{i} + 2\hat{j} + 3\hat{k}$ and

$\vec{Y} = -\hat{i} + 5\hat{j} + a\hat{k}$

Note: If $\vec{X}$ and $\vec{Y}$ are perpendicular to each other, then

$\vec{X}.\vec{Y} = 0$

$\Rightarrow -a + 10 + 3a = 0 \Rightarrow 2a + 10 = 0$ Thus, $a = -5$.

9. **(b)** The lines are $\frac{x}{6} = \frac{y+2}{6} = \frac{z-1}{1}$

and $\frac{x+1}{12} = \frac{y}{6} = \frac{z}{-1}$

Here, $\vec{a}_1 = -2\hat{j} + \hat{k}$, $b_1 + 6\hat{i} + 6\hat{j} + \hat{k}$, $\vec{a}_2 = -\hat{i}$,

$\vec{b}_2 = 12\hat{i} + 6\hat{j} - \hat{k}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 6 & 6 & 1 \\ 12 & 6 & -1 \end{vmatrix} = -12\hat{i} + 18\hat{j} - 36\hat{k}$

Shortest distance $= \frac{|(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 - \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$

$= \frac{|(-\hat{i} + 2\hat{j} - \hat{k}).(-12i + 18\hat{j} - 36\hat{k})|}{\sqrt{(-12)^2 + (18)^2 + (-36)^2}}$

$= \frac{|+12 + 36 + 36|}{\sqrt{1764}} = \frac{84}{42} = 2$

10. **(b)** $|A| = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{vmatrix} = 2$

$\therefore |\text{adj}(\text{adj } A)| = |A|^{(n-1)^2} = |A|^{2^2}$ [$\therefore$ Here n = 3]

$= 2^4 = 16$

11. **(a)** Required probability $= P(P)P(\overline{Q})P(\overline{R}) + P(\overline{P})P(Q)P(\overline{R}) + P(P)P(Q)P(\overline{R})$

$= \left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right) + \left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right)$

$= \frac{12+9}{64} = \frac{21}{64}$

12. **(b)** Clearly order of the differential equation is 2.

Again $\frac{d^2y}{dx^2} + x^{1/4} = -\left(\frac{dy}{dx}\right)^{1/3} \Rightarrow \left(\frac{d^2y}{dx^2} + x^{1/4}\right)^3 = -\frac{dy}{dx}$

which shows that degree of the differential equation is 3.

13. **(d)** Given $\bar{a} = \hat{i} + \hat{j} + \hat{k}, \bar{a}.\bar{b} = 1$

only $\bar{b} = \hat{i}$ satisfies that $\bar{a}.\bar{b} = 1$

14. **(a)** Area of ΔABC = 3 sq. units

$\Rightarrow \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 3 \Rightarrow \begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 6$

$\Rightarrow 1(0-0) - 3(0-k) + 1(0-0) = \pm 6$

$\Rightarrow 3k = \pm 6 \Rightarrow k = \pm 2$

15. **(d)**

16. **(d)** The I.F. of the differential equation

$\frac{dy}{dx} + Py = Q$ is $e^{\int P dx}$. Here P = 5 therefore I.F. = $e^{\int 5dx}$.

Hence A = 5.

17. **(c)** Let 'r' be the radius and V be the volume of the sphere.

Given : Radius increases at the rate of 5cm/sec.

$\therefore \frac{dr}{dt} = 5\text{cm/sec}$

Now, $V = \frac{4}{3}\pi r^3 \therefore \frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt} = 4\pi r^2(5) = 20\pi r^2$

Now, after one second, r = 5

$\therefore \frac{dV}{dt}$ after 1 sec $= 20\pi(5)^2 = 500\pi$.

18. **(b)** **Hint:** Let $\tan^{-1} x^3 = t$

19. **(d)** $l_1 l_2 + m_1 m_2 + n_1 n_2 = 6 - 6 + 2 = 2 \neq 0$

20. **(c)** $\sin\left[\tan^{-1}(-\sqrt{3}) + \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right]$

$= \sin\left[-\tan^{-1}\sqrt{3} + \pi - \cos^{-1}\frac{\sqrt{3}}{2}\right]$

$= \sin\left[-\frac{\pi}{3} + \pi - \frac{\pi}{6}\right] = \sin\frac{\pi}{2} = 1$

Hence, Assertion is correct but Reason is incorrect.

