VECTOR AND THEIR PROPERTIES (XII, R. S. AGGARWAL)

EXERCISE 22 [Pg. No.: 1008]

1. Write down the magnitude of each of the following vectors:

(i) $\vec{a}=\hat{i}+2\hat{j}+5\hat{k}$ (ii) $\vec{b}=5\hat{i}-4\hat{j}-3\hat{k}$

(iii) $\vec{c}=\left(\frac{1}{\sqrt{3}}\hat{i}-\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}\right)$ (iv) $\vec{d}=\left(\sqrt{2}\hat{i}+\sqrt{3}\hat{j}-\sqrt{5}\hat{k}\right)$

Sol. **(i)** Let $\vec{a}=\hat{i}+2\hat{j}+5\hat{k}$; $|\vec{a}|=\sqrt{(1)^2+(2)^2+(5)^2}=\sqrt{1+4+25}=\sqrt{30}$

(ii) Let $\vec{b}=5\hat{i}-4\hat{j}-3\hat{k}$; $|\vec{b}|=\sqrt{(5)^2+(-4)^2+(-3)^2}=\sqrt{25+16+9}=5\sqrt{2}$

(iii) Let $\vec{c}=\left(\frac{1}{\sqrt{3}}\hat{i}-\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}\right)$

$$|\vec{c}|=\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+\left(-\frac{1}{\sqrt{3}}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2}=\sqrt{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}=\sqrt{\frac{1+1+1}{3}}=\sqrt{\frac{3}{3}}=\sqrt{1}=1$$

(iv) Let $\vec{d}=\left(\sqrt{2}\hat{i}+\sqrt{3}\hat{j}-\sqrt{5}\hat{k}\right)$; $|\vec{d}|=\sqrt{\left(\sqrt{2}\right)^2+\left(\sqrt{3}\right)^2+\left(-\sqrt{5}\right)^2}=\sqrt{2+3+5}=\sqrt{10}$

2. Find a unit vector in the direction of the vector

(i) $\left(3\hat{i}+4\hat{j}-5\hat{k}\right)$ (ii) $\left(3\hat{i}-2\hat{j}+6\hat{k}\right)$ (iii) $\left(\hat{i}+\hat{k}\right)$ (iv) $\left(2\hat{i}+\hat{j}+2\hat{k}\right)$

Sol. (i) Let $\vec{a}=3\hat{i}+4\hat{j}-5\hat{k}$

$$\Rightarrow\left|\vec{a}\right|=\sqrt{3^2+4^2+(-5)^2}\Rightarrow\left|\vec{a}\right|=\sqrt{50}\Rightarrow\left|\vec{a}\right|=5\sqrt{2}$$

Unit vector along $\vec{a}$, $\hat{a}=\frac{\vec{a}}{|\vec{a}|}\quad\Rightarrow\hat{a}-\frac{1}{5\sqrt{2}}\left(3\hat{i}+4\hat{j}-5\hat{k}\right)$

(ii) Let, $\vec{a}=3\hat{i}-2\hat{j}+6\hat{k}$

$$\Rightarrow\left|\vec{a}\right|=\sqrt{3^2+(-2)^2+6^2}\Rightarrow\left|\vec{a}\right|=\sqrt{49}\Rightarrow\left|\vec{a}\right|=7$$

Unit vector along $\left|\vec{a}\right|$, $\hat{a}=\frac{\vec{a}}{|\vec{a}|}\quad\Rightarrow\hat{a}=\frac{1}{7}\left(3\hat{i}-2\hat{j}+6\hat{k}\right)$

(iii) Let, $\vec{a}=\hat{i}+\hat{k}$

$$\Rightarrow\left|\vec{a}\right|=\sqrt{1^2+1^2}\Rightarrow\left|\vec{a}\right|=\sqrt{2}$$

Unit vector along $\vec{a}$, $\hat{a}=\frac{\vec{a}}{|\vec{a}|}=\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{k}\right)$

