Sample Question Paper - 9
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

Section A

1. Find the sum of all three-digit natural numbers which are divisible by 13. **[2]**

OR

An A.P. consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the past three terms is 429. Find the A.P.

2. Solve the quadratic equation by factorization: **[2]**

$abx^2 + (b^2 - ac)x - bc = 0$.

3. Two concentric circles of radii a and b (a >b) are given. Find the length of the chord of the larger circle which touches the smaller circle. **[2]**

4. A girl empties a cylindrical bucket full of sand, of base radius 18 cm and height 32 cm, on the floor to form a conical heap of sand. If the height of this conical heap is 24 cm then find its slant height correct to one place of decimal. **[2]**

5. Compute the mode of the following data: **[2]**

Class	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency	25	16	28	20	5

6. Find the values of k for which the given equation has real and equal roots: **[2]**

$(k + 1)x^2 - 2(k - 1)x + 1 = 0$

OR

Find two consecutive positive integers, sum of whose squares is 365.

Section B

7. Data regarding the weights of students of Class X of a school is given below. **[3]**

Weight(in kg)	50 - 52	52 - 54	54 - 56	56 - 58	58 - 60	60 - 62	62 - 64

Number of students	18	21	17	28	16	35	15

Compute the mean weight of the students.

8. Draw two concentric circles of radii 4 cm and 6 cm. Construct a tangent to the smaller circle from a point on the larger circle. Measure the length of this tangent. **[3]**

9. If the median of the following frequency distribution is 32.5, find the values of f_1 and f_2. **[3]**

Class Interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	**Total**
Frequency	f_1	5	9	12	f_2	3	2	40

10. A tower stands vertically on the ground. From a point on the ground 100 m away from the foot of the tower, the angle of elevation of the top of the tower is 45°. Find the height of the tower. **[3]**

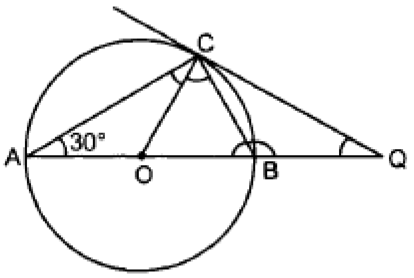
OR

The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle θ with the ground level such that tan $\theta = 15/8$ then find the height of the kite from the ground. Assume that there is no slack in the string.

Section C

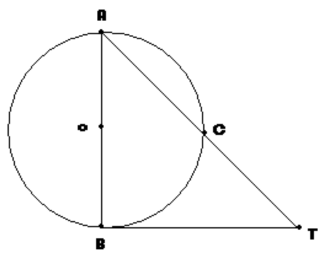
11. Water is flowing at the rate of 7 metres per second through a circular pipe whose internal diameter is 2 cm into a cylindrical tank the radius of whose base is 40 cm. Determine the increase in the water level in 1/2 hour. **[4]**

12. In the figure, AB is diameter of a circle with centre O and QC is a tangent to the circle at C. If ∠CAB = 30°, find ∠CQA and ∠CBA. **[4]**



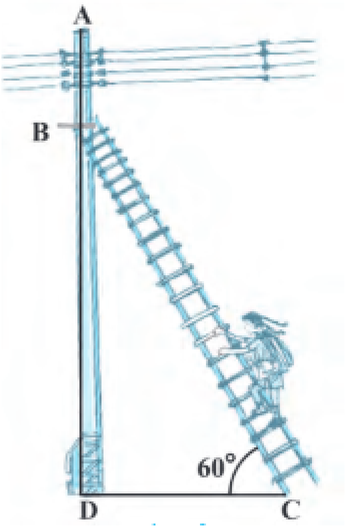
OR

In the given figure, AB is a diameter of the circle. The length of AB = 5 cm. If O is the centre of the length of tangent segment BT = 12 cm, determine CT.