21. Let $\cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \theta$ [1 Mark]

$\cos\left(2\pi + \frac{\pi}{6}\right) = \cos\theta$ [$\because 0 \le \cos^{-1} x \le \pi$]

$\cos\frac{\pi}{6} = \cos\theta \Rightarrow \theta = \frac{\pi}{6}$ [1 Mark]

OR

Let $\theta = \sin^{-1}\left[\sin\frac{5\pi}{3}\right]$

$\Rightarrow \sin\theta = \sin\frac{5\pi}{3} = \sin\left[2\pi - \frac{\pi}{3}\right]$ [1 Mark]

$\Rightarrow \sin\theta = -\sin\frac{\pi}{3} = \sin\left(\frac{-\pi}{3}\right)$ ($\because \sin(-\theta) = -\sin\theta$)

$\theta = \frac{-\pi}{3}$ [1 Mark]

22. Differentiating w.r.t. x; $c'(x) = 3(x-3)(x+1)$ [1 Mark]

or $c'(x) = 0 \Rightarrow 3(x+3)(x+1) = 0 \Rightarrow x = -1, 3$ [½ Mark]

when x = – 1, y = 12 & when x = 3, y = – 20

Hence the points (–1, –12), (3 – 20). [½ Mark]

23. Let $f(x) = \cot x, x \neq n\pi, n \in Z$

Then $f(x) = \frac{\cos x}{\sin x}$ [½ Mark]

Since sin x and cos x are continuous functions, therefore $\frac{\cos x}{\sin x}$ i.e., cot x will be a continuous function at all points where $\sin x \neq 0$ i.e., at all points where $x \neq n\pi,\ n \in Z$.

[1 Mark]

Hence f(x) i.e., cot x is a continuous function in its domain. [½ Mark]

OR

L.H.L. $= \lim_{x \to 0^-} (\sin x - \cos x) = -1$ [1 Mark]

R.H.L. $= \lim_{x \to 0^+} (\sin x - \cos x) = -1$

$f(0) = -1$ [1 Mark]

$\therefore$ L.H.L. = R.H.L. = f(0)

Thus, f is continuous at x = 0

24. Let P (x, y, z) be any point on the locus then 3 PA = 2 PB

$\Rightarrow 9(PA)^2 = 4(PB)^2$

$\Rightarrow 9[(x+2)^2 + (y-2)^2 + (z-3)^2]$ [1 Mark]

$= 4[(x-13)^2 + (y+3)^2 + (z-13)^2]$

$\Rightarrow 5(x^2 + y^2 + z^2) + 140x - 60y + 50z - 1235 = 0$

$\Rightarrow x^2 + y^2 + z^2 + 28x - 12y + 10z - 247 = 0$ [1 Mark]

25. Since, $\left(\vec{a}\times\vec{b}\right)^2+\left(\vec{a}.\vec{b}\right)^2=676$

$\Rightarrow\left(|\vec{a}|.|\vec{b}|\sin\theta\hat{n}\right)^2+\left(|\vec{a}|.|\vec{b}|\cos\theta\right)^2=676$ [1 Mark]

$\Rightarrow|\vec{a}|^2.|\vec{b}|^2\left(\sin^2\theta+\cos^2\theta\right)=676$

$\Rightarrow|\vec{a}|^2(2)^2=676\Rightarrow|\vec{a}|^2=169$

$\Rightarrow|\vec{a}|=13$ [1 Mark]

26. Consider $\int\frac{1}{\cos(x-a)\cos(x-b)}dx$

$=\frac{1}{\sin(a-b)}\int\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}dx$ [1 Mark]

$=\frac{1}{\sin(a-b)}\int\frac{\sin\{(x-b)-(x-a)\}}{\cos(x-a)\cos(x-b)}dx$ [½ Mark]

$=\frac{1}{\sin(a-b)}\int\frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)}dx$

[½ Mark]

$=\frac{1}{\sin(a-b)}\int\{\tan(x-b)-\tan(x-a)\}dx$

$=\frac{1}{\sin(a-b)}\{\log|\sec(x-b)|-\log|\sec(x-a)|\}+c$

[1 Mark]

27. We have

$x=a\cos\theta+b\sin\theta$ ....(i)

$y=a\sin\theta-b\cos\theta$ ....(ii)