(iv) Let, $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$

$$\Rightarrow\left|\vec{a}\right|=\sqrt{2^2+1^2+2^2}=3$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} \Rightarrow \hat{a} = \frac{2\hat{i} + \hat{j} + 2\hat{k}}{3} \Rightarrow \hat{a} = \frac{1}{3}\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$$

3. If $\vec{a} = \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right)$ then find the value of λ so that $\lambda\vec{a}$ may be a unit vector

Sol. Given:- $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$

$\Rightarrow \lambda\vec{a} = 2\lambda\hat{i} - 4\lambda\hat{j} + 5\lambda\hat{k}$

$\because \lambda\vec{a}$ is a unit vector

$\therefore \left|\lambda\vec{a}\right| = 1$

$$\Rightarrow \sqrt{(2\lambda)^2 + (-4\lambda)^2 + (5\lambda)^2} = 1 \Rightarrow 4\lambda^2 + 16\lambda^2 + 25\lambda^2 = 1 \Rightarrow 45\lambda^2 = 1 \Rightarrow \lambda^2 = \frac{1}{45} \Rightarrow \lambda = \pm\frac{1}{\sqrt{45}}$$

$$\Rightarrow \lambda = \pm\frac{1}{3\sqrt{5}}$$

4. If $\vec{a} = \left(-\hat{i} + \hat{j} - \hat{k}\right)$ and $\vec{b} = \left(2\hat{i} - \hat{j} + 2\hat{k}\right)$ then find the unit vector in the direction of $\left(\vec{a} + \vec{b}\right)$

Sol. $\vec{a} = -\hat{i} + \hat{j} - \hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\vec{a} + \vec{b} = \left(-\hat{i} + \hat{j} - \hat{k}\right) + \left(2\hat{i} - \hat{j} + 2\hat{k}\right)$

$= \hat{i} + \hat{k}$

$\left|\vec{a} + \vec{b}\right| = \sqrt{1^2 + 1^2} = \sqrt{2}$

Unit vector $\vec{a} + \vec{b}$ is, $\hat{r} = \dfrac{\left(\vec{a} + \vec{b}\right)}{\left|\vec{a} + \vec{b}\right|} \Rightarrow \hat{r} = \dfrac{1}{\sqrt{2}}\left(\hat{i} + \hat{k}\right)$

5. If $\vec{a} = \left(3\hat{i} + \hat{j} - 5\hat{k}\right)$ and $\vec{b} = \left(\hat{i} + 2\hat{j} - \hat{k}\right)$ then find a unit vector in the direction of $\left(\vec{a} - \vec{b}\right)$

Sol. $\vec{a} = 3\hat{i} + \hat{j} - 5\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$

Now, $\vec{a} - \vec{b} = (3\hat{i} + \hat{j} - 5\hat{k}) - (\hat{i} + 2\hat{j} - \hat{k})$

$= 2\hat{i} - \hat{j} - 4\hat{k}$

Unit vector in the direction of $\vec{a} - \vec{b}$ is, $\vec{r} = \dfrac{\vec{a} - \vec{b}}{\left|\vec{a} - \vec{b}\right|}$

$$= \frac{2\hat{i} - \hat{j} - 4\hat{k}}{\sqrt{2^2 + (-2)^2 + (-4)^2}} = \frac{1}{\sqrt{21}}(2\hat{i} - \hat{j} - 4\hat{k})$$

6. If $\vec{a}=\left(\hat{i}+2\hat{j}-3\hat{k}\right)$ and $\vec{b}=\left(2\hat{i}+4\hat{j}+9\hat{k}\right)$ then find a unit vector parallel to $\left(\vec{a}+\vec{b}\right)$

Sol. $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$

$\vec{b}=2\hat{i}+4\hat{j}+9\hat{k}$

$\vec{a}+\vec{b}=(\hat{i}+2\hat{j}-3\hat{k})+(2\hat{i}+4\hat{j}+9\hat{k})$

$=3\hat{i}+6\hat{j}+3\hat{k}$

$\therefore |\vec{a}+\vec{b}|=\sqrt{3^2+6^2+6^2}=9$

Unit vector parallel to $\vec{a}+\vec{b}$ is, $\hat{r}=\pm\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|}$