13. Basant Pandey noticed that one of the street lights in front of his society gate was not working. He called BSES customer care and registered the complaint. The electricity department of BSES sent an electrician to check the fault. The electrician has to repair an electric fault on the pole of height 5 m. He needs to reach a point 1.3m below the top of the pole to undertake the **[4]**

repair work (see fig.).



i. What should be the length of the ladder that she should use which, when inclined at an angle of 60° to the horizontal, would enable her to reach the required position?
ii. Also, how far from the foot of the pole should she place the foot of the ladder? (You may take $\sqrt{3} = 1.73$)

14. Kamla and her husband were working in a factory in Seelampur, New Delhi. During the pandemic, they were asked to leave the job. As they have very limited resources to survive in a metro city, they decided to go back to their hometown in Himachal Pradesh. After a few months of struggle, they thought to grow roses in their fields and sell them to local vendors as roses have been always in demand. Their business started growing up and they hired many workers to manage their garden and do packaging of the flowers. **[4]**

In their garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row.

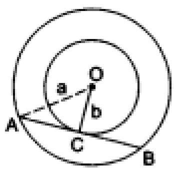
i. How many rows are there of rose plants?
ii. Also, find the total number of rose plants in the garden.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Section A

1. According to question the three-digit numbers which are divisible by 13 are 104, 117, 130, 143,....... 938.
This forms an AP in which a = 104, d = (117 – 104) = 13 and l = 938 (last term)
Let the number of terms be n
Then Tn = 938
$\Rightarrow$a+(n-1)d=988
$\Rightarrow$104+(n-1)$\times$13=988
$\Rightarrow$13n=897
$\Rightarrow$n=69
Therefore required sum=$\frac{n}{2}$(a+l)
=$\frac{69}{2}$[104+988]=69$\times$546=37674
Hence, the sum of all three digit numbers which are divisible by 13 is equal to 37674.
OR
Let the middle most terms of the A.P. be (a - d), a, (a + d)
Given a -d + a + a + d = 225
3a = 225
or, a = 75
and the middle term = $\frac{37+1}{2}$ = 19th term
$\therefore$ A.P. is
(a - 18d),....(a - 2d), {a - d), a, (a + d), (a + 2d),..........(a + 18d)
Sum of last three terms
(a + 18d) + (a + 17d) + (a + 16d) = 429
or, 3a +51d = 429
or, 225 + 51d = 429
or, d = 4
First term a_1 = a - 18d = 75 - 18$\times$4 = 3
a_2 = 3 + 4 = 7
Hence, A.P. = 3, 7, 11 ,.........., 147
2. We have, $abx^2 + (b^2 - ac)x - bc = 0$
$\Rightarrow abx^2 + b^2x - acx - bc = 0$
$\Rightarrow$ bx (ax+b) - c (ax + b) = 0
$\Rightarrow$ (ax + b) (bx - c) = 0
Either ax+b = 0 or bx - c = 0
$\Rightarrow x = -\frac{b}{a}, \frac{c}{b}$
Hence, $x = -\frac{b}{a}, \frac{c}{b}$ are the required solutions.
3. 

Let O be the common centre of the two circles
and AB be the chord of the larger circle which touches the smaller circle at C.
Join OA and OC.
Then, OA = a and OC = b.
Now, $OC \perp AB$ and OC bisects AB [$\because$ the chord of the larger circle touching the smaller circle, is bisected at the point of contact].
In right $\triangle ACO$, we have

$OA^2 = OC^2 + AC^2$ [by Pythagoras' theorem]

$\Rightarrow AC = \sqrt{OA^2 - OC^2} = \sqrt{a^2 - b^2}$.