Squaring and adding (i) and (ii), we get

$x^2+y^2=(a\cos\theta+b\sin\theta)^2+(a\sin\theta-b\sin\theta)^2$

$=a^2\cos^2\theta+b^2\sin\theta^2+2ab\cos\theta\sin\theta+a^2\sin^2\theta$
$+b^2\cos^2\theta-2ab\cos\theta\sin\theta$

$=a^2(\cos^2\theta+\sin^2\theta)+b^2(\sin^2\theta+\cos^2\theta)$

$\Rightarrow\ x^2+y^2=a^2+b^2$ .... (iii) [1 Mark]

Differentiating both sides of (iii) w.r.t x, we get

$2x+2y\frac{dy}{dx}=0$

$\Rightarrow\ 2y\frac{dy}{dx}=-2x$

$\Rightarrow\ \frac{dy}{dx}=-\frac{x}{y}$ .... (iv) [½ Mark]

Differentiating both sides of (iv) w.r.t. x, we get

$\frac{d^2y}{dx^2}=-\left(\frac{y\times1-x\times\frac{dy}{dx}}{y^2}\right)$

$=-\left[\frac{y-x\left(-\frac{x}{y}\right)}{y^2}\right]$ [From (iv)]

$=-\frac{x^2+y^2}{y^3}$ .... (v) [1 Mark]

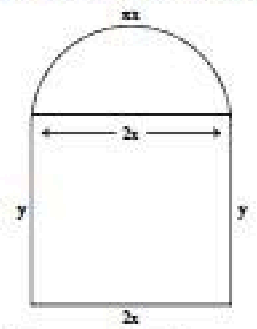
Now, $y^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y$

$=y^2\left(-\frac{x^2+y^2}{y^3}\right)-x\left(-\frac{x}{y}\right)+y$ [From (iv) and (v)]

$=-\frac{x^2+y^2}{y}+\frac{x^2}{y}+y$

$=\frac{-x^2-y^2+x^2+y^2}{y}=0$ [½ Mark]

28. Let 2x m and y m be dimensions of the window



$\because$ 10 m is perimeter of window

$\therefore\ 10=2x+2y+\pi x\Rightarrow y=\frac{10-2x-\pi x}{2}$

Let A be area of the window through which light admit.

$A=2xy+\frac{1}{2}\pi x^2$ [½ Mark]

Put the value of y, we have

$A=2x\frac{(10-2x-\pi x)}{2}+\frac{1}{2}\pi x^2$

$\Rightarrow A=10x-2x^2-\frac{1}{2}\pi x^2$

$\Rightarrow\frac{dA}{dx}=10-4x-\pi x$ [1 Mark]

Put $\frac{dA}{dx}=0\Rightarrow10-4x-\pi x=0\Rightarrow x=\frac{10}{4+\pi}$

Now, $\frac{d^2A}{dx^2}=-4-\pi=-(4+\pi)$

At $x=\frac{10}{4+\pi},\frac{d^2A}{dx^2}<0$

$\therefore$ A is maximum when $x = \frac{10}{4+\pi}$ [½ Mark]

$$y = \frac{10-2x-\pi x}{2} = \frac{10-\frac{20}{4+\pi}-\frac{10\pi}{4+\pi}}{2} = \frac{10}{4+\pi}$$

$\therefore$ Dimensions of window are $\frac{2\times10}{4+\pi}$ and $\frac{10}{4+\pi}$

i.e., $\frac{20}{4+\pi}$ and $\frac{10}{4+\pi}$ [1 Mark]

29. Let $I = \int x\cos^{-1}x\,dx = \int\left(\cos^{-1}x\right)x\,dx$

$= \left(\cos^{-1}x\right)\left(\frac{x^2}{2}\right) - \int\frac{-1}{\sqrt{1-x^2}}\left(\frac{x^2}{2}\right)dx$ [½ Mark]

$= \frac{x^2}{2}\cos^{-1}x + \frac{1}{2}I_1$, where $I_1 = \int\frac{x^2}{\sqrt{1-x^2}}dx$ [½ Mark]

Put $x = \cos\theta$, so that $dx = -\sin\theta\,d\theta$ [½ Mark]

$\therefore\ I_1 = \int\frac{\cos^2\theta(-\sin\theta)}{\sqrt{1-\cos^2\theta}}d\theta$

$= -\frac{1}{2}\int(1+\cos2\theta)d\theta = -\frac{1}{2}\left(\theta+\frac{\sin2\theta}{2}\right)+C_1$ [½ Mark]