$=\pm\frac{3\hat{i}+6\hat{j}+6\hat{k}}{9}=\pm\frac{1}{3}\left(\hat{i}+2\hat{j}+2\hat{k}\right)$

7. Find a vector of magnitude 9 units in the direction of the vector $\left(-2\hat{i}+\hat{j}+2\hat{k}\right)$

Sol. Let, $\vec{a}=-2\vec{i}+\vec{j}+2\vec{k}$

$\Rightarrow |\vec{a}|=\sqrt{(-2)^2+1^2+2^2}\Rightarrow |\vec{a}|=\sqrt{4+1+4}\Rightarrow |\vec{a}|=3$

A vector of magnitude 9 units in the direction of $\vec{a}$ is, $\vec{r}=9\cdot\frac{\vec{a}}{|\vec{a}|}$

$=\frac{9}{3}\left(-2\hat{i}+\hat{j}+2\hat{k}\right)=-6\hat{i}+3\hat{j}+6\hat{k}.$

8. Find a vector magnitude 8 units in the direction of the vector $\left(5\hat{i}-\hat{j}+2\hat{k}\right)$.

Sol. Let, $\vec{a}=5\hat{i}-\hat{i}+2\hat{k}$

$\Rightarrow |\vec{a}|=\sqrt{5^2+(-1)^2+2^2}\Rightarrow |\vec{a}|=\sqrt{30}$

A vector of magnitude 8 units in the direction at $\vec{a}$ is $\vec{r}=8\cdot\frac{\vec{a}}{|\vec{a}|}$

$\Rightarrow \vec{r}=8\cdot\frac{5\hat{i}-\hat{j}+2\hat{k}}{\sqrt{30}}=\frac{8}{\sqrt{30}}\left(5\hat{i}-\hat{j}+2\hat{k}\right)$

9. Find a vector of magnitude 21 units in the direction of the vector $\left(2\hat{i}-3\hat{j}+6\hat{k}\right)$

Sol. Let, $\vec{a}=2\hat{i}-3\hat{j}+6\hat{k}$

$\Rightarrow |\vec{a}|=\sqrt{2^2+(-3)^2+6^2}\Rightarrow |\vec{a}|=7$

A vector of magnitude 21 units in the direction at $\vec{a}$ is, $\vec{r}=21\cdot\frac{\vec{a}}{|\vec{a}|}$

$\Rightarrow \vec{r}=21\cdot\frac{\left(2\hat{i}-3\hat{j}+6\hat{k}\right)}{7}\Rightarrow \vec{r}=6\hat{i}-9\hat{j}+18\hat{k}$

10. If $\vec{a}=(\hat{i}-2\hat{j}), \vec{b}=(2\hat{i}-3\hat{j})$ and $\vec{c}=(2\hat{i}+3\hat{k})$, find $(\vec{a}+\vec{b}+\vec{c})$

Sol. $\vec{a}+\vec{b}+\vec{c}=(\hat{i}-2\hat{j})+(2\hat{i}-3\hat{j})+2\hat{i}+3\hat{k}=5\hat{i}-5\hat{j}+3\hat{k}$

11. If $A(-2,1,2)$ and $B(2,-1,6)$ are two given points, find a unit vector in the direction of $\overrightarrow{AB}$

Sol. Let, O be the origin,

Now, P.V. of A = $\overrightarrow{OA}=-2\hat{i}+\hat{j}+2\hat{k}$

P.V. of B = $\overrightarrow{OB}=2\hat{i}-\hat{j}+6\hat{k}$

$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$

$=(-2\hat{i}+\hat{j}+2\hat{k})-(2\hat{i}-\hat{j}+6\hat{k})=-4\hat{i}+2\hat{j}-4\hat{k}$