$\therefore AB = 2AC = 2\sqrt{a^2 - b^2}$ [$\because$ C is the midpoint of AB]

i.e, required length of the chord AB = $2\sqrt{a^2 - b^2}$

4. Radius of the cylindrical bucket, R = 18 cm.
   Height of the cylindrical bucket, H = 32 cm.
   Volume of sand in the bucket = $\pi R^2 H = (\pi \times 18 \times 18 \times 32)\ cm^3$.
   Let the radius of the conical heap be r.
   Height of the conical heap, h = 24 cm.
   Volume of the conical heap = $\frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\pi \times r^2 \times 24\right)\ cm^3$
   $= (8\pi r^2)\ cm^3$
   Volume of the conical heap = volume of sand in the bucket
   $\Rightarrow 8\pi r^2 = \pi \times 18 \times 18 \times 32 \Rightarrow r^2 = \frac{18 \times 18 \times 32}{8} = 1296$
   $\Rightarrow r = \sqrt{1296} = 36$ cm.
   $l = (r^2 + h^2)^{\frac{1}{2}}$
   $l = (1296 + 576)^{\frac{1}{2}} = 1872^{\frac{1}{2}}$
   $l = 43.2m$
5. Clearly, the modal class is 40-60, as it has the maximum frequency.
   $\therefore x_k = 40, h = 20, f_k = 28, f_{k-1} = 16, f_{k+1} = 20$
   Mode, $M_o = x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$
   $= 40 + 20\left\{\frac{28 - 16}{2(28) - 16 - 20}\right\}$
   $= 40 + 20 \times \frac{12}{20}$
   $= 40 + 12$
   $= 52$
6. We have, $(k+1)x^2 - 2(k-1)x + 1 = 0$.
   $a = k + 1, b = -2(k - 1), c = 1$.
   $D = b^2 - 4ac = 4(k-1)^2 - 4(k + 1) = 4(k^2 - 3k)$
   The given equation will have real and equal roots, if
   $D = 0 \Rightarrow 4(k^2 - 3k) = 0 \Rightarrow k^2 - 3k = 0 \Rightarrow k(k - 3) = 0 \Rightarrow k = 0, 3$

   OR

   Let the two consecutive positive integers be x and x+1
   According to the question
   $x^2 + (x+1)^2 = 365$
   $\Longrightarrow x^2 + x^2 + 2x + 1 = 365$
   $\Longrightarrow 2x^2 + 2x - 364 = 0$
   $\Longrightarrow 2(x^2 + x - 182) = 0$ or $x^2 + x - 182 = 0$
   $\Longrightarrow x^2 + 14x - 13x - 182 = 0$
   $\Longrightarrow x(x+14) - 13(x+14) = 0$
   $\Longrightarrow (x-13)(x+14) = 0$
   Either $x - 13 = 0$ or $x + 14 = 0$
   $\Longrightarrow x = 13, -14$
   Since the numbers are positive. so x = - 14 is rejected.
   Hence the required consecutive positive integers are 13, 13+1 =14.

Section B

7.

Class Interval	**Frequency(f_i)**	**Class mark x_i**	Deviation $d_i = (x_i - A) = (x_i - 57)$	**$(f_i \times d_i)$**
50 - 52	18	51	-6	-108

52 - 54	21	53	-4	-84
54 - 56	17	55	-2	-34
56 - 58	28	57 = A	0	0
58 - 60	16	59	2	32
60 - 62	35	61	4	140
62 - 64	15	63	6	90
	$\Sigma f_i = 150$			$\Sigma\left(f_i \times d_i\right) = 36$

Let the assumed mean A = 57

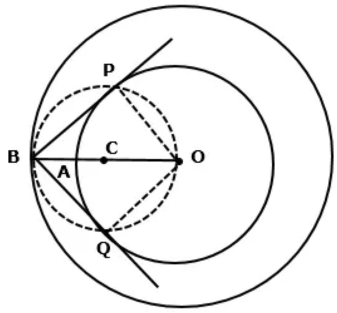
$\therefore \quad \bar{x} = A + \frac{\Sigma(f_i \times d_i)}{\Sigma f_i}$

$= \left(57 + \frac{36}{150}\right)$

= 57 + 0.24

= 57.24

Hence, mean weight = 57.24 kg

8. 