$= -\frac{1}{2}(\theta+\cos\theta\sqrt{1-\cos^2\theta})+C_1$

$= -\frac{1}{2}(\cos^{-1}x+x\sqrt{1-x^2})+C_1$ [½ Mark]

$\therefore\ I = \frac{\cos^{-1}x}{4}(2x^2-1)-\frac{x}{4}\sqrt{1-x^2}+C$ [½ Mark]

OR

Let $I = \int_0^{\pi}\log(1+\cos x)\,dx$ ...(i)

Then, $I = \int_0^{\pi}\log[1+\cos(\pi-x)]\,dx$

$I = \int_0^{\pi}\log(1-\cos x)\,dx$ ...(ii)

Adding (i) and (ii), we get

$2I = \int_0^{\pi}\log(1-\cos^2x)\,dx = 2\int_0^{\pi}\log\sin x\,dx$

$\Rightarrow I = \int_0^{\pi}\log\sin x\,dx = 2\int_0^{\pi/2}\log\sin x\,dx = 2I_1$

[½ Mark]

Now, $I_1 = \int_0^{\pi/2}\log\sin x\,dx$ ... (iii)

$I_1 = \int_0^{\pi/2}\log\sin\left(\frac{\pi}{2}-x\right)dx = \int_0^{\pi/2}\log\cos x\,dx$

... (iv)
[½ Mark]

Adding (iii) and (iv), we get

$2I_1 = \int_0^{\pi/2}\log\left(\frac{2(\sin x\cos x)}{2}\right)dx = \int_0^{\pi/2}\log\left(\frac{\sin2x}{2}\right)dx$

$= \int_0^{\pi/2}\log\sin2x\,dx - \int_0^{\pi/2}\log2\,dx$ [½ Mark]

$= \int_0^{\pi/2}\log\sin2x\,dx - \log2[x]_0^{\pi/2}$

$= \int_0^{\pi/2}\log\sin2x\,dx - \frac{\pi}{2}\log2$ ... (v)

[½ Mark]

Let $I_2 = \int_0^{\pi/2}\log\sin2x\,dx$;

Put $2x = t$, so that $2dx = dt$

When $x = 0$, $t = 0$; when $x = \frac{\pi}{2}$, $t = \pi$ [½ Mark]

$\therefore\ I_2 = \int_0^{\pi}\log\sin t\frac{dt}{2} = \frac{1}{2}\int_0^{\pi}\log\sin t\,dt$

$= \frac{1}{2}.2\int_0^{\pi/2}\log\sin t\,dt = \int_0^{\pi/2}\log\sin x\,dx = I_1$

$\therefore$ From (v), we get; $I_1 = -\frac{\pi}{2}\log2$

$\therefore\ I = 2\times\left(-\frac{\pi}{2}\log2\right) = -\pi\log2.$ [½ Mark]

30. Let us define the events as

E_1 : insured person is a scooter driver
E_2 : insured person is a car driver
E_3 : insured person is a truck driver
A : insured person meets with an accident

To find $P\left(\frac{E_1}{A}\right)$. [½ Mark]

By Baye's theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)+P(E_3)\cdot P\left(\frac{A}{E_3}\right)} \quad ...(i)$$

[½ Mark]

Here, total number of insured vehicles
$= 2000+4000+6000 = 12000$

$P(E_1) = \frac{2000}{12000} = \frac{1}{6}$, $P(E_2) = \frac{4000}{12000} = \frac{1}{3}$

and $P(E_3) = \frac{6000}{12000} = \frac{1}{2}$ [½ Mark]

Given, $P\left(\frac{A}{E_1}\right) = 0.01$

$P\left(\frac{A}{E_2}\right) = 0.03$

$P\left(\frac{A}{E_3}\right) = 0.15$

$$\therefore P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{6}\times 0.01}{\left(\frac{1}{6}\times 0.01\right)+\left(\frac{1}{3}\times 0.03\right)+\left(\frac{1}{2}\times 0.15\right)}$$

[½ Mark]

$$= \frac{\frac{1}{6}\times\frac{1}{100}}{\left(\frac{1}{6}\times\frac{1}{100}\right)+\left(\frac{1}{3}\times\frac{3}{100}\right)+\left(\frac{1}{2}\times\frac{15}{100}\right)}$$

$$= \frac{1/6}{\frac{1}{6}+1+\frac{15}{2}} = \frac{1/6}{\frac{1+6+45}{6}} = \frac{1}{52}$$ [1 Mark]