Unit vector along $\overrightarrow{AB}$ is, $\widehat{AB}=\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$

$$\Rightarrow \widehat{AB}=\frac{-4\hat{i}+2\hat{j}-4\hat{k}}{\sqrt{(-4)^2+2^2+(-4)^2}} \Rightarrow \widehat{AB}=\frac{1}{6}(-4\hat{i}+2\hat{j}-4\hat{k}) \Rightarrow \widehat{AB}=\frac{1}{3}(-2\hat{i}+\hat{j}-2\hat{k})$$

12. Find the direction ratios and the direction cosines of the vector $\vec{a}=(5\hat{i}-3\hat{j}+4\hat{k})$.

Sol. Let $\vec{a}=(5\hat{j}-3\hat{j}+4\hat{k}) \Rightarrow |\vec{a}|=\sqrt{(5)^2+(-3)^2-(4)^2}=\sqrt{25+9+16}=\sqrt{50}=5\sqrt{2}$

Hence, the required direction ratios are $(5,-3,4)$

Direction cosines are $\left(\frac{5}{5\sqrt{2}},\frac{-3}{5\sqrt{2}},\frac{4}{5\sqrt{2}}\right)=\left(\frac{1}{\sqrt{2}},\frac{-3}{5\sqrt{2}},\frac{4}{5\sqrt{2}}\right)$

13. Find the direction ratios and the direction cosines of the vector joining the points $A(2,1,-2)$ and $B(3,5,-4)$.

Sol. Let, $\vec{A}=(2\hat{i}+\hat{j}-2\hat{k})$ and $\vec{B}=(3\hat{i}+5\hat{j}-4\hat{k})$

$\overrightarrow{AB}=$ P.V. of $B-$ P.V. of A

$=(3\hat{i}+5\hat{j}-4\hat{k})-(2\hat{i}+\hat{j}-2\hat{k})=(\hat{i}+4\hat{j}-2\hat{k})$

$\Rightarrow |\overrightarrow{AB}|=\sqrt{(1)^2+(4)^2+(-2)^2}=\sqrt{1+16+4}=\sqrt{21}$

Hence, the required direction ratios are $(1,4,-2)$

Direction cosines are $=\left(\frac{1}{\sqrt{21}},\frac{4}{\sqrt{21}},\frac{-2}{\sqrt{21}}\right)$

14. Show that the points A, B and C having position vectors $(\hat{i}+2\hat{j}+7\hat{k})$, $(2\hat{i}+6\hat{j}+3\hat{k})$ and $(3\hat{i}+10\hat{j}-3\hat{k})$ respectively, are collinear

Sol. $\overrightarrow{AB}$ = P.V. of B – P.V. of A

$=(2\hat{i}+6\hat{j}+3\hat{k})-(\hat{i}+2\hat{j}+7\hat{k})=\hat{i}+4\hat{j}-4\hat{k}$

$\overrightarrow{BC}$ = P.V. of C – P.V. of B

$= \left(3\hat{i}+10\hat{j}-1\hat{k}\right)-\left(2\hat{i}+6\hat{j}+3\hat{k}\right)=\hat{i}+4\hat{j}-4\hat{k}$

$\because \overrightarrow{AB}=\overrightarrow{BC} \Rightarrow \overrightarrow{AB} \parallel \overrightarrow{BC}$

Here, B is common Hence, A, B & C are collinear.

15. The position vectors of the points A, B and C are $\left(2\hat{i}+\hat{j}-\hat{k}\right),\left(3\hat{i}-2\hat{j}+\hat{k}\right)$ and $\left(\hat{i}+4\hat{j}-3\hat{k}\right)$ respectively. Show that the points A, B and C are collinear

Sol. $\overrightarrow{AB}$ = P.V. of B – P.V at A

$= \left(3\hat{i}-2\hat{j}+\hat{k}\right)-\left(2\hat{i}+\hat{j}-\hat{k}\right)$

$=\hat{i}-3\hat{j}+2\hat{k}$

$\overrightarrow{BC}$ = P.V. of C – P.V at B

$= \left(\hat{i}+4\hat{j}-3\hat{k}\right)-\left(3\hat{i}-2\hat{j}-\hat{k}\right)$