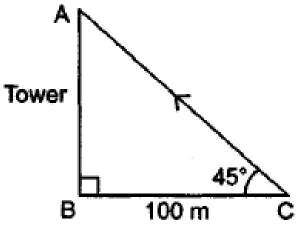
Steps of construction:

1. Take a point O on the plane of the paper and draw a circle of radius OA = 4 cm.
   Also, draw a concentric circle of radius OB = 6 cm.
2. Find the midpoint C of OB and draw a circle of radius OC = BC.
   Suppose this circle intersects the circle of radius 4 cm at P and Q.
3. Join BP and BQ to get the desired tangents from a point B on the circle of radius 6 cm.
   By actual measurement, we find that BP = BQ = 4.5 cm
   Verification:
   In $\triangle$BOQ, we have OB = 6 cm and OP = 4 cm
   $\therefore$ $OB^2 = BP^2 + OP^2$
   $\Rightarrow BP = \sqrt{OB^2 - OP^2} = \sqrt{36 - 16} = \sqrt{20}$ = 4.47 cm = 4.5 cm
   Similarly, BQ = 4.47 cm = 4.5 cm.

9. Let f_1 and f_2 be the frequencies of class intervals 0 - 10 and 40 - 50.
   $f_1 + 5 + 9 + 12 + f_2 + 3 + 2 = 40$
   $\Rightarrow f_1 + f_2 = 9$
   Median is 32.5 which lies in 30 - 40, so the median class is 30 - 40.
   l = 30, h = 10, f = 12, N = 40 and c = $f_1 + 5 + 9 = (f_1 + 14)$
   Now, median $= l + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f}\right]$
   $\Rightarrow \quad 32.5 = \left[30 + \left(10 \times \frac{20 - f_1 - 14}{12}\right)\right]$
   $= \left[30 + \left(10 \times \frac{6 - f_1}{12}\right)\right]$
   $= \left[30 + \left(\frac{30 - 5f_1}{6}\right)\right]$
   $\frac{30 - 5f_1}{6} = 2.5$
   $30 - 5f_1 = 15$

$5f_1 = 15 \Rightarrow f_1 = 3$

$f_1 = 3$ and $f_2 = (9 - 3) = 6$

10. 

$BC = 100\ m$

In right ABC,

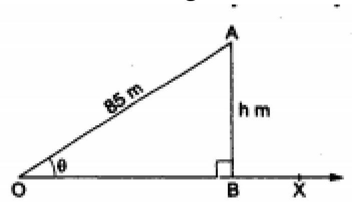
$\frac{AB}{BC} = \tan 45^\circ$

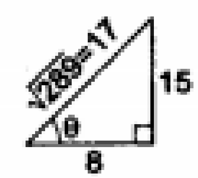
$\Rightarrow \quad \frac{\mathbf{AB}}{100} = 1$

$\Rightarrow AB = 100m$

OR

Let OX be the horizontal ground and let A be the position of the kite. Let O be the position of the observer and OA be the string. Draw $AB \perp OX$.





Then, $\angle BOA = \theta$ such that $\tan\theta = \frac{15}{8}, OA = 85m$ and $\angle OBA = 90^\circ$.

Let AB = h m.

From right ΔOBA, we have

$\frac{AB}{OA} = \sin\theta = \frac{15}{17}\ \left[\because \tan\theta = \frac{15}{8} \Rightarrow \sin\theta = \frac{15}{17}\right]$

$\Rightarrow \frac{h}{85} = \frac{15}{17} \Rightarrow h = \frac{15}{17} \times 85 = 75.$

Section C

11. We have,

Rate of flow of water = 7 m/sec = 700 cm/sec.

Length of pipe in which the water has travelled in half hour h = 700×30×60 = 1260000 cm^3

Internal radius of the pipe r = 2/2 = 1 cm

Internal radius of circular pipe = 1 cm.