OR

$P(\overline{A}\cup\overline{B}) = 1 - P(A\cap B) \Rightarrow \frac{1}{4} = 1 - P(A\cap B)$

$\Rightarrow \quad P(A\cap B) = 1 - \frac{1}{4} = \frac{3}{4},\ P(A) = \frac{1}{2},\ P(B) = \frac{7}{12}$

[1 Mark]

$P(A).P(B) = \frac{1}{2}\times\frac{7}{12} = \frac{7}{24}, P(A\cap B) = \frac{3}{4}$ [1 Mark]

$\Rightarrow \quad P(A\cap B) \neq P(A)\times P(B)$ [1 Mark]

Hence A and B are not independent events.

31. Consider equation $y.e^{x/y}\,dx = (xe^{x/y} + y)\,dy$

$\Rightarrow \quad \frac{dx}{dy} = \frac{xe^{x/y}+y}{ye^{x/y}} \quad \ldots(i)$ [½ Mark]

Let $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ [½ Mark]

Substituting in (i), we get

$v + y\frac{dv}{dy} = \frac{vye^v + y}{ye^v} = \frac{ve^v+1}{e^v}$

$\Rightarrow y\frac{dv}{dy} = \frac{ve^v+1}{e^v} - v = \frac{ve^v+1-ve^v}{e^v}$

[½ Mark]

$\Rightarrow \quad y\frac{dv}{dy} = \frac{1}{e^v} \Rightarrow e^v dv = \frac{dy}{y}$ [½ Mark]

Integrating both sides, we get

$\int e^v dv = \int \frac{dy}{y}$

$\Rightarrow \quad e^v = \log|y| + c$, c is constant of integration

[½ Mark]

$\Rightarrow \quad e^{x/y} = \log y + c$ is the required solution. [½ Mark]

OR

Consider equation,

$(1+y+x^2y)\,dx + (x+x^3)\,dy = 0$

$\Rightarrow \{1+y(1+x^2)\}dx = -(x+x^3)\,dy$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{1+y(1+x^2)}{x(1+x^2)} = -\frac{1}{x(1+x^2)} - \frac{y}{x}$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = -\frac{1}{x(1+x^2)}$ [½ Mark]

$\Rightarrow$ Here $P(x) = \frac{1}{x}, Q(x) = -\frac{1}{x(1+x^2)}$ [½ Mark]

$\therefore$ Integrating factor $= e^{\int\frac{1}{x}dx} = e^{\log x} = x$ [½ Mark]

$\therefore$ Solution is, $x\cdot y = \int\left\{x.\frac{-1}{x(1+x^2)}\right\}dx$

$\Rightarrow \quad x.y = -\int\frac{1}{1+x^2}dx \Rightarrow xy = -\tan^{-1}x + c \quad \ldots(i)$

[½ Mark]

Given $y = 0$, when $x = 1$

$\Rightarrow 0 = -\tan^{-1}1 + c \Rightarrow c = \frac{\pi}{4}$ [½ Mark]

Substituting in (i), we get :

$xy = -\tan^{-1}x + \frac{\pi}{4}$ is the required solution [½ Mark]

32. **One-one/Many-one :** Let $x_1, x_2 \in R - \{3\}$ are the elements such that

$f(x_1) = f(x_2)$: then $f(x_1) = f(x_2)$

$\Rightarrow \quad \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$ [1 Mark]

$\Rightarrow \quad (x_1-2)(x_2-3) = (x_2-2)(x_1-3)$

$\Rightarrow \quad x_1x_2 - 2x_2 - 3x_1 + 6 = x_2x_1 - 2x_1 - 3x_2 + 6$

$\Rightarrow \quad -2x_2 - 3x_1 = -2x_1 - 3x_2$

$\Rightarrow \quad x_2 = x_1,\ \therefore f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$\Rightarrow \quad$ f is one-one function [1 Mark]

Onto/Into : Let $y \in R - \{1\}$ (co-domain)

Then one element $x \in R - \{3\}$ in domain is such that

$f(x) = y \Rightarrow \frac{x-2}{x-3} = y \Rightarrow x - 2 = xy - 3y$

$\Rightarrow \quad x = \left(\frac{3y-2}{y-1}\right)$ [1½ Marks]

$\therefore$ The pre-image of each element of co-domain $R - \{1\}$ exists in domain $R - \{3\}$.