$=-2\hat{i}+6\hat{j}-4\hat{k}$

Hence, $\overrightarrow{BC}=-2\cdot\overrightarrow{AB}$

$\Rightarrow \overrightarrow{AB} \parallel \overrightarrow{BC}$

Here B is common

$\therefore$ A, B & C are collinear

16. if the position vectors of the vertices A, B and C of a ΔABC be $\left(\hat{i}+2\hat{j}+3\hat{k}\right),\left(2\hat{i}+3\hat{j}+\hat{k}\right)$ and $\left(3\hat{i}+\hat{j}+2\hat{k}\right)$ respectively, prove that ΔABC is equilateral

Sol. $\overrightarrow{AB}$ = P.V. at B – P.V at A

$= \left(2\hat{i}+3\hat{j}+\hat{k}\right)-\left(\hat{i}+2\hat{j}+3\hat{k}\right)=\hat{i}+\hat{j}-2\hat{k}$

$\overrightarrow{BC}$ = P.V at C – P.V. at B

$= \left(3\hat{i}+\hat{j}+2\hat{k}\right)-\left(2\hat{i}+3\hat{j}+\hat{k}\right)=\hat{i}-2\hat{j}+\hat{k}$

$\overrightarrow{AB}$ = P.V at C – P.V. at A

$= \left(3\hat{i}+\hat{j}+2\hat{k}\right)\left(\hat{i}+2\hat{j}+3\hat{k}\right)=\left(2\hat{i}-\hat{j}-\hat{k}\right)$

Here, $|\overrightarrow{AB}|=|\overrightarrow{BC}|=|\overrightarrow{AC}|=\sqrt{6}$

Hence, Δ ABC is on equilateral triangle

17. Show that the points A, B and C having position vectors $\left(3\hat{i}-4\hat{j}-4\hat{k}\right)$, $\left(2\hat{i}-\hat{j}+\hat{k}\right)$ and $\left(\hat{i}-3\hat{j}-5\hat{k}\right)$ respectively, form the vertices of a right-angled triangle

Sol. $\overrightarrow{AB}$ = P.V. at B – P.V. at A

$= \left(2\hat{i}-\hat{j}+\hat{k}\right)-\left(3\hat{i}-4\hat{j}-4\hat{k}\right)=-\hat{i}+3\hat{j}+5\hat{k}$

$\overrightarrow{BC}$ = P.V. at C – PV. at B

$= \left(\hat{i}-3\hat{j}-5\hat{k}\right)-\left(2\hat{i}-\hat{j}+\hat{k}\right)=-\hat{i}-2\hat{j}-6\hat{k}$

$\overrightarrow{AC}$ = P.V. at C – P.V. at A

$= \left(\hat{i} - 3\hat{j} - 5\hat{k}\right) - \left(3\hat{i} - 4\hat{j} - 4\hat{k}\right) = -2\hat{i} + \hat{j} - \hat{k}$

$|\overrightarrow{AC}| = \sqrt{(-1)^2 + 3^2 + 5^2} = \sqrt{35}$

$\Rightarrow |\overrightarrow{AB}|^2 = 35$

$|\overrightarrow{BC}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{41}$

$\Rightarrow |\overrightarrow{BC}|^2 = 41$

$|\overrightarrow{AC}| = \sqrt{(-2)^2 + 1^2 + (-1)^2} = \sqrt{6}$

$\Rightarrow |\overrightarrow{AC}|^2 = 6$

$\because |\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 = |BC|^2$ $\quad \therefore \Delta ABC$ is a right angled triangle at A.

18. Using vector method, show that the points $A(1,-1,0)$, $B(4,-3,1)$ and $C(2,-4,5)$ are the vertices of a right-angled triangle

Sol. Given:$\rightarrow$ A (1, -1, 0), B(4, -3, 1) and C (2, -4, 5) are the vertices of a triangle.