Radius of the cylinder R = 40 cm

Let the height of the water in the cylindrical tank is H

Now,

Volume of the water coming out of the pipe = Volume of the water collected in the cylindrical tank

$\implies \pi r^2 h = \pi R^2 H$

or $r^2 h = R^2 H$

or, $1 \times 1260000 = (40)^2 H$

$1260000 = (40)^2 H$

1260000 = 1600H

H = 1260000/1600

H = 12600/16

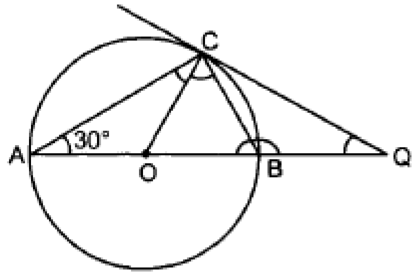
H = 787.5 cm

H = 787.5/100 cm

H = 7.875 m

So, the increase in the water level in 1/2 hour is 7.875 m

12. Given,AB is diameter of a circle with centre O and QC is a tangent to the circle at C.



In $\triangle AOC, OA = OC$ [Radii of the same circle]
$\therefore$ ∠ACO = $\angle CAO = 30^\circ$ [Opp. angles of equal sides are equal.]
Also $\angle ACB = 90^\circ$ [Angle in semicircle]
∠OCB = 90° - 30° = 60°
In △COB, OC = OB [Radii of the same circle]
∠OCB = ∠OBC = 60° [Opposite angles of equal sides]
Now OC ⊥ CQ
$\angle OCQ = 90^\circ$
⇒ ∠BCQ = 90° - 60° = 30°
Also ∠OBC + ∠CBQ = 180°
60° + ∠CBQ = 180°
⇒ ∠CBQ = 120°
In △CBQ,
∠BCQ + ∠CBQ + ∠CQB = 180°
⇒ 30° + 120° + ∠CQB = 180°
⇒ ∠CQB = 30°
∴∠CQA = 30°,
Now ∠CBA = 180° - ∠CBQ
$= 180^\circ - 120^\circ = 60^\circ.$

OR

Since AB is the diameter and BT is the tangent.
So, AB ⊥ BT (Radius of the circle is perpendicular to the tangent through point of contact)
⇒ ABC is a right triangle right angled at B
By applying pythagoras theorem
$AT^2 = AB^2 + BT^2$
$AT^2 = (5)^2 + (12)^2 = 163 = (13)^2$
$AT = 13$
If ACT is a secant to a circle intersecting the circle at A and C and BT is a tangent, then CT × AT = BT^2
So, $(12)^2 = 13 \times CT$
$CT = \frac{144}{13} cm$

13. In Fig, the electrician is required to reach point B on the pole AD. So, BD = AD – AB = (5 – 1.3)m = 3.7 m. Here, BC represents the ladder. We need to find its length, i.e., the hypotenuse of the right triangle BDC. Now, we take sin 60°.
So, $\frac{\text{BD}}{\text{BC}}$ = sin 60° or $\frac{3.7}{BC} = \frac{\sqrt{3}}{2}$
Therefore, BC = $\frac{3.7\times 2}{\sqrt{3}}$ = 4.28 m (approx.) (by taking $\sqrt{3} = 1.73$)
i.e., the length of the ladder should be 4.28 m.
Now, $\frac{DC}{BD} = \cot 60^\circ = \frac{1}{\sqrt{3}}$
i.e., DC = $\frac{3.7}{\sqrt{3}}$ = 2.14 m (approx.)
Therefore, she should place the foot of the ladder at a distance of 2.14 m from the pole.

14. The number of rose plants in the 1^{st}, 2^{nd}, ... are 23,21,19,...........5
a = 23, d = 21 - 23 = - 2 , a_n = 5
$\therefore$ a_n = a + (n - 1)d

or, 5 = 23 + (n - 1)(-2)
or, 5 = 23 -2n + 2
or, 5 = 25 -2n
or, 2n = 20
or, n =10
Total number of rose plants in the flower bed,
$S_n = \frac{n}{2}[2a + (n-1)d]$
$S_{10} = \frac{10}{2}[2(23) + (10-1)(-2)]$
$= 5[46 - 20 + 2]$
S_{10} = 5(46 - 18)
= 5 (28)
S_{10} = 140