$\Rightarrow$ f is onto

Hence f is bijective [1½ Marks]

33. $A=\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}\Rightarrow |A|=-3+4+4=5\neq 0$

[1 Mark]

$\Rightarrow \quad A^{-1}$ exists

$A_{11}=-3 \quad A_{21}=2 \quad A_{31}=2$

$A_{12}=2 \quad A_{22}=-3 \quad A_{32}=2$

$A_{13}=2 \quad A_{23}=2 \quad A_{33}=-3$

$\text{adj}\,A=\begin{bmatrix}-3&2&2\\2&-3&2\\2&2&-3\end{bmatrix}$ [1 Mark]

$A^{-1}=\frac{1}{|A|}\text{adj}\,A=\frac{1}{5}\text{ adj }A$

$A^{-1}=\frac{1}{5}\begin{bmatrix}-3&2&2\\2&-3&2\\2&2&-3\end{bmatrix},$ [1 Mark]

$5A^{-1}=\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}-\begin{bmatrix}4&0&0\\0&4&0\\0&0&4\end{bmatrix}=A-4I$

[1 Mark]

$\Rightarrow \quad 5A^{-1}\cdot A=(A-4I)A\Rightarrow A^2-4A-5I=0.$

[1 Mark]

OR

$AB=\begin{bmatrix}1&-1&0\\2&3&4\\0&1&2\end{bmatrix}\begin{bmatrix}2&2&-4\\-4&2&-4\\2&-1&5\end{bmatrix}$

$=\begin{bmatrix}6&0&0\\0&6&0\\0&0&6\end{bmatrix}=6I$

$\Rightarrow A\frac{1}{6}B=I\Rightarrow A^{-1}=\frac{1}{6}B.$ ...(1)

Matrix form of given equations [1½ Marks]

$\begin{bmatrix}1&-1&0\\2&3&4\\0&1&2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\17\\7\end{bmatrix}$

$AX=D$

$X=A^{-1}D$ [1½ Marks]

$=\frac{1}{6}\begin{bmatrix}2&2&-4\\-4&2&-4\\2&-1&5\end{bmatrix}\begin{bmatrix}3\\17\\7\end{bmatrix}$ [From i]

$=\frac{1}{6}\begin{bmatrix}6+34-28\\-12+34-28\\6-17+35\end{bmatrix}=\frac{1}{6}\begin{bmatrix}12\\-6\\24\end{bmatrix}\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\-1\\4\end{bmatrix}$

$\therefore x=2, y=-1, z=4.$ [2 Marks]

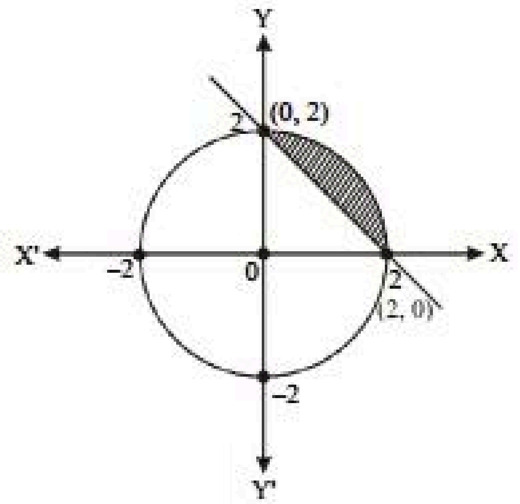
34. Let $x^2+y^2=4$ ...(i)

with centre (0, 0) and radius 2 and line whose equation is

$x+y=2$ ...(ii)

x	0	2
y	2	0

Rough sketch for the above region is given as



[2 Marks]

On solving eqns (i) and (ii), we get

$\Rightarrow \quad x=0$ or 2

Required area of region

$=\int_0^2[\text{ydx from circle})-(\text{ydx from line})]dx$

$=\int_0^2\left[\sqrt{4-x^2}-(2-x)\right]dx$ [1 Mark]

$[\because x^2+y^2=4, y^2=4-x^2, y=\sqrt{4-x^2}]$

$=\int_0^2\sqrt{4-x^2}dx-\int_0^2(2-x)dx$

$=\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2-\left[2x-\frac{x^2}{2}\right]_0^2$

$\left[\because \int\sqrt{a^2-x^2}dx=\left\{\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right\}\right]$