Here, $\overrightarrow{AB} = (4-1)\hat{i} + (-3+1)\hat{j} + (1-0)\hat{k}$

$= 3\hat{i} - 2\hat{j} + \hat{k}$

$\overrightarrow{BC} = (2-4)\hat{i} + (-4+3)\hat{j} + (5-1)\hat{k} = -2\hat{i} - \hat{j} + 4\hat{k}$

$\overrightarrow{AC} = (2-1)\hat{i} + (-4+1)\hat{j} + (5-0)\hat{k} = \hat{i} - 3\hat{j} + 5\hat{k}$

Here, $|\overrightarrow{AB}|^2 = 9 + 4 + 1 = 14$

and, $|\overrightarrow{BC}|^2 = 4 + 1 + 16 = 21$ and, $|\overrightarrow{AC}|^2 = 1 + 9 + 25 = 35$

$\because |\overrightarrow{AB}|^2 + |\overrightarrow{BC}|^2 = |\overrightarrow{AC}|^2$ $\quad \therefore \Delta ABC$ is a right angle triangle.

19. Find the position vector of the point which divides the join of the points $\left(2\vec{a} - 3\vec{b}\right)$ and $\left(3\vec{a} - 2\vec{b}\right)$

(i) internally and (ii) externally in the ratio 2 : 3

Sol. Let, Position vector of A. is $\overrightarrow{a_1} = 2\vec{a} - 3\vec{b}$

and , position vector of B is $\overrightarrow{a_2} = 3\vec{a} - 2\vec{b}$

(i) Let, R divides AB internally in the ratio $m_1 : m_2 = 2 : 3$

P.V. at R = $\dfrac{m_1\overrightarrow{a_2} + m_2\overrightarrow{a_1}}{m_1 + m_2}$

$= \dfrac{2\left(3\vec{a} - 2\vec{b}\right) + 3\left(2\vec{a} - 3\vec{b}\right)}{2+3} = \dfrac{12\vec{a} - 13\vec{b}}{5} = \dfrac{12}{5}\vec{a} - \dfrac{13}{5}\vec{b}$

(ii) Let, S divides AB externally in the ratio 2 : 3

P.V. of R = $\dfrac{2\overrightarrow{a_2} - 3\overrightarrow{a_1}}{2-3}$

$= \dfrac{2\left(3\vec{a} - 2\vec{b}\right) - 3\left(2\vec{a} - 3\vec{b}\right)}{-1} = -\left(6\vec{a} - 4\vec{b} - 6\vec{a} + 9\vec{b}\right) = -5\vec{b}$

20. The position vectors of two points A and B are $\left(2\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-3\vec{b}\right)$ respectively. Find the position vector of a point C which divides AB externally in the ratio 1 : 2. Also, show that A is the mid-point of the line segment CB

Sol. Let, P.V. of A . is $\vec{a_1}=2\vec{a}+\vec{b}$

and, P.V of B is $\vec{a_2}=\vec{a}-3\vec{b}$

$\because$ Since, C divides AB externally in the ratio 1 : 2 $\therefore$ Position vector of C is,

$$\vec{r}=\frac{1\times\vec{a_2}-2\times\vec{a_1}}{1-2}$$

$$\Rightarrow\vec{r}=\frac{\left(\vec{a}-3\vec{b}\right)-2\left(2\vec{a}+\vec{b}\right)}{-1}\Rightarrow\vec{r}=-\{\vec{a}-3\vec{b}-4\vec{a}-2\vec{b}\}\Rightarrow\boxed{\vec{r}=3\vec{a}+5\vec{b}}$$

Now, $\frac{\text{P.V. of B+P.V of C}}{2}=\frac{\vec{a_2}+\vec{r}}{2}=\frac{\vec{a}-3\vec{b}+3\vec{a}+5\vec{b}}{2}=\frac{4\vec{a}+2\vec{b}}{2}$

$=2\vec{a}+\vec{b}=$ P.V. of A Hence, A is the mid-point of BC.

21. Find the position vector of a point R which divides the line joining $A(-2,1,3)$ and $B(3,5,\ 2)$ in the ratio 2:1 (i) internally (ii) externally.

Sol. Here $\hat{a}=-2\hat{i}+\hat{j}+3\hat{k}$ & $\hat{b}=3\hat{i}+5\hat{j}-2\hat{k}$

(i) When R divided PQ internally in the ratio 2:1

Then, P.V. of $R=\frac{\left(m\vec{b}+n\vec{a}\right)}{(m+n)}=\frac{2\left(3\vec{i}+5\vec{j}-2\vec{k}\right)+1\left(-2\hat{i}+\hat{j}+3\hat{k}\right)}{2+1}$

$$\frac{6\hat{i}+10\hat{j}-4\hat{k}-2\hat{i}+\hat{j}+3\hat{k}}{3}=\frac{4\hat{i}+11\hat{j}-\hat{k}}{3}=\frac{4}{3}\hat{i}+\frac{11}{3}\hat{j}-\frac{1}{3}\hat{k}\text{ i.e., }\left(\frac{4}{3},\frac{11}{3},-\frac{1}{3}\right)$$

(ii) When R divided PQ externally in the ratio 2:1

Then position vector of $R=\frac{\left(m\vec{b}-n\vec{a}\right)}{(m-n)}=\frac{2\left(3\vec{i}+5\vec{j}-2\vec{k}\right)-1\left(-2\hat{i}+\hat{j}+3\hat{k}\right)}{2-1}$

$$=\frac{\left(6\hat{i}+10\hat{j}-4\hat{k}\right)-\left(-2\hat{i}+\hat{j}+3\hat{k}\right)}{1}=\left(8\hat{i}+9\hat{i}-7\hat{k}\right)\text{ i.e., }(8,9,-7)$$

22. Find the position vector of the midpoint of the vector joining the points $A\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$ and $B\left(\hat{i}+4\hat{j}-2\hat{k}\right)$.

Sol. The position vector of A and B are given by

$\vec{a}=\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$ and $\vec{b}=\left(\hat{i}+4\hat{j}-2\hat{k}\right)$

$\therefore$ P.V. of midpoint of $AB=\frac{1}{2}\left(\vec{a}+\vec{b}\right)=\frac{1}{2}\left\{\left(3\hat{i}+2\hat{j}+6\hat{k}\right)+\left(\hat{i}+4\hat{j}-2\hat{k}\right)\right\}$

$$=\frac{1}{2}\left(4\hat{i}+6\hat{j}+4\hat{k}\right)=\left(2\hat{i}+3\hat{j}+2\hat{k}\right)$$

23. If $\overrightarrow{AB} = \left(2\hat{i} + \hat{j} - 3\hat{k}\right)$ and $A(1, 2, -1)$ is the given point, find the coordinates of B

Sol. Let, Co-ordinates of B are (α, β, γ)

Now, $\overrightarrow{AB}$ = P.V. of B – P.V. of A

$\Rightarrow 2\hat{i} + \hat{j} - 3\hat{k} = \left(\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}\right) - \left(\hat{i} + 2\hat{j} - \hat{k}\right) \Rightarrow 2\hat{i} + \hat{j} - 3\hat{k} = (\alpha - 1)\hat{i} + (\beta - 2)\hat{j} + (\gamma + 1)\hat{k}$

$\Rightarrow \alpha - 1 = 2, \beta - 2 = 1 \;\&\; \gamma + 1 = -3 \Rightarrow \alpha = 3, \beta = 3 \;\&\; \gamma = -4$

Hence, Co-ordinates of B are (3, 3, -4)

24. Write a unit vector in the direction of $\overrightarrow{PQ}$, where P and Q are the points $(1, 3, 0)$ and $(4, 5, 6)$ respectively

Sol. $\overrightarrow{PQ} = (4 - 1)\hat{i} + (5 - 3)\hat{j} + (6 - 0)\hat{k}$

$\Rightarrow \overrightarrow{PQ} = 3\hat{i} + 2\hat{i} + 6\hat{k} \Rightarrow |\overrightarrow{PQ}| = \sqrt{3^2 + 2^2 + 6^2}$

Unit vector along $\overrightarrow{PQ}$, $\hat{PQ} = \dfrac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} \equiv \dfrac{1}{7}\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ Ans.