[1 Mark]

$=\left[\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}\right]_0^2-\left[2x-\frac{x^2}{2}\right]_0^2$

$=(2\sin^{-1}1-0)-\left[4-\frac{4}{2}\right]=2\sin^{-1}\sin\frac{\pi}{2}-2$

$=2\cdot\frac{\pi}{2}-2=\pi-2$ [1 Mark]

35. Let L be the foot of the perpendicular drawn from

$P\left(2\hat{i}-\hat{j}+5\hat{k}\right)$ on the line

$\vec{r}=11\hat{i}-2\hat{j}-8\hat{k}+\lambda\left(10\hat{i}-4\hat{j}-11\hat{k}\right).$

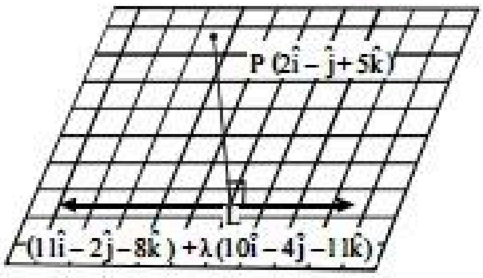
Let the position vector of L be

$=11\hat{i}-2\hat{j}-8\hat{k}+\lambda\left(10\hat{i}-4\hat{j}-11\hat{k}\right)$

$=(11+10\lambda)\hat{i}+(-2-4\lambda)\hat{j}+(-8-11\lambda)\hat{k}$ [½ Mark]

Then, $\overrightarrow{PL}$ = Position vector of L – Position vector of P

$$\Rightarrow \overrightarrow{PL} = \left[(11+10\lambda)\hat{i} + (-2-4\lambda)\hat{j} + (-8-11\lambda)\hat{k}\right] - \left[2\hat{i} - \hat{j} + 5\hat{k}\right]$$ [½ Mark]



[½ Mark]

$$\Rightarrow \overrightarrow{PL} = (9+10\lambda)\hat{i} + (-1-4\lambda)\hat{j} + (-13-11\lambda)\hat{k}$$

[½ Mark]

Since $\overrightarrow{PL}$ is perpendicular to the given line which is parallel to $\vec{b} = 10\hat{i} - 4\hat{j} - 11\hat{k}$ [½ Mark]

$\therefore \quad \overrightarrow{PL} \perp \vec{b} \Rightarrow \overrightarrow{PL}.\vec{b} = 0$

$$\Rightarrow \left[(9+10\lambda)\hat{i} + (-1-4\lambda)\hat{j} + (-13-11\lambda)\hat{k}\right].\left(10\hat{i} - 4\hat{j} - 11\hat{k}\right) = 0$$

$$\Rightarrow \quad 10(9+10\lambda) - 4(-1-4\lambda) - 11(-13-11\lambda) = 0$$

$\Rightarrow \quad 90 + 100\lambda + 4 + 16\lambda + 143 + 121\lambda = 0$

$\Rightarrow \quad 237\lambda = -237 \Rightarrow \lambda = -1$ [1 Mark]

Putting the value of λ, we obtain the position vector of L as $\hat{i} + 2\hat{j} + 3\hat{k}$ [½ Mark]

Now,

$$\overrightarrow{PL} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - \left(2\hat{i} - \hat{j} + 5\hat{k}\right) = -\hat{i} + 3\hat{j} - 2\hat{k}$$

[½ Mark]

$$\Rightarrow \left|\overrightarrow{PL}\right| = \sqrt{1+9+4} = \sqrt{14}$$

Hence, length of the perpendicular from P on the give line is $\sqrt{14}$ units. [½ Mark]

OR

The general points on the given lines are respectively $P(5+3t, 7-t, -2+t)$ and $Q(-3-3s, 3+2s, 6+4s)$.

[½ Mark]

Direction ratios of PQ are

$< -3-3s-5-3t, 3+2s-7+t, 6+4s+2-t >$ i.e.,

$< -8-3s-3t, -4+2s+t, 8+4s-t >$ [½ Mark]

If PQ is the desired line then direction ratios of PQ should be proportional to < 2, 7, –5>, therefore,

$$\frac{-8-3s-3t}{2} = \frac{-4+2s+t}{7} = \frac{8+4s-t}{-5}$$ [½ Mark]

Taking first and second numbers, we get

$-56 - 21s - 21t = -8 + 4s + 2t \Rightarrow 25s + 23t = -48$ ... (i)

[½ Mark]

Taking second and third members, we get

$20 - 10s - 5t = 56 + 28s - 7t$

$\Rightarrow 38s - 2t = -36$ ... (ii) [½ Mark]

Solving (i) and (ii) for t and s, we get

$s = -1$ and $t = -1$. [½ Mark]

The coordinates of P and Q are respectively

$(5+3(-1), 7-(-1), -2-1) = (2, 8, -3)$ and

[½ Mark]

$(-3-3(-1), 3+2(-1), 6+4(-1)) = (0, 1, 2)$ [½ Mark]

$\therefore$ The required line intersects the given lines in the points (2, 8, –3) and (0, 1, 2) respectively.

[½ Mark]

Length of the line intercepted between the given lines

$= |PQ| = \sqrt{(0-2)^2 + (1-8)^2 + (2+3)^2} = \sqrt{78}$ [½ Mark]

36. (i) $Z = 8000x + 12000y$ [1 Mark]

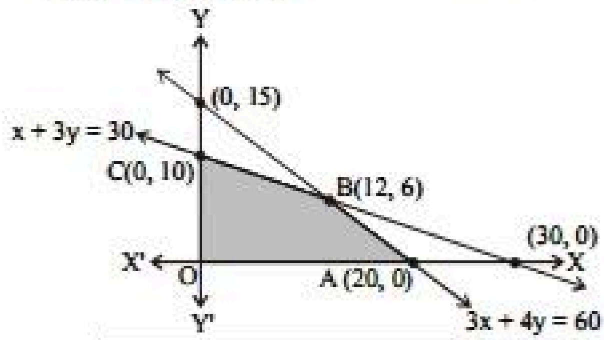
(ii) Since, each piece of model A requires 9 labour hours for fabricating and each piece of model B requires 12 labour hours for fabricating. But for fabricating available hour is 180.

$\therefore 9x + 12y \le 180$ [½ Mark]

For finishing, each piece of model A requires 1 hour and each piece of model requires 3 hours. But, maximum available hour for finishing is 30.

$\therefore x + 3y \le 30$ [½ Mark]

(iii) The value of objective function Z at each corner point of feasible reason is



Corner Point	Z = 8000x + 12000y
O(0, 0)	0
A(20, 0)	160000
B(12, 6)	168000
C(0, 10)	120000

So, Z is maximum corresponding to corner point B(x, y) = B(12, 6).

Hence, company should produce 12 pieces of model A and 6 pieces of model B to realise maximum profit. Since, Z is maximum at (12, 6). [1 Mark]

OR

$\therefore Z(12, 6) = 8000 \times 12 + 12000 \times 6$

$= 168000$

Hence, maximum profit is ₹168000. [2 Marks]

37. (i) $P(\text{getting } 6) = \frac{1}{6}$

$P(\text{Deepa speak truth}) = \frac{3}{4}$

$\therefore$ $P(\text{getting 6 and speak truth}) = \frac{1}{6} \times \frac{3}{4}$

$= \frac{1}{8}$ [1 Mark]

(ii) $P(\text{actually } 6) = \dfrac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}}$

$= \dfrac{\frac{3}{24}}{\frac{3}{24} + \frac{5}{24}} = \frac{3}{8}$ [1 Mark]

(iii) $P[\text{Even prime number } (2)] = \frac{1}{6}$

$P(\text{Radhika speak truth}) = \frac{2}{3}$ [1 Mark]

$P(\text{Even prime number and speak truth})$

$= \frac{1}{6} \times \frac{2}{3} = \frac{1}{9}$. [1 Mark]

OR

$P(\text{actually } 6) = \dfrac{\frac{5}{6} \times \frac{1}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}}$

$= \dfrac{\frac{5}{24}}{\frac{3}{24} + \frac{5}{24}} = \frac{5}{8}$ [2 Marks]

38. (i) $P'(x) = 0.006x - 0.006$ [1 Mark]

$P'(10) = 0.006(10) - 0.006$

$= 0.06 - 0.006$

$= 0.054$ [1 Mark]

(ii) $P'(x) = 0.006(x-1)$

$= 0.006\,(x-1) = 0$

$x = 1$ [1 Mark]

$P''(x) = 0.006 > 0$

$\therefore$ Pollution is minimum when $x = 1$. [1 Mark]